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THE STANDARD MECHANICS' ENCYCLOPEDIA

THE STANDARD
MECHANICS' ENCYCLOPEDIA

A PRACTICAL OUTLINE OF MECHANICAL TRADES

Edited by

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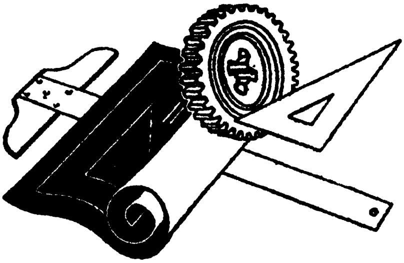
Assisted by a staff of Technical
Specialists and Authorities



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CONTENTS

PART I: BASIC MATHEMATICS

INTRODUCTION xvii

CHAPTER

I FUNDAMENTALS OF ARITHMETIC

1. WEIGHTS AND MEASURES 1

Nature and Importance of Measurement. Kinds of Measures. Standards. Linear, Surface, and Capacity. Measures. Weight. Metric System.

2. COMMON FRACTIONS . . 10

Whole Numbers and Fractions. Reduction of Fractions. Addition and Subtraction. Multiplication and Division. Practical Uses of Fractions.

3. DECIMAL FRACTIONS . . . 18

Use of Decimals. Shop Practice. Addition, Subtraction, Multiplication, and Division of Decimals. Changing Common Fractions to Decimals. Decimal Equivalents. Tolerance.

4. MEASURING INSTRUMENTS 27

Degree of Accuracy. Limits of Accuracy. Errors in Measurement. Construction and Use of the Micrometer Caliper. Theory of the Vernier. Other Types of Calipers. Precision Measurements. Gage Blocks. Using Gage Blocks.

5. PERCENTAGE 40

Finding a Per Cent of a Number. Changing a Per Cent to a Fraction. Changing Common Fractions to Per Cents. Determining the Rate. Finding the Base. Practical Uses. Depreciation. Commercial Discount.

6. Ratio and Proportion 48

Ratio a Comparison. Using Ratios. Scale Drawings. Proportion. Direct Proportion. Inverse Proportion. Pulleys and Gears.

CHAPTER

II ELEMENTS OF ALGEBRA

7. Numbers and Symbols 61

Language of Algebra. Substitution. Negative Quantities. Combining Similar Terms. Operating with Literal Expressions. Factors. Squares. Cubes and Other Powers. Laws of Exponents. Roots. Negative Exponents.

8. Formulas and Equations 72

Meaning of a Formula. Evaluating Formulas. Simple Equations. Solving Equations by Multiplication and Division. Changing the Subject of a Formula. Solving Equations by Addition and Subtraction. Solving Formulas and Equations in General. Practical Use of Formulas.

9. Square Root 82

Use of Square Root. Approximation Methods. Table of Square Roots. The Algebraic Rule. Formulas and Equations involving Square Roots. Solving Quadratic Equations.

10. Variation, Dependence, and Graphs 94

More about Formulas. What a Graph Can Do. Functions. Empirical Curves. Linear Function. Direct Variation. Parabolic Function. Hyperbolic Function. Inverse Variation. Related Variables and Rate of Change. The Exponential Function.

11. Logarithms 106

Logarithms and Exponents. Characteristic and Mantissa. Using a Table. Interpolation. Exponents and Logarithms. Multiplication and Division with Logarithms. Finding Roots and Powers. Exponential Equations.

12. Slide Rule 116

Description of Slide Rule. The Scales. Principle of Proportion. Multiplication and Division. Squares and Square Roots.

CHAPTER

III PRACTICAL GEOMETRY

13. LINES AND ANGLES 125

Lines and Points. Parallels and Perpendiculars. Angle Measurement. The Protractor. Kinds of Angles. Related Angles. Parallels and Transversals. Distance. Projections.

14. TRIANGLES AND POLYGONS 136

Kinds of Triangles. Altitudes and Medians. Bisectors of Sides and Angles of a Triangle. Sum of the Angles of a Triangle. Congruence. Right Triangle Rule. Isosceles and Equilateral Triangles. Polygons and Quadrilaterals.

15. CIRCLES AND TANGENTS 153

Circles. Chords. Central Angles. Inscribed Angles. Circumscribed Circles. Tangents. Angle Bisectors. Inscribed Circles. Regular Polygons. Circumference of a Circle. Length of an Arc.

16. GEOMETRIC CONSTRUCTIONS 167

Bisecting Lines and Angles. Constructing Perpendiculars and Parallels. Dividing a Line into Desired Parts. Constructing Tangents. Constructing Equilateral and Isosceles Triangles. Constructing Inscribed and Circumscribed Squares and Hexagons. Constructing a Regular Pentagon.

17. MEASUREMENT OF AREAS 179

Squares and Rectangles. Parallelogram. Triangle. Rhombus. Trapezoid. Irregular Shapes. Circle. Ring. Sector. Segment. Regular Polygons. Ellipse. Irregular Areas.

18. MEASUREMENT OF SOLID FIGURES . . 198

Rectangular Solids. Area and Volume of a Prism. Right Circular Cylinder. Fillets. Pyramids. Regular Tetrahedron. Frustum of a Pyramid. Right Circular Cone. Sphere. Summary of Formulas.

19. SIMILAR FIGURES 219

Similar Triangles. Proportional Parts. Dividing a Line into a Given Ratio. Constructing a Fourth Proportional. Altitude upon the Hypotenuse. Areas of Similar Figures. Regular Polygons and Circles. Similar Solids of Revolution. Spheres. Section of Pyramid.

CHAPTER

IV SHOP TRIGONOMETRY

20. USING TRIGONOMETRIC FUNCTIONS . . . 233

Similar Triangles and Trigonometric Ratios. Definitions of the Functions. Using a Table of Natural Functions. Co-functions. Using the Tangent. Using Sines and Cosines.

21. PRACTICAL APPLICATIONS OF RIGHT TRIANGLES 240

Solution of Right Triangles. Component Forces and Velocities. The Sine Bar. Setting up an Angle on the Sine Bar. Measuring an Unknown Angle. Spacing Holes on a Circle. Tapers and Taper Angle. Miscellaneous Applications.

22. SOLUTION OF OBLIQUE TRIANGLES . 255

Oblique Triangles. Functions of an Obtuse Angle. Law of Sines. Law of Cosines. Four Types of Problems involving Oblique Triangles. Area of Triangles and Regular Polygons.

PART II: TECHNICAL SUBJECTS

CHAPTER

V MECHANICAL DRAWING

23. TOOLS AND EQUIPMENT: LETTERING . 297

The Drawing Board. The T-square. The Triangles. The Compass. The Scale. The French Curve. The Protractor. The Pencil. Lettering. Slanted Lettering. Upright Gothic. Some Important Rules.

24. ALPHABET OF LINES: CONVENTIONS: REPRODUCTIONS 303

Visible Lines. Invisible Edges. Projection Lines. Center Lines. Dimension Lines. Sections. Conventional Cross-Hatchings. Abbreviations. Tracings and Reproductions.

25. TWO VIEW DRAWINGS . . 309

Purpose. Procedure. Illustrative Plates.

26. THREE VIEW DRAWINGS 323

Purpose. Procedure. Illustrative Plates.

27. ISOMETRIC DRAWINGS 332

Distinctive Features. Procedure. Illustrative Plates.

CHAPTER

VI APPLIED PHYSICS

28. Solids, Liquids, and Gases 339

Properties of Matter. Units of Measurement. Mass and Weight. Cohesion and Adhesion. Surface Tension. Capillarity. Liquid Pressure. Pascal's Law. Hydraulic Press. Hydraulic Brakes. Buoyancy. Flotation Principle. Specific Gravity. Hydrometer. Atmospheric Pressure. Barometers. Boyle's Law.

29. Bodies at Rest 360

Forces. Graphic Representation of a Force. Concurrent Forces. Composition of Linear Forces. Composition of Forces at an Angle. Resolution of Concurrent Forces. Parallel Forces. Principle of Moments. Practical Applications.

30. Bodies in Motion 369

Motion and Force. Uniform Motion. Acceleration. Freely Falling Bodies. Newton's Three Laws of Motion: Inertia, Momentum, and Reaction. Centrifugal and Centripetal Forces.

31. Simple Machines 377

Work and Its Measurement. Simple Machines. Mechanical Advantage. Principle of Work. Lever. Wheel and Axle. Pulleys. Inclined Plane. Wedge. Screw. Friction. Efficiency.

32. Energy, Work, and Power 391

Power. Horsepower. Transmission of Power. Potential Energy. Kinetic Energy. Transformation of Energy. Conservation of Energy. Dissipation of Energy.

33. Heat and Light 396

Nature of Heat. Temperature. Thermometers. Expansion of Solids, Liquids, and Gases. Absolute Temperature. Charles' Law. Boyle's Law. Conduction, Convection, and Radiation. Specific Heat. Change of State. Fusion. Vaporization. Distillation. Refrigeration. Light and Illumination. Reflection. Refraction. Inverse Square Law.

34. Magnetism and Electricity 417

Natural and Artificial Magnets. Magnetic Induction. Lines of Force. Theory of Magnetism. Electromag-

netism. Static Electricity. Electroscope. Condensers. Electrical Potential. Voltaic Cell. Dry Cell. Electromagnetic Induction. Generation of Electricity.

CHAPTER

VII STRENGTH OF MATERIALS

35. PHYSICAL PROPERTIES OF MATERIALS . . 437

Materials in Industry. Hardness. Moh's Scale and Brinell Method. Elasticity. Ductility and Malleability. Plasticity and Brittleness. Detection of Flaws. Thermal Expansion and Heat Conductivity. Summary of Physical Properties.

36. MECHANICS OF MATERIALS 446

Internal Stresses. Unit Stress. Deformation. Hooke's Law. Elastic Limit. Stress and Deformation Diagrams. Ultimate Strength and Yield Point. Young's Modulus. Relative Strength of Materials. Fatigue of Metals. Working Stresses and Factor of Safety. Timber and Concrete. Testing of Materials.

37. THEORY OF BEAMS . . 458

Types of Beams and Loads. Beam Reactions. Strength of a Beam. Flexure of Beams. Beam Design.

CHAPTER

VIII PRACTICAL CHEMISTRY

38. BASIC CONCEPTS OF CHEMISTRY 467

Physical and Chemical Changes. Conservation of Matter. Elements and Compounds. Law of Definite Proportions. Law of Multiple Proportions. Atoms and Molecules. Electrons. X-rays and Radioactivity. Protons. Atomic Numbers. Neutrons.

39. CHEMICAL SUBSTANCES AND CHANGES . . 475

Preparation and Use of Oxygen. Rapid Oxidation and Combustion. Explosions. Slow Oxidation and Corrosion. Spontaneous Combustion. Hydrogen. Electrolysis and Synthesis of Water. Oxidation and Reduction. Hydrogenation.

40. ELEMENTARY CHEMICAL THEORY . 483

Ionization of Atoms. Determination of Atomic Weights. Molecular Weights. Chemical Symbols. Valence. Formulas. Chemical Equations. Solutions. Saturation and Solubility. Suspensions. Electrolysis and Ionization. Acids, Bases, and Salts.

41. SOME IMPORTANT ELEMENTS AND COMPOUNDS . . 495

Common Sodium Compounds. Chlorine. Bleaching. Hydrochloric Acid. Sulfur. Hydrogen Sulfide. Sulfur Dioxide. Sulfuric Acid. Chemical Fire Extinguisher. Nitrogen and Ammonia. Nitric Acid. Calcium Compounds. Hard Water. Types of Chemical Reactions.

42. METALS: THEIR USES AND EXTRACTION 524

Electrochemical Series. Replacement of Hydrogen. Replacement of Metals. Metallurgy of Iron. The Blast Furnace. Pig Iron. Wrought Iron. Copper. Electroplating. Zinc. Dry Cells. Lead. Tin. Nickel. Aluminum. Thermite Process.

43. FUELS 551

Wood. Peat and Lignite. Bituminous Coal. Destructive Distillation of Soft Coal. Anthracite. Water Gas. Producer Gas. Natural Gas. Coal Gas. Acetylene. Petroleum-Refining.

CHAPTER

IX MATERIALS OF TRADE AND INDUSTRY

44. CERAMICS AND RELATED MATERIALS . 565

Grinding Processes. Cement and Concrete. Clay. Firing Processes. Bricks and Tile. Soda Lime Glass. Annealing. Window and Plate Glass. Blown and Pressed Glassware. Lead Glass. Pyrex.

45. IRON AND STEEL 574

Cast Iron. Cupola Furnace. Malleable Cast Iron. Steel. Bessemer Process. Open Hearth Steel. Steel Ingots. Rolling, Forging, and Pressing. Crucible Steel. Electric Steel. Cast Steel.

46. ALLOYS 584

Manganese Steel. Nickel Steels. Silicon and Chromium Steels. Molybdenum and Vanadium Steels. High Speed Steel. Bronzes and Brasses. German Silver. Bearing Metals. Fusible Alloys. Stellite and Carboly.

47. MISCELLANEOUS MATERIALS 591

Abrasives. Diamond. Corundum. Carborundum. Crystallized Alumina. Grinding Wheels. Asbestos. Mica. Celluloid. Galalith. Bakelite. Condensation and Polymerization Products. Natural and Artificial Rubber.

CHAPTER

X MACHINE ELEMENTS

48. TAPERS 605

Machine Parts. Principle of the Taper. Standard Tapers. Morse, Brown and Sharpe, and Jarno Tapers. American Standard Taper. Taper Turning.

49. SCREW THREADS 610

Standard Threads. Terminology. Square Thread. Acme Thread. Worm Threads. Bolts and Nuts. Tap Drills.

50. PULLEYS AND BELTS 625

Power Transmission. Countershafts. Cone Pulleys. Belt Drives. Pulley Speeds. Length of Belts. Transmission Gears. Compound Gear Trains.

51. GEARS . . . 635

Types of Gears and Their Uses. Terminology of Gear Parts. Involute. Pressure Angle. Gear Cutting. Gear Tooth Gauges. Generating Gear Teeth. Spur Gears. Bevel Gears.

52. MACHINE BEARINGS . 657

Bearing Surfaces. Bearing Metals. Anti-friction Bearings. Keys and Keyways. Universal Joint.

PART III: TRADE SUBJECTS

CHAPTER

XI MACHINE SHOP PRACTICE

53. BENCH WORK . . 665

Machine Tools. Shop Operations. Hand Tools. Filing. Draw Filing. Pinning. Care of Files. Hacksaws. Chisels. Drills. Measuring a Drill. Clearance. Reamers. Taps and Dies. Tap Drills. Layout Work and Tools. Theory of Cutting Tools.

54. THE LATHE 680

Types of Lathes. Principal Parts of a Lathe. Feeding Mechanism. General Care and Oiling. Lathe Attachments. Centers. Spindle Sleeves. Chucks. Steady Rests. Lathe Dogs. Spring Collets. Turret Lathes.

55. USING THE LATHE 693

Lathe Turning. Depth of Cut. Straight Turning. Taper Turning. Compound Rests. Shoulder Turning. Parting. Knurling. Boring. Drilling in a Lathe. Thread

Cutting. Feeding the Threading Tool. Cutting the Thread.

56. MILLING AND GRINDING 708

Milling Machines. Types of Millers. Feeds on a Milling Machine. Milling Cutters. Types of Teeth. Direction of Cut. Holding the Work When Milling. Index Heads. Grinding. Grinding Wheels.

57. SPECIAL MACHINE TOOLS 723

The Shaper. The Ram. Shaper Tools. Holding the Work. Squaring Stock in a Shaper. End Cuts. The Planer. Planer Work. Furniture. Broaching Machines. Metal Cutting Band Saws. Jig Borers.

CHAPTER

XII WOODWORKING AND PATTERNMAKING

58. HAND CUTTING TOOLS 735

Bench Knife. Chisel. Gouge. Plane. Saws. Spokeshave. Draw Knife. Brace and Bit.

59. LAYOUT TOOLS 743

Dividers. Calipers. Scales and Rules. Squares. Center Head. Bevel Protractor. Bevel Square. Marking and Panel Gages. Surface Gage. Trammel Points.

60. PRINCIPLES OF PATTERN CONSTRUCTION . . 750

Solid Patterns. Provision for Shrinkage of Metal. Provision for Draft. Allowance for Finishing. Metals Used for Castings. Size and Shape of Castings. Machining and Finishing. Split Patterns. Marking Patterns.

61. MACHINE TOOLS 763

The Band Saw. Discard Spindle Sander. Wood-turning Lathe. Turning Tools and Their Use. Operating the Lathe.

62. PATTERN CONSTRUCTION 774

Solid One-piece Patterns. Split Patterns. Segment Patterns. Straight Round Core Box. Stave Patterns or Core Boxes.

CHAPTER

XIII METAL TRADES

63. PRESS WORK . . . 795

Punch Presses and Dies. Bending. Drawing. Embossing. Swaging. Die Casting. Alloys.

64. FORGING AND WELDING . . . 803

Hand Forging. Anvil and Forging Tools. Upsetting.

Offsetting. Shouldering. Welding. Machine Forging. Power Hammers. Gas Welding. Brazing. Thermite Process. Electric Welding.

65. Heat Treatment of Steel . . 818

Structural Nature of Steel. Tempering. Annealing. Case and Pack Hardening. High Temperatures Measurement. Identification of Iron and Steel. Hardness Testing.

Chapter

XIV ELECTRICAL TRADES

66. Electrical Wiring 829

Types of Circuits. Joints and Splices. Wiring Systems. Bell Circuits. Annunciator Wiring. Burglar Alarm Wiring. Power and Light Wiring. Rigid and Flexible Conduits. Switches. Testing.

67. Direct Current Machinery . . . 853

Theory of Electromagnetism. Electromagnets. The Single Loop Generator. Commutation. Energizing the Fields. Construction and Characteristics of the Generator. Shunt, Compound, and Series Wound Generators. Connecting Generators. Motors. Starting Rheostats. Storage Battery.

68. Alternating-Current Machinery . . 879

Cycle and Frequency. Single-phase Generator. Three-phase Generator. Synchronizing Generator. Transformers. Power Transmission. A.C. Motors. Single-phase Induction Repulsion Motor. Split-phase Motor. Two- and Three-phase Induction Motors. Three-phase Wound Rotor Motors. Starting Equipment.

69. Measuring Instruments 900

Voltmeter. Ammeter. Wattmeter. True Watts and Apparent Watts. Watt-hour Meter.

70. Electrical Illumination 906

Carbon Lamp. Mazda Lamp. Gas-filled Lamp. Cooper-Hewitt Mercury Vapor Lamp. Measurement of Illumination. Direct, Semi-direct and Indirect Lighting. Reflectors. Fluorescent Lamps.

Glossary of Scientific and Technical Terms 917

Tables for Reference 933

Index 945

INTRODUCTION

THE UNPRECEDENTED and constantly increasing need for men trained for work in the mechanical trades and technical industries provides untold opportunities for the highly skilled tool-maker and expert machinist and also for the humblest helper. Those who enter the field with relatively little experience can advance steadily by increasing their knowledge and background through carefully planned study.

Mechanical Trades for Home Study has been prepared for those workers who want to ground themselves thoroughly in one or more of the basic mechanical trades—machine shop, woodworking and pattern making, metal working, and electrical trades—and who want also a sound training in the mathematics and physical sciences which are essential to a proper understanding of mechanical processes and trade operations. Most books dealing with shopwork and specialty trades assume that the reader has some knowledge of the basic mechanical trades and related science and mathematics. This book, however, commences at the very beginning and presents each subject without assuming any previous knowledge or experience.

The purpose of this book, then, is: to offer a sound introductory preparation for the mechanical trades—be they automotive, aviation, carpentry, building, plumbing, or any other specialty trade—a mastery of the fundamentals of trade knowledge, which may be acquired by home study through diligence and systematic reading. Whether the reader starts from scratch, whether he refreshes his memory in subjects once before studied, or whether he seeks to fill in certain gaps in his knowledge or experience —depends, of course, largely upon his individual circumstances; in any of these cases, the scope of this book is broad enough to meet his particular needs.

PLAN OF THE BOOK

The book is divided into three large sections. Part I deals with the basic mathematical processes and skills which are absolutely indispensable in all technical and mechanical work, and which include the fundamentals of arithmetical computation, the essential principles of algebra, a working knowledge of practical geometry, and an understanding of shop trigonometry. Part II includes related subjects also equally indispensable for an adequate preparation in the trades: mechanical drawing, applied physics, strength of materials, practical chemistry, the materials of industry, and machine elements. Without a sound understanding of the content of both these sections the technical aspects of trade practices would rest on an insecure foundation. Part III is devoted to four of the most fundamental mechanical trades: machine shop practice, woodworking and pattern making, metal work, and electrical trades.

SUGGESTIONS FOR STUDY

The following suggestions will prove helpful to the reader:

(1) As all the information given is important, every paragraph should be gone through diligently and thoughtfully, and nothing should be glossed over.

(2) The illustrative examples and the accompanying diagrams should be given close attention, as they aid you in obtaining a clearer understanding of the text.

(3) It is advisable to follow the order of chapters, as well as the individual development of each chapter, substantially as it is given. Many important ideas and salient facts that come up later are anticipated in the earlier sections; one set of facts frequently leads to another; difficulties are minimized; appropriate and intentional repetition, with different emphases, will clarify numerous matters as you go along.

(4) Try to solve the exercises in the sections on mathematics (Ch. I–IV). Conscientious effort will be fully repaid. Make use of the answers given at the end of Part I.

(5) Remember that all through the discussion in both Part I and Part II, the emphasis and illustrative material centers largely around practical trade applications; in fact, the first ten chapters contain much that is so closely related to the "trades" that they must properly be regarded as an essential part of any study of mechanical trades.

(6) Make good use of the various reference lists of books for further reading and study; they have been carefully selected so as to be of greatest value to you.

ABOUT THE AUTHORS

Without exception, each of the several collaborators in the preparation of the book is by training and background thoroughly qualified for his task, having had many years of actual shop experience or teaching experience, or both. Some of them are well-known authorities in their respective fields. All of them have succeeded in achieving the point of view of the reader for whom this volume is intended; with no sacrifice in accuracy, and with no omission of significant material, they have in every instance skillfully managed to bring to the reader the essential content in a simple and readily understandable presentation. The editor takes this opportunity of expressing his sincere appreciation of the generous and thoughtful co-operation they have shown throughout.

ACKNOWLEDGMENTS

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The editor also wishes to record with gratitude the infinite patience and sympathetic understanding of his wife, Jennie C. Schaaf, whose assistance in the preparation of this book has made it an even pleasanter task.

W. L. S.

PART I

BASIC MATHEMATICS

CHAPTER I

FUNDAMENTALS OF ARITHMETIC

William L. Schaaf

---

1. WEIGHTS AND MEASURES

Importance of Measurement. In the mechanized civilization in which we live the vital importance of measurement cannot be overemphasized. Careful measurements are indispensable in business, in almost all trades, in manufacturing and industry, to say nothing of science, technology and engineering. A famous scientist, Lord Kelvin, once said that not until we can measure something can we really understand it. The same is true of design and production; finished products simply could not be made without the use of measurements. One has only to think of the many parts of an auto engine, the delicate mechanism of a watch, the sensitive shutter of a camera, or the precision of a lathe, to realize how utterly dependent modern man is upon the art of measurement. Indeed, the history of the development of units of measure reflects the march of civilization in more ways than one.

Nature of Measurements. To understand fully the nature and use of measurements, two basic principles must be borne in mind:

(1) *All measurements are approximate.*
(2) *All units of measure are arbitrary.*

There is no such thing as an absolute or perfect measurement. An object can be thought of as having an actual, real, or "true" length; but that length can never be found completely, it can only be found *approximately.* How "exact" any particular measurement happens to be depends upon the nature of the instruments used, the skill of the operator, and the conditions under which it is made. The difference between the true length and the measured length is technically known as the *error.* An error is not a mistake. The careless use, or the misuse, of a measuring instrument leads to mistakes. The proper use of a measuring instrument always involves errors. The errors may be large or small; they can never be completely eliminated. The extent of the approximation is known as the *degree of accuracy* of the measurement; a numerical measure of the extent of the error is known as the *precision* of the measurement. What particular degree of accuracy is sought depends chiefly upon the purpose for which the measurement is made, or the use to which the object is to be put. It would be a waste of time, for example, to measure the length of a fence post with the same care, i.e., with the same degree of accuracy, as used in measuring the length of a piston rod or a valve stem. Similarly, the accuracy used in measuring the length of a piece of molding in cabinet work would not suffice when measuring the diameter of needle valve. We shall learn more, however, about degrees of accuracy in Section 4 of this chapter.

Measurements are based upon certain *basic units* which are simply agreed upon by all concerned. In other words, the length of our present *inch* is what it is because it has been fixed by custom, or common consent; it has been *standardized,* so that it is always the same. But it might just as easily have been fixed, or standardized, at half its present length, or twice that amount, etc. In fact, at various times in history, and in different parts of the world, the inch has actually varied considerably from its present length. The same can be said of almost all the other units of measure that man has ever established or used: their values are *arbitrarily* fixed in accordance with conventional standards, and all who use these measures agree to abide by those standards.

Kinds of Measures. The fundamental measures are *length, mass,* and *time.* All other measures are derived from (or related to) one or another of these fundamental quantities. Thus *surfaces* (areas) and *volumes* (capacity) are measured in units derived from linear units; e.g., square inch and cubic inch. *Mass,* which is related to weight, is measured in the same units as weight, e.g., grams, ounces, pounds, etc. *Time* is measured in seconds, minutes, hours, etc. Other units of measure, such as those used for temperature, the electric current, or heat energy, are frequently more complicated. But whatever the unit used, it is always an arbitrary standard involving length, mass or time, or some combination of these. In this chapter we shall deal only with measures of length, area, capacity and weight.

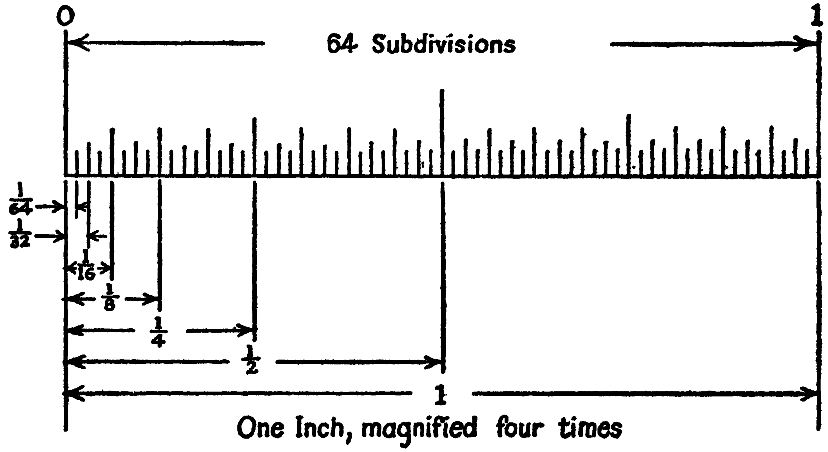
Standards of Measure. Today all basic units of measure used in this country, such as length, weight, area, capacity, temperature, etc., are determined by the United States Bureau of Standards in Washington, D.C. In the main, they agree with similar units used throughout the world. Two major systems are in use,—the *English* system and the *Metric* system, although the latter is little used in industry and trade, being the tool primarily of the scientist.

The best interests of all are served when the measuring instruments and devices used by scientists, engineers and industrial workers are sent to the Bureau of Standards at regular intervals to be checked for accuracy against universally accepted standards. Furthermore, most trades and industries have set up the standards which are used in their own respective fields. These standards concern the choice of measuring units, various specifications, and working conditions; they do not, of course, conflict with the Federal Government's basic standards. But cooperation among various industries, and in various fields of technology, long ago became so important, that the formation of the American Standards Association was inevitable. This organization passes on the standards set in various parts of the country and works with similar organizations in other countries to establish and maintain desirable standards of measures, specifications, and trade practices.

Linear Measure. In the English system, the commonly used linear measures are as follows:

unit = 1 inch
12 inches = 1 foot
3 feet = 1 yard
5½ yards = 1 rod
320 rods = 1 mile
1760 yards = 1 mile
5280 feet = 1 mile

The inch is sometimes subdivided by *halves*, sometimes by *tenths*. Each of these methods is here illustrated; both are commonly used by mechanics and industrial workers.



One Inch, magnified four times

When sub-divided by the decimal system, the subdivisions run as follows:

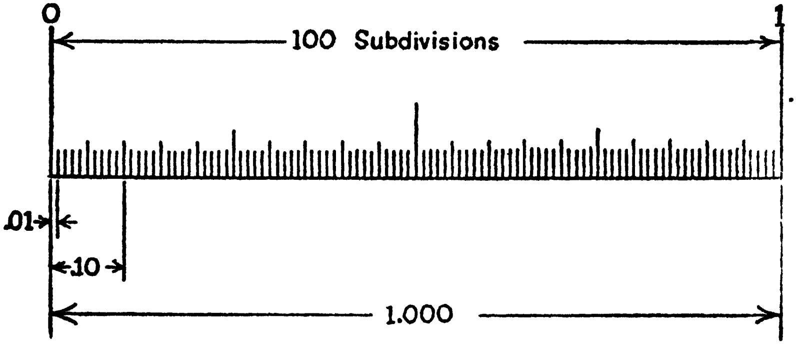
1 = 1.000 (one)

$\frac{1}{10}$ = .10 (one-tenth)

$\frac{1}{100} = .01$ (one-hundredth)

$\frac{1}{1000} = .001$ (one-thousandth)

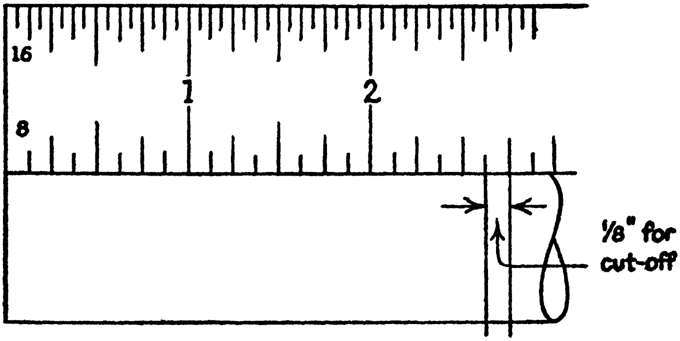
$\frac{1}{10,000} = .0001$ (one ten-thousandth)



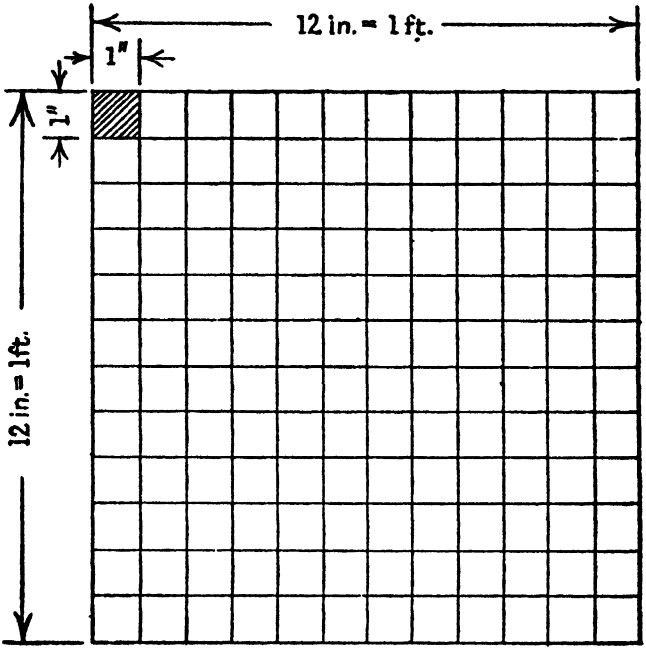
One Inch, magnified four times

Surface Measure. The commonly used measures of area in shop work and trade operations include:

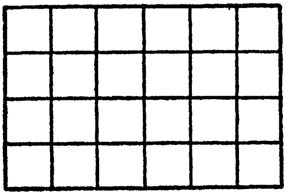
unit = 1 square inch
144 square inches = 1 square foot
9 square feet = 1 square yard



Showing use of scale in measuring a 2½" piece to be cut from stock allowing ⅛" for cut-off.



Pictorial representation, showing how 1 sq. ft. contains 144 sq. in.



Area 4 x 6 = 24 Sq. in.

A square inch is derived from the linear inch by taking as the unit of surface a square one inch on each side. The area of a surface is taken to be the number of square inches it contains. The area of a square is thus found by multiplying the length of one side by the other side, i.e.,

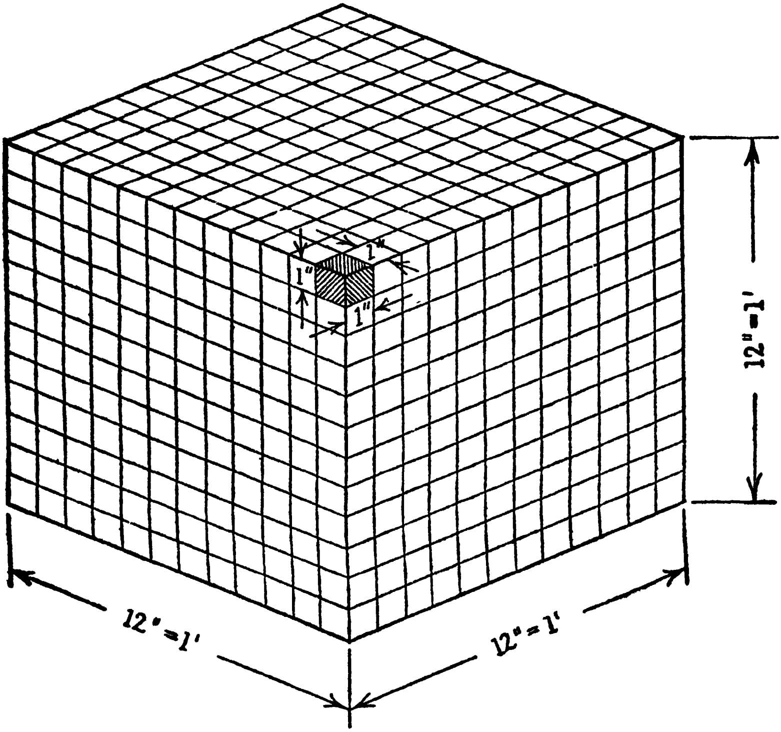
$$A = s \times s, \text{ or } A = s^2,$$

which is read *"area equals* s *squared."* Similarly, the area of a rectangle is found by multiplying its length by its width; i.e.,

$$A = l \times w, \text{ or } A = lw.$$

Volume and Capacity. The *volume* of a solid object such as a block of wood or a slab of metal refers to the amount of space it occupies. The *capacity* of a hollow vessel, such as a glass jar, a wooden box, or a metal bucket, refers to the quantity of material it can contain or hold; capacity is therefore expressed in terms of "inside" measurements, since the thickness of the walls of the container cannot be disregarded (unless they are comparatively thin, or the measure of capacity is to be taken rather approximately). Volume and capacity are often measured in terms of the same units, viz.:

unit = 1 cubic inch
1728 cubic inches = 1 cubic foot
27 cubic feet = 1 cubic yard
1 cubic yard = 1 load



Pictorial representation, showing how 1 cu. ft. contains 1728 cu. in.

The unit of volume is taken as a cube each of whose edges is one inch in length. The volume of a cube is thus given by

$$V = e \times e \times e, \text{ or } V = e^3,$$

which is read *"volume equals e cubed."* Similarly, the volume of a rectangular solid of length l, width w and thickness (or height) h, is given by

$$V = l \times w \times h, \text{ or } V = lwh.$$

For certain purposes, capacity is also measured in other units as well. For liquid measure, we have:

4 gills = 1 pint
2 pints = 1 quart
4 quarts = 1 gallon

Measures of Weight. For measuring the weight of common objects, several systems of measures are in common use, viz., the *Avoirdupois, Troy,* and *Apothecaries'* systems. For all ordinary purposes, and unless otherwise specified, weights are in the Avoirdupois system; Troy weight is used only by jewelers, etc., when weighing gold, silver and gems, while the Apothecaries' system is used only by pharmacists and physicians.

AVOIRDUPOIS WEIGHT

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 hundredweights = 1 ton (T.)
2000 pounds = 1 ton
2240 pounds = 1 long ton
7000 grains (gr.) = 1 pound avoirdupois

Certain common "equivalents" are well worth remembering. These are:

1 gallon = 231 cubic inches
1 cubic foot = 7½ gallons
1 barrel = 31½ gallons
1 cubic foot of water = 62.4 pounds

Exercise 1.

1. A roll of paper is 88″ wide; what is the width expressed in feet?
2. Find the length of the border of a rectangular rug measuring 9′3″× 12′4″.
3. An airplane is traveling at a speed of 270 miles per hour; this is equivalent to how many feet per minute?
4. A piece of metal screening contains 20½ sq. yd.; how many square feet of surface will it cover?
5. A velocity of 88 ft. per second is equivalent to how many miles per hour?
6. A gallon of paint covers 150 sq. ft. with one coat. How many gallons will be required to cover a surface of 750 sq. ft. with two coats of this paint?
7. Linoleum costing 12¢ a sq. yd. was laid on a kitchen floor 12′×21′. What was the cost of the linoleum?
8. A point on the rim of a flywheel is moving at the rate of 1200 ft. per minute; what is the rate in feet per second?

9. A sheet of metal foil measures 2½"×4". How many sq. ft. of foil are there in 1000 such sheets?
10. How many cubic feet are there to a gallon? to the barrel?
11. How much does a gallon of water weigh?
12. If a roof tank contains 1500 gallons of water, what is the weight of its contents?
13. A fuel tank for an oil burner has a capacity of 300 gallons; how many cubic feet is this?
14. A vat in a soap factory has a capacity of 1600 cu. ft. If it is filled to ¾ of its capacity with liquor weighing 70 lb. per cu. ft., what is the weight of its contents?
15. An air blower in a factory has a rectangular cross section, 12"×20". Air is blown through the duct at the rate of 6 ft. per second. At this rate, how many cubic feet of air pass a given point every minute?

Metric System. This is the system of weights and measures used by scientists the world over. While it is legal and permissive in the United States, it has never been widely adopted in industrial and shop practice in this country.

The fundamental standard of length of the metric system is the meter, which is defined as the distance between two scratch-marks on a bar of platinum-iridium, carefully preserved at the International Bureau of Weights and Measures near Paris, when the temperature of the bar is that of melting ice (o° Centrigrade) All other units of length are multiples or submultiples of the meter, as shown below.

METRIC LINEAR MEASURE

10 millimeters (mm.) = 1 centimeter (cm.)
10 centimeters = 1 decimeter (dm.)
10 decimeters = 1 meter (m.)
10 meters = 1 decameter (Dm.)
10 decameters = 1 hectometer (Hm.)
10 hectometers = 1 kilometer (Km.)
10 kilometers = 1 myriameter (Mm.)

It should be added that the most commonly used of these units are the kilometer, the meter, the centimeter and the millimeter. For purpose of converting lengths and distances from one system to the other, the following approximate equivalents are convenient:

APPROXIMATE EQUIVALENTS

1 centimeter = about 0.4 inch
1 meter = about 1.1 yard
1 kilometer = about 0.6 mile

1 inch =about 2.5 centimeters
1 yard =about 0.9 meter
1 mile =about 1.6 kilometers

Metric Surface Measure. The basic units of metric surface measure is the ***square meter,*** which is a square one meter long on a side. Another convenient and commonly used unit of area is the square centimeter, which is a square one centimeter on each side. The relations between the various units of area are given below.

METRIC SURFACE MEASURE

100 square millimeters (sq. mm.) = 1 square centimeter (sq. cm.)
100 square centimeters = 1 square decimeter (sq. dm.)
100 square decimeters = 1 square meter (sq. m.)
100 square meters = 1 square decameter (sq. Dm.)
100 square decameters = 1 square hectometer (sq. Hm.)
100 square hectometers = 1 square kilometer (sq. Km.)

For ordinary purposes and convenience in converting from English into metric units and vice versa, the following equivalents are given:

APPROXIMATE EQUIVALENTS

1 sq. inch=about 6.5 sq. centimeters
1 sq. foot =about .09 sq. meter
1 sq. yard=about .84 sq. meter
1 sq. mile=about 2.6 sq. kilometers
1 sq. centimeter=about .15 sq. inch
1 sq. kilometer =about .39 sq. mi.

Metric Measures of Capacity. In measuring cubical contents by the metric system the fundamental unit of volume is the ***cubic meter.*** Other units commonly employed are as follows:

METRIC UNITS OF VOLUME

1000 cubic millimeters (cu. mm.)=1 cubic centimeter (cu. cm.)
1000 cubic centimeters =1 cubic decimeter (cu. dm.)
1000 cubic decimeters =1 cubic meter (cu. m.)

For measuring both liquids and solids, the *liter* is commonly employed. A liter is the same as a cubic centimeter, or 1000 cu. cm. (also abbreviated *cc.*).

APPROXIMATE EQUIVALENTS

1 cubic inch =about 16 cubic centimeters
1 cubic centimeter=about .06 cubic inch

1 quart (liquid) =about .95 liter
1 liter =about 1.06 quart (liquid)

Metric Units of Weight. Compared to the variety of English units of weight, the metric system is doubtless much simpler. Here the standard unit of weight is the *kilogram,* which is defined as the weight of a certain mass of platinum-iridium kept at the International Bureau of Weights and Measures near Paris, and known as the International Prototype Kilogram. The kilogram and the gram are the two most widely used units, except for very large weights.

METRIC MEASURES OF WEIGHT

10 milligrams (mg.) = 1 centigram (cg.)
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 gram (g.)
1000 grams = 1 kilogram (Kg.)
1000 kilograms = 1 metric ton (T.)

APPROXIMATE EQUIVALENTS

1 ounce = 28.35 grams
1 pound = 454 grams = .45 kilogram
1 gram = .035 ounce
1 kilogram = 2.2 pounds

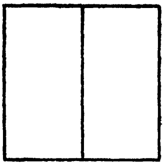
Exercise 2.

1. How many ounces are there in 250 grams?
2. How many inches are there in 45 millimeters?
3. How many centimeters are there in 15 inches?
4. How many kilograms are there in 12 pounds?
5. How many inches are there in 50 centimeters?
6. How many pounds are there in 75 kilograms?
7. How many millimeters are there in 6½ inches?
8. How many grams are there in 8 ounces?
9. What is the weight in kilograms of an athlete weighing 172 pounds?
10. What is the height in feet and inches of a man whose medical record states that he is 178 cm. tall?
11. How many cubic centimeters are there in a glass container capable of holding 2 quarts?
12. How many quarts can a 10-liter jug hold?
13. How many pints are there in a vessel whose capacity is 750 cc.?
14. How many liters are there in a 2-gallon chemical container?
15. How many sq. cm. are there in the surface of a metal plate having an area of 200 square inches?

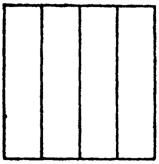
2. COMMON FRACTIONS

Whole Numbers and Fractions. Numbers like 2, 5, 13, 40, etc., are known as whole numbers, or *integers.* When measurements are made, the magnitude is expressed, if possible, in some integral number of units, as:

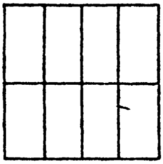
3 inches, 5 feet, 12 pounds, etc. But with many magnitudes this is impossible, and it becomes necessary to make use of a *part of a unit.* To do this, the entire unit is *thought of* as being divided into any convenient number of *equal parts;* one or more of such equal parts into which a whole unit has been divided is known as a *fraction.* Thus $\frac{1}{2}$, $\frac{2}{3}$, $\frac{4}{6}$ and $\frac{3}{10}$ are common fractions. The number below the fraction line, called the *denominator,* states into how many equal parts the whole unit has been divided; the number above the line, known as the *numerator,* tells how many of these equal parts are being considered, or measured, or used.



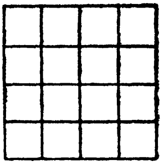
Halves ($\frac{1}{2}$'s)



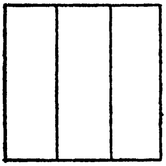
Fourths ($\frac{1}{4}$ths)



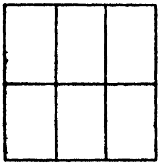
Eighths ($\frac{1}{8}$ths)



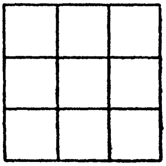
Sixteenths ($\frac{1}{16}$ths)



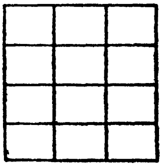
Thirds ($\frac{1}{3}$rds)



Sixths ($\frac{1}{6}$ths)



Ninths ($\frac{1}{9}$ths)



Twelfths ($\frac{1}{12}$ths)

Fractions like $\frac{5}{8}$, $\frac{3}{16}$, $\frac{1}{4}$, and $\frac{15}{32}$, where the numerator is smaller than the denominator, are known as *proper fractions.* Proper fractions always indicate a quantity which is less than 1. On the other hand, fractions like $\frac{4}{3}$, $\frac{3}{2}$, $\frac{9}{8}$, $\frac{25}{12}$, where the numerator is greater than the denominator, are called *improper fractions;* such fractions always indicate an amount larger than 1, or one unit. Fractions like $\frac{2}{2}$, $\frac{3}{3}$, $\frac{8}{8}$, where the numerator equals the denominator, are also called improper fractions, although their value is exactly equal to 1 in each case. Strictly speaking, they do not represent a fractional part of a unit at all; they only have the form of a fraction, and actually represent the *whole unit,* since they say, in effect, "divide the whole unit into a certain number of equal parts, and then consider *all* of those parts."

An improper fraction of the first kind mentioned, e.g., $\frac{5}{3}$, can also be expressed as the sum of a whole number and a proper fraction; thus $\frac{5}{3}=\frac{3}{3}+\frac{2}{3}=1+\frac{2}{3}=1\frac{2}{3}$. When written in this final form, as an integer plus a proper fraction, but with the $+$ sign omitted, it is usually called a *mixed number.*

Example 1: Express $\frac{25}{8}$ as a mixed number.

Solution: $\frac{25}{8}=\frac{24+1}{8}=\frac{24}{8}+\frac{1}{8}=3+\frac{1}{8}=3\frac{1}{8}$, *Ans.*

Example 2: Change $4\frac{7}{16}$ to an improper fraction.

Solution: $4\frac{7}{16}=4+\frac{7}{16}=\frac{4\times16}{16}+\frac{7}{16}=\frac{64}{16}+\frac{7}{16}=\frac{71}{16}$, *Ans.*

Reduction of Fractions. Proper fractions like $\frac{5}{8}$, $\frac{9}{16}$, $\frac{3}{4}$, and $\frac{2}{3}$ are said to be in their *lowest terms.* This means that both the numerator and the denominator cannot be further reduced by dividing each of them by the same number. On the other hand, fractions like $\frac{6}{8}$, $\frac{10}{16}$, $\frac{8}{32}$, and $\frac{4}{64}$ are *not* in their lowest terms; they can be reduced further by dividing *both* numerator and denominator by some number which is an exact divisor of each of them; dividing both the numerator and denominator of a fraction by a common divisor is always permissible, since it does not alter the value of the original fraction. Thus:

$$\frac{6}{8}=\frac{3}{4};\ \frac{10}{16}=\frac{5}{8};\ \frac{8}{32}=\frac{1}{4};\ \frac{4}{64}=\frac{1}{16}.$$

Fractions can also be transformed by *multiplying* both numerator and denominator by any desired number, provided the *same* number is applied to both parts of the fraction; this does not change the value of the fraction either. It enables us, however, to express a fraction in any other denomination. Thus we have the following:

Example 1: Change the fraction $\frac{7}{8}$ to an equivalent fraction having the denominator 32.

Solution: Dividing 32, the new denominator, by 8, the old denominator, gives 4; this is the number by which we must multiply each part of the fraction.

$$\frac{7}{8}=\frac{7\times4}{8\times4}=\frac{28}{32},\ \textit{Ans.}$$

Example 2: Express $\frac{3}{4}$ as 64ths.

Solution: $64\div4=16$

$$\frac{3}{4}=\frac{3\times16}{4\times16}=\frac{48}{64},\ \textit{Ans.}$$

Exercise 3.

1. Change $\frac{3}{8}$ in. to 64ths; $2\frac{1}{4}$ in. to 16ths; $1\frac{5}{8}$ in. to 32nds; $10\frac{3}{16}$ in. to 64ths.

2. The thickness of a brass plate is $\frac{15}{32}''$; express this in eighths of an inch. Is its thickness more than $\frac{1}{2}''$? If so, how much more? Is it less than $\frac{3}{4}''$? If so, how much less?
3. Find the equivalent number of 16ths of an inch in $2\frac{1}{2}$ in.; in $1\frac{5}{8}$ in.; in $3\frac{3}{16}$ in.; in $\frac{3}{4}$ in.
4. A piece of cardboard is $5\frac{3}{4}$ in. wide. If it is to be cut into strips each $\frac{1}{4}$ in. in width, how many such strips will there be?
5. The face of a metal block is $3\frac{5}{8}$ in. wide; this is equivalent to how many 16ths? how many 32nds?
6. Is $\frac{18}{32}$ larger than $\frac{10}{12}$? Is $2\frac{1}{4}$ larger than $\frac{28}{16}$? Arrange the following dimensions in order of magnitude, beginning with the smallest: $\frac{7}{8}''$, $2\frac{1}{16}''$, $2\frac{3}{64}''$, $\frac{3}{4}''$.

Addition and Subtraction of Fractions. When adding two or more fractions having the same denominator, we simply add their numerators and place the result above their *common* denominator. Thus:

$$\frac{3}{8}+\frac{7}{8}+\frac{5}{8}=\frac{3+7+5}{8}=\frac{15}{8}=1\frac{7}{8}.$$

If the denominators of the fractions to be added are not all alike, the fractions must first be changed to equivalent fractions which do have the same denominator. The *least common denominator* (L.C.D.) is the *smallest denominator* that is exactly divisible by *each* of the denominators in question. For example:

(1) to add $\frac{3}{8}$, $\frac{3}{4}$ and $\frac{1}{2}$, the L.C.D.$=8$

(2) to add $\frac{2}{5}$, $\frac{3}{10}$ and $\frac{1}{4}$, the L.C.D.$=20$.

In practical problems, the L.C.D. is readily found by inspection.

Example 1: Add, $\frac{2}{3}+\frac{1}{2}+\frac{3}{4}$.

Solution: L.C.D.$=12$.

$$\frac{2}{3}=\frac{8}{12};\ \frac{1}{2}=\frac{6}{12};\ \frac{3}{4}=\frac{9}{12}$$

$$\frac{8}{12}+\frac{6}{12}+\frac{9}{12}=\frac{8+6+9}{12}=\frac{23}{12}=1\frac{11}{12},\ \textit{Ans.}$$

Example 2: Add, $2\frac{1}{4}+\frac{3}{16}+4\frac{7}{8}+\frac{5}{32}$

Solution: L.C.D.$=32$.

$$\frac{9}{4}+\frac{3}{16}+\frac{39}{8}+\frac{5}{32}$$

$$=\frac{72}{32}+\frac{6}{32}+\frac{156}{32}+\frac{5}{32}$$

$$=\frac{72+6+156+5}{32}=\frac{239}{32}=7\frac{15}{32},\ \textit{Ans.}$$

Example 3: Subtract $4\frac{3}{8}$ from $6\frac{5}{16}$.

Solution: L.C.D.=16.

$$\begin{array}{ll|l} 6\frac{5}{16}=6\frac{5}{16} & & 5\frac{21}{16} \\ 4\frac{3}{8}=4\frac{6}{16} & & 4\frac{6}{16} \\ \hline & & 1\frac{15}{16}, \textit{ Ans.} \end{array}$$

Exercise 4.

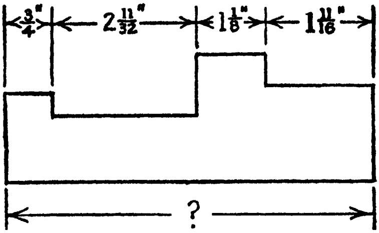
Add the following:

1. $\frac{2}{3}+\frac{3}{4}$
2. $\frac{1}{2}+\frac{5}{8}+\frac{1}{4}$
3. $\frac{1}{8}+\frac{7}{8}+\frac{3}{4}$
4. $\frac{3}{8}+\frac{1}{4}+\frac{5}{16}$
5. $2\frac{1}{2}+5\frac{7}{8}$
6. $3\frac{1}{4}+2\frac{1}{16}+1\frac{5}{8}$
7. $\frac{7}{8}+5\frac{1}{4}+2\frac{3}{16}$
8. $4\frac{1}{2}+\frac{5}{32}+2\frac{1}{8}+1\frac{3}{4}$

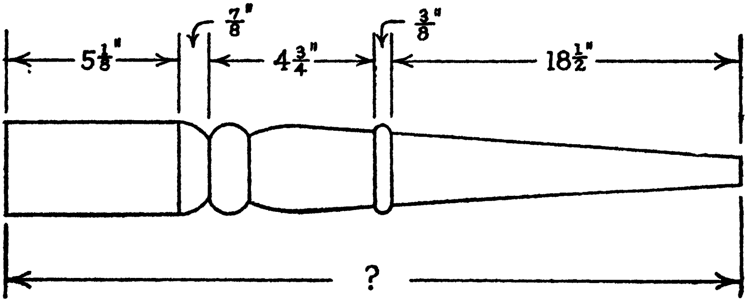
Subtract:

9. $3\frac{1}{2}$ from $5\frac{7}{8}$
10. $\frac{9}{16}$ from $2\frac{3}{4}$
11. $\frac{11}{64}$ from $\frac{9}{32}$
12. $3\frac{5}{8}$ from $5\frac{7}{32}$

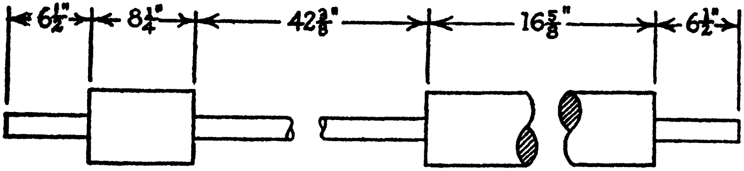
13. Find the overall length of the metal plate shown below:



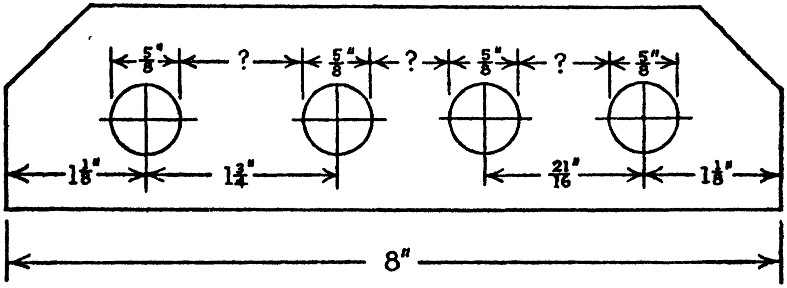
14. Find the total length of the wooden piece to be turned as follows:



15. What is the entire length of the shaft here shown?



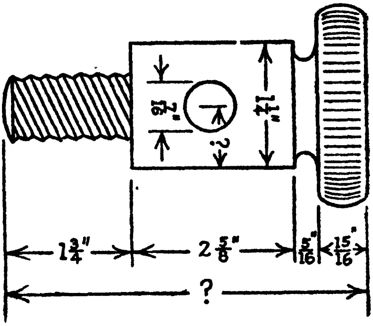
16. A metal plate is 1³⁄₁₆″ thick. If ¹⁄₆₄″ is removed from the top surface and ³⁄₃₂″ from the bottom surface, what is its final thickness?

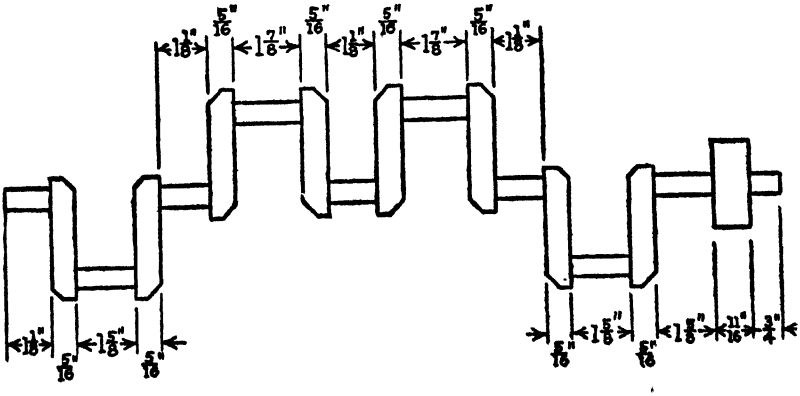


17. Find the missing dimensions in the wooden fixture with four holes drilled as per specification.

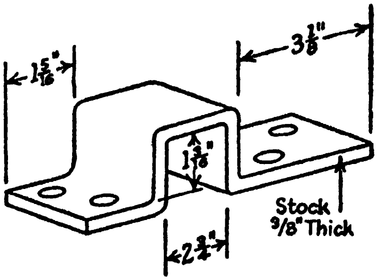
18. Find the overall length of the metal pin here shown; also, the distance from the center of the hole to the edge.

19. Find the overall length of the engine crank shaft here shown.





20. What length of stock is required to make the bent metal fixture here shown? (To allow for the bends, add ½ of the thickness of the stock for each right angle bend to the total of the inside measurements.)



Multiplication and Division of Fractions. In order to multiply two or more fractions together, the respective numerators are multiplied to find the numerator of the product, and their respective denominators are multiplied to find the denominator of the product; the resulting fraction is then reduced to lowest terms, if possible. Should some of the numerators have common factors, these should be divided out first (commonly called "cancellation";) the remaining factors are then multiplied together as before. If mixed numbers are to be multiplied, they are first changed to improper fractions.

EXAMPLE 1: Multiply $\frac{3}{4}\times\frac{5}{8}\times\frac{1}{2}$.

SOLUTION: $\frac{3}{4}\times\frac{5}{8}\times\frac{1}{2}=\frac{3\times5\times1}{4\times8\times2}=\frac{15}{64}$, *Ans.*

EXAMPLE 2: Multiply $\frac{9}{16}\times\frac{2}{3}\times\frac{4}{5}$.

SOLUTION: $\frac{\overset{3}{\cancel{9}}}{\underset{\underset{2}{\cancel{8}}}{\cancel{16}}}\times\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}\times\frac{\overset{1}{\cancel{4}}}{5}=\frac{3\times1\times1}{2\times1\times5}=\frac{3}{10}$, *Ans.*

EXAMPLE 3: Multiply $3\frac{1}{7}\times10\frac{1}{2}$.

SOLUTION: $\frac{\overset{11}{\cancel{22}}}{\cancel{7}}\times\frac{\overset{3}{\cancel{21}}}{\cancel{2}}=33$, *Ans.*

To divide any quantity (a whole number, a fraction, or a mixed number) by a fraction, the divisor is inverted and then multiplied by the dividend; the same holds true even if the divisor is a whole number of an improper fraction. To divide by a mixed number, first change it to an improper fraction, and then proceed as before.

EXAMPLE 1: Divide $\frac{4}{9}$ by $\frac{5}{6}$.

SOLUTION: $\frac{4}{9}\div\frac{5}{6}=\frac{4}{\underset{3}{\cancel{9}}}\times\frac{\overset{2}{\cancel{6}}}{5}=\frac{8}{15}$, *Ans.*

Example 2: Divide $\frac{9}{16}$ by 3.

Solution: $\frac{9}{16}\div 3=\frac{9}{16}\div\frac{3}{1}=\frac{\overset{3}{\cancel{9}}}{16}\times\frac{1}{\cancel{3}}=\frac{3}{16}$, *Ans.*

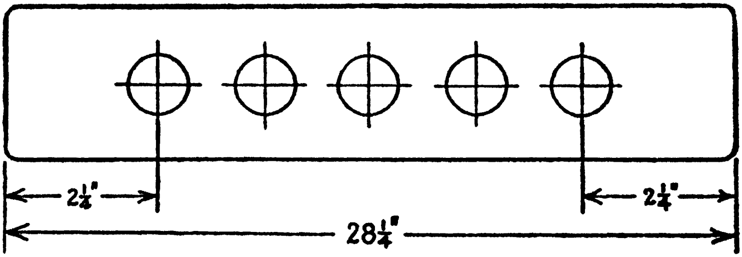
Example 3. Divide $4\frac{2}{3}$ by $2\frac{1}{2}$.

Solution: $4\frac{2}{3}\div 2\frac{1}{2}=\frac{14}{3}\div\frac{5}{2}=\frac{14}{3}\times\frac{2}{5}=\frac{28}{15}=1\frac{13}{15}$, *Ans.*

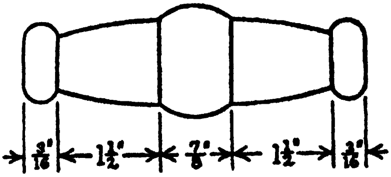
Exercise 5.

Perform the following indicated multiplications or divisions:

1. $1\frac{3}{8}\times 2\frac{1}{2}$
2. $24\times 3\frac{5}{8}$
3. $3\frac{1}{4}\times 1\frac{1}{2}\times 16$
4. $12\frac{3}{8}\div 2\frac{1}{4}$
5. $14\div 1\frac{3}{4}$
6. $3\frac{3}{4}\div 6$
7. If there are $7\frac{1}{2}$ gal. in a cubic foot, how many gallons are there in a tank whose capacity is $28\frac{1}{2}$ cu. ft. when it is $\frac{3}{4}$ filled?
8. If the five holes bored in the wooden strip here shown are to have their centers equally spaced, find the center to center distance between holes.



9. Eight pins each $4\frac{3}{8}''$ in length are cut from a piece of stock $38\frac{1}{4}''$ long. Allowing $\frac{1}{16}''$ waste for each cut, what is the length of the piece left?
10. In turning wooden handles like this one shown, $\frac{3}{4}''$ is allowed for waste on each handle. How many such handles may be made from a piece of stock, if the stock comes in 8 ft. lengths?



11. What is the weight of six $14\frac{1}{2}$-ft. lengths of pipe, if this particular size pipe weighs $3\frac{1}{4}$ lb. per running foot?
12. How many circular discs each $\frac{9}{16}''$ thick can be cut from a metal rod $36\frac{1}{2}''$ long, if $\frac{3}{32}''$ waste is allowed for each cut?

3. DECIMAL FRACTIONS

Meaning and Use of Decimal Fractions. Fractions with a denominator of 10, 100, 1000, etc., may be written as *decimals,* i.e., without a fraction line and without expressing the denominator in numbers; thus .3 is the same as $^{3}/_{10}$, 0.16 is the same as $^{16}/_{100}$, and .247 is the same as $^{247}/_{1000}$. In all decimal fractions the denominators are multiples of 10; they do not have to be written out, since the position of the decimal point takes the place of the denominator. Ordinarily, these decimals would be read as follows:

.3 as "three tenths."
.16 as "sixteen hundredths."
.247 as "two hundred forty-seven thousandths."
.3859 as "three thousand eight hundred fifty-nine ten-thousandths."

In many trades, measurements are sufficiently accurate when expressed in common fractions of an inch. However, not all fractional measurements in shop work are expressed as common fractions; as a matter of fact, in the machine shop, especially in precision work, the machinist uses decimal fractions of an inch more often than common fractions. The machinist, who uses precision instruments to attain the fine measurements called for, has developed a method of reading and designating decimals which differs somewhat from the expressions used by non-technical people. Studying the table given below will show you how to "talk decimals" in the machine-shop manner:

Decimal	*Designation.*
0.0001	One-tenth thousandth.
0.00025	One-quarter thousandth.
0.0005	One-half thousandth.
0.00075	Three-quarter thousandth.
0.001	One thousandth.
0.00125	One and one-quarter thousandths.
0.0015	One and one-half thousandths.
0.002	Two thousandths.
0.0025	Two and one-half thousandths.
0.003	Three thousandths.
0.0075	Seven and one-half thousandths.
0.010	Ten thousandths.
0.0125	Twelve and one-half thousandths.
0.015	Fifteen thousandths.
0.0156	Fifteen and six-tenth thousandths.
0.0312	Thirty-one and two-tenth thousandths.
0.1718	One hundred seventy-one and eight-tenth thousandths.

Exercise 6.

Write each of the following in figures:

1. Two hundred and seventy-six thousandths.
2. Fifteen and four-tenth thousandths.
3. Seven-tenth thousandths.
4. Four and one-quarter thousandths.
5. One hundred thousandths.

Write each of the following in words:

6. 0.3792
7. 0.0006
8. 0.2002
9. 0.0705
10. 0.2816
11. 0.0960
12. 0.4444
13. 0.0158
14. 0.00025

Addition and Subtraction of Decimals. These operations are carried out exactly as in the case of whole numbers, an important feature being the decimal points, which must be kept one under the other, including the decimal point in the sum or difference.

EXAMPLE 1: Add: 0.316+0.0592+1.8034+.26

SOLUTION:
$$\begin{array}{r} 0.316 \\ 0.0592 \\ 1.8034 \\ \underline{0.26} \\ 2.4386 \end{array}$$
Ans.

EXAMPLE 2: From 124.307, subtract 88.092.

SOLUTION:
$$\begin{array}{r} 124.307 \\ \underline{88.092} \\ 36.215 \end{array}$$
Ans.

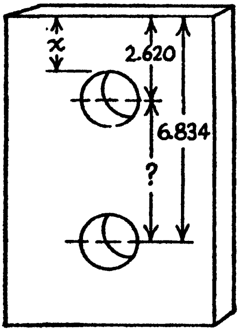
Another point that should be remembered with regard to adding or subtracting *measurements* involving decimals: never add or subtract measurements having different numbers of decimal places. Always "round off" the measurements, as required, to the same number of decimal places as appear in the measurement having the least number of decimal places, as shown below:

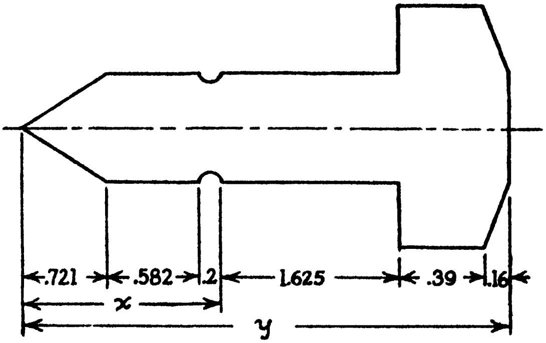
Incorrect		*Correct*	
8.36	inches	8.4	inches
10.082	"	10.1	"
4.5928	"	4.6	"
7.8	"	7.8	"
.749	"	.7	"
31.5838	inches	31.6	inches

If, however, we wish to add a measurement of 14¾″ to another measurement of 23½″, it is correct to say 14.75″+23.5″=38.25″, ***provided that the second measurement is also taken accurately to two decimal places,*** i.e., correct to the nearest .01 inch, in which case it should have been more properly expressed as 23.50″, showing that both the tenth's and hundredth's place had ***actually been measured.*** An exception to this statement may be made wherever dimensions on a drawing, or actual measurements on a piece of work, are understood to be taken to the same degree of accuracy; also when using precision gage blocks, as explained later in Section 4 of this chapter.

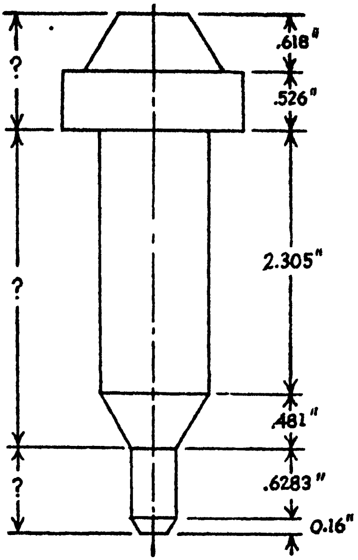
Exercise 7.

1. A hollow metal cylinder has a measured inside diameter of 2.0275″. If the wall of the cylinder is .245″ thick, what is the outside diameter?
2. A round piece of work is supposed to have a diameter of 2.375″. If it was turned .0008″ too large, what was the diameter?
3. Because of a mistake in dimensions on a blueprint a piece of work measuring 1.428 in. in thickness must be reduced 0.236 in. What will the thickness be after the reduction has been made?
4. It is desired to grind down a flat steel plate measuring 0.625″ thick by taking off .0075″. What will it measure after it has been ground?
5. A machinist milled off 0.184″ from each face of a circular brass plate. Before he had done this the plate measured 1.062″ in thickness. What did it measure afterward?
6. A toolmaker in checking a precision measurement uses four gage blocks measuring, respectively, as follows: .141″, .250″, 1.0007″ and 3.000″. Find the combined thickness of the four blocks.
7. Find the inside diameter of a circular tube whose outside diameter is 1.804″ and whose thickness is .216″.
8. Find the center distance between the two holes in the plate shown below. If the diameter of each hole is 1.62″, find the distance *x*.
9. Find the two overall distances *x* and *y* in the pin illustrated below.





10. The actual diameter of a crank shaft is 4.2836″. The inside diameter of the bearing into which this crank shaft fits is found to be 4.285″. What clearance does the shaft have?
11. The inside diameter of a hollow shaft measures 3.026″, and the outside diameter, 3.504″. Find the thickness of the shaft.
12. Find the indicated missing dimensions in the piece shown at the right.



Multiplication of Decimals. When a whole number is to be multiplied by a decimal, or when two decimals are to be multiplied together, the multiplication is first carried out exactly as with whole numbers; then, beginning at the right of the product, point off as many decimal places as there are in both factors together, prefixing ciphers if necessary.

Example 1: Multiply 3.1416 by 32.

Solution:

$$\begin{array}{r} 3.1416 \\ \underline{32} \\ 62832 \\ \underline{94248} \\ 100.5312 \end{array}$$ *Ans.*

Example: 2: Multiply .592 by .013.

Solution:

$$\begin{array}{r} .592 \\ \underline{.013} \\ 1776 \\ \underline{592} \\ .007696 \end{array}$$ *Ans.*

Exercise 8.

1. The length of the side of any square is always equal to the length of its side multiplied by 1.414. How long is the diagonal of a square 4.21 inches on a side?

2. The specific gravity of cast iron is 7.13, which means that cast iron is 7.13 times as heavy as an equivalent volume of water. If water weighs 62.425 lb. per cu. ft., what is the weight of 1 cu. ft. of cast iron?
3. A sheet of newspaper is .0031″ in thickness. What is the approximate thickness of a Sunday newspaper consisting of 220 pages?
4. The horizontal component of a force acting at an angle of 30° to the horizontal is equal to the force multiplied by .866. If the force amounts to 250 lb., what is the amount of the horizontal component?
5. A laminated piece is built up of 45 pieces of metal, each piece having a thickness of .0024″. How thick is the entire piece?
6. Gasoline is .91 times as heavy as water. If a quart of water weighs 2.08 lb., what is the weight of a gallon of gasoline?
7. A certain broach has 36 teeth. If each tooth cuts .018″, how much material is removed by the entire broach?
8. The specific heat of aluminum equals 0.218; this represents the number of calories required to raise 1 gm. of aluminum 1 degree Centigrade. How many calories are required to heat an aluminum block weighing 8.04 gm. from 20.4° C to 100.1° C?
9. A certain size steel bar weighs 7.656 lb. per linear foot. Find the cost of 2000 ft. of this bar, if the price is $2.20 per 100 lb.
10. The coefficient of expansion of iron equals .00000672 per degree Fahrenheit, which means that for each degree rise in temperature it increases in length by that fractional part of its original length. By how much will the length of a 200 ft. iron cable increase if the temperature rises 80° F.?

Division of Decimals. In order to divide a number by a decimal, the decimal point in both the divisor and the dividend must be moved as many places to the right as there are decimal places in the divisor.

EXAMPLE: Divide 92.862 by 2.91.

SOLUTION:

$$\begin{array}{r}
31.91 \\
291\overline{)9286.20} \\
\underline{873} \\
556 \\
\underline{291} \\
2652 \\
\underline{2619} \\
330 \\
\underline{291}
\end{array}$$

Ans., 31.91+

Exercise 9.

1. If one cubic foot contains 7.48 gal., find, to the nearest tenth, the number of cubic feet occupied by 550 gallons.

2. A certain size metal rod weighs 2.84 lb. per linear foot. How many feet of these rods are there in a bundle of various lengths, if the entire bundle weighs 86.75 lb.?
3. There are .3937 inches to one centimeter. How many centimeters long is a wire measuring 6.54″? (Carry the result to two decimal places.)
4. How many metal discs 0.0625″ thick can be stacked to a height of 2¼″?
5. A steel rod 38.26″ long is to be cut into 7 equal parts. Allowing 0.032″ for the thickness of each cut, how long will each piece be? (Remember that only six cuts need be made.)
6. In order to find the number of screws in a box, a mechanic weighs the entire box full of screws and finds the weight to be 2¼ lb. He also finds that a dozen screws weigh .28 lb. Making no allowance for the weight of the box, how many screws does it contain?

Changing Common Fractions to Decimals. Any common fraction may readily be changed to an equivalent decimal fraction simply by annexing zeros to the numerator and dividing by the denominator, as shown below; if the division does not terminate, it may be carried to as many decimal places as desired.

Example 1: Change 7/23 to a decimal.

Solution:

$$\begin{array}{r}
.30434 \\
23\overline{)7.00000} \\
\underline{6\,9} \\
100 \\
\underline{92} \\
80 \\
\underline{69} \\
110 \\
\underline{92}
\end{array}$$

Ans., .30434+

Example 2: Reduce 9/32 to an equivalent decimal.

Solution: .28125, *Ans.*

$$\begin{array}{r}
.28125 \\
32\overline{)9.0000} \\
\underline{6\,4} \\
2\,60 \\
\underline{2\,56} \\
40 \\
\underline{32} \\
80 \\
\underline{64} \\
160 \\
\underline{160}
\end{array}$$

Changing Decimals to Common Fractions. To change a given decimal to a common fraction it is merely necessary to rewrite it with the denominator expressed as 10. 100, 1000, 10,000, etc., and reduce it to lowest terms if possible.

Example 1: Change .042 to a common fraction.

Solution: .042=42/1000=21/500, *Ans.*

Example 2: Reduce .6784 to a common fraction.

Solution: .6784=6784/10,000=1696/2500=424/625, *Ans.*

Exercise 10.

Change each of the following to decimal fractions:

1. 1/8
2. 3/32
3. 7/16
4. 15/64
5. 9/32
6. 15/16
7. 27/32
8. 11/16
9. 3 1/7
10. 5/9
11. 12/13
12. 2/11
13. 11/64
14. 49/64
15. 5/24
16. 25/128

Change each of the following to common fractions:

17. .4375
18. .8125
19. .0625
20. .003125
21. .40625
22. .90625
23. .078125
24. .921875
25. .531250

Table of Decimal Equivalents. Since measurements on blueprints and in the shop are commonly expressed both as decimals and as ordinary fractions, it is important to be able to convert from one to the other quickly and easily. It is therefore highly desirable that some of the decimal equivalents should be memorized, especially the following:

1/2=0.500
1/4=0.250
1/8=0.125
1/16=0.0625
1/32=0.03125
1/64=0.0156

For further convenience the Table of Decimal Equivalents given herewith is constantly used by draftsmen and machinists.

Tolerance. Architects, carpenters and patternmakers generally use eighth's, sixteenth's, and thirty-second's instead of decimals. But they use them to different degrees of accuracy; thus the patternmaker rarely uses fractions of less than 1/16″; the carpenter or cabinetmaker generally does not use fractions of less than 1/8″; and so on. However, when it comes to making dies, tools and machine parts, great accuracy is required; for this purpose decimal fractions are more convenient and more useful. The term ***tolerance*** is used to indicate the limits within which a piece of work is acceptable when it deviates from the dimension indicated.

TABLE OF DECIMAL EQUIVALENTS

1/64	.015625	33/64	.515625
1/32	.03125	17/32	.53125
3/64	.046875	35/64	.546875
1/16	.0625	9/16	.5625
5/64	.078125	37/64	.578125
3/32	.09375	19/32	.59375
7/64	.109375	39/64	.609375
1/8	.1250	5/8	.6250
9/64	.140625	41/64	.640625
5/32	.15625	21/32	.65625
11/64	.171875	43/64	.671875
3/16	.1875	11/16	.6875
13/64	.203125	45/64	.703125
7/32	.21875	23/32	.71875
15/64	.234375	47/64	.734375
1/4	.2500	3/4	.7500
17/64	.265625	49/64	.765625
9/32	.28125	25/32	.78125
19/64	.296875	51/64	.796875
5/16	.3125	13/16	.8125
21/64	.328125	53/64	.828125
11/32	.34375	27/32	.84375
23/64	.359375	55/64	.859375
3/8	.3750	7/8	.8750
25/64	.390625	57/64	.890625
13/32	.40625	29/32	.90625
27/64	.421875	59/64	.921875
7/16	.4375	15/16	.9375
29/64	.453125	61/64	.953125
15/32	.46875	31/32	.96875
31/64	.484375	63/64	.984375
1/2	.5000	1.	1.0000

EXAMPLE 1: If the diameter of a round piece as indicated is 2.138″, with a tolerance of ±.003″; what are the limits within which it is acceptable?

SOLUTION:

$$\left.\begin{array}{l} 2.138+.003=2.141 \\ 2.138-.003=2.135 \end{array}\right\} \textit{Ans.}$$

Thus any dimension less than 2.141″ and more than 2.135″ is acceptable. Such a dimension might be written as 2.138±.003.

Example 2: What are the limits of measurement on a piece that calls for $3\frac{5}{16}''$, plus or minus $.010''$?

Solution: $3\frac{5}{16}=3.3125$

$$\left.\begin{array}{l}3.3125+.01=3.3135''\\ 3.3125-.01=3.3115''\end{array}\right\}\textit{Ans.}$$

Example 3: Using the table of decimal equivalents, change a fractional measurement of $.714''$ to the nearest 32nd of an inch.

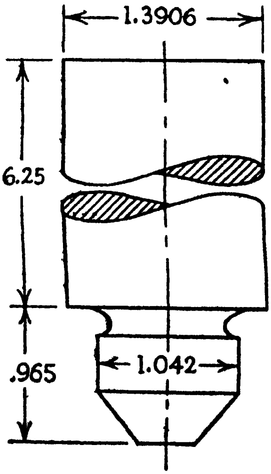
Solution: From the table,

$\frac{23}{32}=.71875$
$\frac{22}{32}=\frac{11}{16}=.6875$
$\frac{24}{32}=\frac{3}{4}=.75$

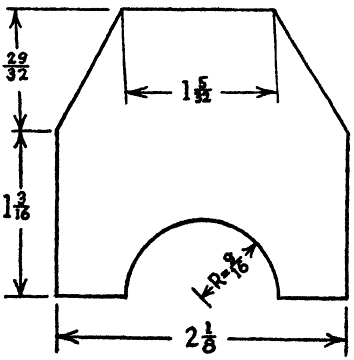
thus $.714=\frac{23}{32}$, approx., *Ans.*

Exercise 11.

1. Using the table, change the following decimal dimensions to the nearest 32nd of an inch:
   a) .216 b) .8391 c) .10123
2. Change the following to the nearest 64th of an inch:
   a) .4869 b) .17235 c) .89268
3. In the round piece shown, find the allowable limits for each of the given dimensions if the tolerance is ±.004.
4. Find the limits for each dimension of the plate shown, if the tolerance allowed is ±.025.



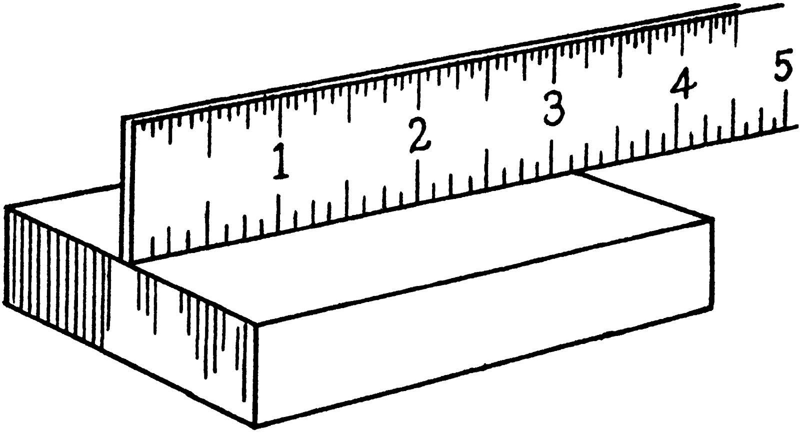
Ex. 3



Ex. 4

4. MEASURING INSTRUMENTS

Degree of Accuracy. As we saw in Section 1, all measurements are approximations, and the degree of accuracy of measurements may vary considerably. Extreme accuracy is not always required in shop work, even in machine shop operations. The greater the degree of accuracy achieved, the greater is the cost of the operation; it is uneconomical to secure greater accuracy than is actually needed. Where ultimate extreme accuracy is desired, the degree of accuracy varies with the successive steps in the operation. Thus rough machining, finishing machining, grinding and lapping are often used in succession to achieve a final high degree of accuracy. For rough machining a steel scale would be used for making the measurement, which would be made to within $\pm\frac{1}{32}''$. For the second, or finishing machining, a micrometer might be used, reading the measurement to $\pm\frac{1}{64}''$. For the grinding operation a precision micrometer would be used, graduated in .0001″, and the measurement would be taken to within ±.0002″. During the lapping operation and for the finished part, gage blocks and indicators would be employed.



Limits of Accuracy. Scientists often use extremely precise measurements; the wave length of sodium light, for example, is 0.00005893 cm., which represents accuracy to 8 decimal places. For most industrial and shop operations, however, the limit of accuracy required is usually to the fourth decimal place. As we have already seen, the limits of accuracy required are referred to as the *tolerances*, and are generally specified on the blueprint or in the specifications which accompany the designs; they are just as important as the dimensions or measurements themselves. The American Standards Association has standardized tolerances for various parts, such as screw threads, cylindrical fits, and surface finish.

The limits of accuracy obtainable in making a measurement depend upon the nature of the particular instrument used, the conditions under which the measurement is made, and the skill of the operator. So far as the possible limitations of the measuring instrument are concerned, the following should be noted. The common steel scale has a limit of 1/64″ or 1/100″. The micrometer caliper will yield an accuracy of .001″, and, with a vernier attachment, to .0001″. These figures represent graduations on the scales of the instruments. The use of a toolmaker's miscroscope, or magnifying glass, is sometimes required when measuring to a graduated line, since the width of the line itself is approximately .006″. Ordinary precision gage blocks measure to .000008 of an inch, and the finest grade blocks to .000002 of an inch. Such blocks have no graduations, but are fixed in measurement; they are used in combination as will be explained below.

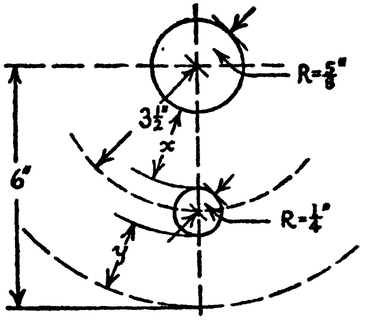
Conditions under Which Instruments Are Used. The degree of accuracy obtainable depends also upon the quality of the instrument and the working conditions. A particular instrument, even when new, will vary in manufactured accuracy, depending upon the quality and cost of the tool; an inexpensive, poorly made scale or calipers is never as accurate as a high grade, carefully made instrument. Again, as a tool becomes worn with constant use, it loses in accuracy. The lines marking the graduations on the scale become obliterated, the edge becomes nicked, and the moving parts (if any) become loose, and bearings are thrown out of alignment by knocking; all of these conditions diminish the accuracy obtainable. Finally, when working to four-place accuracy, temperature variations affect the accuracy of a measurement, since with metals particularly, both the object and the instrument are subject to expansion and contraction with a rise or fall in temperature. Thus for very accurate work the temperature should be ordinary room temperature, i.e., from about 68° to 72° Fahrenheit.

The Human Equation. Personal factors also influence accuracy. These include such considerations as normal eyesight and proper lighting; skill and care in estimating the smallest subdivision of a scale; correct habits, such as reading a meniscus properly, or avoiding parallax; and a delicate sense of touch, the ability to "feel" measure on a measuring instrument. Meticulous care in handling tools, their skilful use, and dependable judgment that comes only with experience—all these are required in making careful measurements. Lathes, shapers, milling machines, drills, taps, etc., are so designed and constructed that tolerances of ±.001″ or better can be achieved if skill and care are used. When extreme accuracy is required, unusual care must be exercised. Thus if a micrometer jaw is set too tightly, as much as .0005″ can be "forced"; or again, when reading a vernier scale, undue pressure against the sliding jaw may cause deflection of the object being measured, or slight clearances in the bearing surfaces of the sliding jaw, either of which will affect the reading of the instrument adversely.

Mechanical Duplication. The possibility of securing mechanical duplication depends not only upon the measuring instruments and the skill of the operator, but also upon the materials used and the machines and tools involved. Stock varies in quality and composition, in texture and finish, according to the particular shipment; such variations increase the difficulties in producing parts exactly alike. Similarly with equipment: the rigidity of a machine, the tightness of the bearings, the solidity of the machine bed, the fastening of fixtures, the wear and support of tools —all these are also involved in mechanical duplication.

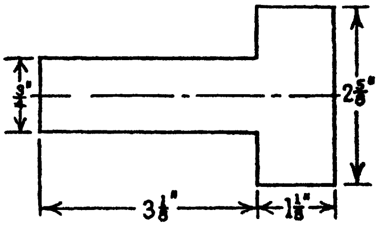
Exercise 12.

1. How much must be removed from a metal piece to be turned, if the original diameter is 3.764″ and the finished diameter required is 2.856″? if the original diameter is 1.827″ and the finished diameter is 1.792″?
2. Find the distances *x* and *y*, respectively, *before* the holes are reamed. If 1/64″ is allowed for reaming, what are these distances *after* reaming?



Ex. 2

3. The piece shown is to be rough turned, allowing 1/16″ for finish machining on all diameters and 1/32″ on all faces; find the corresponding dimensions for rough turning.



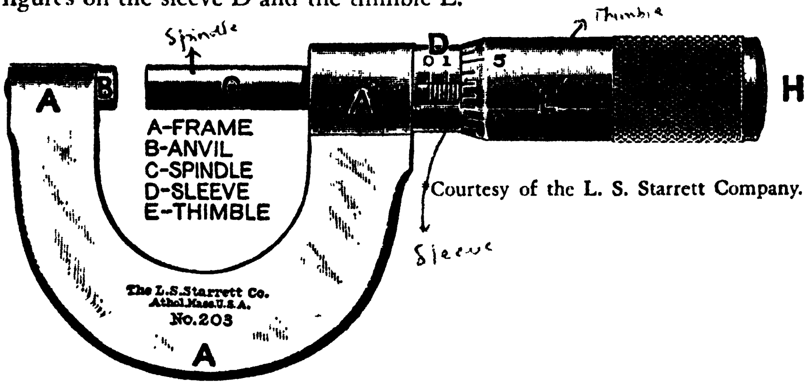
Ex. 3

4. The same piece (Ex. 3) is to be finish turned to allow .012″ for grinding on all diameters and .008″ on all faces; find the corresponding dimensions for finish machining.
5. If the required dimensions of a finished piece of work must be .462±.002, and the piece now measures .467, how much must still be removed?

The Micrometer. The *micrometer caliper,* or "mike," is the most common "precision instrument" used in the machine shop. Every machinist and toolmaker carries a micrometer. This instrument has many advantages:

1. It is small, and easily carried in the pocket.
2. It is convenient to handle and easy to read.
3. It is rugged enough to stand considerable handling.
4. It retains its accuracy, and has adjustments to compensate for wear.
5. It has a practical range of measurement, generally up to one inch.
6. It is comparatively inexpensive.

Construction and Use of the Micrometer.[*] The spindle C is attached to the thimble E, on the inside, at the point H. The part of the spindle which is concealed within the sleeve and thimble is threaded to fit a nut in the frame A. The frame being held stationary, the thimble E is revolved by the thumb and finger, and the spindle C, being attached to the thimble, revolves with it, and moves through the nut in the frame, approaching or receding from the anvil B. The article to be measured is placed between the anvil B and the spindle C. The measurement of the opening between the anvil and the spindle is shown by the lines and figures on the sleeve D and the thimble E.



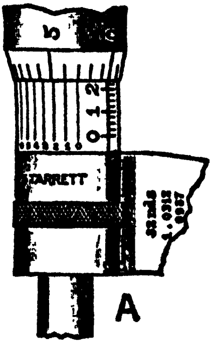
Courtesy of the L. S. Starrett Company.

The pitch of the screw threads on the concealed part of the spindle is 40 to an inch. One complete revolution of the spindle therefore moves it longitudinally one-fortieth (or twenty-five thousandths) of an inch. The sleeve D is marked with 40 lines to the inch, corresponding to the number of threads on the spindle. When the caliper is closed, the beveled edge of the thimble coincides with the line marked 0 on the sleeve, and the 0 line on the thimble agrees with the horizontal line on the sleeve. Open the caliper by revolving the thimble one full revolution, or until the 0 line on the thimble again coincides with the horizonal line on the sleeve; the distance between the anvil B and the spindle C is then 1/40 (or .025) of an inch, and the beveled edge of the thimble will coincide with the second vertical line on the sleeve. Each vertical line on the sleeve indicates a distance of 1/40 of an inch. Every fourth line is made longer than the others, and is numbered 0, 1, 2, 3, etc. Each numbered line indicates a distance of four times 1/40 of an inch, or one-tenth.

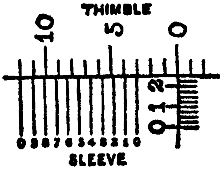
The beveled edge of the thimble is marked in twenty-five divisions, and every fifth line is numbered from 0 to 25. Rotating the thimble from one of these marks to the next moves the spindle longitudinally 1/25 of twenty-five thousandths or one-thousandth of an inch. Rotating it two divisions indicates two thousandths, etc. Twenty-five divisions will indicate a complete revolution, .025 or 1/40 of an inch.

To read the caliper, therefore, multiply the number of vertical divisions visible on the sleeve by 25, and add the number of divisions on the bevel of the thimble, from 0 to the line which coincides with the horizontal line on the sleeve. For example, as the tool is represented in the engraving, there are seven divisions visible on the sleeve. Multiply this number by 25, and add the number of divisions shown on the bevel of the thimble, 3. The micrometer is open one hundred and seventy-eight thousandths. $(7 \times 25 = 175 + 3 = 178.)$

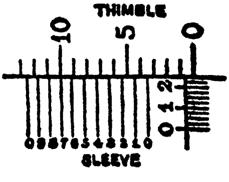
Using a Micrometer Graduated in Ten-Thousandths of an Inch. Readings in ten-thousandths of an inch are obtained by the use of a vernier, so named from Pierre Vernier, who invented the device in 1631 As applied to a caliper this consists of ten divisions on the adjustable sleeve, which occupy the same space as nine divisions on the thimble. The difference between the width of one of the ten spaces on the sleeve and one of the nine spaces on the thimble is therefore one-tenth of a space on the thimble. In engraving B the third line from 0 on thimble coincides with the first line on the sleeve. The next two lines on thimble and sleeve do not coincide by one-tenth of a space on thimble; the next two, marked 5 and 2, are two-tenths apart, and so on. In opening the tool, by turning the thimble to the left, each space on the thimble represents an opening of one-thousandth of an inch. If, therefore, the thimble be turned so that the lines marked 5 and 2 coincide, the caliper will be opened



A



B

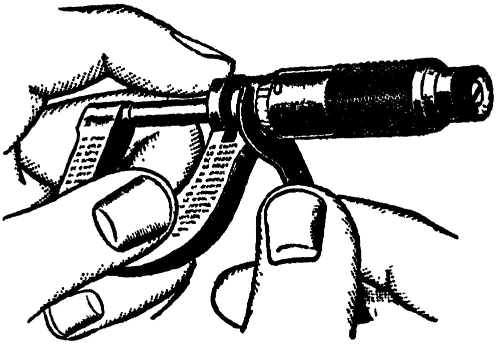


C

two-tenths of one-thousandth or two ten-thousandths. Turning the thimble further, until the line 10 coincides with the line 7 on the sleeve, as in engraving C, the caliper has been opened seven ten-thousandths, and the reading of the tool is .2507.

To read a ten-thousandths caliper, first note the thousandths as in the ordinary caliper, then observe the line on the sleeve which coincides with a line on the thimble. If it is the second line, marked 1, add one ten-thousandth; if the third, marked 2, add two ten-thousandths, etc.

Adjusting the Micrometer. These calipers will read correctly if there is no dirt between the anvil and spindle. When it becomes necessary to readjust the tool to compensate for the wear of screw and nut, this is done, not by the anvil, but by means of our friction sleeve, as follows: Take up the wear of screw and nut, then remove all dirt from face of the anvil and spindle and bring them together carefully. Insert the small spanner wrench in the small hole and turn until the line on the sleeve coincides with the zero line on the thimble.



Exercise 13.

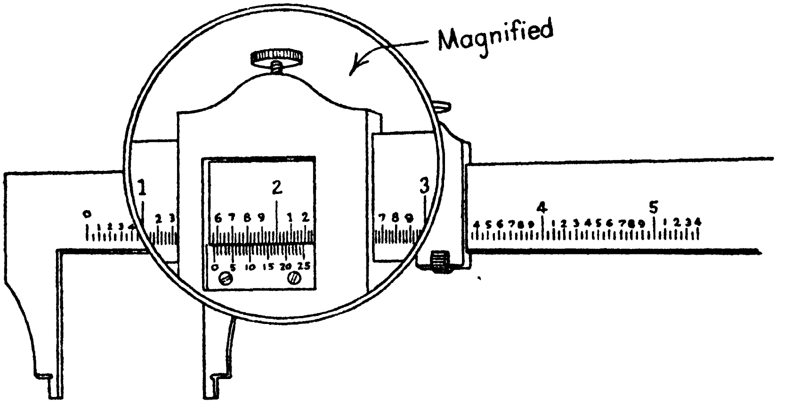
1. The following readings are taken on an ordinary micrometer; complete the table:

	Reading on sleeve is between	Nearest line on thimble	Complete Reading
(a)	.450 and .475	15	
(b)	.900 and .925	4	
(c)	.575 and .600	20	
(d)	.125 and .150	7	
(e)	.000 and .025	22	
(f)	.975 and 1.000	16	

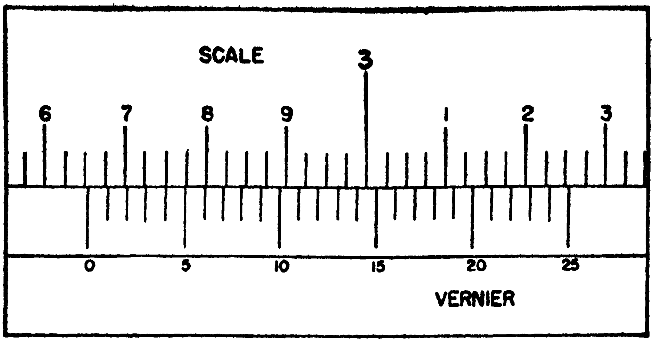
2. The following readings are taken on a micrometer graduated in ten-thousandths of an inch; complete the table:

	Reading on sleeve is between	Reading on thimble between	Vernier line	Complete Reading
(a)	.325 and .350	22 and 23	3	
(b)	.875 and .900	8 and 9	9	
(c)	.450 and .475	16 and 17	2	
(d)	.200 and .225	11 and 12	4	
(e)	.575 and .600	2 and 3	6	
(f)	.600 and .625	24 and 25	1	

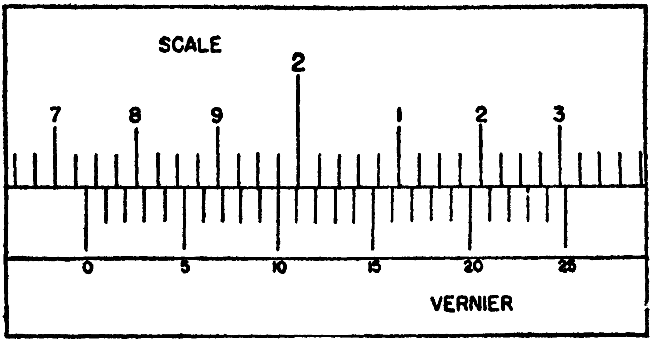
Vernier Instruments. The principle of the vernier has been applied to many kinds of instruments. In the foregoing discussion, it was seen how it was employed to increase the limit of measurement of the micrometer. Where the vernier is employed on an instrument as the sole agent for magnifying ordinarily imperceptible differences in length, it is usually known as a vernier caliper, vernier protractor, etc. It consists of a small auxiliary scale having usually one less or more graduations in the same length as the longer true scale. It is evident therefore that if the whole vernier scale contains one more division than the true scale over an equal length, each division on the vernier scale is proportionally smaller than a corresponding division on the true scale. If 25 divisions on the vernier scale are equal to 24 divisions on the main scale, then each division on the vernier scale is 1/25 of a division smaller than a division on the main scale. If there is an accumulating difference of 1/25 of a division, the effect of going along the vernier scale one division is to subtract 1/25 of a true scale division. By going along the scale 2, 3, 4, 5, etc., divisions of



The Vernier Scale

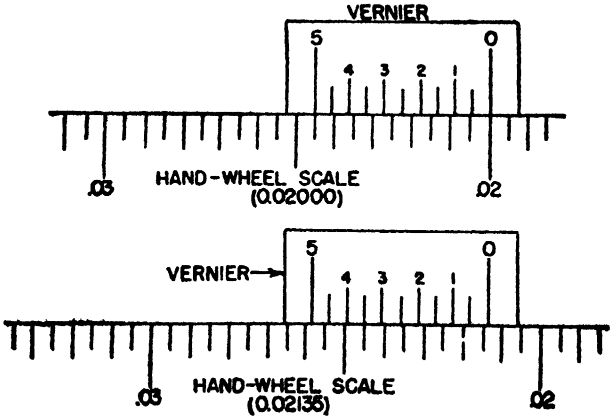


2.654



1.735

FROM A VERNIER CALIPER



FROM HAND-WHEEL OF JIG BORER

VERNIER SCALE READINGS

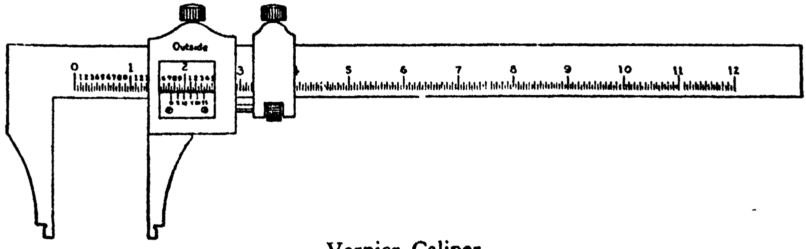
2/25, 3/25, 4/25, 5/25, etc., are subtracted from the original setting until the lines coincide. At this point all of the remaining fraction of a division indicated by the "0" on the vernier scale has been absorbed, and the number of the vernier divisions indicates the number of the 25ths this fraction of a division contains.

Vernier scales are not necessarily 25 units long; they may have any number of units. They may have only ten units, as on the vernier scale of the ten-thousandths micrometer. The graduated hand wheels of a machine tool such as a jig borer often employ the vernier scale for the purpose of indicating "tenths" or "half-tenths" of a thousandth of an inch table travel, etc.

Types of Vernier Instruments. The vernier scale has been applied to a variety of instruments and tools; for example:

1. Vernier Caliper
2. Vernier Height Gage
3. Vernier Depth Gage
4. Vernier Protractor

The typical vernier caliper consists of an L-shaped frame, the end of which is one of the jaws. On the long arm of the "L" is scribed the true scale, which may be 6, 12, 24, 36, or 48 inches long. The sliding jaw carries a vernier scale on either side. The scale on the front side is for outside measurements, whereas the scale on the back side is for inside measurements. It will be noted in the figure that the tips of the jaws have been formed so as to be capable of making an inside measurement. The thickness of the measuring points is automatically compensated for on the inside scale. The sliding jaw assembly consists of two sections joined by a horizontal screw. By clamping the right-hand section at its approximate location, a final fine adjustment of the movable jaw may be obtained by turning the adjusting nut. The sliding jaw may be clamped in any position with the locking screw shown in the figure on top of the jaw. The jaws of all vernier calipers, except the larger sizes, have two center points which are particularly useful in setting dividers to exact dimensions.

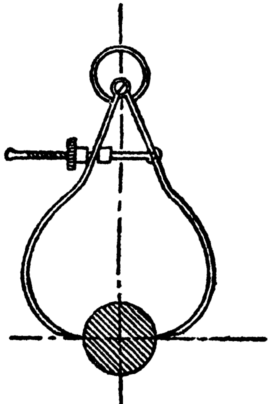


Vernier Caliper

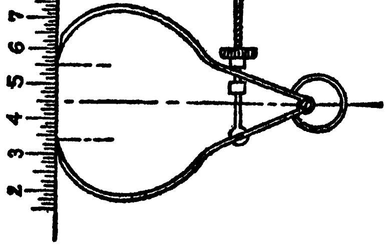
Vernier calipers are made in the standard sizes of 6, 12, 24, 36, and 48 inches, and 150, 300, 600, and 900 millimeters. The length of the jaws will range from 1¼ inches to 3½ inches, and the minimum inside measurement with the smallest caliper is ¼ of an inch or 6 millimeters.

The vernier caliper has a wide range of measurement, and the shape of the measuring anvils and their position with respect to the scale adapts this instrument to certain jobs where a micrometer, for example, could not satisfactorily be applied. It is also capable of being used for both outside and inside measurements—a feature which makes this tool one of the most versatile precision instruments in the shop. However, it does not have the accuracy of a micrometer. In any one inch of its length a vernier caliper should be accurate within .001 of an inch. In any 12 inches it should be accurate within .002, and increase about .001 for every 12 inches thereafter.

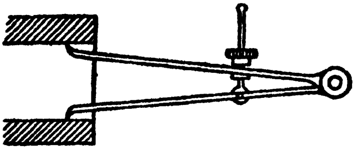
Other Types of Calipers. For outside measurements, such as the thickness of a metal plate or the diameter of a cylinder, the *outside* calipers are used as shown herewith. In using these calipers, the instrument must always be kept square with the work to be measured. For inside measurements, such as the inside diameter of a pipe or a tube, the *inside* calipers are used; when using this, the axis of the calipers must line up with the axis of the work, and the tips of the caliper legs must be square with the largest portion of the diameter being measured. In using the micrometer calipers described in the preceding paragraphs the following points should be observed:



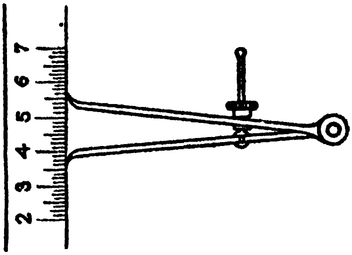
Taking a measurement with an outside calipers.



Transferring the measurement.



Taking a measurement with an inside calipers.



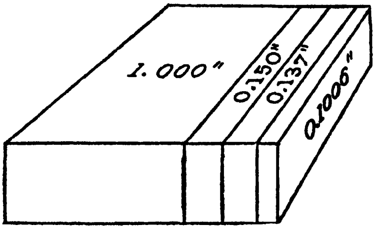
Transferring the measurement.

1. Never force the caliper by using too much pressure.
2. Always take the reading while the micrometer is held on the work.
3. Always open the micrometer before removing it from the part measured.
4. Never use the micrometer on a moving part while a machine is running.

Precision Measurements. Line-graduated measuring instruments, such as the scale, by which measurements up to .001 inch are taken, are "non-precision instruments," despite the fact that careful, accurate measurements may be made with them. But when dimensions are controlled and reproduced to thousandths of an inch or better, whether by a micrometer or by gage blocks, the measurements are known as *precision measurements.* Such precision measuring instruments themselves are calibrated by means of gage blocks, which are taken as the industrial standards of length. For ordinary shop operations, such as patternmaking, forging, stamping, rough machining, etc., precision measurements are not required. But where tolerances are very small, as for example in the manufacture of watches, clocks, delicate instruments, typewriters, firearms or parts of automotive engines, the use of precision gage blocks is indispensable.

Gage Blocks. Gage blocks are rectangular blocks of steel with a measuring surface at each end. They are made of a special alloy steel, heat treated and aged so that internal, molecular stress and strain are at a minimum, thereby decreasing the tendency of the metal to warp or "grow." The surfaces of the blocks are mechanically polished by a special process by which a number of blocks are finished at the same time to identical size. The flat surfaces of the blocks are ground and polished to an extremely high finish resembling that of burnished silver. They are the most accurate pieces of manufactured metal in the world; their errors are generally less than $\frac{5}{1,000,000}$ of an inch per inch of length, and some of them are accurate to within $\frac{2}{1,000,000}$ of an inch. They are probably the nearest approach of a man-made device to a perfect mathematical plane. Since a rise in temperature of 1° causes the blocks to expand 9/1,000,000 of an inch, they are finished, and subsequently used, in a room kept at a constant temperature of 68°F. Gage blocks usually range in length from .010 of an inch up to 20 inches. They are generally obtainable in sets from 5 to as many as 85 blocks of different lengths. With a large set of over 80 blocks more than 100,000 gages in steps of .0001 of an inch may be made by using various combinations of blocks.

To combine them, the surfaces, having first been thoroughly cleaned, are slid one on the other, with a slight inward pressure; this is sometimes spoken of as "wringing" them together. When placed together in this way they stick with remarkable tenacity; when lifted in the air, blocks that have been wrung together properly have been known to support a weight of somewhat over 200 lb., although the precise reason for this amazing adhesion has never been satisfactorily explained.



Showing how gage blocks are used in combination to make up the measurement of 1.3876

As already mentioned, appropriate gage blocks are put together to secure any desired combination necessary for a particular measurement. For example, if a measurement such as 1.3876″ is desired, the following blocks would be selected: 1.000″; .150″; .137″; .1006″; their sum equals 1.3876″, which is the required measurement. In many cases it will be possible to find several combinations of blocks to give the measurement required. A complete set of blocks may contain the following sizes:

1.000″
2.000″
3.000″
4.000″

.010″
.020″
.030″
.040″

.050″
.100″
.150″
.200″
.250″
.300″
.350″
.400″
.450″
.500″

.550″
.600″
.650″
.700″
.750″
.800″
.850″
.900″
.950″

.101″
.102″
.103″
.104″
.105″
.106″
.107″
.108″
.109″
.110″
.111″
.112″
.113″

.114″
.115″
.116″
.117″
.118″
.119″
.120″
.121″
.122″
.123″
.124″
.125″

.126″
.127″
.128″
.129″
.130″
.131″
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.134″
.135″
.136″
.137″

.138″
.139″
.140″
.141″
.142″
.143″
.144″
.145″
.146″
.147″
.148″
.149″

.1001″
.1002″
.1003″
.1004″
.1005″
.1006″
.1007″
.1008″
.1009″

Exercise 14.

Using the above table of sizes, find appropriate combinations of blocks for each of the following measurements:

1.	.3944	5.	.3982	9.	1.8539
2.	.5532	6.	.4338	10.	9.6402
3.	.4265	7.	3.9061	11.	7.2944
4.	.666	8.	2.7072	12.	4.0098

Accuracy of Gage Blocks. The guaranteed accuracy of gage blocks is expressed as ±.000002 of an inch *in an inch.* In other words, a gage block measuring .500 of an inch may vary between .499998 and .500002 and still be acceptable. Furthermore, a block 4.000000 inches long may vary 4 times .000002, or .000008 of an inch, i.e., from 3.999992 to 4.000008 inches and be acceptable. It might be supposed that the accumulated error in a stack of five or six blocks might be considerable; it would be, were it not for the fact that the variations mentioned are distributed according to the laws of probability—some *plus* and some *minus* —so that they counterbalance, and the total error in a stack of blocks rarely exceeds twice that of a single block; frequently it is even *less* than that of a single block.

Uses of Gage Blocks. It should also be pointed out that gage blocks are made and used for various classes of work, i.e., various levels of accuracy. Thus a very high grade of blocks, with a range of error of from 5 to 20 millionths of an inch, would be used for inspecting tools, verifying gages, and calibrating various instruments. A second-grade set of blocks, with a range of error of from 20 to 40 millionths of an inch, might be used in layout work,—dies, jigs, fixtures, etc. A third-rate set, with errors ranging from 40 to 100 millionths of an inch, would be suitable for setting up milling, grinding, and drilling machines, or for the inspection of machine parts, etc.

Standard Sets and Working Sets. Wherever a considerable amount of precision measuring is required in an industrial plant, it is customary to use primary and secondary standards, that is, a master set of gage blocks, and a working set. The master set is carefully preserved, and is used only to check the accuracy of the working sets used in the shop. The master set is usually sent to the Bureau of Standards at Washington, or returned to the manufacturer, once a year or so for certification; each block is then checked for flatness, parallelism and length, and is certified as varying so-and-so many millionths of an inch.

As the working sets get older they become worn through use and handling. It must be remembered that all gage blocks are extremely delicate; even the natural moisture of the hands contains an acid which may stain the blocks if they are handled too much. Hence as the blocks wear out, they are progressively used for less important work. When they become so worn that the error is greater than 100 millionths of an inch they are either discarded or chromium-plated and relapped to size. In recent years, gage blocks have been made from carboloy for use in working sets because of the high resistance of carboloy to wear.

5. PERCENTAGE

Meaning of Per Cent. As we have already seen, a fractional part of any given amount may be expressed either as a common fraction or as a decimal fraction. There is still a third way: by using a per cent. A *per cent* is simply a decimal fraction written without the decimal point, with the (%) sign used instead of the decimal point to indicate the fact that the denominator is "hundredths"; thus

$$\begin{aligned} .28 &= 28\% \\ .125 &= 12.5\% = 12\tfrac{1}{2}\% \\ .003 &= .3\% = \tfrac{3}{10}\% \\ .0425 &= 4.25\% = 4\tfrac{1}{4}\% \end{aligned}$$

Per cents may be added, subtracted, multiplied, etc., just as other numbers having similar units or denominations; thus

$$\begin{aligned} 15\% + 3\% + 10\% &= 28\% \\ 100\% - 16\% &= 84\% \\ 6 \times 3\tfrac{1}{2}\% &= 21\% \\ 24\% \div 3 &= 8\% \end{aligned}$$

Finding a Per Cent of a Number. The commonest problem involving per cents is that of finding a given per cent of a given number. The given number is called the *base;* the per cent required is called the *rate;* and the result of finding the per cent (or taking the rate) is called the *percentage.* Thus, in finding 20% of 750, we say:

$$\begin{aligned} &20\% \text{ of } 750 = ? \\ \text{or } &.20 \times 750 = 150; \end{aligned}$$

here 750=base, .20=rate per cent, and 150=percentage. Finding a per cent of a number is therefore seen to be a simple matter of multiplying a number by a decimal, i.e., using the formula

$$\begin{aligned} &\textit{Percentage} = \textit{Base} \times \textit{Rate} \\ &\text{or, } P = B \times R \end{aligned}$$

EXAMPLE 1: The cost of material for a job is estimated at $48; an additional 15% is allowed for miscellaneous expenses. Find (a) the amount of the allowance, and (b) the total estimated cost.

SOLUTION: (a) \$48×.15=\$7.20, *Ans.*

(b) \$48+\$7.20=\$55.20, *Ans.*

EXAMPLE 2: A specimen of Monel metal consists of 68% of nickel, 28% of copper, and the remainder of small amounts of other metals. Find (a) the number of pounds of nickel and copper in a piece of Monel metal weighing 18½ lb.; (b) the number of pounds of material other than nickel and copper in this specimen.

SOLUTION: (a) 18.5×.68=12.58 lb. nickel, *Ans.*
18.5×.28= 5.18 lb. copper, *Ans.*
(b) 68%+28%=96%
100%—96%= 4%, other metals
18.5×.04 =.74 lb., *Ans.*

Exercise 15.

1. If ice is 91.7% as heavy as water, and water weighs 62.4 lb. per cu. ft., find the weight of a cubic foot of ice.
2. When tested, a gasoline engine actually gave 86% of its rated horsepower. If the engine was rated at 110 H.P., what was the actual horsepower delivered by the engine?
3. A motor is running at 2600 revolutions per minute. If the speed of the motor is increased by 6½%, how many r.p.m. will it then make?
4. If the loss in power due to friction in a certain device is 28%, what amount of power will this device transmit when supplied with 125 horsepower?
5. A machine shop casting weighs 60 lb. Due to an error in dimensioning, it is necessary to remove 12½% of the casting by machine. How many pounds of metal must the machinist remove?
6. The employees in a shop are to receive a wage increase of 12%. If junior mechanics have been getting \$10.75 a day and helpers \$7.80, what is the daily wage rate of each after the raise goes into effect?
7. An alloy used for bearing metal contains 14% tin, 27% antimony, and 59% lead; how much of each of these metals is required to make 150 lb. of bearing metal?
8. A pattern weighs ¼ as much as the casting to be made from the pattern. If the casting weighs 110 lb., what is the weight of the pattern?
9. An inexperienced operator turned out 250 pieces of work on a stamping machine. When inspected, it was found that 2½% of them had to be rejected as imperfect. How many pieces were rejected?
10. In making a certain piece to measurement, an allowance of 1½% either way is permitted. If the dimension called for is 4.8 in., what are the "outside limits" permissible?

11. *In mixing a batch of concrete, about 15% of the weight is cement, 30% is sand, and 55% is gravel. If the dry mixture has a total weight of 1450 lb., how many pounds of each are used?*

Changing a Per Cent to a Fraction. It is sometimes convenient to change a per cent into an equivalent common fraction. To do this, the per cent is first expressed as a decimal fraction, which in turn is then reduced to lowest terms.

Example 1: Change 28% to a common fraction.

Solution: $28\% = .28 = \frac{28}{100}$

$\frac{28}{100} = \frac{14}{50} = \frac{7}{25}$, *Ans.*

Example 2: Express $18\frac{3}{4}\%$ as a common fraction.

Solution: $18\frac{3}{4}\% = .18\frac{3}{4} = .1875 = \frac{1875}{10{,}000} = \frac{75}{400} = \frac{3}{16}$, *Ans.*

Changing a Common Fraction to a Per Cent. In a somewhat similar way, the above process may be reversed, and any common fraction can in turn be expressed as a per cent. Thus the numerator is first divided by the denominator, the quotient being written as a decimal; the decimal is then converted to a per cent by moving the decimal point two places to the right and annexing the % sign.

Example 1: Change $\frac{3}{8}$ to the per cent form.

Solution: $\frac{3}{8} = 3 \div 8$

$$\begin{array}{r} 8\overline{)3.000} \\ \hline .375 \end{array}$$

$.375 = 37.5\%$, *Ans.*

Example 2: Express $\frac{13}{17}$ as a per cent.

Solution:

$$\begin{array}{r} .7647+ \\ 17\overline{)13.000} \\ \underline{11\,9} \\ 1\,10 \\ \underline{1\,02} \\ 80 \\ \underline{68} \\ 120 \\ \underline{119} \end{array}$$

or, $.7647 = 76.47\%+$, *Ans.*

EXAMPLE 3: Change $\frac{6}{5}$ to a per cent.

SOLUTION: $6\div5=1.2=\frac{120}{100}=120\%$, *Ans.*

Determining the Rate Per Cent. Another common problem arising in connection with the use of per cents is that of determining the rate per cent, i.e., finding what per cent one number is of another. It is precisely the same problem as that explained in the preceding paragraph. Its relation to percentage will be seen at once from the following:

$$Percentage=Base\times Rate$$

If $P=B\times R,$

then $R=\frac{P}{B}$

or $Rate=Percentage\div Base.$

EXAMPLE 1: What per cent of \$600 is \$39?

SOLUTION: $\frac{\$39}{\$600}=\frac{39}{600}=\frac{13}{200}=.065=6\frac{1}{2}\%$, *Ans.*

EXAMPLE 2: By changing the plan of a pattern a saving of 5¼ lb. is made in a casting originally weighing 46 lb. What is the per cent of weight thus saved?

SOLUTION:
$$\frac{5\frac{1}{4}}{46}=\begin{array}{r} .114+ \\ 46\overline{)5.25} \\ \underline{4\,6} \\ 65 \\ \underline{46} \\ 190 \\ \underline{184} \end{array}$$

Saving$=.114+=11.4\%+$, *Ans.*

Exercise 16.

1. An automatic production machine turns out 36 pieces of work per hour. After certain adjustments had been made, the machine turned out 42 pieces per hour. What is the per cent of increase in the production rate?
2. A shop hand receiving 80¢ an hour is given an increase in wages. If he now receives 92¢ an hour, by what per cent was his wage rate increased?

3. Before a bronze casting was machined it weighed $31\frac{1}{4}$ lb.; after the machining operations had been performed it weighed $27\frac{1}{2}$ lb. What was the per cent of reduction in weight?
4. A steam pressure of 175 lb. per sq. in. is increased to 220 lb. per sq. in. What is the per cent of increase?
5. If the price of gasoline is increased from $16\frac{1}{2}$¢ a gallon to $17\frac{1}{4}$¢, what per cent of increase is this?
6. A power saw uses a "cutting compound" made by mixing 5 quarts of lard oil with water enough to fill a 10-gallon tank. What per cent of the compound is oil?
7. A beam is expected to support a maximum load of 40 lb. per linear foot. If it is designed to withstand a load of 56 lb. per ft., what factor of safety was allowed?
8. In making a pattern, a designer allows $\frac{3}{16}$ in. per foot for shrinkage. What per cent is this?
9. A bottle contains 250 gm. of potassium chloride "analyzed reagent." The label states that it contains 0.083 gm. of magnesium chloride impurity. What per cent is this?
10. A carpenter added $1\frac{1}{4}$ pints of alcohol to $2\frac{1}{2}$ quarts of shellac. By what per cent did he "thin" the shellac?
11. In a printing of 2500 leaflets the press operator spoiled 45 copies. What was the per cent of spoilage?
12. By tuning up a Diesel engine an operator saves an average of 15 gal. of fuel per day. If the average consumption of fuel had been 175 gal. per day, what was the per cent of saving in fuel?

Finding the Base. A less frequently occurring problem is the following: having given the percentage and the rate per cent, what was the original base? When expressed by means of the formula, the method of answering this type of question may readily be seen; thus

$$\text{since } P=B\times R,$$
$$\text{then } B=P\div R,$$

or the base is found by dividing the percentage by the rate.

Example 1: A factory "let out" 240 of its employees. If this meant that 15% of its employees were laid off, how many were originally employed?

Solution: 15% = 240
1% = 240 ÷ 15 = 16
100% = 100 × 16 = 1600, *Ans.*

Example 2: By increasing the amount of "filler" in a certain grade of paper stock the weight of the paper was increased by 10%. If the stock now weighs $16\frac{1}{2}$ lb., what did it weigh originally?

SOLUTION: $100\%+10\%=110\%$

$110\%=16.5$ lb.

$1\%=16.5\div110=.15$ lb.

$100\%=100\times.15=15$ lb., *Ans.*

Exercise 17.

1. In normal times a shop produces a certain number of finished pieces per day. When production is stepped up 25% by working overtime, 39 additional pieces per day are produced. What is the daily production under normal conditions? when overtime work is done?
2. A certain material shrank, on account of moisture, 4% in size. If it now measures 21.6″, what was its original size?
3. The inventory clerk of a factory has on hand 300 ft. of round bars of a certain size. If this is 30% of the average stock of that size, how much of it is usually kept on hand?
4. The weight of zinc in a casting made of Lumen metal is 74.8 lb. If Lumen metal consists of 5% aluminum, 10% copper, and the rest zinc, find the weight of the casting.
5. The overhead in a manufacturing plant is 24% of the value of the goods produced. If in a certain month the overhead expense amounted to $38,400, what was the value of the goods produced?
6. A foreman's weekly wage is increased by 12%. If his raise amounts to $5.82 per week, what was his original weekly wage?

Practical Uses of Percentage. Many practical applications of percentage problems arise in the trades, especially in connection with the business aspects of industrial practice. Thus profit is expressed as a certain per cent of the volume of sales, or sometimes as a per cent of the cost; so also, are labor costs, overhead, and cost of materials expressed as per cents. Other items frequently expressed in per cents are the cost of maintenance of physical plant and equipment; the cost of repairs and replacements; taxes; insurance, such as fire insurance, flywheel insurance, boiler explosion, plate-glass insurance, etc., the premiums on accident policies, workmen's compensation, unemployment insurance, pensions, old-age security benefits and the like; all these are usually figured in terms of percentages.

Depreciation. All equipment, such as machinery, tools, fixtures, trucks, even buildings, decrease in value as time goes on. This decrease in value is known as *depreciation.* It is usually due to the actual wearing out of the equipment so that it is no longer serviceable, although it sometimes becomes necessary to discard equipment even before it is worn out completely because of new inventions or improved styles. Frequently the equipment to be discarded has a certain junk value, or scrap value, when it is disposed of. The number of years that the equipment remains in use is called its estimated or "useful life". The amount of annual depreciation can be computed in several ways. One of the simplest and commonest methods is to suppose that it depreciates in value in ***equal amounts*** each year of its life. This is not actually the case with some types of equipment. However, when this method is used, the computation is as follows:

$$\textit{Annual depreciation} = \frac{C - S}{n}$$

where C=original cost, S=scrap value, and n=number of years of estimated life. On this basis (called the "constant-value" method), the ***rate of depreciation*** is given by:

$$\textit{Rate of depreciation} = \frac{\textit{Annual depreciation}}{\textit{Original cost}}$$

EXAMPLE: A power drill worth \$875 when new has an estimated life of 15 years, and its scrap value is \$50. Using the constant-value method, find
(a) the annual depreciation charge, and
(b) the annual rate of depreciation.

SOLUTION: (a) Amount of annual depreciation $= \frac{\$875 - \$50}{15} = \frac{\$825}{15}$
$= \$55$, *Ans.*

(b) Rate of annual depreciation $= \frac{\$55}{\$875} = .0629 = 6.29\%$, *Ans.*

Commercial Discount. When material or equipment is purchased, it is usually subject to *discount.* This means that a certain per cent of the quoted price is allowed, either for immediate cash payment, or because of a trade allowance on the list price, or because it is bought in large quantity. The following examples will illustrate such discounts.

EXAMPLE 1: A bill for lumber amounted to \$176; a discount of 2½% for cash was allowed. If prompt payment was made, what did the lumber actually cost the purchaser?

SOLUTION: $\$176 \times .025 = \$4.40$, discount
$\$176 - \$4.40 = \$171.60$, net cost, *Ans.*

EXAMPLE 2: A wrench listed in the catalog at \$2.75 is subject to a discount of 33⅓%. What is the net cost?

SOLUTION: $\$2.75 \times 33\frac{1}{3}\% = \$.916 = \$.92$, discount
$\$2.75 - \$.92 = \$1.83$, net cost, *Ans.*

Example 3: A supply house lists a rotary pump at \$75, subject to a discount of 20% and 10%. Find the net price.

Solution:
\$75×20%=\$15, first discount
\$75—\$15 =\$60, first "net price"
\$60×10%=\$ 6, second discount
\$60—\$6 =\$54, net price, *Ans.*

Note: Either discount can be computed first; the result will be the same. The two discounts *cannot be added,* however; "20% and 10%" is *not equivalent* to a 30% discount. This is the short cut, if you want one:

100%—20%=80%
100%—10%=90%
(.9)×(.8)=.72=72%
\$75×.72=\$54, *Ans.*

Exercise 18.

1. Brass fittings are offered by a manufacturer at \$7.20 a hundred. At a discount of 25%, what is the cost of 200 fittings?
2. Micrometers are quoted at \$32 a doz., less 25% and 20%. What is the net cost of one micrometer?
3. A wood plane was purchased at a net cost of \$3.80. If the catalog price was \$4.75, what per cent of discount was offered?
4. A lathe costing \$1200 has an estimated life of 15 years. If it has a scrap value of \$150, what is the annual depreciation charge?
5. If a piece of equipment costing \$240 has no scrap value and has a useful life of 8 years, what is the annual rate of depreciation?
6. A factory building cost \$40,000 to erect. If the depreciation is figured at 2% of the original cost each year, what is the "book value" of the building after 22 years?
7. A solution of a solid in water contains 24% of the solid by weight. How much of this solid is dissolved in 15 oz. of the solution?
8. As calculated, the speed of a pulley is 300 revolutions per minute. When checked with a tachometer, however, it actually made only 280 r.p.m., due to belt slippage. What is the per cent of slippage?
9. Ordinary air contains about 19.5% of oxygen by volume. How many cubic feet of oxygen are there in 2000 cu. ft. of air?
10. Because of a leaky valve, 2½ quarts of oil were lost out of every 20 gallons. What per cent was lost?
11. A factory regularly employing 320 men increases its force by 15%. How many men are now working in this factory?
12. The diameter of a rod when measured by a micrometer is found to be 1.23 in. If the blueprint called for a diameter of 1.18 in., what is the per cent of error?

13. If the working hours are increased from 40 hours per week to a 44-hour week, what is the increase in a weekly pay roll amounting to $5600, if the same hourly wage rates are maintained?
14. An upholsterer allows 10% extra on the cost of material for nails, thread, glue, sandpaper, etc. If the wood and fabric for an upholstered stool come to $4.38, what is the total estimated cost of material for 2 doz. such stools?
15. When making a bid on an installation job, a contractor allows 30% of the estimated cost for additional "overhead." If he offers a bid of $156, what did he estimate the cost to be?

6. RATIO AND PROPORTION

Ratio as a Comparison. A *ratio* is a device for comparing two quantities of the same kind. For example, if two strips of metal are 8 in. and 10 in. long, respectively, we could say that the second is 2 in. longer than the first, or 25% longer than the first. This is a *difference* method of comparing them; telling how much more or less. Another way of comparing them would be to say that one is ⅘ as long as the other, or the second is 5/4, i.e., 1¼, times as long as the first. This is the *ratio* method of comparison: telling how many times as much. We say that the lengths of the boards are "in the ratio of 4 to 5, or 5 to 4," which may be written as 4:5, or 5:4. A ratio, then, is simply a *fraction* which gives the comparison at a glance; the above ratio might also be written as ⅘ or 5/4 instead of "4:5" or "5:4"; in fact, the colon (:) is really an abbreviation for "÷" with the horizontal line omitted. Notice that a ratio is independent of the units of measure; i.e., the two lengths mentioned above are in the ratio of 4:5 whether we express them in inches, feet, or yards. The units "cancel out," and the ratio remains 4:5. When comparing two quantities by the ratio method, however, care should be taken that the numbers to be compared are always expressed in the same units of measure to begin with.

EXAMPLE 1: A spindle is 3″ high and ¾″ in diameter. What is the ratio of its diameter to its height?

SOLUTION: $\frac{3}{4} \div 3 = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4} = 1:4$, *Ans.*

EXAMPLE 2: A rectangular sheet of tin measures 12′6″ in length by 8′4″ in width. Find the ratio of the length to the width.

SOLUTION: 12′6″ = 12½ ft.
8′4″ = 8⅓ ft.
$12\frac{1}{2} \div 8\frac{1}{3} = \frac{25}{2} \times \frac{3}{25} = \frac{3}{2} = 3:2$, *Ans.*

Exercise 19.

1. Two ladders are 12 ft. and 18 ft. long. What is the ratio of their lengths?
2. What is the ratio of the lengths of a 6″-pocket rule and a yardstick?
3. Two near-by office buildings are 24 stories and 36 stories high. Assuming that the "stories" in each building are the same height, what is the ratio of the heights of the buildings?
4. A certain style "legal size" envelope measures 4″×9½″. What is the ratio of its width to its length?
5. A photographic print is 3¼″×4¼″. What is the ratio of its dimensions?
6. A rectangle is said to have the most pleasing appearance when the ratio of its width to its length is 0.7. According to this standard, what should be the width of a rectangular placard that is 25 in. long?
7. A boy is 3 ft. 9 in. tall, and his father stands 5 ft. 9 in. Find the ratio of their heights.
8. A drawing of a flower in a biology textbook is 5¼ in. high. If the caption under the drawing reads "¾ actual size," what is the actual height of the flower?
9. The micro-photograph of a textile fibre is 2.4 cm. long. If the magnification is 1:60, how long is the actual specimen?
10. A mechanic constructed a miniature model of a machine part which was actually 3 ft. 6″ long. If he used a scale of "1 inch=½ foot," how long did he make the model?

Using Ratios. Ratios are very useful, and can be employed in many ways, as the following illustrative problems will show.

EXAMPLE 1: A board is 16 ft. long; if it is to be divided into two pieces in the ratio of 3:5, how long should each piece be?

SOLUTION: 3+5=8

one piece =⅜ of entire length, or 6 ft.
other piece=⅝ of entire length, or 10 ft., *Ans.*

EXAMPLE 2: Muntz metal consists of 6 parts of copper and 4 parts of zinc by weight. How many pounds of each metal are there in a block of Muntz metal weighing 72 lb.?

SOLUTION: 6+4=10
Ratio of copper to Muntz=6:10=.6
Ratio of zinc to Muntz =4:10=.4
72×.6=43.2 lb. copper,
72×.4=28.8 lb. zinc, *Ans.*

Example 3: The ratio of the diagonal of a square to the side of the square is 1.4. Find (a) the diagonal of a square whose side is 30 inches; (b) the side of a square whose diagonal is 28 inches.

Solution: A ratio of 1.4 is the same as 14:10, or (1.4):(1).

(a) diagonal$=1.4\times$side

$=1.4\times30=42$ in., *Ans.*

(b) side$=$diagonal$\div1.4$

$=28\div1.4=20$ in., *Ans.*

Since a ratio is always a fraction, ratios are frequently expressed as per cents. The specific gravity of a substance is the ratio of its weight to the weight of an equal volume of water. Thus ether, being only about 70% as heavy as water, has a specific gravity of 0.7; ice, 0.92; air, 0.0013; aluminum, 2.6; lead, 11.37; etc.

Exercise 20.

1. One inch is equivalent to 2.5 cm. What is the ratio of an inch to a centimeter? of a centimeter to an inch?
2. One liter is equivalent to 1.06 quarts. What is the ratio of a quart to a liter?
3. Divide a 42″-rod into two pieces in the ratio of 5:7.
4. The angles of a triangle are in the ratio of 1:2:3. If the sum of the three angles equals 180°, how large is each angle?
5. One quart equals approximately .95 liters. How many liters of acid are there in a 5-gallon acid carboy?
6. The ratio of the altitude of an equilateral triangle to its side is .866. What is the altitude of such a triangle if its side is 20 inches? What is the length of the side if the altitude is 2.598 inches?
7. The smaller of two connected pulleys makes 180 revolutions per minute while the larger one makes 45 revolutions. What is the ratio of their speeds? If the smaller one is speeded up to 220 r.p.m., what will be the speed of the larger, assuming the same speed ratio?
8. Aluminum metal expands .000013 of its length per Fahrenheit degree rise in temperature. If the original length of an aluminum bar is 200 cm., what is its length when raised 100°F?
9. Monel metal consists of 68½% of nickel, 1½% of iron, and the rest, copper. How many pounds of copper are there in a Monel metal casting weighing 60 lb.?
10. A ton of ready-mix concrete consists of cement, sand and gravel in the ratio of 1½:3½:5. How many pounds of each ingredient are there in the mixture?
11. If a sample of petroleum weighs 55 lb. per cu. ft., and water weighs 62.5 lb. per cu. ft., find the specific gravity of the petroleum.

12. If the specific gravity of ice is 0.92, what is the weight of 8 cu. ft. of ice, assuming that water weighs 62.5 lb. per cu. ft.
13. Brazing metal is an alloy made up of 20% zinc and 80% copper. What is the ratio of zinc to copper?
14. German silver (white metal) consists of 2 parts zinc, 3 parts nickel, and 5 parts copper. Find (a) the ratio of zinc to copper; (b) of copper to nickel; (c) what per cent of the alloy is nickel?
15. A commonly used mixture for concrete is made up of 1 part of cement, 2½ parts of sand, and 4 parts of stone. Find (a) the ratio of sand to stone; (b) the ratio of cement to sand; (c) what per cent of the concrete mixture is sand?

Scale Drawings. In representing distances on a map or dimensions on a plan or blueprint, it is necessary to use a scale, or to "scale down" the quantities, all in the same ratio. Thus on a given map, an inch might represent 300 miles, in which case two cities located 2½ inches apart on the map would actually be 750 miles distant from each other. Or the floor plan of a house might be drawn to a scale of 1″=10 ft.; in that case a room which on the plan is ¾″ wide is actually 7½ ft. wide, and a room 18 ft. long would be represented by a line 1.8 in. long.

EXAMPLE 1: A catalog picture of a machine part is labeled as being "⅖ actual size." If the length of the part in the picture is 3.8 in., what is its actual length?

SOLUTION: $3.8 \times \frac{5}{2} = 9.5$ in., *Ans.*

EXAMPLE 2: The working model of a machine is to be on a scale of 1:50. If a connecting piece of this machine is actually 16′8″ long, how long should the corresponding piece of the model be made?

SOLUTION: 16′8″=200″
200″$\times \frac{1}{50}$=4″, *Ans.*

EXAMPLE 3: On a blueprint the scale used is 1″=10′. Find (a) the actual size of a distance of 2¼″ on the blueprint; (b) how long on the blueprint an actual distance of 36 ft. ought to be.

SOLUTION: 1″=10 ft.
(a) 2¼″=10×2¼=22.5 ft., *Ans.*
(b) 1 ft.=$\frac{1}{10}$ in.
36 ft.=$36 \times \frac{1}{10}$=3.6 in., *Ans.*

Exercise 21.

1. The dimensions of the top of a rectangular workbench are 4′ by 8¼′. What should be its dimensions on a scale drawing, if the scale is ¼″=1′?

2. A living room is 20′×14′. On the architect's plan, what are its dimensions, if the scale used is ⅛″=1′?
3. If the scale used in the following is 1″=1′, fill in the missing values:

Actual Length	2′	5′	8′	20′		6½′
Scale Length	2″	5″			15″	

4. If the scale used in the following is ½″=10′, fill in the missing values:

Actual Length		20′	5′	24′		
Scale Length	⅛″				2″	3⅝″

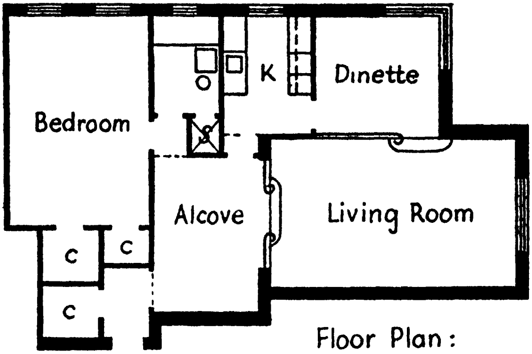
5. Complete the following table:

Scale Used	*Actual length*	*Scale length*	*Actual length*	*Scale length*
(a) 1″=20 ft.	35 ft.	?	?	3¼″
(b) 1″= 6″	1½ ft.	?	?	10″
(c) ⅛″= 1 ft.	12 ft.	?	?	2⅜″
(d) ½″= 1 mile	25 mi.	?	?	5¾″
(e) ¼″=10 ft.	45 ft.	?	?	4½″

6. The floor space of a storage bin is 14 ft. by 24½ ft. Using a scale of 1″=1′, what are the dimensions of the floor space on the architect's plan?
7. The dimensions of a metal plate for a machine are 6½ ft.×9 ft. What should its dimensions be on a blueprint, if the scale used is ½″=1′?
8. The scale of miles on a map is 1″=150 mi. How far apart on the map are two cities that are actually 1225 miles from each other?
9. On the plan of an apartment house a bedroom is shown, measuring 3½″ by 5¾″. If the floor plan is drawn to a scale of ¼″=1′, what are the actual dimensions of the room?
10. The detailed plan of a working model is represented on a draftsman's drawing by a scale of 1″=6″. What are the dimensions of a rectangular part measuring 4½″×6¼″ on the drawing?
11. A surveyor's map is drawn on a scale of 1″=10 feet. How far is it actually from one point to another that is 3¾ inches from it on the map?
12. On the layout of a camp site 1″=0.5 mile. It is 1¼ miles from the mess hall to a certain cottage. How far apart are these two places on the diagram?
13. Complete the blank spaces:

Length of object	1′	5′	6″		5″	4 yd.
Length of drawing	1″	½″		3′		6″
Scale used			½	¼	1′=6″	

14. A carpenter is using a blueprint with a scale of ½″=1′. What are the dimensions of a door that is 1¾″×3½″ on the blueprint?
15. Find the dimensions of (a) the living room, (b) the dinette, (c) the bedroom, and (d) the alcove.



Floor Plan:
3½ Room Apartment
Scale 1/16″ = 1′ 0″

Proportion. The word "proportion" is one which is often used carelessly, or with only a vague notion of what is really meant. Strictly speaking, when we compare two quantities, we cannot speak of their "proportion"; we can refer to the ratio between them, or what part or what per cent one is of the other. When we speak of a proportion, we have in mind *four* quantities, and are comparing them *in pairs,* as ratios. In other words, if two ratios are equal to each other, they are said to form a proportion. Thus, 3:5=12:20 is a proportion. Putting it another way, if the ratio between any two quantities is numerically equal to the ratio between two other quantities, then the four quantities are in proportion. For example, a nickel bears the same ratio to a dime that a half-dollar does to a dollar, since

$$\frac{5}{10}=\frac{50}{100}, \qquad \text{or } \frac{1}{2}=\frac{1}{2}$$

Notice that in the first case, all four quantities are expressed in terms of the same units, *viz.,* cents, although the units don't appear; in the second case, the two ratios are reduced to lowest terms to show their equality. Or again, if a man 6 ft. tall casts a shadow 8 ft., then a pole 18 ft. high will cast a shadow 24 ft. long; or, 6:8=18:24.

$$\frac{6}{8}=\frac{18}{24} \qquad \frac{3}{4}=\frac{3}{4}$$

In other words, a proportion is simply an equation stating that two fractions are equal. For example,

$$\text{if } 2:3=10:15, \qquad \text{then } \frac{2}{3}=\frac{10}{15},$$

or $2\times 15=3\times 10$, which might be called "cross-multiplying" the four *terms of the proportion. If only three of the four terms of a proportion are known, the remaining term can easily be found by the principle of "cross-multiplication,"* as shown below.

EXAMPLES:

1. If $\frac{2}{3}=\frac{8}{n}$, find the missing number "n."
Multiplying "crisscross,"
$2\times n=8\times 3$
$2n=24$
$n=12$
Check:
$\frac{2}{3}=\frac{8}{12}$

2. If $\frac{4}{5}=n/20$, find the missing quantity "n."
Cross-multiplying:
$4\times 20=5\times n$
$5n=80$
$n=16$
Check:
$\frac{4}{5}=\frac{16}{20}$

3. If $3/n=\frac{5}{8}$, find n.
Multiplying:
$3\times 8=5\times n$
$5n=24$
$n=\frac{24}{5}=4\frac{4}{5}$
Check:
$3\div 4\frac{4}{5}=\frac{5}{8}$

4. If $n/4=\frac{2}{7}$, find n.
$7\times n=2\times 4$
$7n=8$
$n=\frac{8}{7}=1\frac{1}{7}$
Check:
$1\frac{1}{7}\div 4=\frac{2}{7}$

Exercise 22.

Find the missing term in each of the following proportions:

1. $\frac{3}{4}=15/n$
2. $\frac{2}{5}=16/n$
3. $\frac{2}{3}=14/n$
4. $\frac{5}{6}=n/24$
5. $\frac{3}{8}=n/32$
6. $\frac{9}{4}=n/12$
7. $6/n=\frac{2}{3}$
8. $7/n=\frac{1}{6}$
9. $n/16=\frac{3}{8}$
10. $n/40=\frac{3}{5}$
11. $n/180=\frac{24}{720}$
12. $88/n=3\frac{1}{7}$

Direct Proportion. A proportion in which the ratios vary in the same order is called a *direct proportion.* For example, the volume of a gas (under constant pressure) *varies directly* with the temperature: as the temperature increases, the volume increases; as the temperature decreases, the volume decreases. This may be expressed mathematically as follows:

$$\frac{V_1}{V_2}=\frac{T_1}{T_2};$$

note that the "subscripts" of the letters are in the same order.

Example 1: If 8 bolts weigh 10 oz., how much will 48 similar bolts weigh?

Solution: Let x represent the required weight.

$$8/48 = 10/x$$

$$8x = 10 \times 48$$

$$x = \frac{10 \times 48}{8} = 60 \text{ oz., } \textit{Ans.}$$

Example 2: The elongation of a certain spring varies directly with the weight applied. If a weight of 48 oz. causes an elongation of 2 inches, (a) what will the elongation be when a weight of 60 oz. is applied? (b) what weight will produce an elongation of 1½ inches?

Solution:

$$\frac{e_1}{e_2} = \frac{w_1}{w_2}$$

(a)

$$2/x = 48/60$$

$$48x = 2 \times 60$$

$$x = \frac{2 \times 60}{48} = 2\tfrac{1}{2} \text{ in., } \textit{Ans.}$$

(b)

$$\frac{2}{1\frac{1}{2}} = \frac{48}{x}$$

$$2x = \tfrac{3}{2} \times 48$$

$$x = \frac{\frac{3}{2} \times 48}{2} = 36 \text{ oz., } \textit{Ans.}$$

Exercise 23.

1. If 100 pages of a book measure ⅜″ in thickness, what will be the thickness of a book of 480 pages of the same quality of paper?
2. If 7½ gallons of paint cost $10.50, what will 40 gallons of paint cost?
3. Eight stamping machines turn out 560 pieces of work in one hour. How many pieces will 5 of these machines turn out in an hour and a half?
4. If a bomber flies 840 miles in 3 hours, how far will it fly in 8½ hours at the same rate?
5. If the electrical resistance of 250 ft. of a certain wire is 150 ohms, how many ohms resistance will 875 ft. of the same wire have?
6. Metal castings are often sold by the pound. If a casting weighing 240 lb. costs $12.80, what is the weight of a similar casting that costs $41.60?

7. A section of a steel girder 18 ft. in length weighs 450 lb. How long is another section of the same girder if it weighs 1050 lb.? What is the weight of a piece 10 ft. long?
8. The volume of a gas under constant pressure varies directly as the absolute temperature. If $V=420$ when $T=225$, what is the value of V when $T=175$? For what value of T will V equal 1200?
9. At 75 lb. pressure per sq. in., a certain exhaust pipe discharges 270 cu. ft. of gas per minute. Assuming direct variation, how many cu. ft. are discharged at 80 lb. pressure? What pressure is required to discharge 450 cu. ft. per minute?
10. The distance traveled by sound varies directly as the time required to hear the sound. A storm is 1½ miles away, and the sound of the thunder reaches an observer 7.2 seconds after the lightning flash is seen. Some time later the thunder is heard 4.5 seconds after the flash. How much nearer is the storm the second time?

Inverse Proportion. A proportion in which the ratios vary in the opposite order is called an *inverse proportion.* For example, the volume of a gas (under constant temperature) *varies inversely* as the pressure: as the pressure increases, the volume *decreases,* and as the pressure decreases, the volume *increases.* This may be expressed mathematically as follows:

$$\frac{V_1}{V_2}=\frac{P_2}{P_1};$$

note that in this case the subscripts of the letters are in *reverse* order.

EXAMPLE: The current (C) in an electric circuit varies inversely as the resistance (R). If $C=2$ amperes when $R=55$ ohms, find (a) the current when $R=220$; (b) find the resistance R when $C=5$ amperes.

SOLUTION: $\frac{C_1}{C_2}=\frac{R_2}{R_1}$

(a) $\frac{2}{C_2}=\frac{220}{55}$

$220\ C_2=2\times55$

$C_2=110\div220=\frac{1}{2}$ ampere, *Ans.*

(b) $\frac{2}{5}=\frac{R_2}{55}$

$5R_2=2\times55$

$R_2=110\div5=22$ ohms, *Ans.*

Inverse proportion is well illustrated by the relation of the diameters of pulleys and gears. Whenever two pulleys having different diameters are connected, the smaller pulley always rotates more times than the larger. Likewise, when two gears with different diameters are in mesh, the smaller (having the lesser number of teeth) turns more rapidly than the larger one.

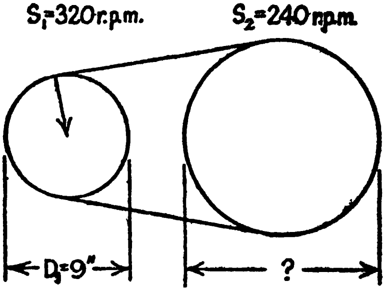
EXAMPLE 1: Two pulleys are connected with a belt; the smaller, having a diameter of 9″, makes 320 revolutions per minute, while the larger makes 240 r.p.m. Find the diameter of the larger pulley.

SOLUTION: $\frac{D_1}{D_2}=\frac{S_2}{S_1}$

$\frac{9}{D_2}=\frac{240}{320}$

$240\ D_2=9\times 320$

$D_2=16''$, *Ans.*



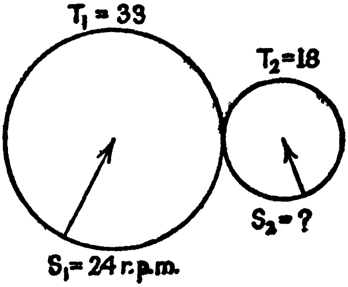
EXAMPLE 2: Two gears in mesh have 33 teeth and 18 teeth. When the larger makes 24 r.p.m., how many r.p.m. does the smaller gear make?

SOLUTION: $\frac{T_1}{T_2}=\frac{S_2}{S_1}$

$\frac{33}{18}=\frac{S_2}{24}$

$18\ S_2=24\times 33$

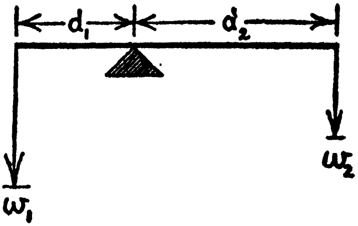
$S_2=44$ r.p.m., *Ans.*



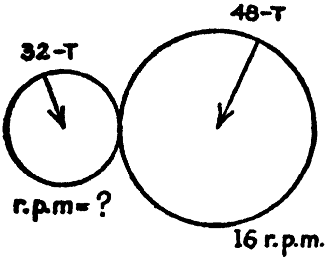
Exercise 24.

1. The resistance (R) in an electric circuit varies inversely as the current (C). If $R_1=40$ and $C_1=54$, find C_2 when $R_2=180$; find R_2 when $C_2=36$.
2. The volume of a gas at constant temperature varies inversely as its pressure. If the volume of the gas in a certain cylinder is 720 cu. in. at a pressure of 20 lb. per sq. in., what will be its volume under a pressure of 25 lb. per sq. in.? What pressure will be required to reduce its volume to 200 cu. in.?

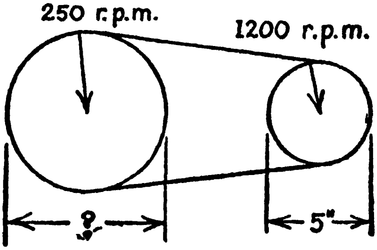
3. The so-called principle of moments, or balanced turning tendencies, follows the law of inverse proportion; for $\frac{w_1}{w_2}=\frac{d_2}{d_1}$, or $w_1d_1=w_2d_2$. Find w_2 if $w_1=4$ lb., $d_1=6$ in., and $d_2=8$ in.; find d_2 if $w_1=24$ lb., $w_2=30$ lb., and $d_1=10$ in.



4. The density of a substance varies inversely as its volume when the mass (weight) is constant, or $D_1 : D_2=V_2 : V_1$. If the density of a gas is .0015 when its volume is 8000 cu. cm., what will its density be when it has been compressed to a volume of 5000 cu. cm.?

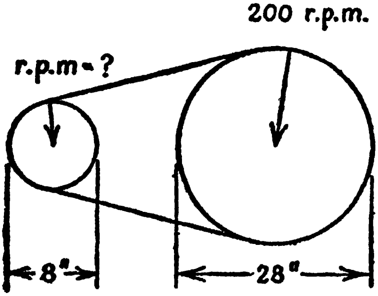


Ex. 5

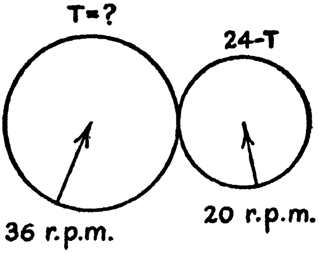


Ex. 6

5. Find the number of r.p.m. of the smaller gear.
6. Find the diameter of the larger pulley.



Ex. 7



Ex. 8

7. Find the speed of the smaller pulley.
8. Find, to the nearest whole number, the number of teeth in the larger gear.
9. If a driving pulley has a diameter of 20″ and its speed is 750 r.p.m., what is the speed of a 6″ driven pulley?

10. A circular buffer 6″ in diameter should revolve at 1800 r.p.m. If it is driven by a line shaft revolving at 400 r.p.m., find the diameter of the driving pulley on the line shaft that will be needed to obtain the desired speed of the buffer.

CHAPTER II

ELEMENTS OF ALGEBRA

William L. Schaaf

7. NUMBERS AND SYMBOLS

The Language of Algebra. The methods of algebra are essentially an extension of arithmetic. In other words, the numbers, symbols and operations used in algebra are the same as those used in arithmetic, only they are *more general* in character. This means (1) that *letters* as well as numbers are used to represent quantities, and (2) that numbers are regarded as having *quality* as well as quantity.

Consider the use of letters as symbols of quantity. In arithmetic, we say a force of 10 lb.; in algebra, we say a force of F lb., and we think of the letter F as having *various numerical values,* either one after another, or even "all at once." Or again, in arithmetic we say a bolt has a diameter of 1½″; in the language of algebra we say it has a diameter of d inches; etc.

The following will illustrate how *verbal statements* are translated into *algebraic symbols:*

Verbal statement	*Algebraic formulation*
(1) Four times a number.	$4n$
(2) Sum of two numbers decreased by a third number.	$a+b-c$
(3) Three times a number increased by twice a second number.	$3k+2m$
(4) Sum of two numbers divided by 5 times a third number.	$\frac{x+y}{5z}$
(5) Twice the product of two numbers increased by ⅔ of a third number.	$2ab+\frac{2}{3}c$
(6) Ten times a number decreased by six.	$10n-6$

Exercise 25.

"Translate" into words each of the following algebraic expressions:

1. $2a+3b$
2. $5x-\frac{1}{2}y$
3. $\frac{a-b+c}{3}$
4. $\frac{2}{3}k+8$
5. $P-\frac{1}{N}$
6. $5Rh$
7. $\frac{m-n}{2p}$
8. $\frac{9}{5}C+32$
9. $2lw+2lh+2wh$
10. $\frac{H-h}{T-t}$

Substitution. The process of finding the numerical value of an algebraic expression for certain specific values of the letters that occur in the expression is known as *substitution.*

Example 1: Find the value of $\frac{\pi D}{4}h$, when $h=12$, $D=3.5$, and $\pi=3\frac{1}{7}$.

Solution: $\frac{1}{4} \cdot \frac{22}{7} \cdot \frac{7}{2} \cdot 12=33$, *Ans.*

Example 2: Find the value of $2ak-\frac{3}{4}m$, when $a=20$, $k=2.5$, and $m=1.6$.

Solution: $(2)(20)(\frac{5}{2})-(\frac{3}{4})(1.6)=100-1.2=98.8$, *Ans.*

Exercise 26.

Find the numerical value of each of the following:

1. $2x+3y$, when $x=4\frac{1}{2}$ and $y=0.8$.
2. $\frac{3}{4}h-2$, when $h=16.44$.
3. $\frac{p+q}{3r}$, when $p=10.3$, $q=8.6$, and $r=.07$.
4. $2\frac{1}{4}D+1\frac{3}{4}$, when $D=.816$.
5. $\frac{4}{3}\pi R^3$, when $\pi=3\frac{1}{7}$ and $R=3.5$.
6. $0.3707p-0.0052$, when $p=\frac{1}{6}$.
7. $\frac{9}{5}C+32$, when $C=99.5$.

Complete each of the following tables of values as indicated:

(8)

D	$D/2$
4	?
6	?
8	?
10	?
15	?

(9)

k	$8k$
0	
1	
2	
3	
4	

(10)

H	$.75H$
4	
8	
10	
12	
24	

(11)

N	$\frac{1}{8}N$
0	
2	
4	
6	
10	

(12)

R	$\frac{4}{3}R$
2	
4	
6	
8	
12	

(13)

p	$\frac{2}{3}p$
1	
3	
5	
7	
10	

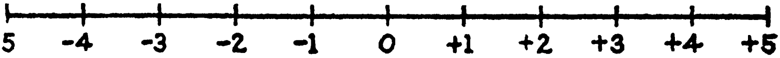
(14)

h	$\frac{h}{3.14}$
0	
1	
2	
10	

(15)

x	$4\frac{1}{2}x$
0	
1	
2	
4	
10	

Positive and Negative Quantities. The second feature which distinguishes algebra from arithmetic is the use of *negative* numbers as well as positive numbers. Accordingly, all numbers and letters are assumed to be either positive or negative (except zero), and are designated, respectively, as + or —; thus +5, —2, +½, —0.15, etc. The essential significance of these signs is to denote *oppositeness,* i.e., oppositeness of direction on a number scale, as suggested below:



Addition of signed numbers in algebra is equivalent to combining "steps" or "intervals" along a number scale, the signs of the numbers indicating the "direction" from the zero point taken in each step. The "sum" of an algebraic addition is thus the *net result* of combining two or more such steps. For example, by algebraic addition, we get:

+3	+4	—5	—3	+2	—1	+4
+2	—3	+2	—2	—3	+3	—4
+5	+1	—3	—5	—1	+2	0

If no sign appears before a quantity, it is assumed to be positive.

RULE I: *To add two signed numbers whose signs are* alike, *find their arithmetic sum and prefix the same sign that both have.*

RULE II: *To add two signed numbers whose signs are* opposite, *find their arithmetic difference, and then prefix the sign of the (numerically) larger quantity.*

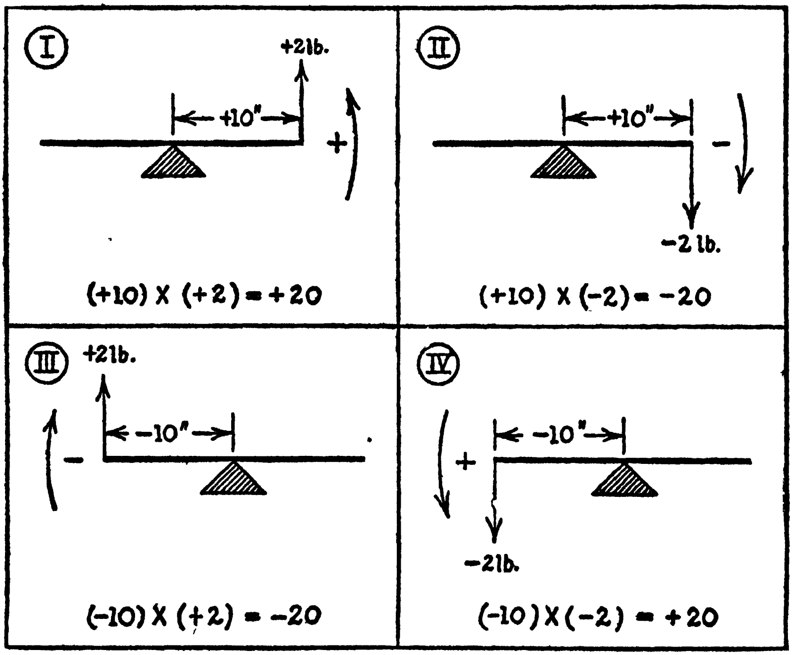
Subtraction being the opposite or inverse process of addition, it might be expected that we also reverse something when we subtract in algebra; we do. To subtract a quantity in algebra, we add the same quantity *with the opposite sign.* Hence

RULE III: *To subtract one signed number from another, change the sign of the subtrahend; then add them algebraically.*

For example, when subtracting the lower quantity from the upper in each case, we get:

$$\begin{array}{ccccccc} +6 & +4 & -7 & -8 & +3 & -5 & -6 \\ +4 & -3 & +2 & -8 & -7 & +9 & -3 \\ \hline +2 & +7 & -9 & 0 & +10 & -14 & -3 \end{array}$$

Multiplication and Division with Signed Numbers. The rules concerning signs when multiplying and dividing signed numbers are simple, and, while they may seem a bit arbitrary, their reasonableness may be seen by studying the accompanying diagrams of a lever (or see-saw), where it is agreed that distance to the right of P are +, and to the left, —; where it is also agreed that downward pulls are negative (—) and upward pushes positive (+); and finally, that clockwise turning is negative and counterclockwise, positive.



Rule IV: *In multiplication, like signs give a positive product, and unlike signs give a negative product.*

For division, the same rule holds true: if the signs of the dividend and divisor (or numerator and denominator) are alike, the quotient is positive; if unlike, the quotient is negative.

Exercise 27.

1. Add:

$+14$	$+22$	$+17$	-31	-12	$+15$	-18
-8	$+16$	-17	$+12$	-10	-27	30

2. Subtract:

$+15$	$+18$	-16	$+32$	-14	-18	-25
-8	$+5$	-10	$+32$	-12	18	-25

3. Multiply:

$(+3)(-21)$
$(-6)(-10)$
$(-15)(+4)$
$(+16)(+\frac{1}{8})$
$(-\frac{1}{2})(-12.4)$
$(-6)\times(12k)$
$(-10m)\times(-3)$
$(4)\times(-3a)$
$(-8)\times(6r)$

4. Divide:

$(+18)\div(-2)$
$(-27)\div(+3)$
$(-15)\div(-5)$
$(+24)\div(+2)$
$(+50)\div(-25)$
$(-16)\div(-16)$
$(+12)\div(-6)$
$24\div(-3a)$
$(-48x)\div 6$

Addition and Subtraction of Similar Terms. In algebra, quantities to be added or subtracted are known as *terms*. *Similar terms* are terms similarly composed of the same letters; thus, for example, $3a$ and $5a$ are similar terms; so are $3xy$, $-2xy$ and $6xy$. *Only similar terms can be added and subtracted.* The sum of $4n$, $-3n$, and $5n$ is $6n$; but $4n$ and $3p$ cannot actually be added (until numerical values are substituted for both n and p). In terms like these the number before a letter (or group of letters mutiplied together) is known as the numerical coefficient of the term.

RULE V: *To add (or subtract) similar terms, add (or subtract) their numerical coefficients, and annex the same letters to the new coefficient.*

Exercise 28.

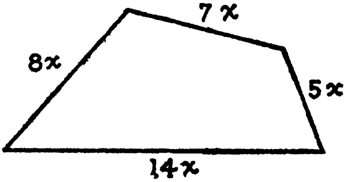
1. Add:

$15p$	$+28r$	$-3m$	$+16.4h$	$-8\frac{1}{2}x$
$6p$	$-2r$	$12m$	$-7.8h$	$-14x$

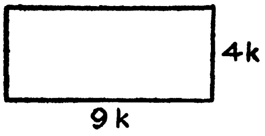
2. Subtract:

$12k$	$-20a$	$+15xy$	$-18lw$	$+\frac{1}{2}ab$
$8k$	$-14a$	$-11xy$	$+18lw$	$-2ab$

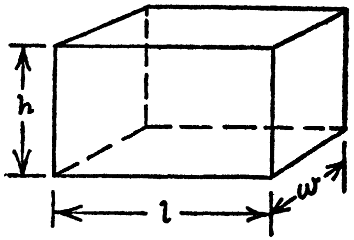
3. Find the perimeter of the figure at the right.

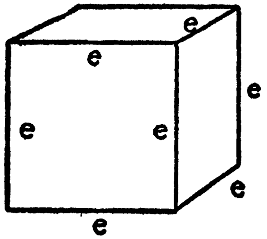


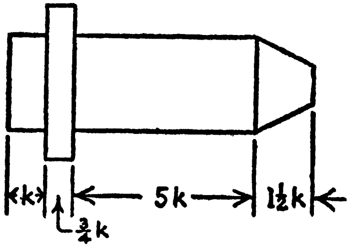
4. What is the perimeter of the rectangle shown below.



5. Find the sum of the twelve edges of the rectangular solid shown below.



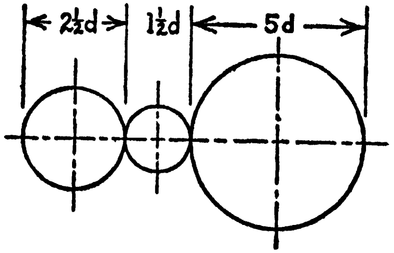




6. What is the sum of all the edges of the cube?

7. What is the overall length of the piece of work shown?

8. Find the overall diameter of this train of gears.



Multiplying and Dividing Literal Numbers. Although only similar terms can be added and subtracted, the operations of multiplication and division can be performed on any number of terms, whether similar or not. Thus, $2a \times 3b = 6ab$; $5mh \times 3r = 15mhr$. Or again, $12a \div 4a = 3$; $16xy \div 8x = 2y$; etc. Furthermore, to show how similar terms are multiplied and divided, study the following examples:

$$a \times a = a^2$$
$$b \times b \times b = b^3$$
$$3a \times 5a = 15a^2$$
$$4p \times 3pq = 12p^2q$$
$$5h^2r \times 2hr^3 = 10h^3r^4$$

$$\frac{6a^2}{2a} = 3a$$
$$\frac{-12m^2x}{2mx^3} = \frac{-6m}{x^2}$$
$$\frac{-8a^2b^3}{-4ab^2} = 2ab$$

NOTE: In algebra, the multiplication sign ($\times$) is frequently omitted between two letters to be multiplied together; instead, a dot is placed between them, or nothing at all, to indicate multiplication. Thus

$$(2m)\times(r)=2m\cdot r=2mr.$$

Exercise 29.

1. Find the product of:

$8\times 4m$
$10\times 3k$
$5p\times 3q$
$6h\times 2ab$
$4a^2\times 3ab$
$3\frac{1}{7}\times 21h$
$\frac{2}{3}s\cdot\frac{1}{2}t$
$\frac{1}{3}r^2\cdot 15h$
$4\ R^2\cdot\frac{1}{3}R$
$3.8at\times 2at^2$
$4a^2b\cdot 3ab^2$
$5x^3\cdot x^4$
$m^2\cdot m^3$
$12xy\cdot\frac{1}{2}xy$
$(\frac{1}{2}a^3)\cdot(\frac{1}{3}a^2)$

2. Divide:

$14m\div 2$
$32x\div 4x$
$24p\div p$
$15\div 5x$
$240k\div 32$
$35y^2\div 5y$
$12a^3\div ab$
$20xy\div 4xy$
$15r^2\div 5rs^3$
$-8cd^3\div c^2d$
$2\pi R^2h\div\pi R$
$-10a^3b^6\div 5ab^4$

Terms and Factors. The distinction about to be made between *terms* and *factors* is very important. Thus in the expression $(2)(a+b)$, one factor is "2" and the other is the *entire* quantity "$a+b$." *Factors* are quantities which are to be multiplied or divided; *terms* are quantities that are to be added or subtracted. Thus, in the expression $2a+2b$ there are *two terms,* viz., "$2a$" and "$2b$"; each of these terms consists of two factors, to wit, 2 and a, and 2 and b, respectively. In the first instance the parenthesis ( ) indicates that the quantity enclosed within it is to be regarded as *all one quantity* so far as the operations of multiplication (or division) are applied to it. In short, whatever the "a" is multiplied by, the "b" must also be multiplied by. Or

$$3(a+b)=3a+3b;$$
$$\tfrac{1}{2}(p+q-m)=\tfrac{1}{2}p+\tfrac{1}{2}q-\tfrac{1}{2}m;\ \text{etc.}$$

Exercise 30.

Write each of the following without the parentheses:

1. $2(l+w)$
2. $k(a+b-c)$
3. $a+(n-1)d$
4. $I(R+r)$
5. $P(1+RT)$
6. $S=\frac{n}{2}(a+l)$
7. $\frac{h}{2}(B+b)$
8. $\pi(R^2-r^2)$
9. $L(t_0-t_1)$
10. $2\pi r(l+r)$
11. $\frac{n}{2}[2a+(n-1)d]$
12. $V=\frac{h}{6}(b_1+b_2+4M)$

Factor each of the following expressions:

13. $p+pi$
14. $hp-hq$
15. I^2R+I^2r
16. $2ab-2ac$
17. $P+Prt$
18. $\frac{1}{2}B_1h+\frac{1}{2}B_2h$
19. $\pi Dl+\pi D$
20. $2\pi Rl+2\pi R^2$

Find the numerical value of each of the following:

21. $\frac{5}{9}(F-32)$, when $F=212$; when $F=68$.
22. $\frac{1}{2}h(B_1+B_2)$, when $h=6\frac{1}{2}$, $B_1=10.4$, and $B_2=5.6$.
23. $\dfrac{12(D-d)}{L}$, when $D=8.5$, $d=6.25$, and $L=48$.
24. $2\pi h(R^2-r^2)$, when $R=10$, $r=6$, $h=21$, and $\pi=3\frac{1}{7}$.

Squaring a Number. In order to find the area of a square we simply multiply the length of the side *by itself*. This is called squaring a number. The square, or second power, of a number is the product of the number multiplied by itself (not multiplied by 2); thus $3^2=3\times3=9$, or $10^2=10\times10=100$. If an indicated product contains two equal factors, it is the "square" of either one of them.

The Cube of a Number. Similarly, if a number is multiplied *by itself 3 times* (not multiplied by 3), the product is called the *cube* of the number; this is suggested by the fact that the volume of a cube equals the edge multiplied by itself three times. An indicated product having three equal factors is called the "cube" of any one of them. Thus, $2^3=2\times2\times2=8$; $5^3=5\times5\times5=125$; $10^3=10\times10\times10=1000$; etc.

Exercise 31.

Find the value of each of the following:

1. $8^2=?$
2. $16^2=?$
3. $6^3=?$
4. $12^3=?$
5. $1^3=?$
6. $3^3=?$
7. $20^2=?$
8. $7^3=?$
9. $(\frac{1}{2})^2=?$
10. $(\frac{2}{3})^2=?$
11. $(\frac{1}{4})^3=?$
12. $(\frac{1}{10})^2=?$
13. $(\frac{3}{4})^3=?$
14. $(.01)^2=?$
15. $(.01)^3=?$
16. $(.001)^2=?$

Other Powers. A number may be "raised to any power" desired. Thus:

$3^4=3\times3\times3\times3=81$ ("fourth power of 3")
$2^5=2\times2\times2\times2\times2=32$ ("fifth power of 2")
$10^6=10\times10\times10\times10\times10\times10=1{,}000{,}000$ ("sixth power of 10")

As a matter of fact, using various "powers" of 10 is a convenient device employed by scientists and engineers. Let us first study the following table of powers of the base 10:

$10^1=10$	$10^{-1}=0.1$
$10^2=100$	$10^{-2}=0.01$
$10^3=1000$	$10^{-3}=0.001$
$10^4=10{,}000$	$10^{-4}=0.0001$
$10^5=100{,}000$	$10^{-5}=0.00001$
$10^6=1{,}000{,}000$, etc.	$10^{-6}=0.000001$, etc.

Using these values, we can "abbreviate" a number like 290,000 as 29×10^4, since $29\times10^4=29\times10{,}000=290{,}000$. Similarly, a large number like 3,920,000,000 may be expressed more conveniently as 3.92×10^9, since $10^9=1{,}000{,}000{,}000$. Very small numbers, like .00000057 may be written as 57×10^{-8}; and so on.

Exercise 32.

Express each of the following in full:

1. 34×10^5
2. 62×10^8
3. 91×10^9
4. 43×10^{10}
5. 5.2×10^6
6. 13.8×10^{11}
7. 8.62×10^8
8. 24.63×10^{12}
9. 35×10^{-6}
10. 23×10^{-8}
11. 4.9×10^{-9}
12. 32.6×10^{-11}

Express each of the following as a power of 10:

13. 37,000,000
14. 5,800,000
15. 12,400,000
16. 4,900,000,000
17. 0.00004
18. 0.00000028
19. 0.000392
20. 0.00000000076

21. Astronomers use a unit known as the "light year," which is the distance traveled by light in one year. If this distance is equal to 5.8825×10^{12} miles, express this distance without using a power of 10.
22. Scientists have measured the wave length of sodium light and found it to be 0.0005893 millimeters. Express this in abbreviated standard form.

Laws of Exponents. When a quantity, whether a number or a letter, is raised to a power, the small number which indicates the power to which it is to be raised, and which is written to the upper right of it, is called an *exponent.* Thus in 25^2, the exponent is *2;* in $3x^5$, the exponent is *5.* Letters, too, may be used as exponents; thus, in 10^m, $4a^x$, and P^{k+1}, the exponents are *m, x,* and $(k+1)$, *respectively.* When powers are to be combined by multiplication or division, the following principles must be observed:

Principle	*Illustration*
(1) $a^m\cdot a^n=a^{m+n}$	(1) $x^4\cdot x^3=x^{4+3}=x^7$
(2) $a^m\div a^n=a^{m-n}$	(2) $y^5\div y^2=y^{5-2}=y^3$
(3) $(a^m)^n=a^{mn}$	(3) $(p^3)^2=p^{(3)\,(2)}=p^6$
(4) $(ab)^m=a^mb^m$	(4) $(ar)^4=a^4r^4$

Meaning of a Root. The operation which is the inverse (opposite) of raising to a power is called "taking the root." In other words, finding the square root of a quantity means finding the two equal factors which multiplied give that quantity; to find a cube root means finding the three equal factors of that product; etc.

EXAMPLES

Raising to a Power	*Expressing the Root*
$a^2=a\times a=P$	$\sqrt{P}=a$
$x^3=x\,.\,x\,.\,x=Q$	$\sqrt[3]{Q}=x$
$k^5=k\,.\,k\,.\,k\,.\,k\,.\,k=R$	$\sqrt[5]{R}=k$

Roots may also be expressed by using *fractional exponents,* as shown:

$$\sqrt{m}=m^{1/2} \qquad \sqrt[4]{x}=x^{1/4}$$
$$\sqrt[3]{a}=a^{1/3} \qquad \sqrt[5]{p}=p^{1/5}$$
$$\sqrt[3]{y^2}=y^{2/3}$$
$$\sqrt{A^3}=A^{3/2}$$

Negative Exponents. We have already seen that $10^{-1}=1/10$ or 0.1; $10^{-2}=1/100$, or 0.01; etc. A quantity with a *negative exponent* indicates that *that quantity with the same positive exponent is to be divided into one,* i.e., it indicates the *reciprocal* of that power; thus

$$a^{-1}=\frac{1}{a};\quad x^{-2}=\frac{1}{x^2};\quad p^{-5/3}=\frac{1}{\sqrt[3]{p^5}}$$

This may be better understood by studying the following summary:

$a^5=aaaaa$	$a^{-1}=\frac{1}{a}$
$a^4=aaaa$	$a^{-2}=\frac{1}{aa}=\frac{1}{a^2}$
$a^3=aaa$	$a^{-3}=\frac{1}{aaa}=\frac{1}{a^3}$
$a^2=aa$	$a^{-4}=\frac{1}{aaaa}=\frac{1}{a^4}$
$a^1=a$	$a^{-5}=\frac{1}{aaaaa}=\frac{1}{a^5}$

The reason why a negative exponent causes a factor to appear in the denominator may be seen by applying principle No. 2 above. Thus

$$\frac{a^3}{a^5}=a^{3-5}=a^{-2};$$

$$\text{but } \frac{a^3}{a^5}=\frac{\overset{1}{\cancel{a}}\overset{1}{\cancel{a}}\overset{1}{\cancel{a}}}{\cancel{a}\cancel{a}\cancel{a}aa}=\frac{1}{a^2};$$

$$\text{therefore } a^{-2}=\frac{1}{a^2}.$$

$$\text{Similarly, } \frac{x}{x^4}=x^{1-4}=x^{-3};$$

$$\text{but } \frac{x}{x^4}=\frac{\overset{1}{\cancel{x}}}{\cancel{x}xxx}=\frac{1}{x^3};$$

$$\text{therefore } x^{-3}=\frac{1}{x^3}.$$

The same principle also shows why any quantity with a ***zero exponent*** must always be equal to 1, no matter what the quantity itself may be; thus

$$\frac{a^5}{a^5}=a^{5-5}=a^0;\ \text{but } \frac{a^5}{a^5}=1;\ \text{hence } a^0=1$$

$$\frac{x^{13}}{x^{13}}=x^{13-13}=x^0;\ \text{but } \frac{x^{13}}{x^{13}}=1;\ \text{hence } x^0=1.$$

Roots and Powers of Fractions. Just to make sure, the reader is reminded that in raising a fraction to a power, both numerator and denominator are raised to that power; thus

$$(2/3)^2=\frac{2^2}{3^2}=4/9;\ (3/10)^3=.027;\ \left(\frac{2\pi R^2}{h}\right)^2=\frac{4\pi^2R^4}{h^2}.$$

Likewise when taking a root:

$$\sqrt{\frac{9}{16}}=\frac{\sqrt{9}}{\sqrt{16}}=\frac{3}{4};\ \sqrt[3]{\frac{1}{8}}=\frac{1}{2};\ \sqrt{\frac{2E}{m^2}}=\frac{\sqrt{2E}}{m}$$

Exercise 33.

1. Write the following without using exponents:
$a^{1/3}$; $x^{1/4}$; $(5a)^{1/3}$; $p^{2/3}$; $A^{3/2}$; $(mn)^{1/2}$.

2. Write the following without using radicals:

$$\sqrt{p};\quad \sqrt[4]{k};\quad \sqrt[3]{a^2};\quad \sqrt[5]{10x};\quad \sqrt{\frac{2s}{g}};\quad \sqrt{\frac{A}{\pi}}.$$

3. Find the value of the following:

$x^{1/2}$, when $x=144$.
$k^{1/3}$, when $k=27/8$.
$a^{2/3}$, when $a=8$.
$x^{2/3}$, when $x=8$.
$p^{1/2}$, when $p=36a^2$.
$3a^2b^3$, when $a=2$ and $b=1$.
$2x^3y^2$, when $x=3$ and $y=2$.
$(2x^3)^2$, when $x=10$.

4. What is the numerical value of each of the following?

$25^{-1/2}$
$4^{3/2}$
$27^{2/3}$

$(.04)^{1/2}$
$64^{2/3}$
$32^{1/5}$

$(.027)^{1/3}$
$16^{3/4}$
$8^{-1/3}$

10^{-3}
$(5x)^{-2}$
$(16a^4)^{-1/2}$

8. FORMULAS AND EQUATIONS

Meaning of a Formula. Many of the computations used in shop problems are either simplified or more clearly understood by the use of formulas. A *formula* is simply a mathematical statement of a principle or a rule describing the relation between two or more quantities. This mathematical statement shows that there is an *equality* between certain quantities; in other words, the formula translates a verbal rule into algebraic symbols. Thus a formula is very similar to an *equation*. For example, since there are 12 inches to every foot, this verbal rule can be stated as a formula by writing $I=12F$; or, since the percentage equals the base multiplied by the rate, we have the formula $P=BR$; or again, if in the lever, one weight multiplied by its distance from the support *balances* (i.e., equals) the other weight multiplied by *its* distance, then we can write this as a formula by saying: $w_1d_1=w_2d_2$.

Evaluating Formulas. It will be seen that a formula may involve two three, four or even more quantities. If the formula expressing the relation or connection between these quantities is known, and if a particular value is known for every quantity in the formula *except one*, then th value of that remaining one is easily found. This is sometimes called "evaluating a formula," or "substituting in a formula."

EXAMPLE 1: If $P=2(l+w)$, find P when $l=7.9$ and $w=5.2$.

SOLUTION: $P=2(7.9+5.2)$
$=2(13.1)=26.2$, *Ans.*

EXAMPLE 2: If $A=\frac{1}{2}h(B_1+B_2)$, what is the value of A when $h=8$, $B_1=4\frac{1}{2}$, and $B_2=6\frac{1}{2}$?

SOLUTION:

$$A=\tfrac{1}{2}(8)(4\tfrac{1}{2}+6\tfrac{1}{2})$$
$$=\tfrac{1}{2}(8)(11)=44, \textit{ Ans.}$$

EXAMPLE 3: If $S=\frac{1}{2}at^2$, find S when $a=32.2$ and $t=3$; also when $a=980$ and $t=10$.

SOLUTION:

$$S=\tfrac{1}{2}(32.2)\ (3)^2$$
$$=\tfrac{1}{2}(32.2)\ (9)=144.9, \textit{ Ans.}$$
$$S=\tfrac{1}{2}(980)\ (10)^2=49{,}000, \textit{ Ans.}$$

EXAMPLE 4: If $C=\frac{5}{9}(F-32)$, find C when $F=212$; when $F=32$.

SOLUTION:

$$C=\tfrac{5}{9}(212-32)=\tfrac{5}{9}\ (180)=100, \textit{ Ans.}$$
$$C=\tfrac{5}{9}(32-32)=\tfrac{5}{9}\ (0)=0, \textit{ Ans.}$$

Exercise 34.

1. If $V=lwh$, find V when $l=14''$, $w=6\frac{1}{2}''$ and $h=4''$.
2. If $A=6e^2$, find A when $e=2\frac{1}{2}''$.
3. In the formula $V=v_0+gt$, find the value of V when $v_0=200$, $g=32$, and $t=8$.
4. Given the relation $F=\frac{9}{5}C+32$, what is the value of F when $C=18^0$?
5. A formula for the amount of money due on a loan at simple interest is: $A=P\left(1+\dfrac{nr}{12}\right)$. Find A when $P=\$500$, $r=.04$, and $n=6$ months.
6. An electric current (I) is expressed by the formula $I=\dfrac{E}{R+r}$; find I when $R=85$, $r=3$, and $E=220$.
7. Under certain conditions the energy of a moving body is given by $E=\dfrac{Mv^2}{2g}$; find E when $M=2000$, $v=80$, and $g=32$.
8. If $I=\dfrac{\pi abC^2}{6}$, what is the value of I when $a=3.6$, $b=2.1$, $C=2$, and $\pi=3\frac{1}{7}$?
9. In designing modern automobile highways, the following formula relating to motor trucks is sometimes used: $W=k(L+40)$, where W = total gross weight of truck with its load, L = distance in feet between first and last axles of the truck, or truck and trailer. If the value of k in a certain state $=720$, and $L=42$ ft., find the value of W.

The heat generated in an electric circuit is expressed by the formula $H=0.24Ri^2t$. If R, the resistance, equals 55 ohms; i, the current, equals 2 amperes; and t, the time, equals $\frac{1}{2}$ hour, find the amount of heat (H) produced (expressed in calories).

11. A formula sometimes used for finding the horse power rating of a gas engine is: $\text{H.P.}=\frac{D^2n}{2.5}$, where D=diameter of the cylinder in inches, and n=number of cylinders, provided the piston speed is 1000 ft. per min. Find the H.P. of an engine having 12 cylinders with a 4″-bore, running at 2000 ft. per min.
12. Marine engineers find the "wetted surface" of a ship by using the formula $A=15\sqrt{Dl}$, where D=displacement in tons and l=length of the ship in feet. If a ship 900 ft. long displaces 81,000 tons, find the wetted surface A (in sq. ft.).

Simple Equations. As already stated, an equation is very similar to a formula. The chief difference, as far as we are concerned here, is this: in a formula there are at least two "quantities," or literal numbers, whereas in a simple equation there is only one such literal number. Thus:

Formulas	*Equations*
$A=lw$	$12x=5-3x$
$V=\frac{1}{3}Bh$	$\frac{3}{6}x=42$
$\frac{1}{p}+\frac{1}{q}=\frac{2}{R}$	$\frac{x}{3}+\frac{x}{2}=12\frac{1}{2}$

It turns out, therefore, for reasons that will become clearer as we go along, that in a simple equation there is *only one* numerical value which will hold true for the literal number; in the case of a formula, however, there are indefinitely many combinations of values for the literal quantities which will make the formula hold true. The process of finding the particular value which holds true for the literal quantity in any equation is called "solving the equation"; that numerical value of the letter is called the "solution," and it is said to "satisfy" the equation. For example, if it is known that $6x-17=4x+13$, it is possible to find a value which *satisfies* this equation; the desired value is 15. For if $x=15$, then substituting 15 for x in the equation we obtain:

$$\begin{gathered}(6)(15)-17=(4)(15)+13\\ 90-17=60+13\\ 73\equiv 73\end{gathered}$$

This shows that the equality expressed by the original equation really holds true when x has the numerical value 15, since 73 is identical with 73, or both sides of the equation have been shown to be identical.

Solving an Equation by Division. Certain types of equations are readily solved by dividing both sides of the equation by an appropriate number.

EXAMPLE 1: Solve the equation $6x=51$.

SOLUTION: Dividing each "side" of the equation by 6 we obtain

$$x=\frac{51}{6}$$

or $x=8\frac{1}{2}$, *Ans.*

EXAMPLE 2: Solve for y: $3\frac{1}{2}y=28$

SOLUTION: Dividing both sides by $3\frac{1}{2}$, we obtain

$$y=\frac{28}{3\frac{1}{2}}$$

or $y=(28)(\frac{2}{7})$

$y=8$, *Ans.*

Solving an Equation by Multiplication. Sometimes an equation may be conveniently solved by multiplication instead; each side of an equation may be multiplied by any number, provided it is the ***same*** **number.**

EXAMPLE 1: Solve: $\frac{x}{14}=3$

SOLUTION: Multiplying both sides by 14 we obtain

$x=(14)(3)$

or $x=42$, *Ans.*

EXAMPLE 2: Solve for n the equation $\frac{n}{.6}=25$

SOLUTION: Multiplying both sides by .6 we obtain

$n=(25)(.6)$

or $n=15$, *Ans.*

Exercise 35.

Solve each of the following equations for the "unknown" letter:

1. $5n=65$
2. $\frac{x}{4}=8$
3. $100x=35$
4. $\frac{1}{8}p=2.5$
5. $84=7k$
6. $\frac{y}{9}=1\frac{2}{3}$
7. $3\frac{1}{2}x=49$
8. $.05k=16$
9. $\frac{n}{.4}=30$
10. $\frac{2x}{3}=24$
11. $.3x=4.8$
12. $\frac{3}{4}h=120$

Changing the Subject of a Formula by Multiplication or Division. The same procedure as used for equations can also be applied to solving a formula for any particular letter desired. The other letters are simply regarded as numbers. This is called "changing the subject of the formula," or solving for a particular letter "in terms of the other letters."

Example 1: Solve the formula $F=ma$ for m.

Solution: Dividing both sides by a:

$$\frac{F}{a}=m, \text{ or } m=\frac{F}{a}, \textit{ Ans.}$$

Example 2: In the formula $\frac{E}{I}=nR$, express I in terms of E, n and R.

Solution: Multiplying both sides by I:

$$E=InR$$

Dividing both sides by nR:

$$\frac{E}{nR}=I, \text{ or } I=\frac{E}{nR}, \textit{ Ans.}$$

Exercise 36.

Solve each of the following formulas for the quantity specified:

1. $A=lw$; solve for l.
2. $D=\frac{M}{V}$; solve for M.
3. $D=RT$; solve for T.
4. $I=\frac{E}{R}$; solve for R.
5. $C=2\pi R$; solve for R.
6. $d=1.4s$; solve for s.
7. $PV=kT$; solve for P.
8. $I=PRT$; solve for T.
9. $A=\frac{1}{2}bh$; solve for h.
10. $S=2\pi rh$; solve for r.
11. $l_1 w_1=l_2 w_2$; solve for w_2
12. $S=\frac{1}{2}gt^2$; solve for t^2.

Solving an Equation by Addition or Subtraction. Frequently an equation is of such a form that its solution may be effected by adding or subtracting appropriate numbers, as shown below. Any number may be added to, or subtracted from, both sides of an equation; but it must be the same number.

Example 1: Solve for x: $x+10=17$

Solution: Subtracting:

$$\begin{array}{r@{\,}l} x+10 &= 17\\ 10 &= 10\\ \hline x &= 7, \textit{ Ans.}\end{array}$$

Example 2: Solve for k: $k-6\frac{1}{2}=12$

Solution: Adding:

$$\begin{array}{r@{\,}l} k-6\frac{1}{2} &= 12\\ 6\frac{1}{2} &= 6\frac{1}{2}\\ \hline k &= 18\frac{1}{2}, \textit{ Ans.}\end{array}$$

Exercise 37.

Solve each of the following equations for the unknown letter:

1. $n+12=34$
2. $30=16+k$
3. $x-25=13$
4. $60=y-35$
5. $28+p=42$
6. $h-16=20$
7. $14=x+10\frac{1}{2}$
8. $n-3.5=7.2$
9. $y+6.4=8\frac{1}{2}$
10. $x-3\frac{1}{2}=5\frac{3}{4}$
11. $18=x-4\frac{1}{2}$
12. $40.2=78.5-x$

Solving Any Kind of a Simple Equation. By "any kind" of a simple equation we simply mean an equation in which several operations may be necessary to find the solution. The procedure is illustrated by the following:

EXAMPLE: Solve: $5x+8=2x+20$

SOLUTION:
$$\begin{aligned} 5x+8-2x&=20\\ 3x+8&=20\\ 3x&=20-8\\ 3x&=12\\ x&=4,\ \textit{Ans.} \end{aligned}$$

NOTE 1: These steps can be shortened by "transposing" terms; this means that any term of an equation can be "brought" from either side to the other side, ***provided its sign is reversed.*** Thus, in one step, we could write:

$$\begin{aligned} 5x-2x&=20-8\\ \text{then}\qquad 3x&=12\\ x&=4 \end{aligned}$$

NOTE 2: The solution of an equation should always be checked by substituting the value obtained for the letter ***in the original equation;*** this should yield an ***identity.*** Thus:

$$\begin{aligned} (5)(4)+8&=(2)(4)+20\\ 20+8&=8+20\\ 28&\equiv 28,\ \text{Check.} \end{aligned}$$

Changing the Subject of Any Simple Formula in General. The same procedure described in the foregoing paragraph applies to simple formulas as well as to simple equations.

EXAMPLE 1: Solve for F the temperature formula: $C=\frac{5}{9}(F-32)$.

SOLUTION:
$$\begin{aligned} C&=(\tfrac{5}{9})(F-32)\\ \tfrac{9}{5}C&=F-32\\ F-32&=\tfrac{9}{5}C\\ F&=\tfrac{9}{5}C+32,\ \textit{Ans.} \end{aligned}$$

Example 2: Solve for R: $C=\frac{E}{R+nr}$

Solution:

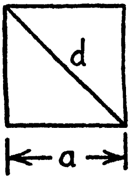
$$C=\frac{E}{R+nr}$$
$$C\,(R+nr)=E$$
$$CR+Cnr=E$$
$$CR=E-Cnr$$
$$R=\frac{E-Cnr}{C},\ \textit{Ans.}$$

Exercise 38.

Solve the following equations:

1. $8.5x+5=22$; find x.
2. $28-4y=3(2y-4)$; find y.
3. $n=p+kr$; solve for r.
4. $C=4p+q$; solve for p.
5. $P=2(l+w)$; solve for w.
6. $V=E+Ir$; solve for I.
7. $T=\frac{12(D-d)}{L}$; solve for d.
8. $V=v_0+gt$; solve for t.
9. $A=\frac{1}{2}h(B+b)$; solve for B.
10. $A=P+PRT$; solve for R.
11. $l=a+(n-1)d$; solve for d.
12. $C=\frac{N-n}{2P}$; solve for n.
13. $D=\frac{2\pi R}{L}$; solve for R.
14. $N=\frac{S-W}{W}$; solve for S; also for W.
15. $F=\frac{Wv^2}{gR}$; solve for W; for R.
16. $Q=\frac{w(H-h)}{t_1-t_0}$; solve for H.

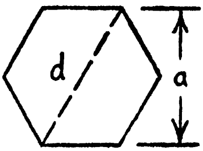
Practical Use of Formulas in the Shop. It should be pointed out that formulas of all kinds are constantly used in the various trades, in the shop, and in industrial work. For illustrative purposes as well as for reference and for self-practice a number of typical formulas are given below; these and many others will be found from time to time throughout the rest of the book.



(A) *Nuts and Bolts.*

(1) Diameter of blank for square bolt:
$d=1.414a$

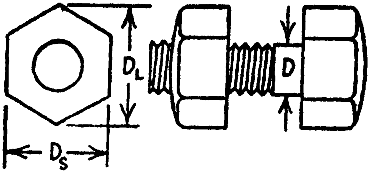
(2) Diameter of blank for hexagonal bolt:
$d=1.155a$



(3) Bolt and nut dimensions:

$D_S = \frac{3}{2} D + \frac{1}{8}$

$D_L = \frac{7}{4} D + \frac{1}{8}$

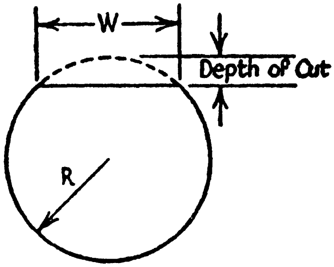


(B) *Depth of Cut.*

(4) When milling flats on round stock:

Depth $= R - \sqrt{R^2 - \frac{1}{4}W^2}$,

where R = radius of round bar, and W = width of flat surface.

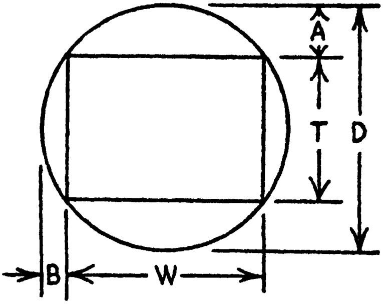


(5) When machining round stock with rectangular bars:

Depth $A = \dfrac{D-T}{2}$,

Depth $B = \dfrac{D-W}{2}$,

where D = diameter of round stock, W = width of rectangular bar, T = thickness of rectangular bar, and A and B = depths of cut.



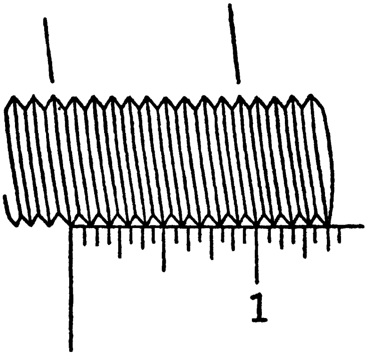
(C) *Screw Threads.*

(6) Pitch of screw (P) and number of threads per inch (N):

$$P = \frac{1}{N};\ N = \frac{1}{P}$$

(7) Depth (d) of sharp V-thread:

$$d = .866p = \frac{.866}{N}$$



Screw Thread : 10 Threads per inch

(8) Depth of American National thread:

$$d=.6495p=\frac{.6495}{N}$$

(9) Tap drill for sharp V-thread:

$$\text{Tap drill size } S=T-\frac{1.733}{N}$$

(10) Tap drill for American National thread:

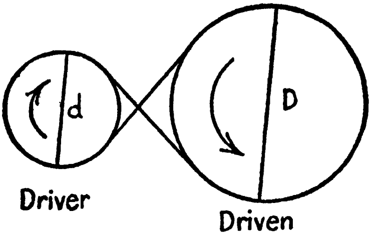
$$\text{Tap drill size } S=D-\frac{1}{N}$$

(D) *Pulley and Gear Speeds.*

(11) Speeds of belt-driven pulleys (s, S=speed, r.p.m.):

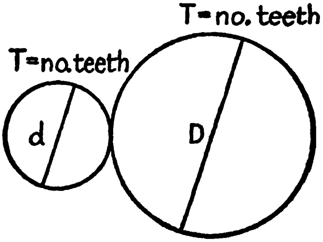
$$s=\frac{D\times S}{d}$$

$$d=\frac{D\times S}{s}$$



(12) Speed of gears in mesh (t, T= no. of teeth):

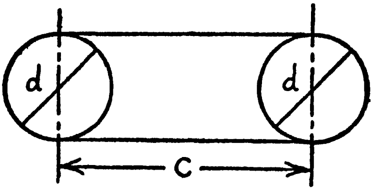
$$S=\frac{t\times s}{T}$$



(E) *Belting.*

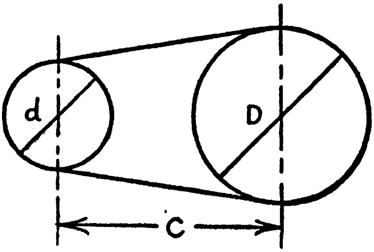
(13) Length of open belt (equal pulleys):

$$L=\pi d+2c$$



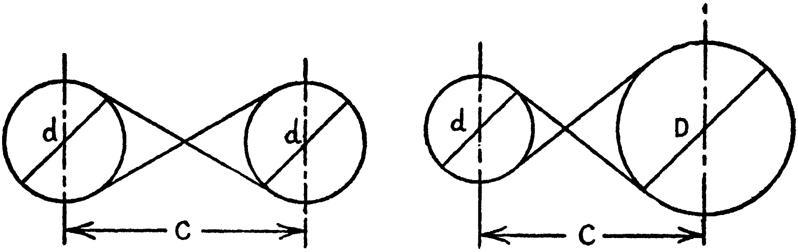
(14) Length of open belt (unequal pulleys):

$$L=\frac{\pi}{2}(D+d)+2\sqrt{c^2+(R-r)^2}$$



(15) Length of crossed belt, (equal or unequal pulleys):

$$L=\frac{\pi}{2}(D+d)+2\sqrt{c^2+(R+r)^2}$$



(16) Horsepower transmitted by belts:

$$\text{H.P.}=\frac{S\times W\times T}{33{,}000}\text{, where}$$

S =speed of belt (ft./min.)
W=width of belt (inches)
T =tension in belt (40 or 70 lb.)

(F) *Energy and Power.*

(17) Work=Force×Time:

$$W=FT$$

(18) Power=Work÷Time:

$$P=\frac{W}{T}$$

(19) Horsepower:

$$\text{H.P.}=\frac{\text{ft.-lb. per min.}}{33{,}000}=\frac{\text{ft.-lb. per sec.}}{550}$$

(20) Kinetic energy of moving object:

$$\text{K.E.}=\frac{wv^2}{2g}\text{, where}$$

w=weight (lb.)
v =velocity (ft. per min.)
g =acceleration of gravity (32 ft./sec./sec.)

(21) Indicated horsepower of steam engine:

$$\text{I.H.P.}=\frac{PLAN}{33{,}000}\text{, where}$$

P =mean effective pressure (lb./sq. in.)
L =length of stroke (ft.)
A =area of piston (sq. in.)
N=no. of strokes (2×r.p.m.)

(22) Horsepower rating of gas engine:

$$H.P. = \frac{D^2N}{2.5}, \text{ where}$$

D=diameter of cylinders (in.)
N=number of cylinders
(assumed piston speed=1000 ft. per min.)

9. SQUARE ROOT

Use of Square Root. For some reason or other, learning to find the square root of a number seems to be a stumbling block for most folks. As Stephen Leacock puts it, square root is as "obdurate as a hardwood stump in a pasture—nothing but years of effort can extract it. You can't hurry the process." Yet there is nothing really difficult about it. And it is a very practical matter, for, as we shall soon see, many problems in mensuration require the determination of the square root of a number. Finding a square root is simply the opposite of squaring a number; i.e., given the product of any number multiplied by itself, to find the original number. Several methods for extracting the square root of a number are available:

1. By inspection and approximation.
2. By using a table.
3. By the algebraic rule.
4. By means of logarithms.
5. By using the slide rule.

The last two methods will be explained in subsequent sections of the present chapter; the first three will now be discussed. Which method you use will depend largely upon the accuracy required and the availability of tables or a slide rule.

Approximation Methods. A fundamental method based upon successive trials and approximations is the following; it is somewhat laborious and therefore not particularly convenient, but will be found useful if not required too frequently, and if no other method is available. It has the advantage, however, of being simple to understand. Suppose we wish to find the square root of 44. Obviously, the value of $\sqrt{44}$ must be greater than 6, but less than 7, since 44 lies between 36 ($=6^2$) and 49 ($=7^2$). Suppose we *guess* the value to be 6.6, since 44 is somewhat closer to 49 than it is to 36. Now by actual multiplication we find that $(6.6)^2=43.56$, which is already quite close to 44; in fact, so close that it is safe to say that, correct to the nearest tenth, the value of $\sqrt{44}=6.6$ (the actual value=6.633). To make sure, we find by multiplication once more that $(6.7)^2= 44.89$, which is too large by more than 43.56, is too small;

hence 6.6 is correct to the nearest tenth. If we wish to find the value correct to the nearest hundredth, we would continue the trial guesses and approximations until we were certain of the second decimal place. To be sure, the labor becomes a bit cumbersome; fortunately, for many purposes the result to one decimal place suffices. For greater accuracy, other methods are frankly superior. If the original guess of 6.6 had been less fortunate, say 6.5 or 6.7, the amount of trial multiplication needed to determine the first decimal place is not prohibitive. This method is not at all suited, obviously, for three-place figures or greater, e.g., to find $\sqrt{273}$ or $\sqrt{4136}$.

Another approximation method is based on the algebraic formula:

$$\sqrt{a^2+b}=a+\frac{b}{2a};$$

applying it to the same problem, we regard $\sqrt{44}$ as equal to $\sqrt{36+8}$, so that $a=6$ and $b=8$. Therefore $\sqrt{44}=\sqrt{6+8/12}=6\frac{2}{3}=6.67$, which is also fairly close, although when "rounded off" it would give 6.7 for the answer. This method is fairly useful for numbers somewhat larger than two-place figures. It has the advantage, moreover, of not requiring a number of tedious multiplications.

Using a Table of Square Roots. The values of squares and square roots of many numbers have been carefully worked out and are usually available in the form of *tables* in most handbooks and reference manuals. When such tables are accessible, their use is doubtless the most convenient method of finding the desired square root, unless, of course, it is required to find the value to more decimal places than given in the table; in that case, either the "algebraic" method or logarithms should be used. A convenient table of square roots is given on pages 89–93.

Using this table we find the $\sqrt{44}$ directly to be 6.633, as already mentioned. However, it should be noted that the table can be used for many more numbers than appear in the column under the heading "N". Thus $\sqrt{44.8}=6.693$; also the $\sqrt{4400}=66.33$; and $\sqrt{4480}=66.93$. In other words, moving the decimal point *two places* in N means that it must be moved *one place* in the root. Furthermore, to find $\sqrt{4.4}$, we look on page 89 under "N" for 4.4 and find that the required value of $\sqrt{4.4}=2.098$; similarly, $\sqrt{4.45}=2.110$; $\sqrt{445}=21.10$; and $\sqrt{440}=20.98$, while $\sqrt{44{,}000}=209.8$.

Square Root by the Algebraic Rule. This method is a "rule of thumb" procedure based on the algebraic relation that $(a+b)^2=a^2+2ab+b^2$. Without stopping to explain the theory in full, we shall simply illustrate the procedure.

Example 1: Find the $\sqrt{44}$.

Solution:

$44.000000\ )\underline{6.633+}$, *Ans.*

36

$\boxed{12}$ 6 $\quad$ 8 00

$\qquad\qquad$ 7 56

$\boxed{132}$ 3 $\quad$ 4400

$\qquad\qquad$ 3969

$\boxed{1326}$ 3 $\quad$ 43100

$\qquad\qquad$ 39789

Point off in blocks of two, beginning at the decimal point.

Largest square in 44=36; $\sqrt{36}$= 6, first digit in root.

Double 6=12; "trial divisor"= 126. Multiplying 126 by 6=756.

Double root so far obtained; twice 66=132. Second "trial divisor"=1323. Multiply 1323 by 3=3969.

Double root so far obtained; twice 663=1326. Third "trial divisor"=13263; 13263×3=39789.

And so on, to as many decimal places as desired.

Example 2: Find the value of $\sqrt{79,328}$ correct to the nearest hundredth.

Solution: $79328.0000\ )\underline{281.65+}$, *Ans.*

4

48 $\quad$ 3 93

$\qquad$ 3 84

561 $\quad$ 9 28

$\qquad$ 5 61

5626 $\quad$ 3 67 00

$\qquad$ 3 37 56

56325 $\quad$ 29 44 00

$\qquad$ 28 16 25

It is clear that to find the square root of a number of 4 or 5 places, or more, neither the approximation method nor the table is of very much help; for that matter, neither is the slide rule. Only an extensive table of logarithms would do. Hence the "algebraic" method is quite useful, even though it may be slightly annoying at first.

Exercise 39.

Find, to the nearest tenth, the square root of each of the following by an approximation method; then check your result by means of the table:

1. 18
2. 30
3. 12
4. 41
5. 60
6. 21
7. 55
8. 74
9. 129
10. 150
11. 172
12. 200

Find, to the nearest tenth, the square root of each of the following by using the algebraic rule; check your result, by referring to the table:

13. 86
14. 54.8
15. 625
16. 382
17. 567
18. 2933
19. 69.43
20. 426.5

Evaluation of Formulas Involving Square Root. It so happens that formulas in mensuration, shop work and science problems frequently involve the square root of a constant or of one of the variables. If we wish to evaluate such a formula, it is useful to be able to find the numerical value of a square root quickly and easily; the following problems will afford practice in such computation.

Exercise 40.

1. Find the value of d in the formula $d=\sqrt{2hr}$ when $r=4.2$ and $h=6.5$.
2. Find the value of D in the formula $D=\sqrt{\frac{1}{3}RS}$ when $R=28$ and $S=6\frac{1}{4}$.
3. Find the value of t in the formula $t=\frac{44}{7}\sqrt{\frac{l}{g}}$ when $l=44.1$ and $g=9.80$
4. Find the value of A when $S=8.6$ in the formula $A=\frac{S^2}{4}\sqrt{3}$.
5. If $s=650$ and $g=32$, find the value of t to the nearest tenth in the formula $t=\sqrt{\frac{2s}{g}}$
6. Find b to the nearest hundredth from the formula $b=\sqrt{c^2+a^2}$ when $c=10.4$ and $a=4.2$.
7. The volume of a frustum of a pyramid is given by $V=\frac{1}{3}h(B_1+B_2+\sqrt{B_1B_2})$. Find V if $B_1=34$, $B_2=22$, and $h=6$.
8. The maximum distance at which an object on the earth may be seen from a point above the earth's surface is given by the formula $D=1.22\sqrt{E}$, where D is the distance of the object in miles and E is the elevation of the observer in feet. Find D when $E=840$ ft.
9. The "effective" area of a smokestack is expressed by the formula $E=A-\frac{3}{5}\sqrt{A}$, where A is its measured area in square feet. Find E when $A=44$ sq. ft.
10. The amount of sag (d) in a rope or wire suspended between two points is given by

$$d=\sqrt{\frac{3l(L-l)}{8}},$$

where l=length of the rope in feet when taut, L=its actual length in feet, and d equals the maximum sag in inches. Find d when $L=50$ and $l=45$.

Solving Equations Containing Radicals. Formulas and equations frequently involve radicals, particularly the square root of a quantity, whether a constant or a radical; or they contain terms that are raised to the second power. The procedure in such cases is illustrated by the following.

Example 1: If $A=\frac{\pi d^2}{4}$, find d.

Solution: $4A=\pi d^2$

$$d^2=\frac{4A}{\pi}$$

$$d=\sqrt{\frac{4A}{\pi}}=2\sqrt{\frac{A}{\pi}},\ \textit{Ans.}$$

Example 2: Solve for V: $F=\frac{wV^2}{gr}$

Solution: $grF=wV^2$

$$V^2=\frac{grF}{w}$$

$$V=\sqrt{\frac{grF}{w}},\ \textit{Ans.}$$

Example 3: If $a^2=b^2+c^2-2cp$, find b.

Solution: $b^2=a^2-c^2+2cp$

$$b=\sqrt{a^2-c^2+2cp},\ \textit{Ans.}$$

Example 4: In the formula $t=\sqrt{\frac{2s}{g}}$, solve for s.

Solution: $t^2=\frac{2s}{g}$

$$gt^2=2s$$

$$s=\frac{gt^2}{2},\ \textit{Ans.}$$

Example 5: Solve for r: $p=\frac{a}{2\pi\sqrt{nr}}$

Solution: $p^2=\frac{a^2}{4\pi^2nr}$

$$4\pi^2nrp^2=a^2$$

$$r=\frac{a^2}{4n\pi^2p^2},\ \textit{Ans.}$$

Exercise 41.

1. The kinetic energy of a body in motion is given by the formula $E=\frac{1}{2}mv^2$, where m is its mass and v its velocity. Solve for v.
2. The velocity in feet per second of a freely falling body at t seconds after it began falling is found from the formula $v=\frac{1}{2}gt^2$, where $g=$ the acceleration of gravity (32 ft. per sec.). Solve for t.

3. If m is the maximum "visibility" in miles from an elevation above the earth of h feet, we have $m=\sqrt{\frac{3h}{2}}$. Solve for h.

4. If h is the altitude of an equilateral triangle with side s, then $h^2=\frac{3s^2}{4}$. Solve for h.

5. Solve the formula $A=\frac{s^2}{4}\sqrt{3}$ for s.

6. The volume of a circular cone is given by $V=\frac{1}{3}\pi R^2 h$. Solve for R.

7. In dealing with a radio circuit, the formula $f=\frac{1}{2\pi\sqrt{LC}}$ is used. Solve for C.

8. In computing the strength of a certain beam, the formula $\frac{I}{C^2}=\frac{\pi ab}{6}$ is needed. Solve for C.

9. The formula $P=\frac{Wv^2}{550\times 2g}$ is used in the determining horsepower. Solve for v.

10. A formula of electrical engineering is $C=\frac{E}{\sqrt{R^2+(2\pi nL)^2}}$. Find R.

11. The area in sq. ft. of the cross-section of a smoke stack required to carry off the smoke is given by $A=\frac{.6P}{\sqrt{h}}$, where P=no. of lb. of coal burned per hr., and h=height of stack in ft. Solve for h.

12. In Ex. 11, find A if $h=75$ ft. and $P=550$ lb.

Solving a Simple Quadratic Equation. A simple quadratic formula or equation is one which contains second-degree (squared) terms as well as first-degree terms, or only squared terms, involving one of the variables only, and no higher powers of any other variable; examples of such quadratic equations are:

(1) $y=3x^2$
(2) $y=5x^2+3x-10$
(3) $E=\frac{1}{2}mv^2$
(4) $H=I^2R$
(5) $V=at+\frac{1}{2}at^2$
(6) $V=\frac{1}{3}\pi R^2 h$

The general type of a simple quadratic function may be written as

$$y=ax^2+bx+c,$$

and the general form of a simple quadratic equation is

$$ax^2+bx+c=o,$$

where a, b and c are the numerical coefficients. Thus, if the equation to be solved is $3x^2+5x-7=0$, then $a=3$, $b=5$, and $c=-7$

The solution of such an equation is found by using the formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}\cdot$$

Without explaining how this formula is derived, we shall illustrate its use in solving quadratic equations.

Example 1: Solve for x: $3x^2+5x-7=0$.

Solution:

$$x=\frac{-5\pm\sqrt{25-(4)(3)(-7)}}{(2)(3)},$$

$$\text{or } x=\frac{-5\pm\sqrt{25+84}}{6},$$

$$x=\frac{-5\pm\sqrt{109}}{6}=\frac{-5\pm10.44}{6}.$$

$$\left.\begin{array}{r}\text{Thus } x\frac{-5+10.44}{6}=\frac{5.44}{6}=+.91;\\ \text{and also } x\frac{-5-10.44}{6}=\frac{-15.44}{6}=-2.57\end{array}\right\}\textit{Ans.}$$

It should be noted from the above that there are *two values* for x; each of them satisfies the original equation. Every quadratic equation thus has *two roots,* although in the case of formulas, one of them might have no *practical* significance in the relation represented by the formula.

Example 2: Solve to the nearest hundredth for t:

$$5t^2-10t+2=0$$

Solution: In this case $a=5$, $b=-10$, and $c=2$.

$$\text{Then } t=\frac{+10\pm\sqrt{100-(4)(5)(2)}}{(2)(5)}$$

$$t=\frac{10\pm\sqrt{20}}{10}=\frac{10\pm4.472}{10}$$

$t=+1.45$ and $+.55$, *Ans.*

Note: Before substituting in the formula, always transpose *all* terms to the left side of the equation; otherwise the signs of a, b and c will be incorrect.

Exercise 42.

Solve the following, finding the roots correct to the nearest tenth:

1. $3x^2-4x-11=0$
2. $x^2-30=10x$
3. $y^2+y=14$
4. $4k^2-k=8$
5. $2p^2-2=7p$
6. $32t+16t^2=240$

SQUARE ROOTS OF NUMBERS*

N	0	1	2	3	4	5	6	7	8	9
1.0	1.000	1.005	1.010	1.015	1.020	1.025	1.030	1.034	1.039	1.044
1	1.049	1.054	1.058	1.063	1.068	1.072	1.077	1.082	1.086	1.091
2	1.095	1.100	1.105	1.109	1.114	1.118	1.122	1.127	1.131	1.136
3	1.140	1.145	1.149	1.153	1.158	1.162	1.166	1.170	1.175	1.179
4	1.183	1.187	1.192	1.196	1.200	1.204	1.208	1.212	1.217	1.221
1.5	1.225	1.229	1.233	1.237	1.241	1.245	1.249	1.253	1.257	1.261
6	1.265	1.269	1.273	1.277	1.281	1.285	1.288	1.292	1.296	1.300
7	1.304	1.308	1.311	1.315	1.319	1.323	1.327	1.330	1.334	1.338
8	1.342	1.345	1.349	1.353	1.356	1.360	1.364	1.367	1.371	1.375
9	1.378	1.382	1.386	1.389	1.393	1.396	1.400	1.404	1.407	1.411
2.0	1.414	1.418	1.421	1.425	1.428	1.432	1.435	1.439	1.442	1.446
1	1.449	1.453	1.456	1.459	1.463	1.466	1.470	1.473	1.476	1.480
2	1.483	1.487	1.490	1.493	1.497	1.500	1.503	1.507	1.510	1.513
3	1.517	1.520	1.523	1.526	1.530	1.533	1.536	1.539	1.543	1.546
4	1.549	1.552	1.556	1.559	1.562	1.565	1.568	1.572	1.575	1.578
2.5	1.581	1.584	1.587	1.591	1.594	1.597	1.600	1.603	1.606	1.609
6	1.612	1.616	1.619	1.622	1.625	1.628	1.631	1.634	1.637	1.640
7	1.643	1.646	1.649	1.652	1.655	1.658	1.661	1.664	1.667	1.670
8	1.673	1.676	1.679	1.682	1.685	1.688	1.691	1.694	1.697	1.700
9	1.703	1.706	1.709	1.712	1.715	1.718	1.720	1.723	1.726	1.729
3.0	1.732	1.735	1.738	1.741	1.744	1.746	1.749	1.752	1.755	1.758
1	1.761	1.764	1.766	1.769	1.772	1.775	1.778	1.780	1.783	1.786
2	1.789	1.792	1.794	1.797	1.800	1.803	1.806	1.808	1.811	1.814
3	1.817	1.819	1.822	1.825	1.828	1.830	1.833	1.836	1.838	1.841
4	1.844	1.847	1.849	1.852	1.855	1.857	1.860	1.863	1.865	1.868
3.5	1.871	1.873	1.876	1.879	1.881	1.884	1.887	1.889	1.892	1.895
6	1.897	1.900	1.903	1.905	1.908	1.910	1.913	1.916	1.918	1.921
7	1.924	1.926	1.929	1.931	1.934	1.936	1.939	1.942	1.944	1.947
8	1.949	1.952	1.954	1.957	1.960	1.962	1.965	1.967	1.970	1.972
9	1.975	1.977	1.980	1.982	1.985	1.987	1.990	1.992	1.995	1.997
4.0	2.000	2.002	2.005	2.007	2.010	2.012	2.015	2.017	2.020	2.022
1	2.025	2.027	2.030	2.032	2.035	2.037	2.040	2.042	2.045	2.047
2	2.049	2.052	2.054	2.057	2.059	2.062	2.064	2.066	2.069	2.071
3	2.074	2.076	2.078	2.081	2.083	2.086	2.088	2.090	2.093	2.095
4	2.098	2.100	2.102	2.105	2.107	2.110	2.112	2.114	2.117	2.119

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*From Mechanical Engineers' Handbook, by Lionel S. Marks (1941). Courtesy of the McGraw-Hill Book Co.

SQUARE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9
4.5	2.121	2.124	2.126	2.128	2.131	2.133	2.135	2.138	2.140	2.142
6	2.145	2.147	2.149	2.152	2.154	2.156	2.159	2.161	2.163	2.166
7	2.168	2.170	2.173	2.175	2.177	2.179	2.182	2.184	2.186	2.189
8	2.191	2.193	2.195	2.198	2.200	2.202	2.205	2.207	2.209	2.211
9	2.214	2.216	2.218	2.220	2.223	2.225	2.227	2.229	2.232	2.234
5.0	2.236	2.238	2.241	2.243	2.245	2.247	2.249	2.252	2.254	2.256
1	2.258	2.261	2.263	2.265	2.267	2.269	2.272	2.274	2.276	2.278
2	2.280	2.283	2.285	2.287	2.289	2.291	2.293	2.296	2.298	2.300
3	2.302	2.304	2.307	2.309	2.311	2.313	2.315	2.317	2.319	2.322
4	2.324	2.326	2.328	2.330	2.332	2.335	2.337	2.339	2.341	2.343
5.5	2.345	2.347	2.349	2.352	2.354	2.356	2.358	2.360	2.362	2.364
6	2.366	2.369	2.371	2.373	2.375	2.377	2.379	2.381	2.383	2.385
7	2.387	2.390	2.392	2.394	2.396	2.398	2.400	2.402	2.404	2.406
8	2.408	2.410	2.412	2.415	2.417	2.419	2.421	2.423	2.425	2.427
9	2.429	2.431	2.433	2.435	2.437	2.439	2.441	2.443	2.445	2.447
6.0	2.449	2.452	2.454	2.456	2.458	2.460	2.462	2.464	2.466	2.468
1	2.470	2.472	2.474	2.476	2.478	2.480	2.482	2.484	2.486	2.488
2	2.490	2.492	2.494	2.496	2.498	2.500	2.502	2.504	2.506	2.508
3	2.510	2.512	2.514	2.516	2.518	2.520	2.522	2.524	2.526	2.528
4	2.530	2.532	2.534	2.536	2.538	2.540	2.542	2.544	2.546	2.548
6.5	2.550	2.551	2.553	2.555	2.557	2.559	2.561	2.563	2.565	2.567
6	2.569	2.571	2.573	2.575	2.577	2.579	2.581	2.583	2.585	2.587
7	2.588	2.590	2.592	2.594	2.596	2.598	2.600	2.602	2.604	2.606
8	2.608	2.610	2.612	2.613	2.615	2.617	2.619	2.621	2.623	2.625
9	2.627	2.629	2.631	2.632	2.634	2.636	2.638	2.640	2.642	2.644
7.0	2.646	2.648	2.650	2.651	2.653	2.655	2.657	2.659	2.661	2.663
1	2.665	2.666	2.668	2.670	2.672	2.674	2.676	2.678	2.680	2.681
2	2.683	2.685	2.687	2.689	2.691	2.693	2.694	2.696	2.698	2.700
3	2.702	2.704	2.706	2.707	2.709	2.711	2.713	2.715	2.717	2.718
4	2.720	2.722	2.724	2.726	2.728	2.729	2.731	2.733	2.735	2.737
7.5	2.739	2.740	2.742	2.744	2.746	2.748	2.750	2.751	2.753	2.755
6	2.757	2.759	2.760	2.762	2.764	2.766	2.768	2.769	2.771	2.773
7	2.775	2.777	2.778	2.780	2.782	2.784	2.786	2.787	2.789	2.791
8	2.793	2.795	2.796	2.798	2.800	2.802	2.804	2.805	2.807	2.809
9	2.811	2.812	2.814	2.816	2.818	2.820	2.821	2.823	2.825	2.827
8.0	2.828	2.830	2.832	2.834	2.835	2.837	2.839	2.841	2.843	2.844
1	2.846	2.848	2.850	2.851	2.853	2.855	2.857	2.858	2.860	2.862
2	2.864	2.865	2.867	2.869	2.871	2.872	2.874	2.876	2.877	2.879
3	2.881	2.883	2.884	2.886	2.888	2.890	2.891	2.893	2.895	2.897
4	2.898	2.900	2.902	2.903	2.905	2.907	2.909	2.910	2.912	2.914

SQUARE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9
8.5	2.915	2.917	2.919	2.921	2.922	2.924	2.926	2.927	2.929	2.931
6	2.933	2.934	2.936	2.938	2.939	2.941	2.943	2.944	2.946	2.948
7	2.950	2.951	2.953	2.955	2.956	2.958	2.960	2.961	2.963	2.965
8	2.966	2.968	2.970	2.972	2.973	2.975	2.977	2.978	2.980	2.982
9	2.983	2.985	2.987	2.988	2.990	2.992	2.993	2.995	2.997	2.998
9.0	3.000	3.002	3.003	3.005	3.007	3.008	3.010	3.012	3.013	3.015
1	3.017	3.018	3.020	3.022	3.023	3.025	3.027	3.028	3.030	3.032
2	3.033	3.035	3.036	3.038	3.040	3.041	3.043	3.045	3.046	3.048
3	3.050	3.051	3.053	3.055	3.056	3.058	3.059	3.061	3.063	3.064
4	3.066	3.068	3.069	3.071	3.072	3.074	3.076	3.077	3.079	3.081
9.5	3.082	3.084	3.085	3.087	3.089	3.090	3.092	3.094	3.095	3.097
6	3.098	3.100	3.102	3.103	3.105	3.106	3.108	3.110	3.111	3.113
7	3.114	3.116	3.118	3.119	3.121	3.122	3.124	3.126	3.127	3.129
8	3.130	3.132	3.134	3.135	3.137	3.138	3.140	3.142	3.143	3.145
9	3.146	3.148	3.150	3.151	3.153	3.154	3.156	3.158	3.159	3.161
10	3.162	3.178	3.194	3.209	3.225	3.240	3.256	3.271	3.286	3.302
1	3.317	3.332	3.347	3.362	3.376	3.391	3.406	3.421	3.435	3.450
2	3.464	3.479	3.493	3.507	3.521	3.536	3.550	3.564	3.578	3.592
3	3.606	3.619	3.633	3.647	3.661	3.674	3.688	3.701	3.715	3.728
4	3.742	3.755	3.768	3.782	3.795	3.808	3.821	3.834	3.847	3.860
15	3.873	3.886	3.899	3.912	3.924	3.937	3.950	3.962	3.975	3.987
6	4.000	4.012	4.025	4.037	4.050	4.062	4.074	4.087	4.099	4.111
7	4.123	4.135	4.147	4.159	4.171	4.183	4.195	4.207	4.219	4.231
8	4.243	4.254	4.266	4.278	4.290	4.301	4.313	4.324	4.336	4.347
9	4.359	4.370	4.382	4.393	4.405	4.416	4.427	4.438	4.450	4.461
20	4.472	4.483	4.494	4.506	4.517	4.528	4.539	4.550	4.561	4.572
1	4.583	4.593	4.604	4.615	4.626	4.637	4.648	4.658	4.669	4.680
2	4.690	4.701	4.712	4.722	4.733	4.743	4.754	4.764	4.775	4.785
3	4.796	4.806	4.817	4.827	4.837	4.848	4.858	4.868	4.879	4.889
4	4.899	4.909	4.919	4.930	4.940	4.950	4.960	4.970	4.980	4.990
25	5.000	5.010	5.020	5.030	5.040	5.050	5.060	5.070	5.079	5.089
6	5.099	5.109	5.119	5.128	5.138	5.148	5.158	5.167	5.177	5.187
7	5.196	5.206	5.215	5.225	5.235	5.244	5.254	5.263	5.273	5.282
8	5.292	5.301	5.310	5.320	5.329	5.339	5.348	5.357	5.367	5.376
9	5.385	5.394	5.404	5.413	5.422	5.431	5.441	5.450	5.459	5.468
30	5.477	5.486	5.495	5.505	5.514	5.523	5.532	5.541	5.550	5.559
1	5.568	5.577	5.586	5.595	5.604	5.612	5.621	5.630	5.639	5.648
2	5.657	5.666	5.675	5.683	5.692	5.701	5.710	5.718	5.727	5.736
3	5.745	5.753	5.762	5.771	5.779	5.788	5.797	5.805	5.814	5.822
4	5.831	5.840	5.848	5.857	5.865	5.874	5.882	5.891	5.899	5.908

SQUARE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9
35	5.916	5.925	5.933	5.941	5.950	5.958	5.967	5.975	5.983	5.992
6	6.000	6.008	6.017	6.025	6.033	6.042	6.050	6.058	0.066	6.075
7	6.083	6.091	6.099	6.107	6.116	6.124	6.132	6.140	6.148	6.156
8	6.164	6.173	6.181	6.189	6.197	6.205	6.213	6.221	6.229	6.237
9	6.245	6.253	6.261	6.269	6.277	6.285	6.293	6.301	6.309	6.317
40	6.325	6.332	6.340	6.348	6.356	6.364	6.372	6.380	6.387	6.395
1	6.403	6.411	6.419	6.427	6.434	6.442	6.450	6.458	6.465	6.473
2	6.481	6.488	6.496	6.504	6.512	6.519	6.527	6.535	6.542	6.550
3	6.557	6.565	6.573	6.580	6.588	6.595	6.603	6.611	6.618	6.626
4	6.633	6.641	6.648	6.656	6.663	6.671	6.678	6.686	6.693	6.701
45	6.708	6.716	6.723	6.731	6.738	6.745	6.753	6.760	6.768	6.775
6	6.782	6.790	6.797	6.804	6.812	6.819	6.826	6.834	6.841	6.848
7	6.856	6.863	6.870	6.877	6.885	6.892	6.899	6.907	6.914	6.921
8	6.928	6.935	6.943	6.950	6.957	6.964	6.971	6.979	6.986	6.993
9	7.000	7.007	7.014	7.021	7.029	7.036	7.043	7.050	7.057	7.064
50	7.071	7.078	7.085	7.092	7.099	7.106	7.113	7.120	7.127	7.134
1	7.141	7.148	7.155	7.162	7.169	7.176	7.183	7.190	7.197	7.204
2	7.211	7.218	7.225	7.232	7.239	7.246	7.253	7.259	7.266	7.273
3	7.280	7.287	7.294	7.301	7.308	7.314	7.321	7.328	7.335	7.342
4	7.348	7.355	7.362	7.369	7.376	7.382	7.389	7.396	7.403	7.409
55	7.416	7.423	7.430	7.436	7.443	7.450	7.457	7.463	7.470	7.477
6	7.483	7.490	7.497	7.503	7.510	7.517	7.523	7.530	7.537	7.543
7	7.550	7.556	7.563	7.570	7.576	7.583	7.589	7.596	7.603	7.609
8	7.616	7.622	7.629	7.635	7.642	7.649	7.655	7.662	7.668	7.675
9	7.681	7.688	7.694	7.701	7.707	7.714	7.720	7.727	7.733	7.740
60	7.746	7.752	7.759	7.765	7.772	7.778	7.785	7.791	7.797	7.804
1	7.810	7.817	7.823	7.829	7.836	7.842	7.849	7.855	7.861	7.868
2	7.874	7.880	7.887	7.893	7.899	7.906	7.912	7.918	7.925	7.931
3	7.937	7.944	7.950	7.956	7.962	7.969	7.975	7.981	7.987	7.994
4	8.000	8.006	8.012	8.019	8.025	8.031	8.037	8.044	8.050	8.056
65	8.062	8.068	8.075	8.081	8.087	8.093	8.099	8.106	8.112	8.118
6	8.124	8.130	8.136	8.142	8.149	8.155	8.161	8.167	8.173	8.179
7	8.185	8.191	8.198	8.204	8.210	8.216	8.222	8.228	8.234	8.240
8	8.246	8.252	8.258	8.264	8.270	8.276	8.283	8.289	8.295	8.301
9	8.307	8.313	8.319	8.325	8.331	8.337	8.343	8.349	8.355	8.361
70	8.367	8.373	8.379	8.385	8.390	8.396	8.402	8.408	8.414	8.420
1	8.426	8.432	8.438	8.444	8.450	8.456	8.462	8.468	8.473	8.479
2	8.485	8.491	8.497	8.503	8.509	8.515	8.521	8.526	8.532	8.538
3	8.544	8.550	8.556	8.562	8.567	8.573	8.579	8.585	8.591	8.597
4	8.602	8.608	8.614	8.620	8.626	8.631	8.637	8.643	8.649	8.654

SQUARE ROOTS (*continued*)

N	0	1	2	3	4	5	6	7	8	9
75	8.660	8.666	8.672	8.678	8.683	8.689	8.695	8.701	8.706	8.712
6	8.718	8.724	8.729	8.735	8.741	8.746	8.752	8.758	8.764	8.769
7	8.775	8.781	8.786	8.792	8.798	8.803	8.809	8.815	8.820	8.826
8	8.832	8.837	8.843	8.849	8.854	8.860	8.866	8.871	8.877	8.883
9	8.888	8.894	8.899	8.905	8.911	8.916	8.922	8.927	8.933	8.939
80	8.944	8.950	8.955	8.961	8.967	8.972	8.978	8.983	8.989	8.994
1	9.000	9.006	9.011	9.017	9.022	9.028	9.033	9.039	9.044	9.050
2	9.055	9.061	9.066	9.072	9.077	9.083	9.088	9.094	9.099	9.105
3	9.110	9.116	9.121	9.127	9.132	9.138	9.143	9.149	9.154	9.160
4	9.165	9.171	9.176	9.182	9.187	9.192	9.198	9.203	9.209	9.214
85	9.220	9.225	9.230	9.236	9.241	9.247	9.252	9.257	9.263	9.268
6	9.274	9.279	9.284	9.290	9.295	9.301	9.306	9.311	9.317	9.322
7	9.327	9.333	9.338	9.343	9.349	9.354	9.359	9.365	9.370	9.375
8	9.381	9.386	9.391	9.397	9.402	9.407	9.413	9.418	9.423	9.429
9	9.434	9.439	9.445	9.450	9.455	9.460	9.466	9.471	9.476	9.482
90	9.487	9.492	9.497	9.503	9.508	9.513	9.518	9.524	9.529	9.534
1	9.539	9.545	9.550	9.555	9.560	9.566	9.571	9.576	9.581	9.586
2	9.592	9.597	9.602	9.607	9.612	9.618	9.623	9.628	9.633	9.638
3	9.644	9.649	9.654	9.659	9.664	9.670	9.675	9.680	9.685	9.690
4	9.695	9.701	9.706	9.711	9.716	9.721	9.726	9.731	9.737	9.742
95	9.747	9.752	9.757	9.762	9.767	9.772	9.778	9.783	9.788	9.793
6	9.798	9.803	9.808	9.813	9.818	9.823	9.829	9.834	9.839	9.844
7	9.849	9.854	9.859	9.864	9.869	9.874	9.879	9.884	9.889	9.894
8	9.899	9.905	9.910	9.915	9.920	9.925	9.930	9.935	9.940	9.945
9	9.950	9.955	9.960	9.965	9.970	9.975	9.980	9.985	9.990	9.995

10. VARIATION, DEPENDENCE AND GRAPHS

Another Use for the Formula. By studying a formula carefully it is possible to tell how changes in the value of one of the quantities will affect the values of another quantity. Thus we note that in the formula

$$D=\frac{M}{V},$$

(1) if V remains constant and M is doubled, then D is doubled.
(2) if M remains constant and V is doubled, D becomes $\frac{1}{2}$ as large.
(3) if M and V are both doubled, then D remains the same.
(4) if M is doubled and V is halved, then D becomes 4 times as large.

Or again, consider the formula $A=6e^2$;

(1) if e is doubled, A becomes 4 times as large.
(2) if e is halved, A becomes $\frac{1}{4}$ as large.
(3) if e is multiplied by 3, A becomes 9 times as large.
(4) if e is divided by 4, A becomes $\frac{1}{16}$ as large.
(5) if e is multiplied by 10, A becomes 100 times as large.

Exercise 43.

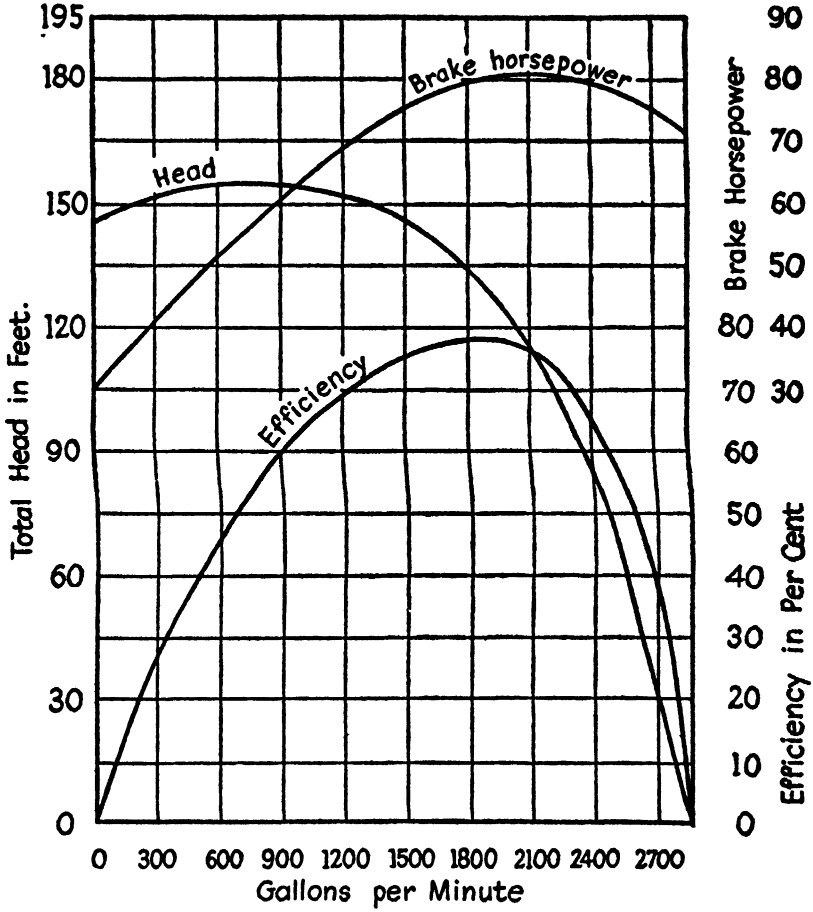
1. In the formula $D=RT$, how is D changed when
   (a) R is doubled and T remains constant.
   (b) T is halved and R remains constant.
   (c) R is divided by 10 and T remains constant.
   (d) T is tripled and R is halved.
   (e) R is divided by 4 and T is divided by 2.

2. In the formula $A=\frac{1}{2}bh$, how is A affected when
   (a) b and h are both tripled.
   (b) h remains constant and b is divided by 6.
   (c) b is doubled and h is halved.
   (d) h is multiplied by 4 and b remains constant.

3. In the formula $PV=k$, if k is always constant, what change takes place when
   (a) P increases.
   (b) V increases.
   (c) V is halved.
   (d) P is tripled.
   (e) V is multiplied by 5.

4. In the formula $A=\pi R^2$, what happens to A when
   (a) R is doubled.
   (b) R is divided by 3.
   (c) R is multiplied by 5.
   (d) R is divided by 10.

What a Formula Does. By this time it will be seen that a formula has a number of advantages over the verbal statement of the relationship between two or more quantities. It is briefer and simpler; it emphasizes the nature of the relationship; it enables us to compute the value of one of the variables when specific values of the other variables are known; it permits the relationship to be restated so that any particular variable may be expressed or "described" in terms of the other variables, which is generally a convenient device. In short, a formula is a powerful mathematical tool, since it implicitly represents *all the infinite sets of corresponding values of the variables,* all at once, so to speak.



Characteristic curves of a single-stage centrifugal pump operating at a constant speed of 1,200 r.p.m.

A Graph also Shows Dependence. Consider the three graphs shown in the accompanying figure. Study the curve labeled "efficiency"; referring to the horizontal scale of "gallons per minute" and the vertical scale (at the lower right) of "efficiency in per cent," we can read directly from the efficiency at any particular load, or the load at any particular efficiency. Thus when pumping 2700 gal. per minute the efficiency is 40%; when the efficiency is 60%, it may be pumping a little over 900, or about 2500 gal. per minute (the 60% line crosses the curve in *two places*). Also the maximum (greatest) efficiency is reached when pumping a little under 1950 gal. per minute. And in the same way, the other two curves might be studied and analyzed. This could not be done conveniently with a formula.

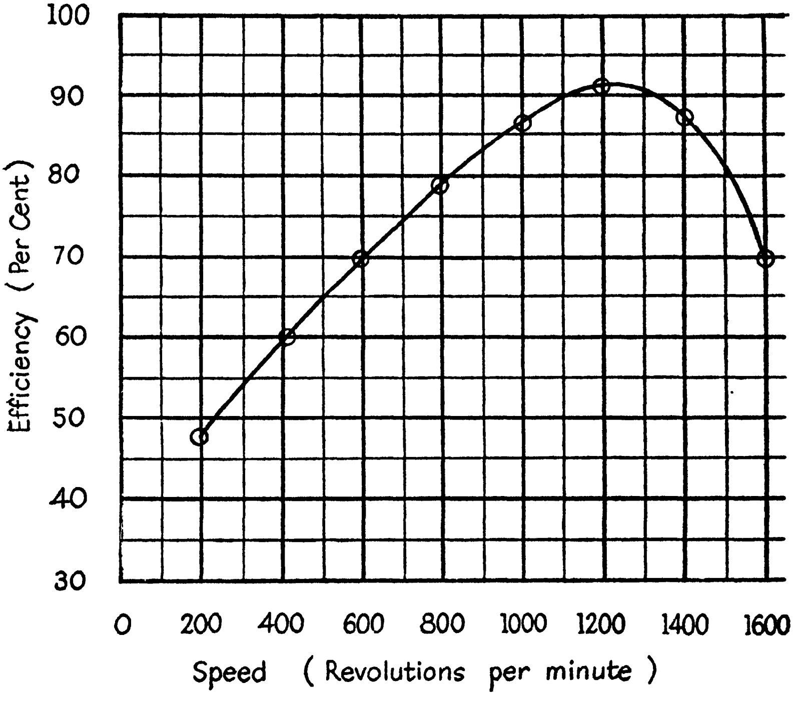
What a Graph Can Do. A graph, as the name itself suggests, can go a step further than the formula—it can make visible what the formula represents—it can give an actual *picture* of the mathematical relationship. The relationship literally becomes more graphic; the relative magnitudes of the variables become apparent to the eye, as do extreme maximum and minimum values, if any; so do the rates at which they change; trends become clear; extrapolation and interpolation become more meaningful; any special features of the relationship are emphasized; general types of relationships are recognizable; two or more relationships can frequently be directly compared with one another.

Mathematical Functions. In mathematics, a definite quantitative relation between two or more variables, whether expressed verbally, by a formula, or by a graph, is called a *functional relationship,* or simply a mathematical function. Each variable is said to be a function of the other. The word function, as used here, has nothing to do with use or purpose; it simply calls attention to the fact that the quantities in question are quantitatively related to each other in a definite manner. Each, in other words, depends upon the other (or others) for its numerical values. Thus if $V=32t^2$, then V is a function of t, and t is a function of V; or, the value of V depends upon the particular value taken for t; and the value of t depends upon the value of V.

It should be noted that in mathematics the word *curve* is used to designate any sort of a graph between two related variables, even though that graph may be a straight line, and not really "curved" at all. It also frequently happens that two variables, representing physical quantities or scientific measurements, may indeed be mutually dependent one upon the other, i.e., functionally related, and yet there is no simple or known formula to represent the relation between them. Such a relationship is called an *empirical function,* and its graph is called an empirical curve. Before discussing formula graphs (i.e., mathematical functions), we shall study empirical graphs a little further in order to understand mathematical graphs better.

Empirical Curves. Consider the accompanying table of values, which were determined by experimental measurements. It represents certain particular pairs of corresponding values of the two variables in question, which in this case are the speed of a certain machine (expressed in revolutions per minute) and the efficiency of the machine (expressed in per cents). Using the horizontal scale or axis for values of the speed (reading from left to right) and using the vertical axis for the efficiencies (reading from the bottom up), we then plot the graph as here shown.

Speed	Efficiency (%)
200	47.5
400	60.0
600	70.0
800	79.0
1000	86.0
1200	90.5
1400	87.5
1600	70.0



Exercise 44.

1. The solubility (S) of a certain chemical, as shown by the number of grams dissolved in a given amount of water, varied with the temperature (T) as shown in Table I. Plot the graph, using the horizontal scale for the values of the S. How many grams dissolved at 55°? At what temperature would 27 gm. be dissolved?

Table I

T	*S*
10	10 gm.
20	12
30	16
40	21
50	30
60	50
65	65

Table II

T	*W*
5	6.8
10	9.4
15	12.8
20	17.2
25	22.9
30	29.2

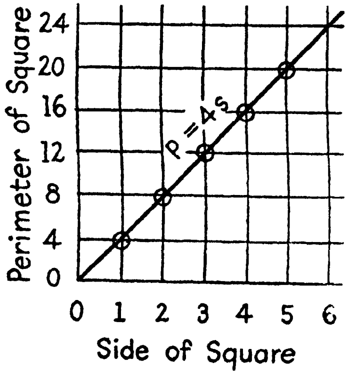
Table III

H	*P*
0	30.0
5,000	24.8
10,000	21.2
15,000	17.0
20,000	14.5
25,000	11.4
30,000	9.5
35,000	7.8

2. The weight (W gm.) of water contained in a cubic meter of saturated air at various temperatures (T) is given in Table II. Plot the graph with the temperature along the horizontal scale. How many grams are contained in a cubic meter of air at 18°? at 28°? At what temperature does one cubic meter of saturated air contain 20 gms.? 25 gms.?
3. The atmospheric pressure (P) expressed in inches of mercury varies at different altitudes above sea level as shown in Table III. Plot the graph, using the vertical scale for the pressure. What is the pressure at the top of Mount Everest, 29,000 ft. high? How high is an airplane when its pressure gage indicates 22.5 inches?

The Linear Function. When two variables are so related that their graph is a straight line, the relation is said to be a *linear function.* Consider, for example, the relation between the side of a square and its perimeter. Whatever the length of its side, its perimeter is always 4 times as great; or, $P=4s$.

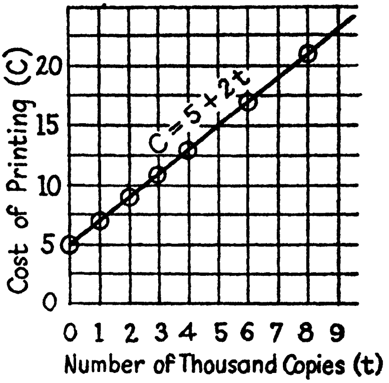
s	*P*
0	0
1	4
2	8
2½	10
3	12
4	16
5	20



When $s=0$, $P=0$; since, if there is no square, there can be no perimeter. Plotting these values on ruled paper, we obtain the accompanying linear graph. From this graph it is possible to "read off" many pairs of values of s and P that do not appear in the original table.

Or again, consider the cost of printing circulars, quoted at $5 for setting the type, and $2 per 1000 copies run off. The cost can be represented by the equation $C=5+2t$, where t=the number of thousand copies printed, and C=the total cost. The following set of values can readily be tabulated from the formula. When the graph of this table is plotted, the curve is again seen to be a straight line. This time, however, it does not pass through the "0,0 point," but at the point where $t=0, C=5$. This, of course, mathematically represents the fact that even if no copies are printed the cost of setting type is $5.

t	C
0	5
1	7
2	9
3	11
4	13
6	17
8	21



Exercise 45.

1. Plot the graph of $I=2.5C$, representing the relationship between inches (I) and centimeters. Use values of $C=0$, 2, 4, 6, 10. Use the horizontal axis for values of C.
2. Draw the graph of $E=IR$, considering a constant value of $R=20$; i.e., plot the graph of $E=20I$. Use values of $I=0$, 1, 2, 3, 4, 5, plotting them along the horizontal axis.
3. Plot the graph of $v=gt$, where v is the velocity of a freely falling body in ft. per second, and t is the time in seconds. Consider g as constant and equal to 32 ft. per sec. Use values of $t=0$, 1, 2, 3, 5, 8, 10; use the horizontal axis for t.
4. Draw the graph of $D=RT$, where R is constant and equal to 60 miles per hr., T is the time in hours, and D is the distance covered. Plot T along the horizontal axis.
5. The circumference (C) of a circle is related to its diameter (D) as given by the formula $C=\pi D$, where $\pi={}^{22}\!/_{7}$. Plot the graph, using values of $D=0$, 7, 14, 21, 28, 35, along the horizontal axis.
6. The amount due at simple interest on $100 at 3% a year for various periods of time is given by the formula $A=100+3n$, where n is the number of years. Plot the graph from $n=0$ to $n=6$; use the horizontal axis for n.

7. Plot the graph of $F={}^9\!/_5C+32$, where C=the temperature expressed in Centrigrade degrees, and F=the temperature in Fahrenheit degrees. Use the horizontal axis for values of C.
8. Plot the graph of $V=v_0+at$, where V is the final velocity in ft. per sec. and t is the time in seconds, and where v_0, the initial velocity, is 600, and a, the acceleration, is 50 ft. per sec. Use values of $t=0, 2, 4, 6, 8, 10$ along the horizontal axis.

Direct Variation. Dependence or variation of this type, where the graph of the relation is a straight line, is called *direct variation.* The significance of the word "direct" is that as one variable increases, the other also increases in direct proportion, just as we saw in the section on ratio and proportion (Chap. I, Sect. 6). Whenever two variables *vary directly,* the relation may be expressed as

$$y=kx, \text{ or } \frac{y}{x}=k;$$

in this latter form it is seen that the *ratio* of the variables has a constant value, whatever their *individual* values may be. Recalling that the volume and temperature of a gas (at constant pressure) were found to be in *direct proportion;* thus

$$\frac{V_1}{T_1}=\frac{V_2}{T_2}, \text{ which can also be written as}$$

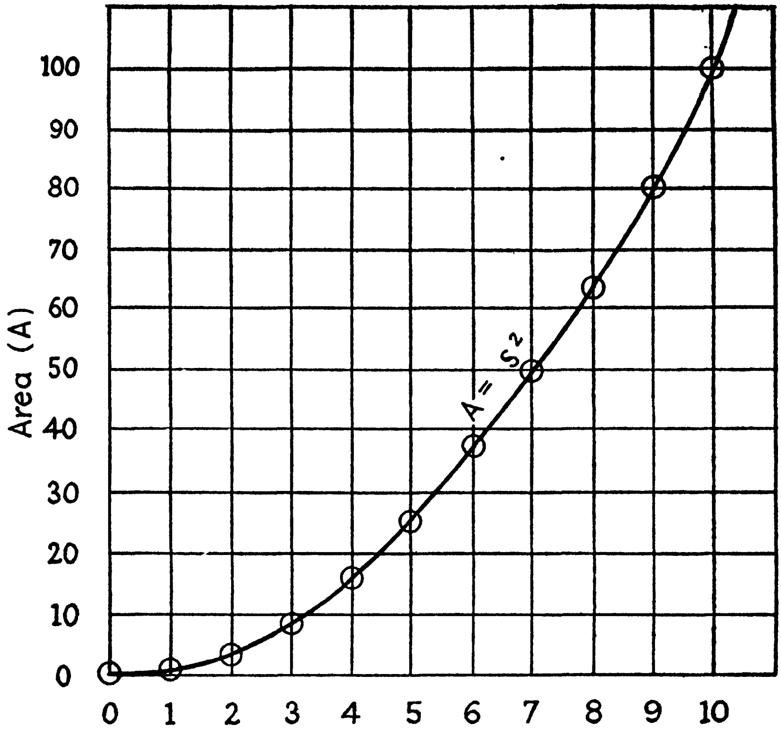
$$\frac{V_1}{T_1}=\frac{V_2}{T_2}=\frac{V_3}{T_3}=\frac{V_4}{T_4}=\ldots\ldots=k;$$

this means that the ratio of *any* volume to its *corresponding* temperature is the same ratio as *any other* volume is to *its* corresponding temperature; or, the *ratio being constant,* the relation is seen as an example of direct variation if it is simply rewritten as

$$\frac{V}{T}=k, \text{ or } V=kT.$$

The Parabolic Function. Some variables are related by a mathematical dependence which involves the square of one of them, as e.g., the relation between the area of a square and the length of its side, or $A=s^2$. Thus, from this formula we get the following table of values; then, from this tables of values we can plot the graph shown below. It is called a *parabolic* curve.

s	0	1	2	3	4	5	8	10
A	0	1	4	9	16	25	64	100

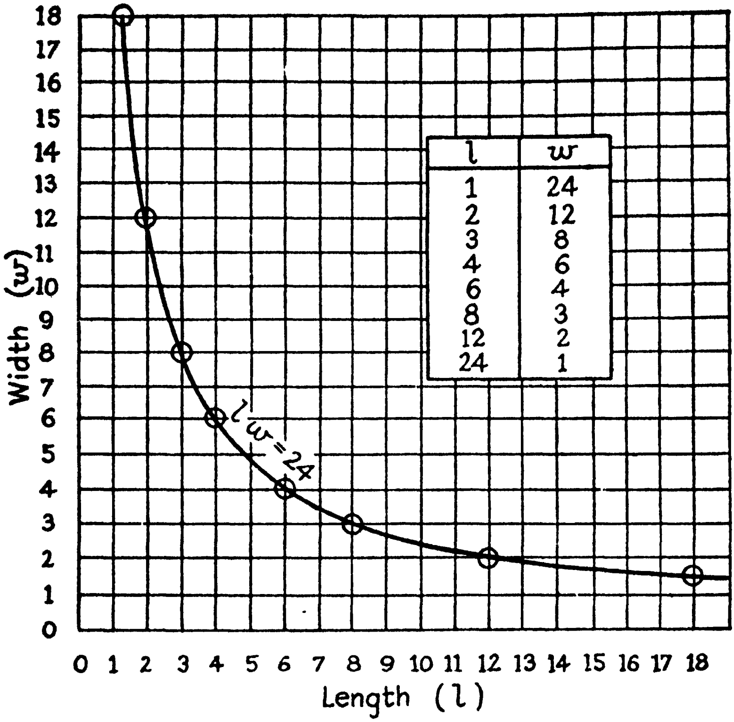


Exercise 46.

Plot the graph of each of the following formulas; in each case use the horizontal axis for the variable that is squared.

1. $S=at^2$, where S is the distance (S) in feet moved by a body in t seconds at a constant acceleration $a=10$ ft. per sec. Use values of $t=0, 1, 2, 3, 4$.
2. $A=\pi R^2$, where A is the area of a circle, R=radius, and $\pi=22/7$. Use values of $R=0, 3\frac{1}{2}, 7, 10\frac{1}{2}, 14$.
3. $H=I^2R$, where H is the heat produced by an electric current (I) that varies; consider the R constant and equal to 10. Use values of $I=0, 1, 2, 3, 4, 5$.
4. The kinetic energy (E) of a moving automobile weighing 2,000 lb. is $E=\frac{1}{2}mv^2$, where $m=$ the weight of the car and v its velocity in ft. per sec. Plot the graph for values of $v=0, 20, 40, 60, 80$.
5. The wind pressure (P) on a moving plane at 90° inclination, expressed in lb. per sq. ft., is given by the formula $P=.003\ v^2$, where v is the wind velocity expressed in miles per hour. Plot the graph for values of $v=0, 10, 20, 30, 40, 50$.

The Hyperbolic Function. Another type of function frequently encountered is the *hyperbolic* relation, expressing inverse variation, such as the relations $xy=k$, $A=lw$, or $R=\frac{D}{T}$. In each case, if one of the variables is held constant, the graph takes the characteristic form shown above. Consider first, the formula $A=lw$, where l and w are the length and width, respectively, of a rectangle having the area A. If we assume that the area A is constant, equal say to 24 sq. in., then the graph of the formula lw 24 is found as follows, since $l=\frac{24}{w}$, or $w=\frac{24}{l}$. This is the type of problem encountered by photoengravers in determining the possible dimensions for a rectangle of a given, constant area.



l	w
1	24
2	12
3	8
4	6
6	4
8	3
12	2
24	1

Inverse Variation. In this type of relation, neither variable can ever equal zero, although it can become as small as you please. The smaller either variable becomes, the larger is the value of the other, since their product is a constant; that is what is meant by *inverse variation.* It is exactly the same situation as was seen in the case of inverse proportion (Chap. I, Sect. 6), where the pressure and volume of a gas at constant temperature were in *inverse* proportion, $\frac{P_1}{P_2}=\frac{V_2}{V_1}$ This can also be written as follows:

$$P_1V_1=P_2V_2=P_3V_3=P_4V_4=\ldots=k;$$

i.e., *any* P multiplied by its *corresponding* V gives the *same constant product* as *any other* P multiplied by *its* corresponding V. Hence the formula is $PV=k$, which is, as we now know, a hyperbolic curve.

Exercise 47.

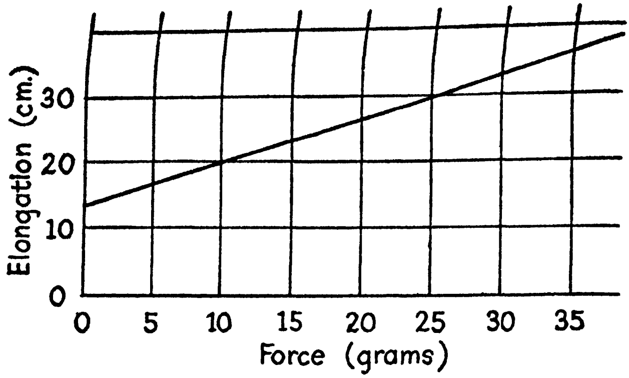
Plot the graph of each of the following formulas:

1. $xy=20$
2. $y=\frac{144}{x}$
3. $IR=120$
4. $PV=1000$
5. $T=\frac{100}{R}$
6. $500=Fs$

Related Variables. The dependence of varying quantities upon one another has already been discussed in connection with the formula. The value of either of two variables may properly be said to "depend" upon the other for its value. Quantities depending upon each other in this way are said to bear a functional relationship to each other; each is said to be a function of the other. Thus the volume occupied by a gas at constant temperature depends upon the pressure exerted; conversely, the pressure exerted depends upon the volume occupied by the gas. Hence V is a function of P, or P is a function of V. Similarly, the amount of expansion due to heat depends upon the temperature; the population of a community, upon the time; the fuel consumed in driving a locomotive, upon the speed; the quantity of a commodity sold, upon the price. In each instance the latter variable is conveniently regarded as the independent, and the former as the dependent, variable.

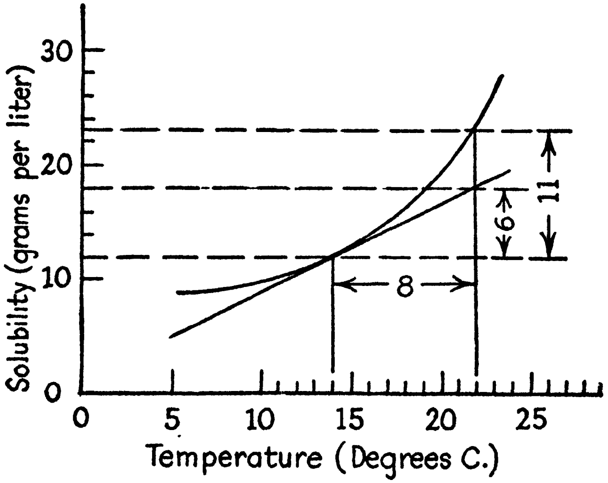
According to accepted convention, the independent variable is denoted by x and its values are always measured along the horizontal scale, while the dependent variable is denoted by y, and its values are measured along the vertical scale. Hence, the independent variable may be regarded as freely assuming all values along the range of the horizontal axis, while the dependent variable necessarily varies in some definite way to correspond, as shown by the *changing height* of the graph.

The Rate of Change Concept. It is clear that for any given point on a graph, its horizontal distance from the vertical scale (abscissa) represents the magnitude of the independent variable, while the vertical distance above or below the horizontal scale (ordinate) represents the corresponding magnitude of the dependent variable. Thus the position of the curve with respect to the axes depicts the actual magnitudes of the variables. But in studying changing variables and functional relationships, it is frequently desirable to inquire as to the *rate* at which a quantity



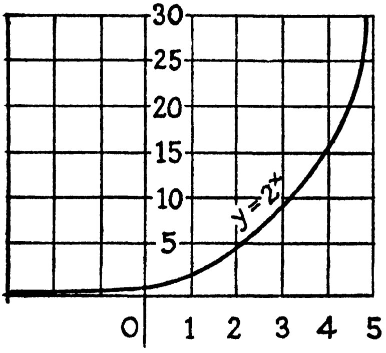
is changing, i.e., how fast it is increasing or decreasing, rather than how large or how small it is. Rate implies a ratio; a rate of change means the amount of change in the function (or dependent variable) per unit change in the independent variable. On the graph this means the amount of vertical rise or fall in the curve per horizontal unit, so that when discussing rates of change we are concerned with the *steepness* of the graph rather than with its actual height at any particular point. If the graph of a given function is a straight line, the function is obviously increasing at a constant rate, since the increase (or decrease) in the dependent variable is the same for every horizontal unit. Most quantities, however, change at varying instead of at constant rates, so that it becomes necessary to distinguish between an *average* rate and an *instantaneous* rate.

Average Rate of Change. An average rate during a definite interval does not necessarily imply that the rate during that interval need be uniform;



indeed, it may fluctuate considerably. An average rate during an interval merely means *the ratio of the net increase (or decrease) of the dependent variable during a given interval to the net change of the independent variable.* To find an average rate graphically, it is only necessary to read the amount of increase (or decrease) at the end of a given convenient interval and divide by the length of that interval. For example, the amount by which the solubility of a certain substance changed during the interval from 14° to 22° is seen to be 11 grams; hence the average rate is $^{11}/_{8}$ or 1.38 grams per degree. It is clear that the average rate of change will depend upon the length of the interval during which the average rate is observed, as well as where the interval is chosen.

The Exponential Function. One of the most interesting and important of all functions is the *exponential function,* $y=k^x$. It will be observed that it differs markedly from the ordinary *power function,* $y=x^k$; in the latter the exponent is merely a constant, while in the former it is the independent variable itself. Let us study carefully the graph of $y=2^x$. At first the curve rises very slowly, but gradually its rate of increase becomes greater and greater; as a matter of fact, the greater the value of x, the more rapid is the rate of growth of y. In this respect it resembles the graph of $A=P(1+i)^n$, which is the formula for the amount at compound interest, and is also of the type $y=k^x$. In both these curves, the *rate of increase of the dependent variable at any instant is proportional to the magnitude of the independent variable at the particular instant.* This is an extremely important characteristic of every exponential function. Otherwise expressed, the percentage of increase is constant throughout. Consequently any function of the type $y=ak^x$ is said to follow the *compound interest* law, or more picturesquely, the "snowball law," since it resembles the growth of a snowball rolling down a hill; it gathers more and more snow the farther down it rolls, and at any given instant is growing at a rate which is approximately proportional to the magnitude which it has already attained at that instant.



Graph of $y = 2^x$

Exponential functions arise in connection with the speed of some chemical reactions, the rate of decomposition of radium, the reduction in speed of revolving wheels, the amount of belt friction on drums, the flow of electric currents, the transmission of light under certain conditions, changes in atmospheric pressure with increase in elevation, and many other physical and technical phenomena.

11. LOGARITHMS

Logarithms Are Exponents. Consider the exponential equation $N=10^x$. How large is N? That depends upon the particular value selected for x; thus

$$\begin{aligned} &\text{if } x=1, \quad N=10 \\ &\text{if } x=2, \quad N=100 \\ &\text{if } x=3, \quad N=1000 \\ &\text{if } x=4, \quad N=10{,}000 \\ &\qquad\qquad \textit{etc.} \end{aligned}$$

What would be the value of N if $x=1.5$? Clearly, since 1.5 is greater than 1, N would be more than 10; and since 1.5 is less than 2, N would be less than 100. In other words, when $x=1.5$, the value of N must be *between* 10 and 100. For any value of x equal to 1 plus some decimal, the value of N must be between 10 and 100. Or again, if we know that the value of N is some number between 100 and 1000, we know at once that x must have a value somewhere between 2 and 3.

In the exponential form of the equation $N=10^x$, we refer to N as the *number;* to 10, as the *base;* and to the exponent (x) as the *logarithm of the number N to the base 10.* This may be written as $x=\log_{10}N$. When expressed in this way, we speak of it as being in the "logarithmic form"; when written as $N=10^x$ we speak of it as the "exponential form." But the two equations are equivalent and interchangeable—two different ways of writing the same relationship.

Any number could be used as a base; thus

$$\begin{aligned} y&=2^x; \text{ or } \log_2 y=x \\ P&=5^y; \text{ or } \log_5 P=y \\ A&=k^t; \text{ or } \log_k A=t \end{aligned}$$

For practical computational purposes, however, the base 10 is universally used, and for this reason the base is commonly omitted. Thus we write

$$\log 10{,}000=4, \text{ instead of } \log_{10}10{,}000=4;$$

$$\text{or} \quad \log P=x, \text{ instead of } \log_{10}P=x.$$

In other words, the logarithm of a given number is the exponent to which 10 must be raised to give that number. Logarithms are thus usually not whole numbers, but mixed numbers, and are invariably expressed in decimal form. Every number has a logarithm; also, every logarithm corresponds to some number.

Characteristic and Mantissa. From what has been said thus far, it is clear that very few logarithms are whole numbers—by far most of them are decimals whose approximate values are to be found in a table.

The part of the logarithm to the *left* of the decimal point, i.e., the integral part, is known as the *characteristic;* the decimal part is called the ***mantissa.*** Thus, the value of $\log 634 = 2.8021$; the characteristic is "2" and the mantissa is ".8021."

Rule 1: If the number whose logarithm is being found is greater than 1, the characteristic is positive, and numerically ***one less than the number of figures to the left of the decimal point*** in the given number.

For example:

$$\begin{aligned} \log 7.92 &= .8987 \\ \log 79.2 &= 1.8987 \\ \log 792 &= 2.8987 \\ \log 79{,}200 &= 4.8987 \end{aligned}$$

Rule 2: If the number whose logarithm is to be found is less than 1, the characteristic is then negative and numerically ***one greater than the number of zeros immediately following the decimal point;*** but the mantissa is still positive. The two parts of the logarithm are not actually added together, but are written as shown below.

For example:

$$\begin{aligned} \log .792 &= -1.0 + .8987 = 9.8987 - 10 \\ \log .0792 &= -2.0 + .8987 = 8.8987 - 10 \\ \log .00792 &= -3.0 + .8987 = 7.8987 - 10 \end{aligned}$$

Finding a Logarithm from the Table. From these rules and illustrations it will be seen that the characteristic of a logarithm is determined solely by the position of the decimal point in the given number, and not by the particular sequence of digits; the mantissa, however, is quite independent of the position of the decimal point, depending instead upon the actual sequence of digits in the given number. Thus, for example, the mantissas of the respective logarithms of 264,500, 26.45, and .0002645 are identical; their logarithms differ only in the characteristics, and are, respectively, 5.4224, 1.4224, and $6.4224-10$. In the "table of logarithms" only the mantissas are shown, the decimal point in front of each figure in the table being understood; the characteristic is supplied mentally, by inspection.

EXAMPLE 1: Find from the table the value of log 467.

SOLUTION: Look under the column headed "N," running down to "46"; then run across horizontally until reaching the column headed "7." The figure found is "6693." Hence $\log 467 = 2.6693$, since the 467 has 3 digits, and the required characteristic is 2.

EXAMPLE 2: Find log .0803.

SOLUTION: Look under "N" for 80; opposite 80, under "3," find "9047." Hence $\log .0803 = 8.9047 - 10$.

TABLE OF LOGARITHMS

N	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2801	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
N	0	1	2	3	4	5	6	7	8	9

TABLE OF LOGARITHMS (*continued*)

N	0	1	2	3	4	5	6	7	8	9
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
N	0	1	2	3	4	5	6	7	8	9

TABLE OF LOGARITHMS (*continued*)

N	0	1	2	3	4	5	6	7	8	9
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996
N	0	1	2	3	4	5	6	7	8	9

Interpolation. If the number whose logarithm is sought should contain more than three significant figures, the logarithm is found by a method known as *interpolation*. This is based upon the idea of direct proportion; i.e., it is assumed that the logarithm of a number "halfway between" two numbers is halfway between their logarithms; etc. Although such direct variation between numbers and their logarithms is not strictly true, the approximation is accurate enough for ordinary purposes of calculation.

EXAMPLE 1: Find log 2834.

SOLUTION: $\log 2830 = 3.4518$
$\log 2840 = 3.4533$
Interval between numbers = "10"
Difference between logarithms = .0015
Given number $N = 2834$ is .4 of the interval 10;
$.4 \times .0015 = .0006$.
Hence, $\log 2834 = 3.4518 + .0006 = 3.4524$, *Ans.*

EXAMPLE 2: Find log .06178.

SOLUTION: $\log .06170 = 8.7903 - 10$
$\log .06180 = 8.7910 - 10$
Difference between logs = .0007
Given $N = .06178$ is .8 of "interval" between .06170 and .06180; thus $.8 \times .0007 = .00056$, or .0006, is added to the mantissa .7903, giving .7909.
Hence $\log .06178 = 8.7909 - 10$, *Ans.*

EXAMPLE 3: Find log 12.476

SOLUTION: $\log 12.4 = 1.0934$
$\log 12.5 = 1.0969$
Difference between logs = .0035
$.76 \times .0035 = .0027$.
Hence $\log 12.476 = 1.0961$, *Ans.*

Exercise 48.

Find the logarithms of the following numbers:

1. 932
2. 408.5
3. .504
4. 71.68
5. .000526
6. 643,000
7. 8.037
8. .00341
9. .6932
10. .00045
11. 4,806
12. 236.96

Finding the Number When Its Logarithm Is Given. If the logarithm of a number is given and we wish to determine the number itself, this reverse process is called finding the *antilogarithm*. The principle of interpolation is used, but in the reverse way; the procedure follows:

(1) In the table, find the two mantissas between which the given mantissa lies, and write the three figures corresponding to the smaller of these two mantissas.

(2) Next find the difference between these two consecutive mantissas, and also the difference between the smaller of them and the given mantissa whose antilog we are seeking.

(3) Divide the latter difference by the former, carrying the division to the nearest tenth.

(4) Annex the figure so found to the three figures already found, making it the fourth figure of the antilogarithm.

(5) Locate the decimal point in the antilogarithm by inspection of the given logarithm.

It sounds far more difficult than it really is.

Example 1: Find the antilog of 2.3942.

Solution: The given mantissa .3942 lies between 3927 and 3945; thus the first three figures of the antilog are "247."

$$\begin{array}{r} .3945\\ .3927\\ \hline .0018 \end{array}\qquad \begin{array}{r} .3942\\ .3927\\ \hline .0015 \end{array}\qquad \frac{.0015}{.0018}=.8$$

hence antilog$=247.8$, *Ans.*

Example 2: Find the antilog of 7.5029—10.

Solution: Antilog lies between 318 and 319.

$$\begin{array}{r} .5038\\ .5024\\ \hline .0014 \end{array}\qquad \begin{array}{r} .5029\\ .5024\\ \hline .0005 \end{array}\qquad \frac{.0005}{.0014}=.4$$

hence antilog$=.003184$, *Ans.*

Exercise 49.

Find the number whose logarithm is:

1. 1.3806
2. .6432
3. 2.4237
4. 9.6145—10
5. 1.3989
6. 9.3001—10
7. .7625
8. 8.4182—10
9. 2.4416
10. 4.7215
11. 7.9103—10
12. 3.0462

Laws of Exponents Applied to Logarithms. By applying the laws of exponents to logarithmic expressions, we find that if M and N are any two numbers, then:

Law I. $\log (MN)=\log M+\log N$.

Suppose we let $\log M=x$,

and $\log N=y$.

then $10^x=M$, and $10^y=N$.

But $(MN)=(10^x)\cdot(10^y)=10^{(x+y)}$;

hence $\log (MN)=x+y$,

or $\log (MN)=\log M+\log N$.

In other words, *the logarithm of a product equals the sum of the logarithms of its factors.*

For example: $\log 21 = \log 7 + \log 3$

$\log 5000 = \log 1000 + \log 5$

$\log (372 \times 28.1) = \log 372 + \log 28.1$

$\log Prt = \log P + \log r + \log t$

Note: This property of logarithms, viz.

$$\log (PQ) = \log P + \log Q,$$

should not be confused with the following:

$$\log P + \log Q \neq \log (P+Q).$$

In short, the *sum of two logarithms does not* equal the logarithm of their sum; neither does the logarithm of the *sum of two numbers* equal the sum of their logarithms.

Law II. $\log\left(\frac{M}{N}\right) = \log M - \log N$

That is, ***the logarithm of a quotient equals the logarithm of the numerator diminished by the logarithm of the denominator.***

Note: This should not be misinterpreted; $\log (M-N) \neq \log M - \log N$, and $\frac{\log M}{\log N} \neq \log M - \log N$. In short, the logarithm of the difference of two numbers is the logarithm of *their* difference, not the difference of their logarithms. Again, the ratio of two logarithms is simply a number; it is not the same as the logarithm of a ratio, and is therefore not equal to the difference of two logarithms.

Law III. $\log (N)^p = p(\log N)$

Law IV. $\log (N)^{\frac{1}{p}} = \log \sqrt[p]{N} = \frac{1}{p}\,(\log N)$

Thus, for example, we have:

$\log r^2 = 2 \log r$

$\log v^3 = 3 \log v$

$\log \sqrt{d} = \frac{1}{2} \log d$

$\log \sqrt[3]{k} = \frac{1}{3} \log k$

$\log p^{3/2} = \frac{3 \log p}{2}$

$\log \sqrt{\pi r^2} = \frac{1}{2} \log \pi + \log r$

Multiplying and Dividing by Using Logarithms. The chief practical value of logarithms is in shortening the multiplication and division of numbers. When skill in their use has been achieved, the time and labor required by longhand multiplication and division in extensive computations is very materially lessened, as the following examples will show.

Example 1: Multiply $396.2 \times 8.735 \times .07434$.

SOLUTION: log 396.2 = 2.5979
log 8.735 = .9413
log .07432 = 8.8712—10

$$\overline{12.4104-10}$$

antilog 2.4104 = 257.3, *Ans.*

EXAMPLE 2: Find the value of

$$\frac{.376\times 4.018\times 57.64}{.0675\times 3.1416}.$$

SOLUTION:

log .376 = 9.5752—10
log 4.018 = .6040
log 57.64 = 1.7607

$$\overline{11.9399-10}$$
$$9.3264-10$$
$$\overline{2.6135}$$

log .0675 = 8.8293—10
log 3.1416 = .4971

$$\overline{9.3264-10}$$

antilog 2.6135 = 410.8, *Ans.*

Exercise 50.

Find, by using logarithms, the value of:

1. $\dfrac{622.5\times .0317}{14.8}$
2. $\dfrac{.00905}{2.54\times .3614}$
3. $\dfrac{46.22\times 309.4}{162\times .084}$
4. $2\times \frac{22}{7}\times 18.6\times 35.8$
5. $\frac{1}{365}\times .0525\times 1650$
6. $\dfrac{26.8\times .00048\times 127.5}{709.2\times .486}$

Using Logarithms to Find Roots and Powers. Another extremely practical use of logarithms is in finding roots and powers of numbers.

EXAMPLE 1: Find $\sqrt[5]{62.4}$.

SOLUTION: Let $x=(62.4)^{\frac{1}{5}}$
$\log x = \frac{1}{5}(\log 62.4)$
$\log x = (\frac{1}{5})(1.7952) = .3590$
$x = 2.286$, *Ans.*

EXAMPLE 2: Compute the value of $(1.055)^{12}$.

SOLUTION: Let $x=(1.055)^{12}$
$\log x = 12\ (\log 1.055)$
$\log x = (12)\ (.0232) = .2784$
$x = 1.898$, *Ans.*

EXAMPLE 3: What is the value of $x=\dfrac{.0046\times\sqrt[3]{85.7}}{(.063)^2}$.

SOLUTION: $\log .0046 = 7.6628-10$

$\frac{1}{3}\log 85.7 = \underline{.6443}$

$8.3071-10$

$2\log .063 = \underline{17.5986-20}$

$\log x = .7085$

$x = 5.111$, *Ans.*

EXAMPLE 4: In the formula $t=\sqrt{\frac{2s}{g}}$, find t when $s=1155$ and $g=32.2$.

SOLUTION: $t=\left(\frac{2s}{g}\right)^{\frac{1}{2}}$

$$\log t = \tfrac{1}{2}\,[\log 2+\log s-\log g]$$
$$= \tfrac{1}{2}\,[\log 2+\log 1155-\log 32.2]$$

$\log 2 = .3010$

$\log 1155 = \underline{3.0626}$

3.3636

$\log 32.2 = \underline{1.5079}$

$2\overline{)1.8557}$

$\log t = .9279$

$t = 8.47$, *Ans.*

Exercise 51.

Using logarithms, compute each of the following:

1. $\sqrt[3]{756}$
2. $\sqrt[5]{895}$
3. $(.562)^5$
4. $\sqrt[3]{9.47}$
5. $(6.29)^{\frac{2}{3}}$
6. $\sqrt[4]{24.38}$
7. $(1.045)^8$
8. $(40.21)^3$
9. $\sqrt{.000394}$
10. $(8.9)^3\times(27.84)$
11. $\sqrt[3]{162.4}\times(85.36)$
12. $(.364)^2\times 29.25\times\sqrt[5]{162.4}$
13. $\sqrt[3]{\dfrac{215.2}{3.14\times 4.57}}$
14. The formula for the volume of a cylinder is $V=\pi R^2H$; find R when $V=906.0$, $H=14.6$ and $\pi=3.142$.
15. The volume of a sphere is given by $V=\frac{4}{3}\pi R^3$; find to the nearest hundredth the diameter of a sphere whose volume is 85.4 cu. in., using $\pi=3.142$.

Solving Exponential Equations by Use of Logarithms. Still another useful application of logarithms is in connection with the solution of *exponential equations,* i.e., an equation in which the variable (or unknown quantity) is an exponent. Such equations frequently arise in engineering and technical problems.

Example 1: Solve for x: $18=(4.5)^x$.

Solution: $\log 18=(x)\ (\log 4.5)$

$$x=\frac{\log 18}{\log 4.5}=\frac{1.2553}{.6532}=1.92, \textit{ Ans.}$$

Example 2: Solve: $5^k=10^{k+1}$.

Solution: $k(\log 5)=(k+1)\ (\log 10)$

$$\log 5= .6990$$
$$\log 10=1.0000$$
$$.699k=k+1$$
$$k=-\frac{1}{.301}=-3.33, \textit{ Ans.}$$

Exercise 52.

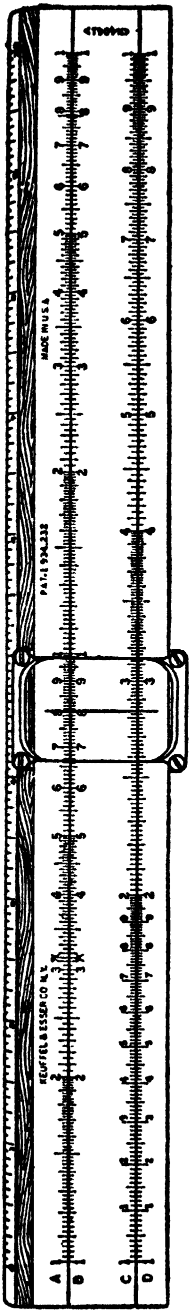
Solve the following exponential equations:

1. $5^x=18$
2. $1.5=(1.03)^x$
3. $12.8=75^{x^2}$
4. $(3.02)^x=100$
5. $(3.4)^x=22.84$
6. $(2^x)(10^x)=30$
7. $\sqrt[x]{12}=4$
8. $(1.04)^{t+1}=6.8$
9. A principal of \$400 at compound interest for a certain length of time at 3½% a year, compounded annually, amounted to \$750. Find the time.
10. A machine originally costing \$1800 depreciates in value each year 15% of its value at the beginning of the year. What is its value at the end of 6 years? [*Hint:* $V=1800\ (.85)^6$]. In how many years will it be worth \$900?

12. THE SLIDE RULE

Mechanical Computation. Historically speaking, man has made use of various sorts of mechanical aids to computation throughout the ages. The slide rule is one such mechanical device. It was invented some 300 years ago by an English mathematician, William Oughtred; subsequently improved, in its modern form it was invented about 1850 by Amédée Mannheim, and is now known by his name. In addition to the simple Mannheim rule, there are available today some ten or twelve other types, more elaborate and more complicated, and designed for a variety of mathematical computations as well as for special purposes. In the present discussion we shall confine our attention to the simple Mannheim rule.

Description of the Rule. Generally made of wood, xylonite or celluloid, the slide rule is from 5 to 10 inches in length. It consists of three parts: the rule, which is grooved; the slide, which is carefully fitted so that it slides easily in the grooved rule from left to right and vice versa; and the hairline runner. The slide and the rule are both faced with accurately graduated scales.



In all, there are four rows of figures on the front of the slide rule:

Row A: two complete logarithmic scales, on the rule, giving directly squares and square roots.
Row B: two complete logarithmic scales, exactly like A, but on the slide.
Row C: a single logarithmic scale, also on the slide.
Row D: another single logarithmic scale, exactly like C, but on the rule.

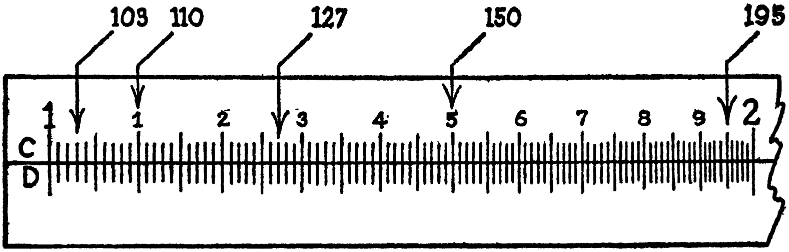
On the back of the slide there are to be found three more scales:

Row S: a trigonometric scale of sines, used in conjunction with the A and B scales.
Row T: a trigonometric scale of tangents, used in conjunction with the C and D scales.
Row L: a scale of equal parts, used to find the common logarithms of numbers.

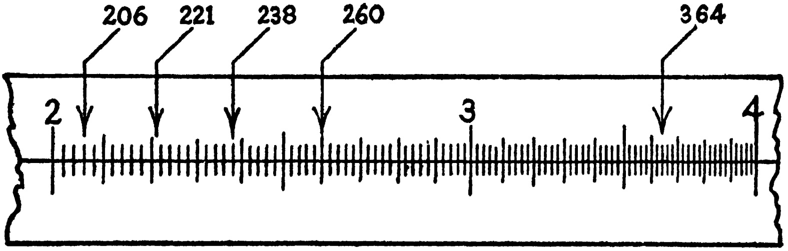
The C and D Scales. Confining ourselves to the front of the slide rule, and noting the two scales marked C and D, respectively, we find that the graduations in each case begin with 1 at the extreme left and are numbered 1, 2, 3, 4, etc., from the left to 8, 9, 1 at the right. The reason that the figure at the extreme right is 1 and not 10 is that *all* the figures on the scales, as marked, are *arbitrary*. Thus the initial 1 at the left index may represent 10, 100, 0.1, 0.01, etc., depending upon the particular computation, but once the value of the left index has been chosen, the same ratio must be observed throughout; e.g., if we start reading it as 10, then the other figures are to be read 20, 30, 40, etc.; if we commence with .01, then the other figures are read as .02, .03, .04, etc. It will also be observed that the *actual* space or distance from 1 to 2 is the same as that from 2 to 4, and also as that from 4 to 8; that is what makes it a *logarithmic* scale.

The segment between 1 and 2 is subdivided into 10 parts, each succeeding part decreasing slightly in size; each of these 10 subdivisions is again divided into 10 smaller divisions. If space permitted, these *subdivisions would be carried out all along, but, because of their decreasing* size, they are later subdivided only into halves or fifths, which must be interpreted as decimal divisions and not as fractional ones. Nor are they all marked with numerals, since this would tend to overcrowd the rule and make it difficult to read; moreover, as one acquires skill and facility in using the slide rule, these additional numbers are not really needed at all.

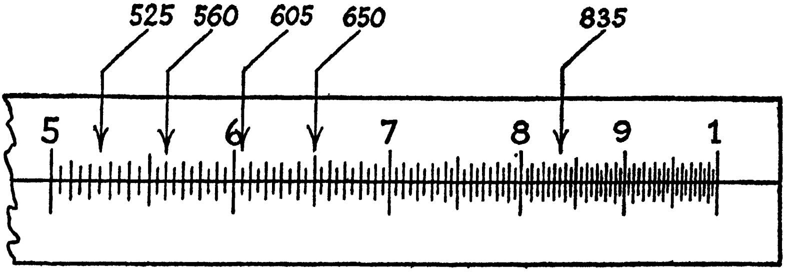
To learn to read these scales, study the following sketches given of portions of the rule and slide, noting especially the "sample readings" indicated.



It will be seen that this portion of the C and D scales (between 1 and 2) indicates accurately numbers which contain three digits, if the first digit is 1. If a number contains four digits (the last one not being zero), the number can be read accurately to three digits, but the fourth digit must be estimated. Do not be concerned about this; *always estimate to the maximum degree the smallest subdivision when reading a slide rule.*



Turning our attention next to the portion of the C and D scales between 2 and 4, it will be shown how these figures are read. Any number beginning with 2 and 3 is read from this part of the scale. In this portion of the scale each large unnumbered division represents 1/10 of 100, or 10; furthermore, since there are five unnumbered divisions between any two large unnumbered divisions, each small unnumbered division represents 2. Thus all even numbers between 200 and 400 can be read accurately, but all odd numbers must be estimated.



Finally, the portion of the C and D scale from 4 to 10 is used to locate any number whose first digit is 4, 5, 6, 7, 8 or 9. The first two digits can always be read accurately; the third digit can be read accurately only if it is a 5, otherwise it must be estimated.

The Principle of Proportion. The slide rule is not designed for addition and subtraction; its unique adaptability is for multiplication, division, proportions, powers and roots. It is a basic principle of the slide rule that, no matter where the slide is placed, *all* the numbers on the slide bear the same ratio to their corresponding or coinciding numbers on the rule. This holds true of the C and D scales, and also of the A and B scales. For example, pull the slide out to the right until 1 on C corresponds exactly with, or is "over," 2 on D; you will then note that the ratio 1:2 exists between every pair of coinciding numbers on the C and D scales, respectively. Thus if we set the first two terms of a proportion against each other on the slide and the rule, we find the third term on the slide coinciding with the fourth term on the rule. Hence:

$$\left(\begin{matrix}\text{any number}\\ \text{on C}\end{matrix}\right):\left(\begin{matrix}\text{the number}\\ \text{under it on D}\end{matrix}\right)=\left(\begin{matrix}\text{any other}\\ \text{number on C}\end{matrix}\right):\left(\begin{matrix}\text{the number}\\ \text{under it on D}\end{matrix}\right)$$

This principle of proportion can also be expressed as follows, giving a rule of procedure for using the slide rule to solve for the fourth term of a proportion:

C	set first term	under third term
D	over second term	find fourth term

EXAMPLE: Solve for x: $\frac{3}{8\frac{1}{2}}=\frac{15}{x}$.

SOLUTION:

C	set 3	under 15
D	over 8.5	find 42.5

Ans., 42.5

Multiplication. Since the multiplication of two factors, say 4×7, is the same as finding x in the proportion $1:4=7:x$, the rule for multiplication on the slide rule is simply this:

C	set 1	under the other factor
D	over one factor	find their product

Example 1: Multiply 14×6½.

Solution:

C	set 1	under 65
D	over 14	find 91

Ans., 91

Note: The location of the decimal point in the answer is most readily achieved by inspection of the original numbers rather than by any other rule; thus it is obvious that 14×6½ is approximately equal to 100, so that when reading the value "9—1" it would be read as 91 rather than as 9.1 or 910.

Example 2: Find the product of 5¼×24.

Solution:

C	set 1	under 24
D	over 525	find 126

Ans., 126

Note: If when drawing the slide out *to the right* in order to set 1 on C over one of the factors on D the other factor on C extends beyond the right terminal 1 on D, then the slide is drawn to the left and the right terminal 1 on C is set over the first factor on D.

Example 3: Multiply: 17.6×.0428.

Solution:

C	set 1	under 428
D	over 176	753

Ans., 0.753

Note 1: Since 18×.04=.72, the decimal point is placed before the 7 in reading "753."

Note 2: In setting 428, the terminal "8" is *estimated* as nearly as possible between "425" and "430"; likewise, when reading the product "753" on the D scale, the terminal "3" is also estimated as closely as possible. In this case, the product might have been read as .752 or .754 instead of .753, which would be an error of only one part in a thousand; the actual product is .75328. But since the factors are only given to three significant figures each, the product is adequately expressed as .753.

Division. The procedure for dividing one number by another is quite similar, as might be expected, since division is the reverse of multiplication. Thus the rule is:

C	set divisor	under 1
D	over dividend	find quotient

Example 1: Divide 64 by 4½.

Solution:

C	set 45	under 1
D	over 64	find 1425

Ans., 14.25

Example 2: Find $0.256 \div 33.2$.

Solution:

C	set 332	under 1
D	over 256	find 771

Ans., 0.00771

Example 3: Divide 80.3 by .049.

Solution:

C	set 49	under 1
D	over 803	find 1639

Ans., 1639

Combined Multiplication and Division. Here the extreme utility of the slide rule becomes even more evident, as the following illustrations will make abundantly clear.

Example 1: Find: $18 \times 4.2 \times .17$.

Solution:

C	set 1	bring runner to 42	bring 1 to runner	under 17
D	on 18			find 1285

Ans., 12.85

Example 2: Find: $\frac{27.2 \times 44 \times .88}{.73 \times 6.4}$

Solution:

C	set 73	runner to 44	64 to runner	under 88
D	on 272			find 2255

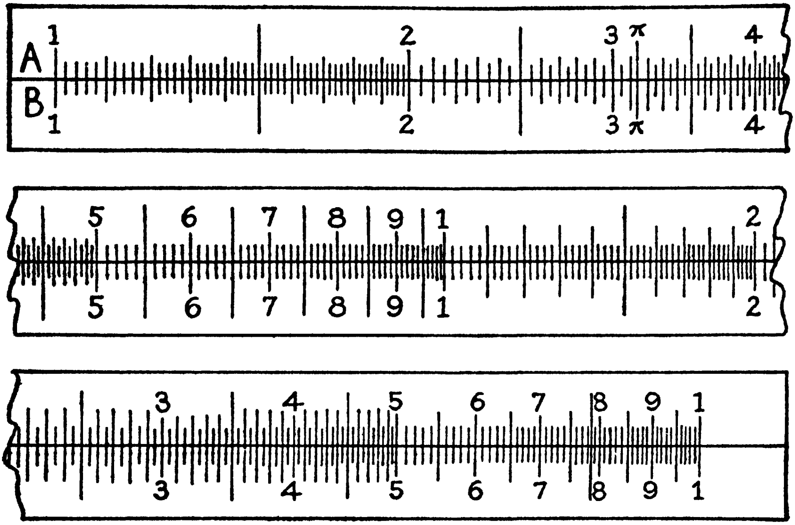
Ans., 225.5

Exercise 53.

Using the slide rule, find the following:

1. 39.6×24
2. 87×6.5
3. 6.21×9.24
4. $7.4 \div 32$
5. $60.5 \div 0.49$
6. $4.62 \times .866 \times 29$
7. $3.08 \times 144 \times 1.6$
8. $28.5 \times 3.14 \times 0.045$
9. $835 \div 21.8$
10. $.32 \div 0.077$
11. $\frac{63 \times 22}{17.5}$
12. $\frac{8.02 \times 30.6}{68 \times 2.5}$
13. $\frac{46.5 \times 3.14}{80.2 \times .35}$
14. $\frac{.707 \times 18.4}{2.54}$

Squares and Square Roots. Upon examining the upper scales A and B, it will be found that all the numbers on both the A and B scales are the *squares* of their coinciding numbers on the C and D scales; conversely, all the numbers on the C and D scales are the *square roots* of their coinciding numbers on the A and B scales. Thus, if the runner is set to 3 on the D scale, the hairline coincides with 9 on the A scale; this may be read as $3^2=9$, or $\sqrt{9}=3$; also as $30^2=900$, or $\sqrt{900}=30$; also as $300^2=90000$, or $\sqrt{90000}=300$; etc. Furthermore, if the runner is set to 5 on the D scale, the hairline will coincide with 25 on the A scale (not 2.5); i.e., $5^2=25$, or $\sqrt{25}=5$; also, $50^2=2500$, or $\sqrt{2500}=50$; etc. It should further be observed that the A scale is not graduated in the same manner as the D scale, since the A scale consists of *two parts,* the left- and right-hand scales, each running from "1" to "1." But if the left index of the left half of the A scale is "1," then the right terminal index of that half of the scale is "10," and the "2" in the right half of the A scale is then "20," etc.



In other words, to find the square root of a number, we locate that number on the A scale, observing the following rules; then set the runner on the number (on the A scale) whose root is to be found, and under it, on the D scale, read the desired square root:

Rule 1: To find the square root of a number having an *even* number of digits to the left of the decimal point, use the *right half* of the A scale; e.g., to find $\sqrt{62}$, $\sqrt{38.4}$, $\sqrt{6531.5}$, etc.

Rule 2: To find the square root of a number having an *odd* number of digits to the left of the decimal point, use the *left half* of the A scale; e.g., to find $\sqrt{6}$, $\sqrt{182}$, $\sqrt{3.1416}$, $\sqrt{856.2}$, etc.

Rule 3: To find the square root of a decimal having an *even* number of zeros to the right of the decimal point, use the *right half* of the A scale; e.g., $\sqrt{0.35}$, $\sqrt{0.0042}$, $\sqrt{0.831}$, $\sqrt{0.0000562}$, etc.

Rule 4: To find the square root of a decimal having an *odd* number of zeros to the right of the decimal point, use the *left half* of the A scale; e.g., $\sqrt{0.0269}$, $\sqrt{0.00048}$, $\sqrt{0.086}$, etc.

Example 1: Find the square root of 39.5.

Solution: Since 39 contains an even number of digits to the left of the decimal point, we use the *right half* of the A scale. Set runner on 395 on right side of A scale; under hairline on D scale read "628."

Ans., $\sqrt{39.5}=6.28$.

Example 2: Find $\sqrt{431}$.

Solution: Since 431 contains an odd number of digits to the left of the decimal point, we use the *left half* of the A scale. Set runner on 431 on left side of A scale; under hairline on D scale read "2075."

Ans., $\sqrt{431}=20.75$.

Example 3: Find $\sqrt{0.88}$.

Solution: Since the decimal has no zeros, i.e., an even number of zeros, to the right of the decimal point, we use the *right half* of the A scale. Set runner on 88 on right side of A scale; under hairline read "938."

Ans., $\sqrt{0.88}=0.938$.

Example 4: Find $\sqrt{0.0825}$.

Solution: Since the decimal has an odd number of zeros to the right of the decimal point, we use the *left half* of the A scale. Set runner on 825 on left side of A scale; under hairline read "287."

Ans., $\sqrt{0.0825}=0.287$.

Exercise 54.

Find the square root of each of the following by using the slide rule:

1. 44
2. 24.8
3. 70.5
4. 137
5. 8.25
6. .846
7. 0.00032
8. 0.0643
9. 3450
10. 0.005
11. 3.07
12. 0.0076

SELECTED REFERENCES FOR FURTHER STUDY

General Mathematics

Breneman, J. W. Mathematics. McGraw-Hill Book Co.
Brown, W. L. Related Mathematics. J. Wiley & Sons.
Douglass, H. R. & Kinney, L. B. Everyday Mathematics. H. Holt & Co.
Kanzer, E. M. & Schaaf, W. L. Essentials of Business Arithmetic. D. C. Heath & Co.
Lasley, S. J. & Mudd, M. F. The New Applied Mathematics. Prentice-Hall, Inc.
Lennes, N. J. Senior Practical Mathematics. Macmillan Co.
Mueller, C. H. Geometric Concepts. J. Wiley & Sons.
Schaaf, W. L. Mathematics for Everyday Use. Garden City Publishing Co.
Stone, J. C. & Mallory, V. S. Mathematics for Everyday Use. Benj. H. Sanborn & Co.
Van Tuyl, G. H. Mathematics at Work. American Book Co.

CHAPTER III

PRACTICAL GEOMETRY

William L. Schaaf

13. LINES AND ANGLES

Nature of Geometry. The subject of geometry may be studied in two different ways—either as a system of logical demonstrations, or as a body of practical facts and relationships concerning geometric figures and forms. In this book geometry will be presented from the second point of view, since it is the practical applications of geometric principles with which the mechanic and vocational student needs to be familiar. Scarcely a single shop operation or trade process will be encountered which does not involve some geometric relationship or formula. These basic geometric relations can readily be learned without the conventional logical treatment. Most of them are simple to understand on the basis of common sense (intuition) and practical measurements—especially if approached through everyday illustrations from shop problems.

Lines and Points. Geometrically we think of a *point* simply as representing a certain position only; it has no magnitude. A *line* may be regarded as the path of a moving point, or the series of positions traced by a point in motion; it has length only—no width or thickness. Lines are either straight, curved, or broken, or a combination of these. A *straight* line may be thought of as an "imaginary" string stretched as tightly as possible (taut); a straight line may also be represented by the intersection of two plane surfaces, such as the edge of a

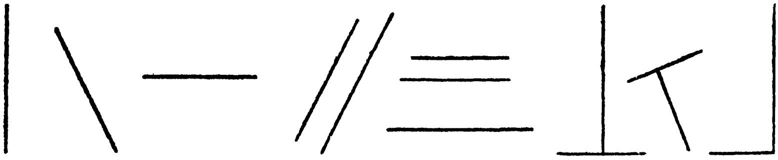
Straight Line

Broken Line

Curved Line

cube. Any line that is not straight is *curved,* unless, of course, it consists of a series of connected straight lines, i.e., a *broken* line.

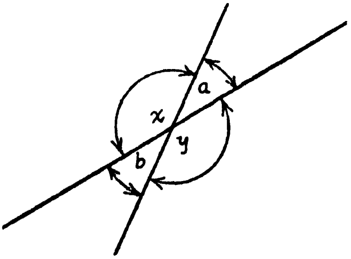
If the length of a straight line is not specified, but only its direction, it is to be thought of as extending indefinitely in both directions. If, on the other hand, its terminal, or end points, are specified, it is more properly called a *line segment,* or simply a segment. The various positions in which straight lines may lie are indicated by familiar names, as suggested in the accompanying figure. If the word "line" is used without an adjective before it, the reader is to assume that a straight line is meant.



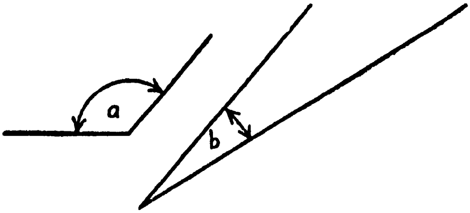
Parallel lines are two or more lines, all lying in the same plane surface, and which never meet however far they are extended (prolonged) in either direction. Any two parallel lines are equally distant from each other throughout their entire length; in a series of three or more parallels, however, some may be closer to one another than others.

Perpendicular lines are lines forming right angles with each other, where by a right angle is meant a quarter of a complete rotation. A vertical line (plumb line) meeting a horizontal line is an illustration of two perpendiculars; perpendicular lines need not, however, be horizontal and vertical, as shown above. When two lines form right angles with each other, each line is said to be perpendicular to the other.

It simply remains to point out that the shortest distance between any two specified points is a straight line joining those points. Also, that two straight lines can intersect in one point only; indeed, a point might be described as the *position* where two lines intersect. When two straight lines intersect, two pairs of equal angles are formed; the opposite angles, called *vertical angles,* are equal to each other; i.e., $\angle a = \angle b$, and $\angle x = \angle y$.



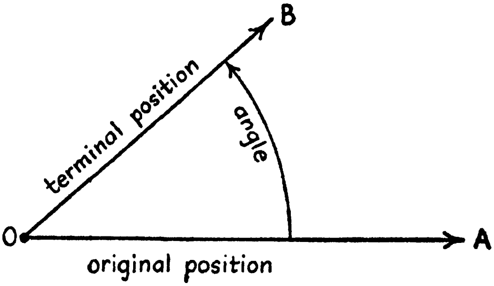
Angular Measurement. An *angle* may be described as the "opening" between two lines that intersect or meet. The lines are called the *sides* of



Angle a is larger than angle b, even though its sides are shorter.

the angle, and the point where they intersect or meet is called the *vertex* of the angle. The *size* of the angle in no way depends upon the length of its sides, but only upon the "amount" of opening or divergence between them.

Another way of thinking of an angle is as follows. If a line is rotated about any fixed point on the line, thus taking the line out of its original position to a new terminal position, the two positions are said to form an angle with each other; the point around which the rotation took place is the vertex of the angle. In other words, an angle is really a certain amount of *rotation* or turning. Thus an angle cannot be said to have length, or width, or area, etc., but simply rotation.



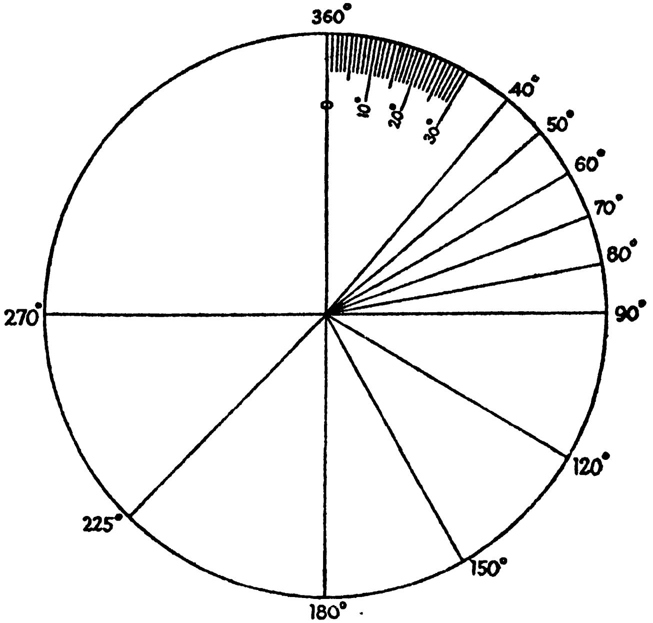
If a line is rotated until it returns to its original position, i.e., through one complete revolution, it is said to have described or formed a *round angle,* or to have turned through 360 degrees. If we consider any particular point on a line, except the pivotal or turning point, it will have described a *circle* when the line has made one complete revolution. Thus there are 360 degrees of angle in every circle, regardless of the size of the circle. For convenience, the standard unit of angular measurement is the *degree,* or 1/360 of a complete revolution.

Degrees, Minutes and Seconds. Angles are measured in terms of the following units:

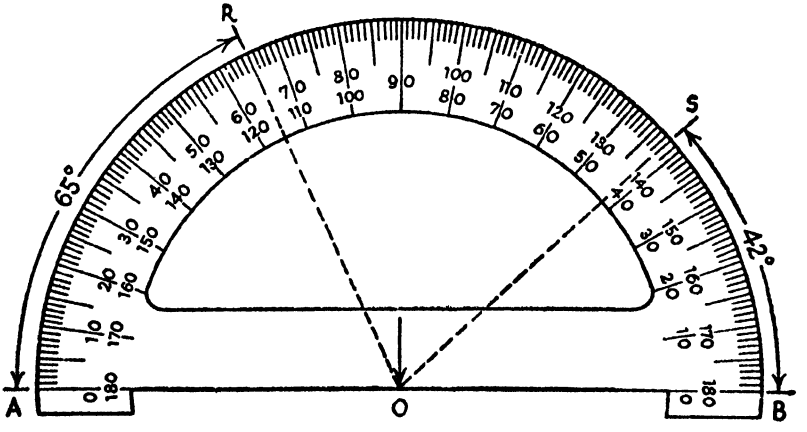
TABLE OF ANGULAR MEASUREMENT

60 seconds = 1 minute
60 minutes = 1 degree
90 degrees = 1 quadrant
360 degrees = 1 circumference

Degrees are designated by (°); minutes by (′); and seconds by (″); thus an angle of 32 degrees, 45 minutes, 20 seconds is written as 32°45′20″. The accompanying diagram shows a circle divided into degrees; each of the smallest subdivisions represents 1°.

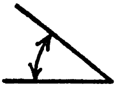


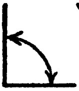
The Protractor. This is the instrument commonly used to lay out or measure angles. There are two sets of scales, each from 0° to 180°, one running clockwise from A to B, the other counterclockwise from B to A. Whenever an angle is to be measured or laid out, the vertex is always put

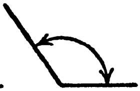


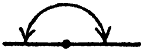
at point 0 on the protractor. The figure shows how an angle AOR (65°) and another angle BOS (42°) can be measured or laid out with the aid of a protractor.

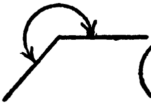
Kinds of Angles. A *right* angle is ¼ of a revolution, or 90°. An *acute* angle is an angle less than a right angle, or one containing less than 90°; an *obtuse* angle is one that is greater than a right angle, or containing more than 90° (but less than 180°). If a line has been rotated through *half* a revolution, or 180°, it is said to form a straight angle; thus any straight line may be regarded as a "straight angle" with its vertex at any point on the line we may wish to choose. Again, if a line is rotated through more than 180° but less than 360°, it is said to form a *reflex* angle; however, when two lines form angles as in (e), *unless otherwise specified,* "the angle" between them is the angle *less than* 180°, rather than the reflex angle ("reflex" means, literally, "bending back"). Finally, a complete revolution of 360° is sometimes called a *round angle;* similarly a line making two successive complete revolutions is said to have described an angle of 720°; three revolutions, 1080°; 1½ revolutions, 540°; etc.

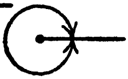

Acute Angle (a)


Right Angle (b)


Obtuse Angle (c)

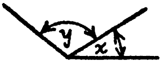

Straight Angle (d)


Reflex Angle (e)

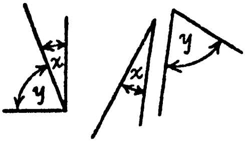

Round Angle (f)

Related Angles. A few common relations between angles should be understood. Vertical angles, always occurring in pairs, have already been mentioned; such angles are equal to each other (oppositely) in pairs. Other important relations are as follows:

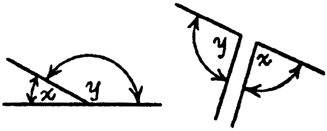
(1) All right angles are equal.
(2) A straight angle equals two right angles.
(3) Two angles are *adjacent* if they have a side in common.
(4) Two angles are *complementary* if their sum equals 90°. Each is the complement of the other; they may or may not be adjacent.
(5) Two angles are *supplementary* if their sum equals 180°. Each is the supplement of the other; they need not be adjacent.



Adjacent Angles



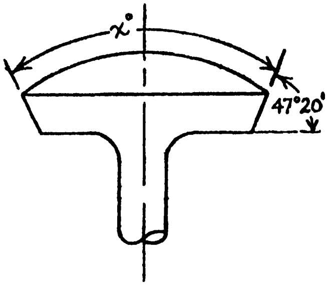
Complementary Angles

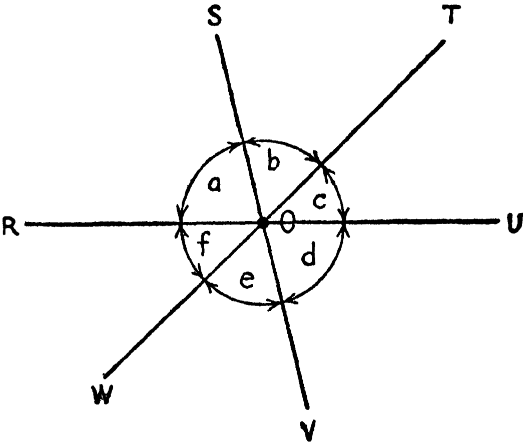


Supplementary Angles

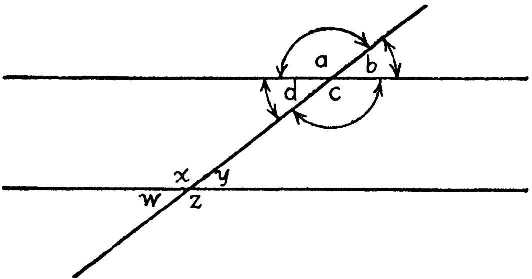
Exercise 55.

1. What is the complement of 39°? of 22½°? of 44°20′? of 67°41′12″?
2. What is the supplement of 32°? of 45°? of 90°? of 122°? of 57°29′? of 160°44′48″?
3. Find the angle x of the valve seat of the gasoline engine valve shown.
4. If $\angle$ROT=140°, and $\angle$TOV=120°, find the number of degrees in $\angle$SOU.
5. If $\angle$WOS=140°, and $\angle$ROT=125°, find $\angle d$.
6. Two angles are complementary. The greater exceeds the less by 22°. Find the angles.
7. Find the number of degrees in an angle the sum of whose supplement and complement is 202°.
8. The supplement of a certain angle exceeds three times its complement by 10°. Find the angle.





Parallel Lines and Transversals. When two parallel lines are cut by a third line, the latter, which is said to be a *transversal,* forms four pairs of angles with the two parallels. These are known as follows:



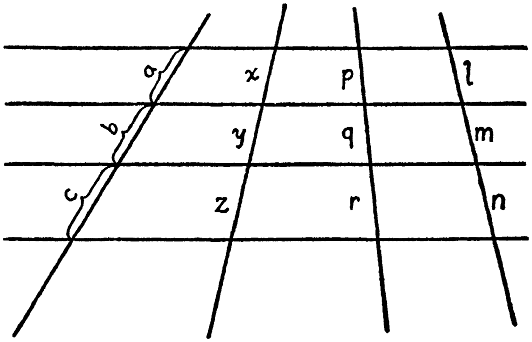
$\angle a$ and $\angle x$
$\angle b$ and $\angle y$
$\angle c$ and $\angle z$
$\angle d$ and $\angle w$
are *corresponding* angles.

$$\left.\begin{array}{l}\angle d \text{ and } \angle y\\ \angle c \text{ and } \angle x\end{array}\right\}\text{are } \textit{alternate-interior} \text{ angles.}$$

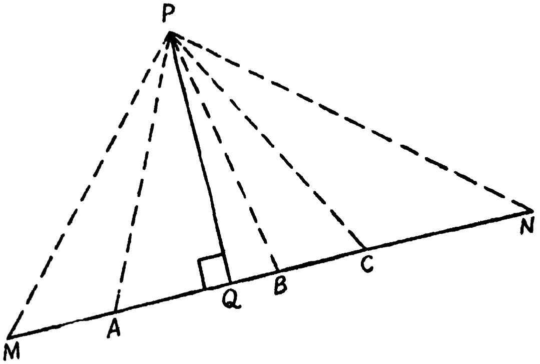
$$\left.\begin{array}{l}\angle a \text{ and } \angle z\\ \angle b \text{ and } \angle w\end{array}\right\}\text{are } \textit{alternate-exterior} \text{ angles.}$$

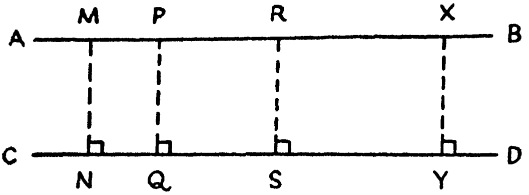
By studying the figure carefully, it will be seen that whenever a transversal intersects a pair of parallel lines, all the pairs of corresponding angles, as well as all the pairs of alternate angles, whether "interior" or "exterior," are equal. Thus: $\angle a=\angle x$; $\angle d=\angle y$; $\angle a=\angle z$; etc. Angles that lie on the *same* side of the transversal and are *included* between the parallels, are not equal, but *supplementary* to each other; thus $\angle d+\angle x=180°$, and $\angle c+\angle y=180°$.

If a series of parallel lines is cut by two or more transversals, and the segments cut off on any one of these transversals are equal to each other, then the segments on every other transversal are also equal to each other. On the diagram, this means that if $a=b=c$, then $x=y=z$; $p=q=r$; and $l=m=n$. It does *not* mean, however, that $a=x=p=l$, or that $b=y=q=m$, etc.; whether this is true depends upon the angles at which any two transversals cut the parallels.



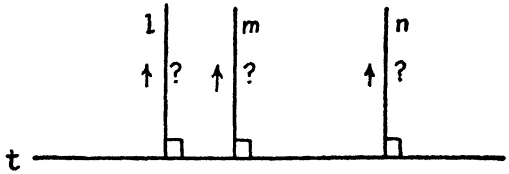
Meaning of Distance. In geometry, when we speak of the *distance* from a point to a line (or from a line to a point) it is understood to mean the *shortest* distance, or the *perpendicular* drawn from the point to the line. Thus, while PM, PA, PB, PC, and PN do represent "distances" from P *to various points on MN,* the distance from P *to the line MN* is PQ, and no other. Similarly, the distance between two parallel lines refers to the *perpendicular* distance be-



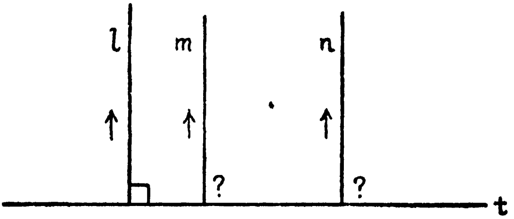


tween them, anywhere along the line; this perpendicular distance is the same at any point, since two parallel lines are everywhere equidistant.

Further Properties of Parallels and Perpendiculars. The following relationships are not only fundamental, but frequently quite useful as well.

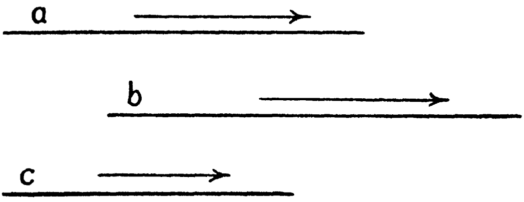


(1) If two or more lines are all perpendicular to another line, then they are parallel to one another. For example, if *l*, *m*, and *n* are each ⊥ *t*, then *l*, *m* and *n* are ‖ each other.



(2) If any one of two or more parallel lines is perpendicular to another line, then all of them are perpendicular to that line. For example, if *l*, *m* and *n* are ‖ each other, and *l* ⊥ *t*, then *m* and *n* are also ⊥ *t*.

(3) If a line is perpendicular to one of two or more parallels, then it is perpendicular to all of them. For example, if *l*, *m*, and *n* are ‖, and if *t* ⊥ *l*, then *t* is also ⊥ *m* and *n*.

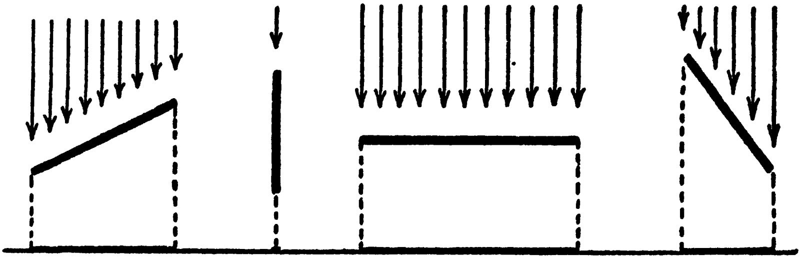


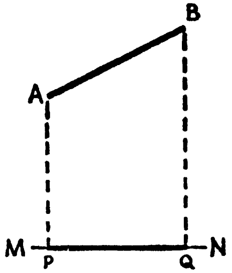
(4) If two lines are both parallel to a third line, then they are parallel to each other. For example, if *a* ‖ *x*, and *b* ‖ *x*, then *a* ‖ *b*.

(5) If a line is parallel to one of two other parallel lines, then it is parallel to the other. For example, if *a* ‖ *b*, and *x* ‖ *a*, then *x* is also ‖ *b*.

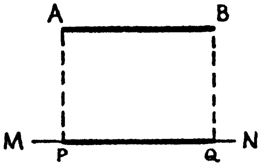
Projections. Everybody knows that when the sun is directly overhead, the shadow cast by a stick held obliquely will be shorter than the stick. The more nearly vertically it is held, the shorter the shadow becomes;

when it is exactly vertical, its shadow is shortest and approximates a point. The more nearly horizontal its position, the longer its shadow becomes; when exactly horizontal, the shadow is longest, being equal to the length of the stick itself. In all these cases, the shadow of the stick is called its *projection* (technically, its orthographic projection). Or, expressed somewhat differently, the projection of one line upon a second line is the segment included between the feet of the perpendiculars drawn to the second line from the extremities of the first line. This is illustrated in the figures below:

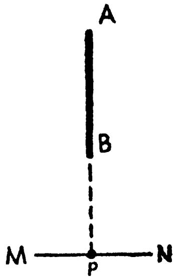




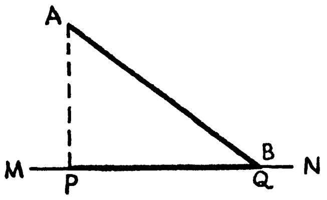
(1)

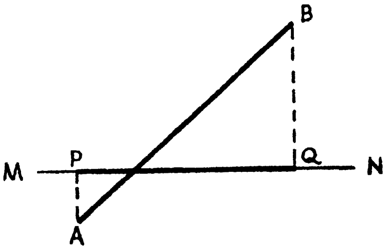


(2)



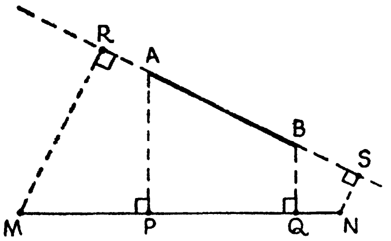
(3)



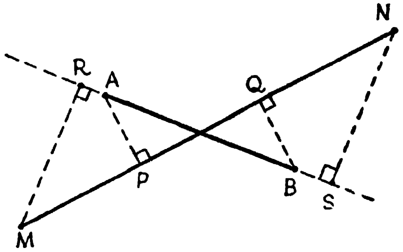


In each case, the projection of line AB upon line MN is the segment PQ; even in case (3), where the projection is the point P, this point may be regarded as a segment PQ of zero length (the points P and Q having "come together," or coincided).

Furthermore, given any two lines, either line may be projected upon the other, although one of them may have to be prolonged first. Thus in



(1)

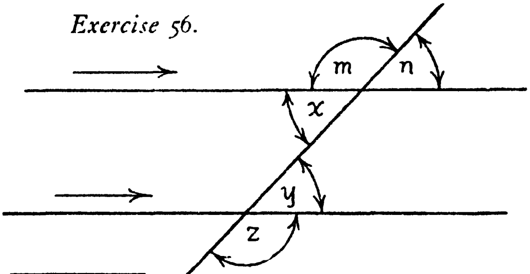


(2)

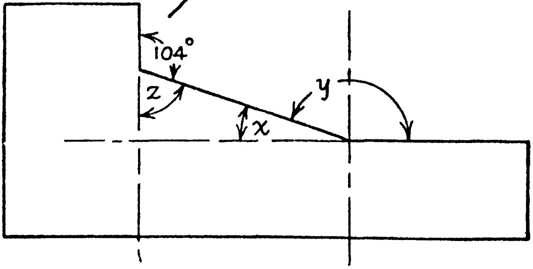
(1), the projection of AB upon MN is the segment PQ, while the projection of MN upon AB is the segment RS (where AB was first extended in both directions). Similarly in (2), the projection of AB upon MN is PQ; and the projection of MN upon AB is RS.

Exercise 56.

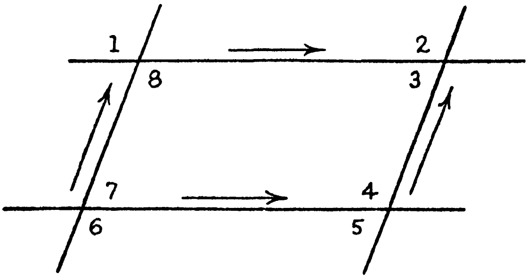
1. (a) If ∠ x = 38°, find ∠ z.
   (b) If ∠ m = 112°, find ∠ y.
   (c) If ∠ z = 155°, find ∠ n.



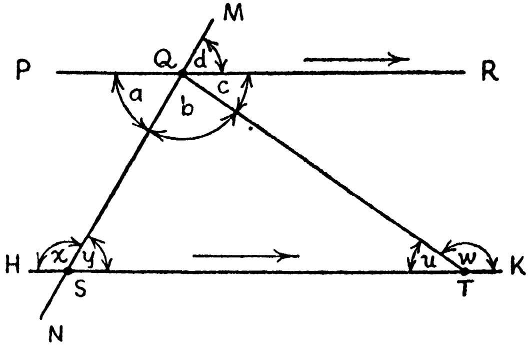
2. In the metal plate shown, find the number of degrees in each of the angles x, y and z.



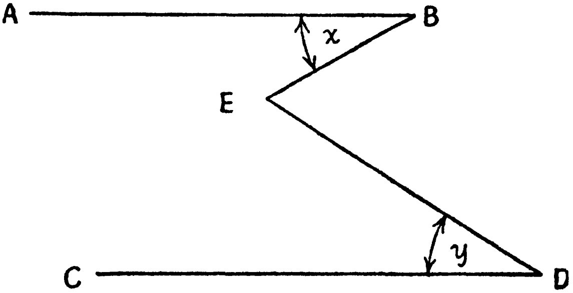
3. (a) If ∠1 = 120°, find ∠4.
   (b) If ∠7 = 58°, find ∠3.
   (c) If ∠8 = 115°, find the sum of ∠3 + ∠4 + ∠7 + ∠8.



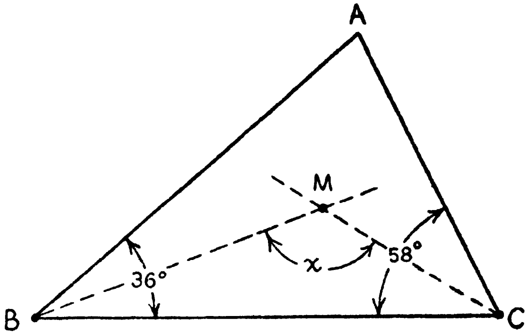
4. (a) If $\angle y = 50^\circ$, and $\angle u = 65^\circ$, find $\angle MQT$.
   (b) If $\angle y = 70^\circ$, and $\angle u = 42^\circ$, find $\angle b$.
   (c) If $\angle b = 85^\circ$, and $\angle u = 45^\circ$, find $\angle x$.
   (d) If $\angle a = 48^\circ$, and $\angle w = 160^\circ$, find $\angle TQM$; also $\angle PQM$.



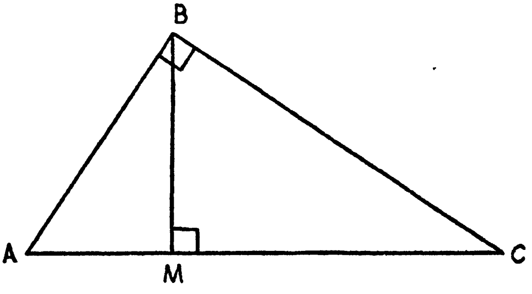
5. Line AB is parallel to CD; if $\angle x = 26^\circ 30'$, and $\angle y = 38^\circ 15'$, how many degrees are there in $\angle BED$? (*Hint:* Draw a line through E, parallel to AB.)



6. If two angles of a triangle are 58° and 36°, respectively, what is the angle (x) formed by their bisectors?

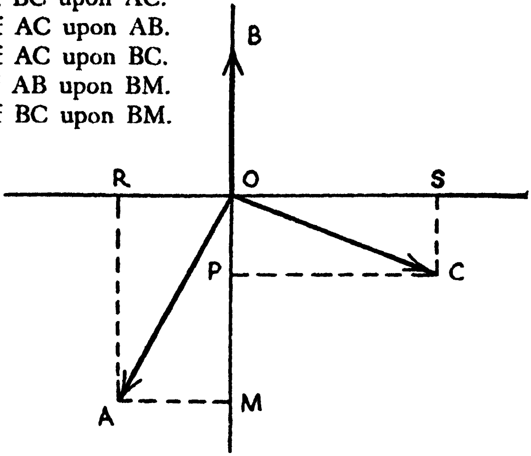


7. If $\angle ABC$ is a right angle, and $\angle AMB$ is also 90°, name the following:
   (a) the projection of AB upon AC.



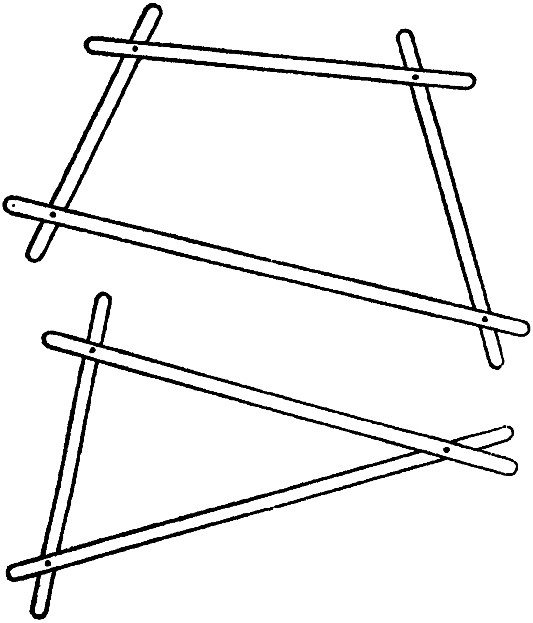
(b) the projection of BC upon AC.
(c) the projection of AC upon AB.
(d) the projection of AC upon BC.
(e) the projection of AB upon BM.
(f) the projection of BC upon BM.

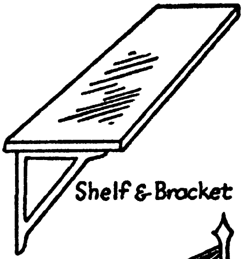
8. In this diagram of forces, state (a) the projection of OA, OB and OC, respectively, upon the horizontal axis; (b) the projections of OA, OB and OC, respectively, upon the vertical axis.

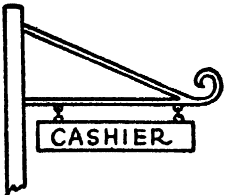


14. TRIANGLES AND POLYGONS

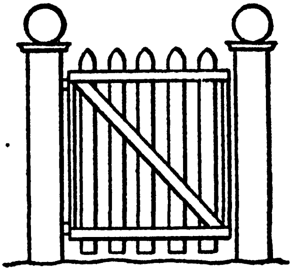
Rigid Frames. If four strips of wood are pinned together with nails, everyone knows that such a frame can easily be deformed, i.e., changed in shape. But if only *three* sticks be similarly fastened together, the resulting triangular frame cannot be changed in shape. In other words, triangles, or three-sided figures, are rigid frames; their *shape* cannot be altered if the lengths of the sides are fixed. For this reason triangular frames are used in structures to secure greater strength and rigidity. In fact, frame structures and trusses are commonly used in engi-

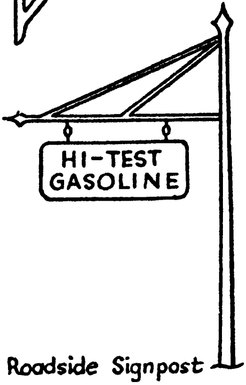



Shelf & Bracket

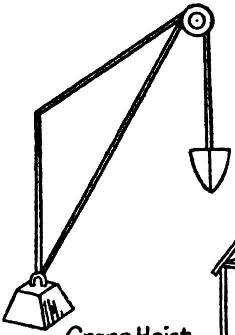


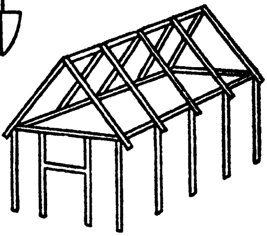
Wall Bracket

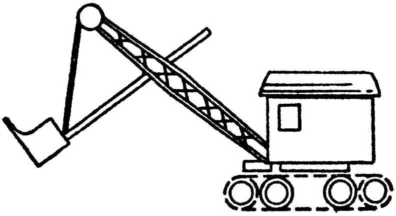

Garden Gate



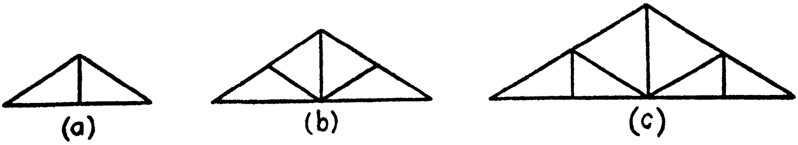
Roadside Signpost


Crane Hoist

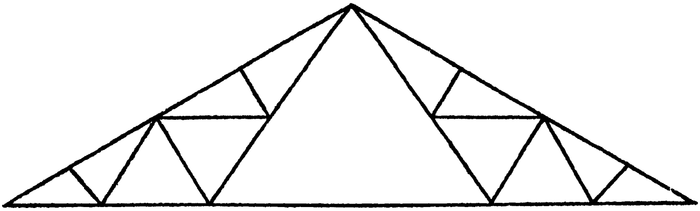

Frame House


Steam Shovel

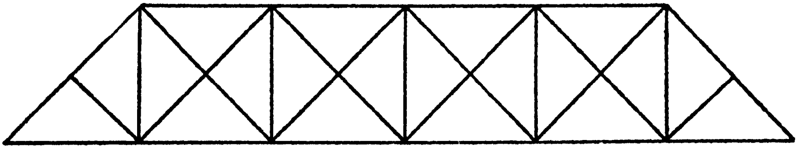
neering and structural work, such as bridges, roofs, pillars, towers, and supporting members and frameworks. A few types of trusses are shown below.


(a) (b) (c)

Simple Trusses

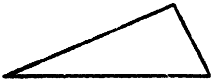


Roof Truss

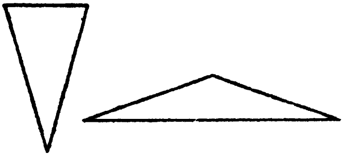


Railroad Bridge

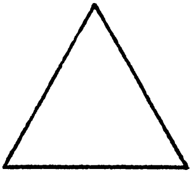
Kinds of Triangles. A triangle, then, is a straight-line figure having three sides and three angles. The vertices of the angles are also called the vertices of the triangle. If the lengths of the sides have been chosen, the "shape," or angles of the triangle, are automatically determined (fixed). Triangles may be classified according to the lengths of their sides, as follows:



(1) Scalene Triangle



(2) Isosceles Triangles



(3) Equilateral triangle

In an *equilateral* triangle, all three sides are equal in length.

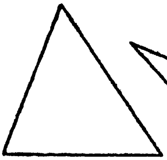
In an *isosceles* triangle, only two sides are equal in length.

In a *scalene* triangle, no two sides are equal in length. In an isosceles triangle, the two equal sides are called the *arms,* and the third remaining side is called the *base.* An equilateral triangle is obviously also isosceles; any one of its sides may be regarded as the base.

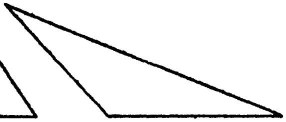
Triangles are also classified according to their angles, as follows:

In an *acute* triangle, each of the three angles is less than 90°.

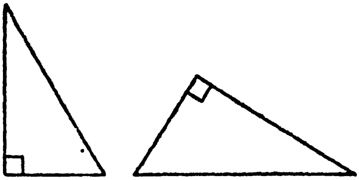
In an *obtuse* triangle, one of the three angles is obtuse, and the two



(1) Acute Triangle



(2) Obtuse Triangle



(3) Right Triangles

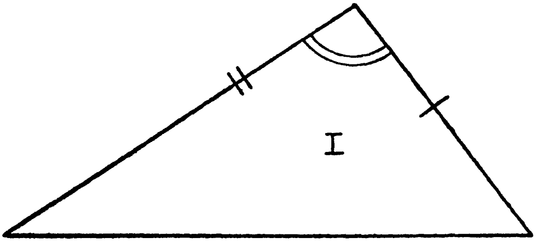
remaining angles are both acute. A triangle cannot have more than one obtuse angle.

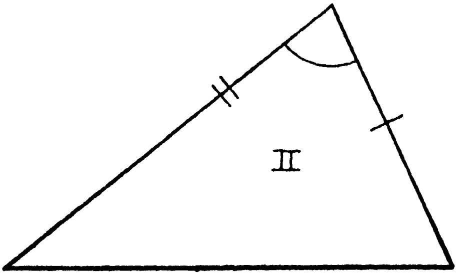
In a *right* triangle, one of the angles is a right angle, and the two remaining angles are acute angles. A triangle cannot have more than one right angle.

The sum of any two sides of a triangle is always greater than the third side, since a straight line is the shortest distance between any two given points.

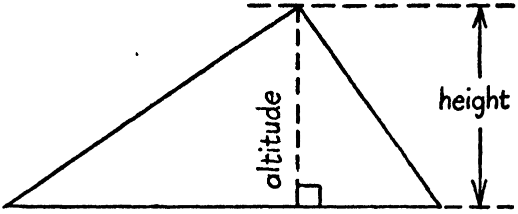
If the angles of a triangle are unequal, the longest side is opposite the largest angle, and *vice versa;* also the shortest side is opposite the smallest angle, and *vice versa.* In a right triangle, the side opposite the right angle is thus the longest side; it is known as the ***hypotenuse.***

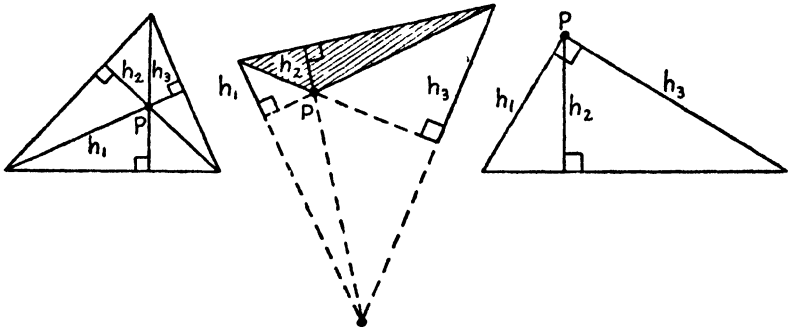
If two sides of one triangle are respectively equal to two sides of another, but the included angle of the first is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle. Conversely, if two pairs of sides are respectively equal, but the third side of one triangle is longer than the third side of a second triangle, then the angle opposite the longer side is greater than the angle opposite the shorter side.





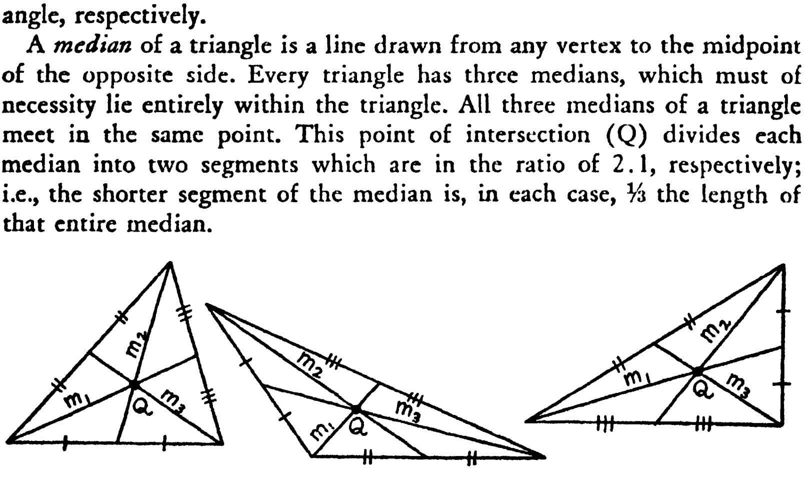
Altitudes and Medians. An *altitude* of a triangle is the perpendicular distance from any vertex to the opposite side. Every triangle therefore has three altitudes, one from each of the three vertices. In an acute triangle, all three altitudes fall *inside* the triangle; in an obtuse triangle, two of the altitudes fall *outside* the triangle; and in a right triangle, two of the altitudes coincide with the two sides.



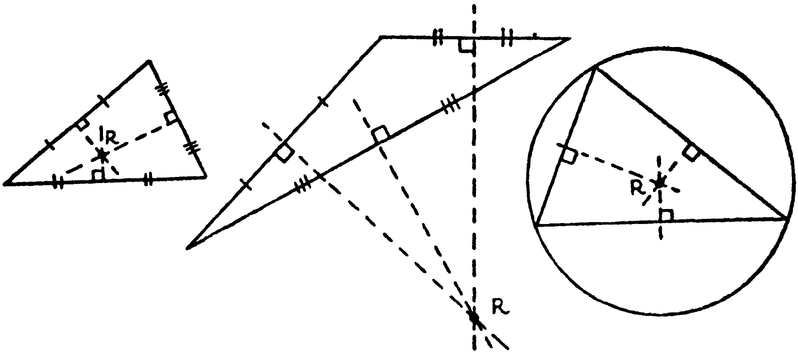


All three altitudes of a triangle (prolonged, if necessary) meet in the same point; this point of intersection (P) lies within, without, or on the triangle, according as the triangle is an acute, an obtuse, or a right triangle, respectively.

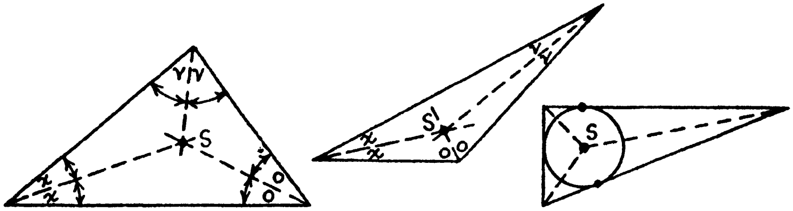
A *median* of a triangle is a line drawn from any vertex to the midpoint of the opposite side. Every triangle has three medians, which must of necessity lie entirely within the triangle. All three medians of a triangle meet in the same point. This point of intersection (Q) divides each median into two segments which are in the ratio of 2.1, respectively; i.e., the shorter segment of the median is, in each case, ⅓ the length of that entire median.

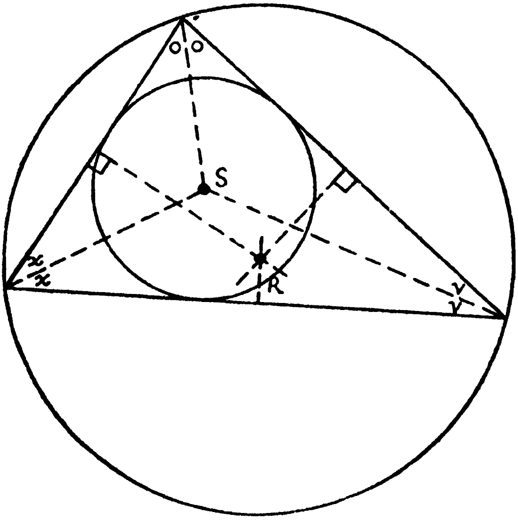


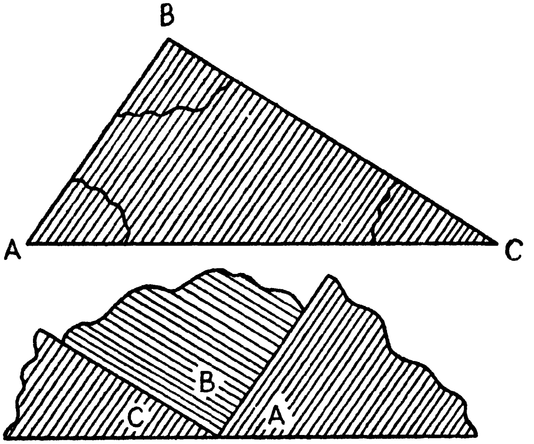
Bisectors of the Sides and Angles. The perpendicular bisectors of the sides of a triangle also meet in a single point. This point is equally distant from the three vertices; hence if it is used as a center, a circumscribed circle can be drawn which will pass through all three vertices.



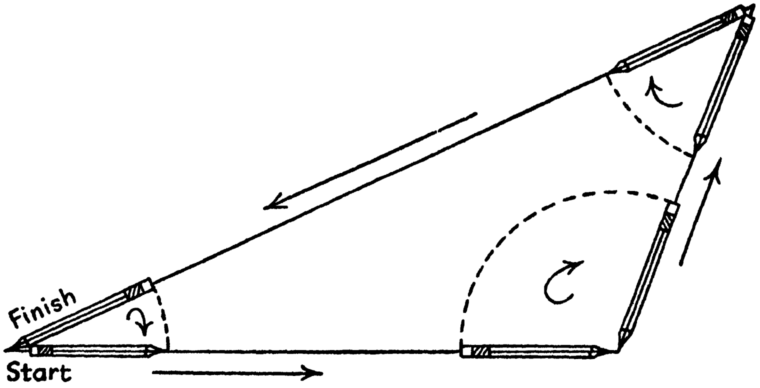
Similarly, the bisectors of the angles of a triangle also meet in a point, but this point is equally distant from all three sides of the triangle. Hence it is the center of the circle *inscribed* in the triangle, i.e., touching each of the three sides of the triangle in just one point. Only in one exceptional case, as we shall see later, are the centers of the inscribed and circumscribed circles the same, viz., in the case of an equilateral triangle; the equilateral triangle is a "regular polygon," and a circle may be both inscribed within and circumscribed about any regular polygon, both circles having a common center.





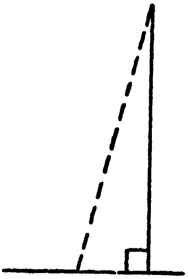


Sum of the Angles of a Triangle. One of the most important properties of a triangle is the fact that, irrespective of its shape, the sum of the three angles always equals 180°. This may be seen by tearing off the three "corners" of a triangular piece of cardboard and fitting the pieces together as shown; when rearranged, the three angles taken together form a straight angle, or 180°. Another way of showing that the angle sum equals 180° is by tracing the sides of a triangle, by sliding and rotating a pencil as suggested in the accompanying sketch. When the triangle has been completely traversed, the pencil will be found to have been reversed in direction; in other words, it has been turned through half a revolution, or 180°, again showing that the sum of the three angles of the triangle equals a straight angle.



Many properties of geometric figures depend upon this fact. Some of these are:

(1) That no triangle may have more than one right angle or one obtuse angle.
(2) That only one perpendicular may be drawn to a line from a point outside.
(3) That the acute angles of a right triangle are complementary.
(4) That an exterior angle of a triangle (formed by prolonging any side) is equal to the sum of the two remote interior angles.
(5) That angles included between parallels, and on the same side of a transversal, are supplementary.



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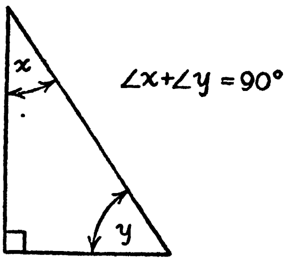

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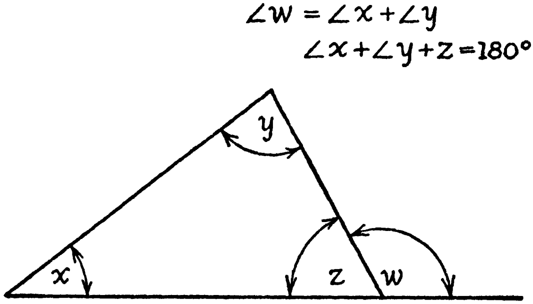

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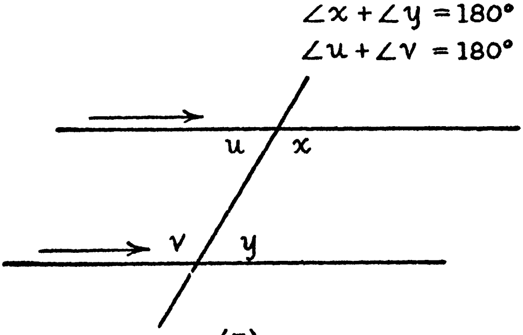
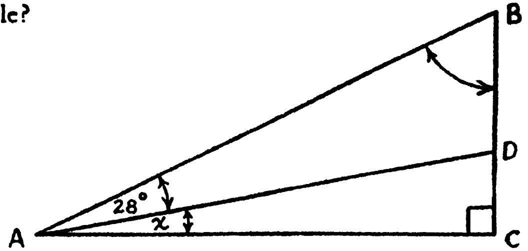


(5)

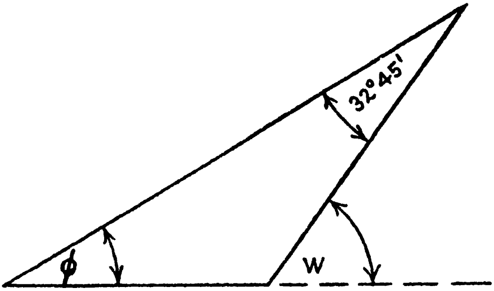
Exercise 57.

1. If one acute angle of a right triangle equals $14^\circ 36'$, what is the value of the other acute angle?

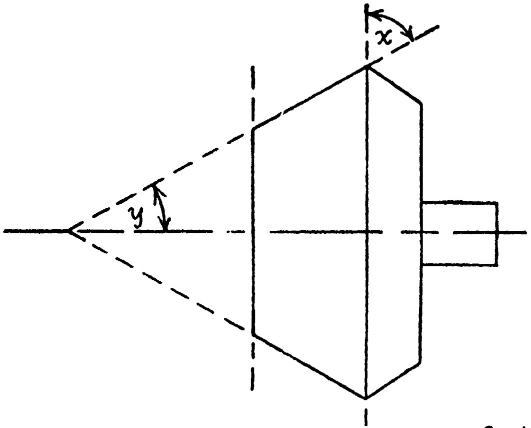
2. In the right triangle ABC, find $\angle x$ if $\angle B$ equals 46°.



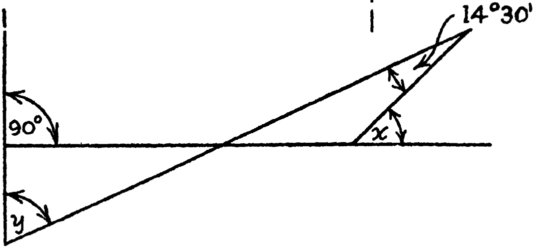
3. If $\angle w = 78^\circ 12'$, find the value of $\angle \phi$.



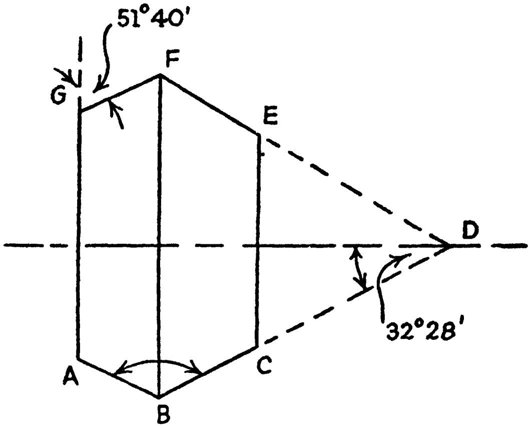
4. In this sketch of a bevel gear blank, if $\angle y = 61^\circ 24' \, 10''$, find $\angle x$.



5. Find the value of $\angle x$, if $\angle y$ equals $39^\circ 52'$.

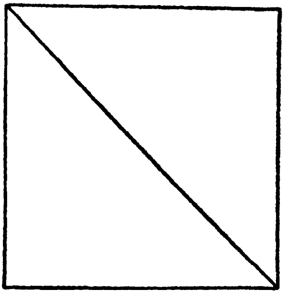


6. Find ∠ABC, if the face angle at G is 51°40′ and the half-angle at D equals 32°28′.

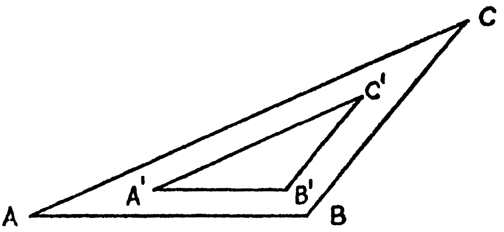


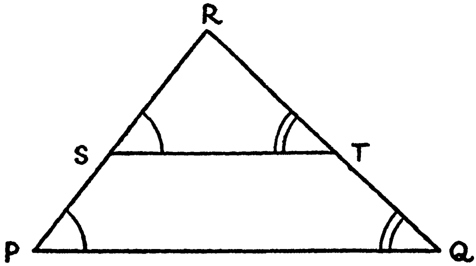
Congruence, Similarity, and Equivalence. If two triangles can be made to coincide or "fit exactly," they are said to be *congruent*. Congruent triangles might be said to have the same "shape" and the same "size." Any two triangles are congruent if:

(1) two sides and the included angle of one triangle are equal respectively to two sides and the included angle of the other;
(2) two angles and the included side of one triangle are equal respectively to two angles and the included side of the other;
(3) if a side and any two angles of one triangle are equal respectively to a side and the corresponding angles of the other;
(4) if all three sides of one triangle are respectively equal to the three sides of the other (even though nothing is known about the angles);
(5) if they are right triangles, and the hypotenuse and one side of one triangle are equal respectively to the hypotenuse and corresponding side of the other.



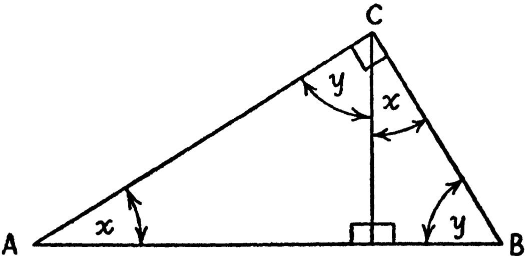
Two triangles are not necessarily congruent, however, merely because the three angles of the one are equal respectively to the three angles of the other; in this case they are said to be *similar*



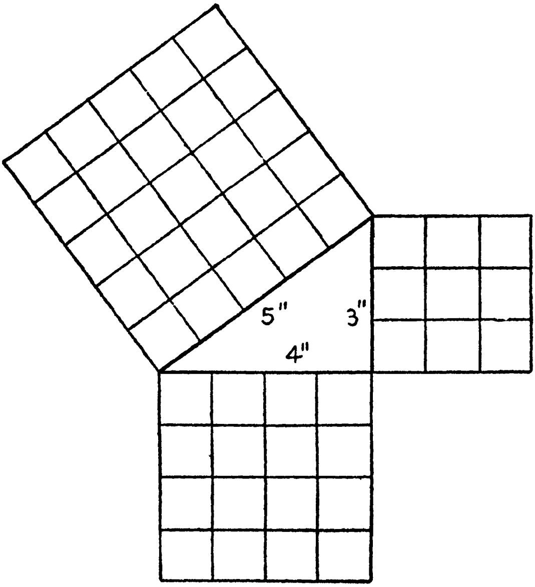


triangles. Such triangles have the same "shape," but not the same "size." We shall learn more about similar triangles later.

Triangles which are neither congruent nor similar may nevertheless be equivalent in area (i.e., cover the same surface), just as, for example, rectangle 4″ ×9″ covers the same surface as a square 6″ ×6″; the rectangle and the square are not congruent, neither are they similar in shape, yet they are *equivalent* in area. We shall learn more about this, too, later on.

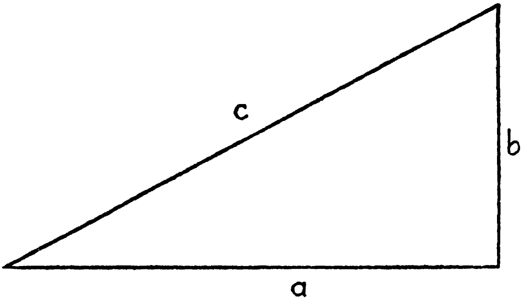


The Right Triangle Rule. One of the most famous and useful geometric relations is the relation between the lengths of the three sides of a right



Since $3^2+4^2=5^2$, or $9+16=25$, it is clear that if we *square* the number of units in each side and add these results, the sum obtained will be the square of the number of units in the hypotenuse.

triangle, viz., that $a^2+b^2=c^2$. Put into words: the square of the hypotenuse equals the sum of the squares of the other two sides. By means of this relation we can find the length of any side of a right triangle if we know the lengths of the other two sides. We simply use one of the following formulas:



$c^2=a^2+b^2$ or $c=\sqrt{a^2+b^2}$
$a^2=c^2-b^2$ or $a=\sqrt{c^2-b^2}$
$b^2=c^2-a^2$ or $b=\sqrt{c^2-a^2}$

Rule 1. To find the hypotenuse of a right triangle, find the square root of the sum of the squares of the other two sides.

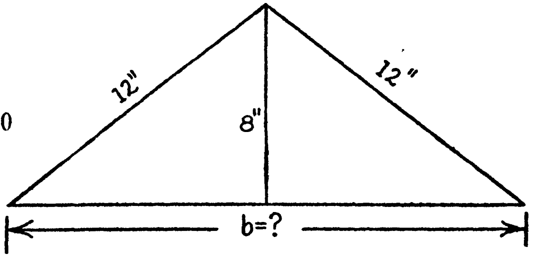
Rule 2. To find either side of a right triangle, if the other side and the hypotenuse are known, find the square root of the difference between the square of the hypotenuse and the square of the known side.

EXAMPLE 1: Find the length of the base of the isosceles triangle shown.

SOLUTION: The altitude, 8″, divides the base in half. Call the half-base x.

$x^2+8^2=12^2$
$x^2=12^2-8^2=144-64=80$
$x^2=80$
$x=\sqrt{80}=8.94+$,

Therefore the base $=$
$2\times8.94=17.9''$, *Ans.*



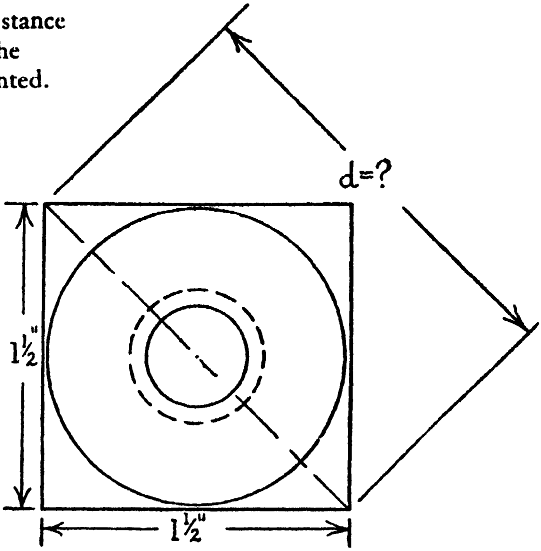
Example 2: Find the distance across the corners of the squarehead nut represented.

Solution:

$d^2 = (3/2)^2 + (3/2)^2$

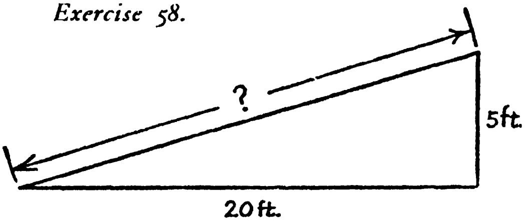
$d^2 = 9/4 + 9/4 = 18/4 = 4.5$

$d = \sqrt{45} = 2.1''$, *Ans.*

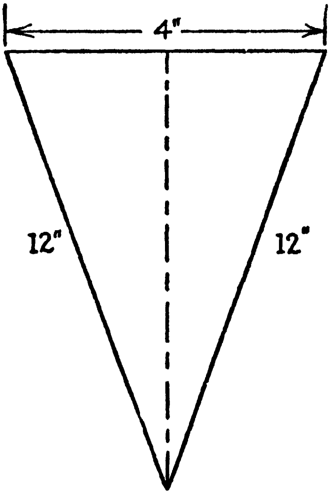


Exercise 58.

1. If the base of an isosceles triangle is 16" and the altitude is 9", find the length of the equal sides.



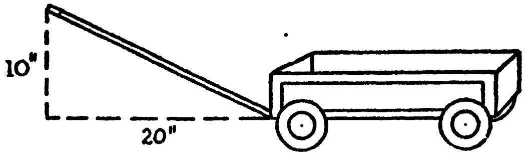
2. The runway of a garage ramp rises 5 ft. in a horizontal distance of 20 ft. Find the length of the ramp.



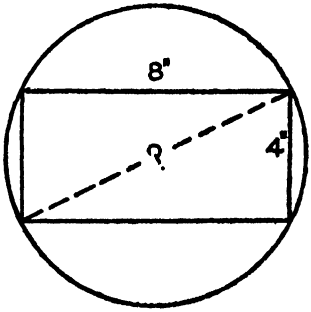
3. A wedge in the shape of an isosceles triangle has the dimensions shown. Find the length of the center line.

4. If the normal lens used in a camera has a focal length equal to the diagonal of the film which the camera accommodates, what should be the focal length of the lens for a camera using 4″×5″ plates?

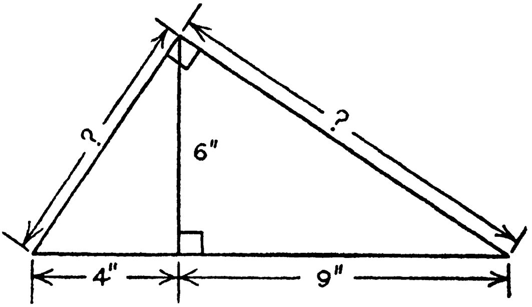
5. A child pulling a cart stands 20″ in front of it. One end of the handle is 10″ higher than the other. How long is the handle?



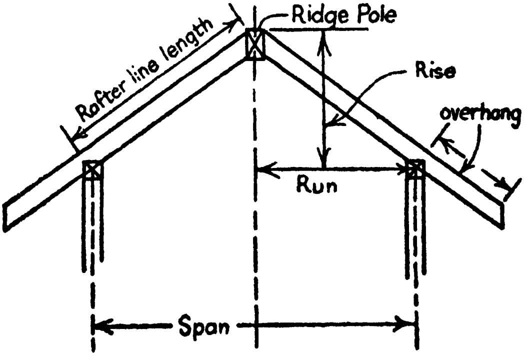
6. A wooden beam 4″×8″ is to be cut from a log. What is the diameter of the smallest log that could be used?



7. The altitude of a right triangle drawn to the hypotenuse is 6″. If the altitude divides the hypotenuse into segments of 4″ and 9″, find the lengths of the other two sides. How could you check your result?

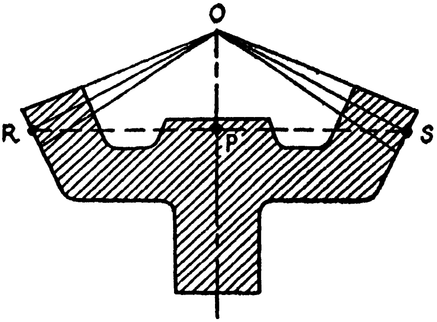


8. With an overhang of 18 in., a rise of 16 ft. and a span of 28 ft., find the overall length of rafter required.

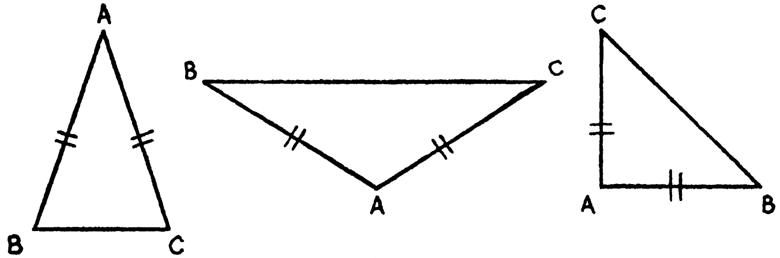


9. A square is inscribed in a circle whose radius is 4″. Find the length of the side of the square.

10. The distance OR in a bevel gear is called the apex distance, and RS is called the pitch diameter. If OP is $2\frac{1}{2}''$ and OR is $5\frac{1}{2}''$, find the pitch diameter.



Isosceles Triangle. Any triangle having at least two of its sides of equal length is called an *isosceles* triangle. The third side (BC) is referred to as the *base,* although it need not necessarily be the side on which the triangle is "standing." That vertex (A) of the triangle which lies opposite the base is called *the vertex* of the isosceles triangle. The altitude drawn from the vertex to the base has the following properties:

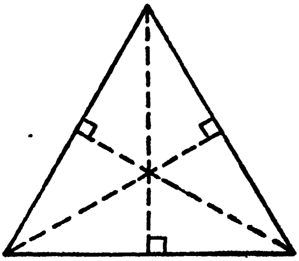


Isosceles Triangle

(1) It bisects the base (it is thus a median).
(2) It bisects the angle at the vertex (the "vertex angle").
(3) It divides the isosceles triangle into two congruent triangles.

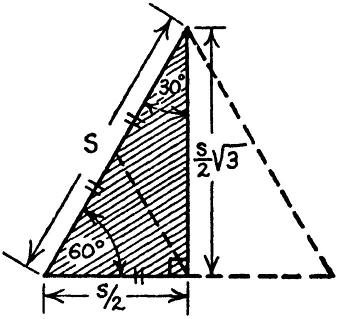
Another property of the isosceles triangle is that its *base angles* are equal; these are the two angles opposite the equal sides, i.e., adjacent to the base. It is also true that if any two angles of a triangle are equal, then the sides opposite them are equal, i.e., the triangle is isosceles.

Special Triangles. An *equilateral triangle* has special properties, not found in other triangles. Thus its three altitudes are all equal in length;

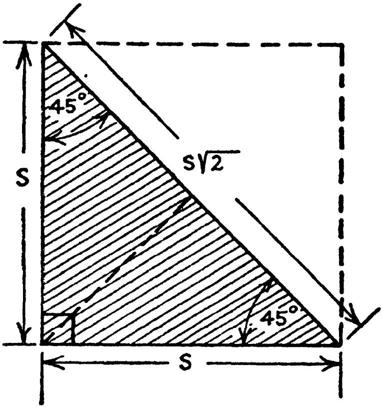


so are the three medians. In fact, the medians and altitudes are identical with one another; they are also identical with the angle-bisectors and with the perpendicular bisectors of the sides. A general formula for the altitude of an equilateral triangle is

$$h=\frac{s}{2}\sqrt{3}, \text{ where } s=\text{length of a side.}$$

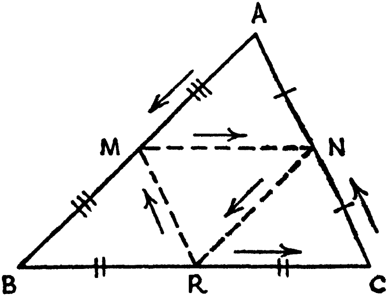


Another "special" triangle is the so-called *30°—60°—90° triangle,* or half an equilateral triangle. It is clear that in such a triangle, the hypotenuse is twice as long as the side opposite the 30° angle, or the shortest side. Moreover, the median drawn from the vertex of the right angle to the hypotenuse equals half the hypotenuse in length.



The *isosceles right triangle* is another special triangle often encountered. It is simply half of a square. In an isosceles right triangle, the altitude drawn to the hypotenuse equals

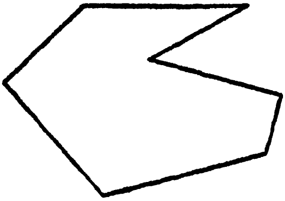
$$\frac{s}{2}\sqrt{2}.$$

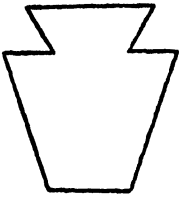


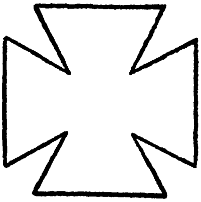
Mid-join of a Triangle. A line joining the midpoint of any side of a triangle with the midpoint of any other side is sometimes called the *mid-join* of the triangle. The mid-join has two important properties: (1) it is parallel to the third side of the triangle; (2) it is equal in length to *half* the third side.

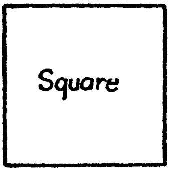
Thus if BM=MA and AN=NC, then MN (the mid-join) is parallel to BC and equal to ½ (BC). If the other two mid-joins be drawn, the original triangle is divided into four congruent triangles and three equivalent parallelograms.

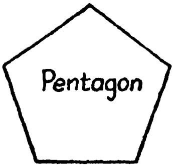
Polygons. Any rectilinear figure having four or more sides is called a *polygon*. Polygons may be irregular or regular. Irregular polygons may be *convex* or *reëntrant*. A *regular* polygon is a convex polygon all of whose sides are equal and all of whose angles are equal. Examples of regular polygons are shown below; both kinds of polygons are frequently encountered in shop practice and trade operations.

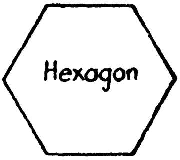


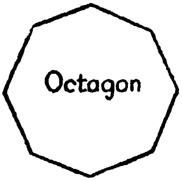




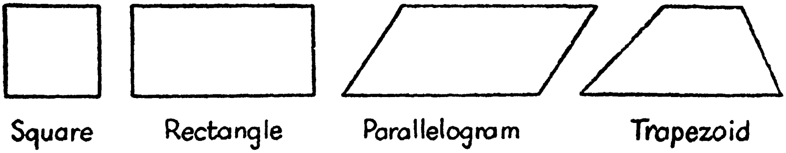






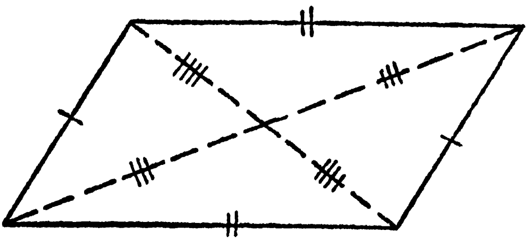


Quadrilaterals. Four-sided polygons are called *quadrilaterals*. Such figures are not rigid, as in the case of the triangle. In other words, the shape of a quadrilateral is *not* determined by the lengths of its sides; a figure of four fixed-length sides may be distorted by changing its angles *without* changing the lengths of its sides. The sum of the four angles of every quadrilateral, however, is constant, and equals 360°. Important types of quadrilaterals are shown below.

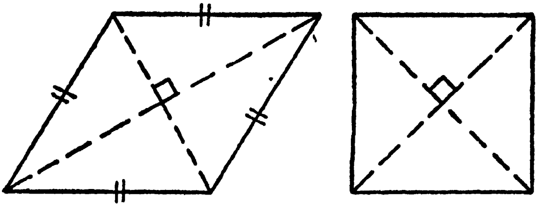


Parallelograms. A quadrilateral having its opposite sides parallel in pairs is called a *parallelogram*. Some of the most important properties of all parallelograms are:

(1) the opposite sides are equal.
(2) the opposite angles are equal.
(3) the diagonals bisect each other



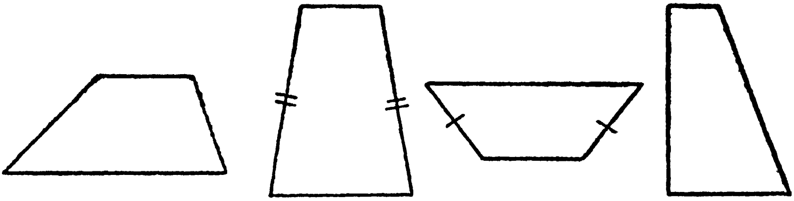
The *square* and the *rectangle* are special cases of the parallelogram; in both cases, all four angles are equal, each being 90°; the square differs from the rectangle in being equilateral, i.e., all four sides are equal. A parallelogram (other than a square) which is equilateral is called a *rhombus*. A special feature of the rhombus, which does not hold for non-equilateral parallelograms, is the fact that its diagonals not only bisect each other, but intersect at right angles as well, a feature that holds for squares also. The *altitude* of any parallelogram is the perpendicular distance between a pair of bases; either pair of parallel sides may be regarded as bases.



The Trapezoid. Any quadrilateral having two and only two of its sides parallel is known as a *trapezoid;* if the remaining two sides are equal in length it is an *isosceles trapezoid.* In either case, the two parallel sides are called the *bases* of the trapezoid. Trapezoidal shapes are frequently encountered in various machine parts and in construction work. The mid-join of a trapezoid, also commonly called the *median* of the trapezoid, is a line joining the mid-points of the two non-parallel sides. The median of a trapezoid is

(1) parallel to the two bases; and

(2) equal to half the sum of the two bases.

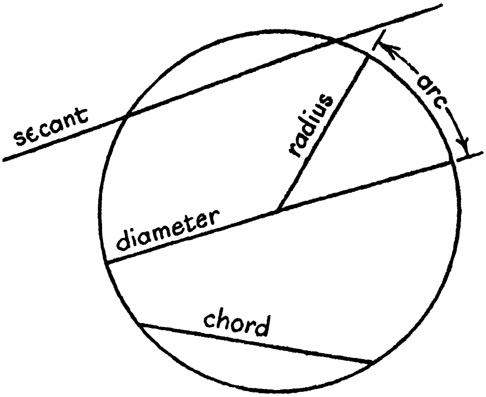


The *altitude* of a trapezoid is the perpendicular distance between the two bases.

15. CIRCLES AND TANGENTS

Circles. The circle is one of the earliest geometric forms known to primitive man. It was doubtless suggested by the rims of his pottery or the edge of spherical fruit when cut in half.

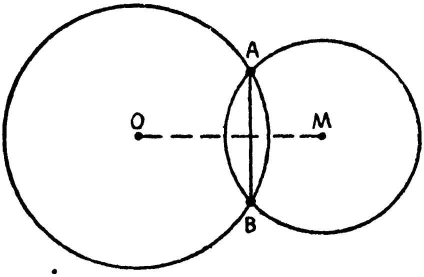
A *circle* is simply a closed line, every point of which is equally distant from a point within called the *center.* The distance from the center to the circle, or a line from the center to any point on the circle, is called the *radius.* Obviously, all the radii of any given circle are equal to one



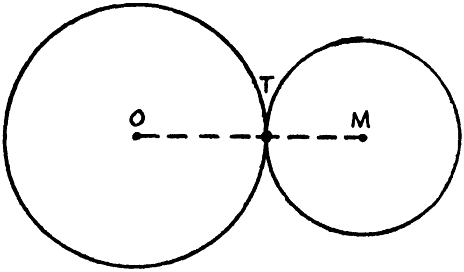
another. A straight line can intersect a circle in two points and only two; such a line is called a *secant*. If the ends of a line terminate in a circle, the line is called a *chord*. A *diameter* is thus also a chord; it is the longest chord that can be drawn in a circle, and is exactly twice the length of a radius. Naturally, all diameters of the same circle are equal.

An *arc* of a circle is that part of the circle included between any two particular points. The ends of a diameter divide the circle into two equal arcs. If an arc is less than half the circle, or *semicircle,* it is a *minor* arc; if greater than a semicircle, it is a *major* arc. The straight line joining the ends of an arc is called the *chord* of that arc; a chord is said to subtend the arc. A diameter is thus a chord subtending a semicircle.

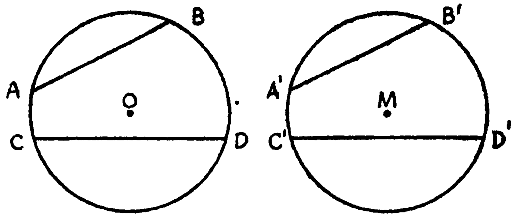
Two circles can intersect each other in only two points; the chord joining these points is called a common chord. Circles may also be *tangent* to each other, which means that they touch each other in one point and only one point; this common point is called the *point of tangency*. The line joining their centers, or line of centers, passes through the point of tangency. In the case of intersecting circles, the line of centers is the perpendicular bisector of the common chord.

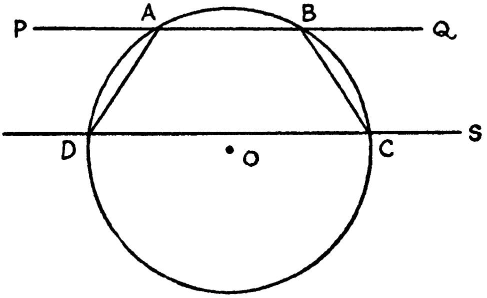


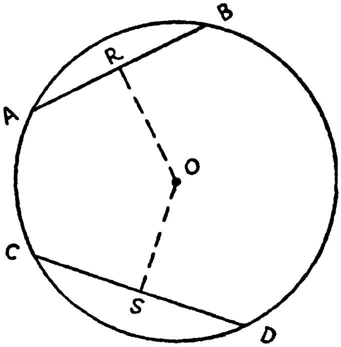
Chords. It is readily seen that in the same circle (or in two equal circles), if two chords are equal in length, then their arcs are equal; conversely, if two arcs are equal in length, then their subtended chords are equal. Furthermore, if in the same circle (or in two equal



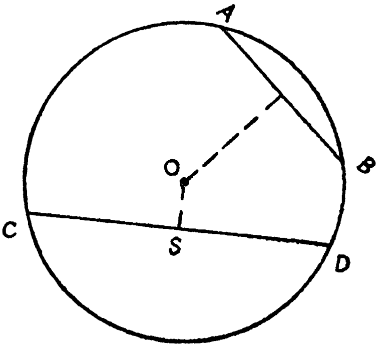
circles) two chords are equal in length, they must be equally distant from the center of the circle; and conversely, if two chords are equally distant from the center, then the chords are equal. If, on the other hand, two chords are *unequal,* then their distances from the center are unequal, and the greater chord is at the smaller distance. In other words, the nearer a chord approaches the center, the longer it becomes, and vice versa. Conversely, if two chords are at unequal distances from the center, the chords are unequal, and the one farthest from the center is the shorter chord. Any line passing through the center of a circle and perpendicular to a chord will bisect both the chord and its subtended arc; conversely, any perpendicular bisector of a chord must pass through the center of the circle.





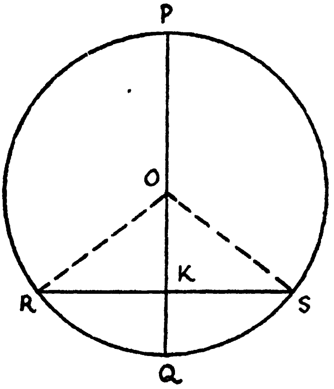


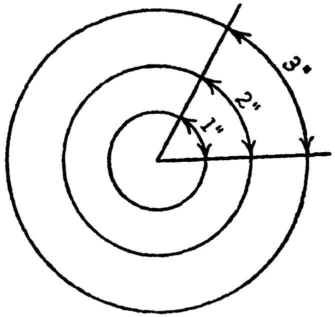
Circles and Angles. Any angle formed by two radii is called a *central angle;* its vertex is, of course, at the center. A central angle is said to be measured by its *intercepted arc;* this means that the number of degrees in the arc is the same as the number of degrees in its central angle, and conversely. An arc, in other words, may be described as being of so-and-so many degrees, or as so many inches, etc., long. How to measure an arc in *linear* units instead of *angular* units will be described later; here it should be

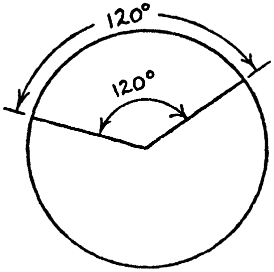


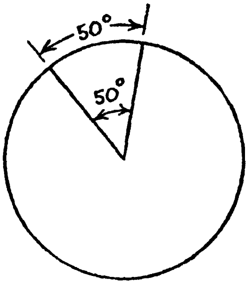
noted that in a series of *concentric circles,* or circles having the same center but different radii, a central angle of a fixed number of degrees will cut off arcs of different *absolute* lengths, even though all of these arcs are of the same *relative* length, i.e., contain the same number of angular units, or degrees. Or, putting it another way, the *actual* length of an arc depends not only upon the number of degrees of arc, but also on the radius of the circle.

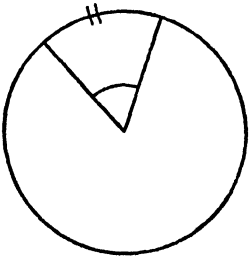
In any given circle, therefore, or in any two equal circles, equal central angles intercept equal arcs, and equal arcs are intercepted by equal central angles. Furthermore, if two central angles are unequal, their intercepted arcs are unequal, and the greater angle intercepts the greater arc; conversely, if two arcs are unequal, their central angles are unequal, and the greater arc is intercepted by the greater angle.

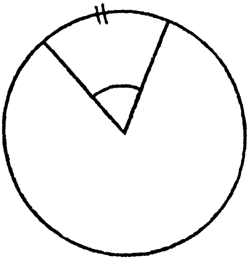




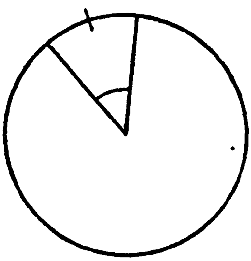


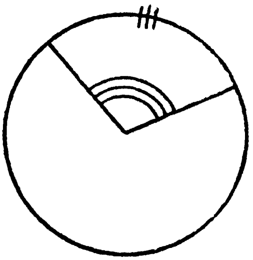




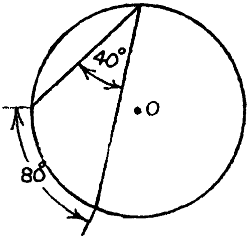


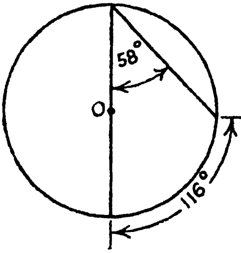
An inscribed angle of a circle refers to

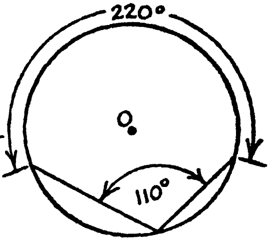


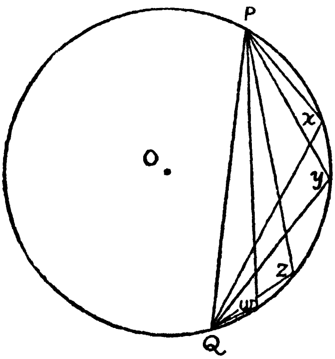


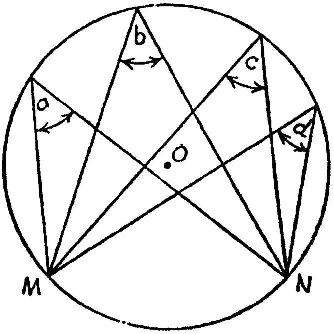
any angle formed by two chords (or a chord and a diameter) which intersect on a point on the circle. An inscribed angle is always measured by *half* its intercepted arc; or, the arc contains *twice* as many degrees as the inscribed angle which intercepts it. From this it will be seen that *all* angles inscribed in the same *segment* are equal, a segment being a figure bounded by an arc and its subtended chord. Thus in the figure, ∠x= ∠y= ∠z= ∠w; also, ∠a= ∠b= ∠c= ∠d, etc. As a special case of this, we note that all angles inscribed in a semicircle must be right angles. This is the relation a carpenter uses to test the accuracy of a semicircular groove; if the vertex of the carpenter's square touches every point of the groove as the square slides around, the groove is a true semicircle.

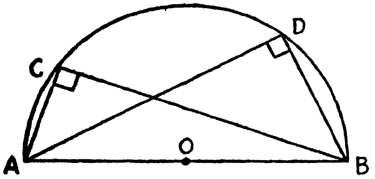


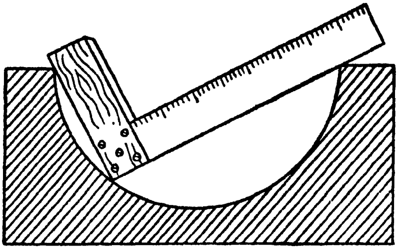


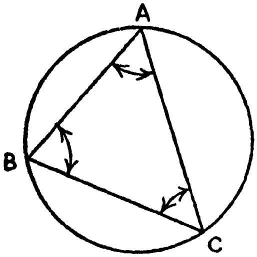


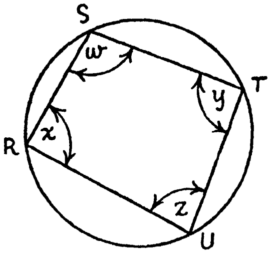


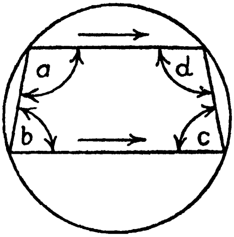












Other interesting and significant relations also become clear.

(1) Verification of the sum of the angles of a triangle;

$$\angle A = \tfrac{1}{2}\ \overset{\frown}{BC}$$
$$\angle B = \tfrac{1}{2}\ \overset{\frown}{AC}$$
$$\underline{\angle C = \tfrac{1}{2}\ \overset{\frown}{AC}}$$
$$\angle A + \angle B + \angle C = \tfrac{1}{2}\,(\overset{\frown}{BC} + \overset{\frown}{AC} + \overset{\frown}{AC}) = \tfrac{1}{2}(360°) = 180°.$$

(2) The opposite angles of an inscribed quadrilateral are supplementary;

$$\angle w = \tfrac{1}{2}\ \overset{\frown}{RUT}$$
$$\underline{\angle z = \tfrac{1}{2}\ \overset{\frown}{RST}}$$
$$\angle w + \angle z = \tfrac{1}{2}\overset{\frown}{RUT} + \tfrac{1}{2}\overset{\frown}{RST} = \tfrac{1}{2}(\overset{\frown}{RUT} + \overset{\frown}{RST}) = 180°.$$

Similarly, $\angle x + \angle y = \tfrac{1}{2}\overset{\frown}{STU} + \tfrac{1}{2}\overset{\frown}{SRU} = \tfrac{1}{2}(360°) = 180°$.

(3) A trapezoid inscribed in a circle must be an isosceles trapezoid;

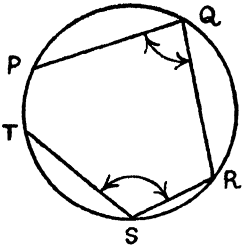
$\angle b + \angle a = 180°$, or $\angle b = 180° - \angle a$

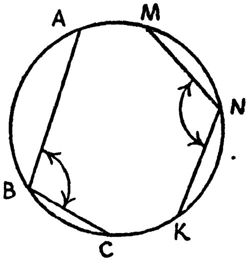
$\angle c + \angle a = 180°$, or $\angle c = 180° - \angle a$

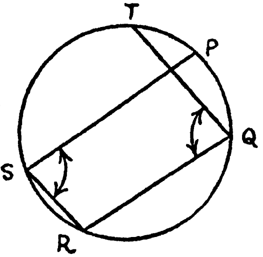
therefore $\angle b = \angle c$.

Exercise 59.

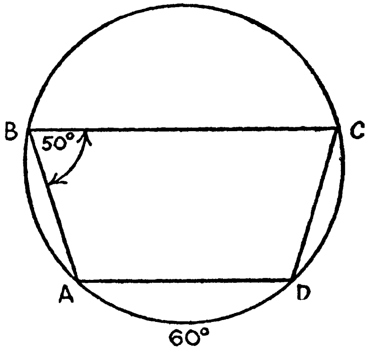
1. Find the sum of $\angle Q + \angle S$, if arc PT=40°.
2. How many degrees are there in $\angle B + \angle N$, if arc AM=30° and arc CK=40°?



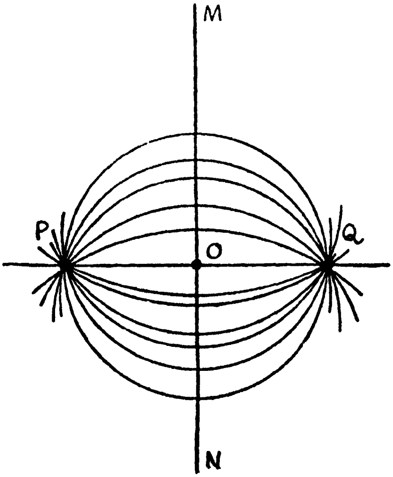


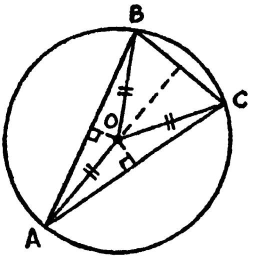


3. Find the sum of $\angle S + \angle Q$, if arc PT=44°.
4. Show why an oblique parallelogram cannot be inscribed in a circle.
5. If ABCD is an inscribed trapezoid, and $\angle ABC = 50°$ and arc AD=60°, find the number of degrees in each of the other arcs.
6. A quadrilateral PQRS is inscribed in a circle, and the two diagonals are drawn; arc PQ=80°, arc QR =150°, and arc RS=40°. Find all the angles in the figure.

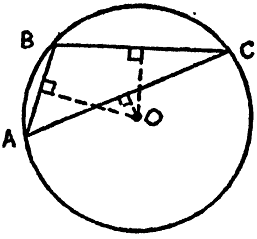


Circumscribed Circles. Geometric figures whose vertices lie on a circle and whose sides are chords of the circle, are said to be *inscribed* in the circle; or, the circle is circumscribed about the figure. A little consideration will show that through any *two* given points it is possible to draw an infinite number of circles, each of which has its center somewhere on the perpendicular bisector of the line PQ joining the two given points. But if it is attempted to draw a circle through any *three* given points, only one circle is possible. Thus, one and only one circle may be circumscribed about a given triangle. As we have already seen, the perpendicular bisectors of the three sides of any triangle all meet in a point which is equidistant from the vertices of the triangle. Hence this common point of intersection is the center of the circumscribed circle; its radius is the distance from this point to any one of the vertices (OA=OB=OC).

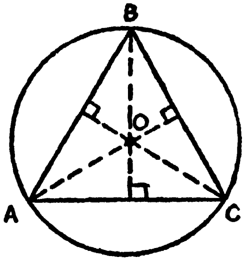




Acute Triangle

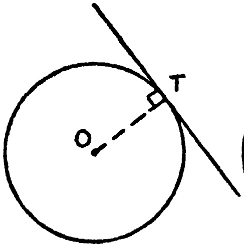


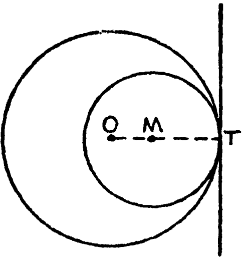
Obtuse Triangle

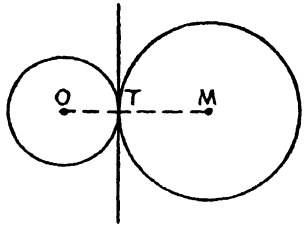


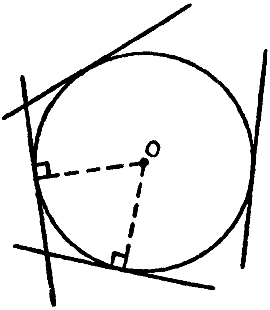
Equilateral Triangle

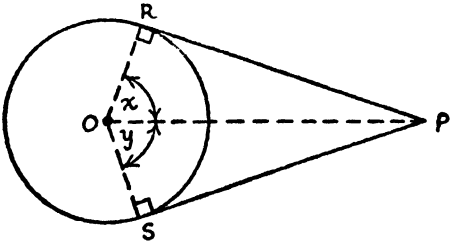
Circles and Tangents. A straight line is said to be *tangent* to a circle if the line touches the circle in one and only one point, no matter how far it is extended. Two circles are said to be tangent if they have but one point in common; they may be *internally* tangent or *externally* tangent. The point of tangency is also called the point of contact. A radius drawn to a tangent at the point of contact is always perpendicular to the tangent; likewise, any line perpendicular to a tangent at the point of contact will pass through the center. In the case of tangent circles, the line of centers passes through the point of tangency.



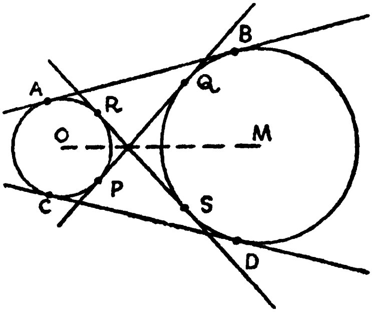


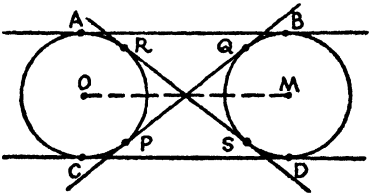


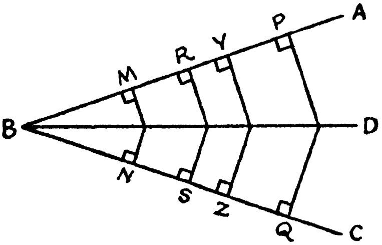




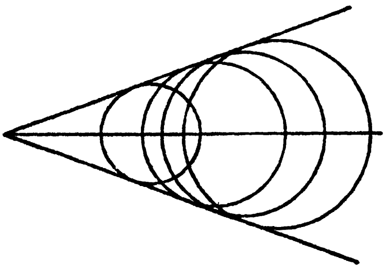
Any number of tangents may be drawn to a given circle, but at different points on the circle; at any *particular* point on a circle *only one* tangent can be drawn, however. From a point outside a circle, exactly two tangents may be drawn to the circle and no more. These tangents are equal, i. e., PR=PS. Moreover, the line from the outside point (P) to the center bisects the angle between the tangents, i.e., ∠RPO=∠SPO; also, ∠*x*=∠*y*. Where two non-tangent, non-intersecting circles are given, two external and two internal tangents may be drawn. These pairs of tangents are respectively equal, i.e., AB=CD, and RS=PQ. If the circles are equal, the external tangents are parallel as well. In both instances the line of centers passes through the point of intersection of the pair of internal tangents.



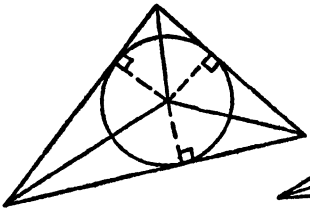




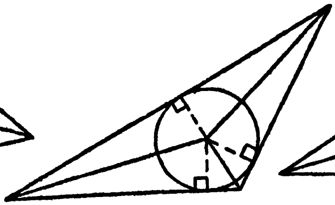
The Bisector of an Angle. Every point on the bisector of an angle lies at equal distances from the sides of the angle. Thus a series of circles could be "inscribed" within the angle, i.e., the sides of the angle would be tangent to the respective circles. This principle is employed when inscribing a circle in a triangle.



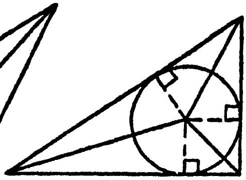
Inscribed Circles. A circle is said to be inscribed in a figure if it is tangent to every side of the figure. If a circle is inscribed in a triangle, the center of the circle is always the common point of intersection of the three angle-bisectors, and its radius equals the perpendicular distance from the center to the respective sides.



Acute Triangle

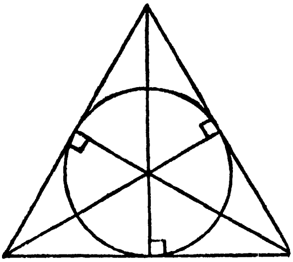


Obtuse Triangle

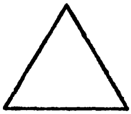


Right Triangle

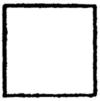
Regular Polygons. If all the sides and all the angles of a polygon are equal, the figure is said to be a *regular polygon.* A polygon may have equal angles, but unequal sides; or, it may have equal sides, but unequal angles; in neither case, however, is it a regular polygon. A circle may always be both inscribed in, and also circumscribed about, any regular polygon, no matter how many sides it may have; the inscribed and circumscribed circles in each case have the same center. The radius of the circumscribed circle is called the radius of the polygon (R); the radius of the inscribed circle is called the *apothem* of the polygon (r). In the case of a square, R=half the diagonal, and r=half the side. In the case of a regular hexagon, R equals the side of the hexagon. In the case of the equilateral triangle, R=$2r$. In all cases, the central angle of a regular



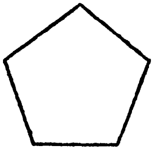
Equilateral Triangle



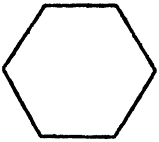
Equilateral Triangle



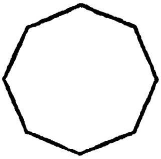
Square



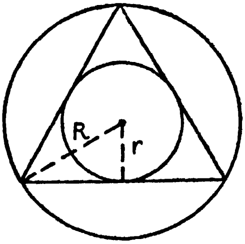
Regular Pentagon

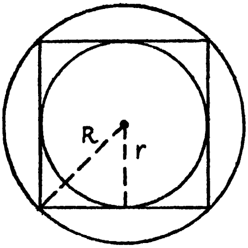


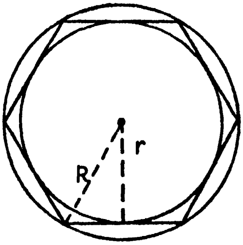
Regular Hexagon



Regular Octagon







polygon is the angle included between any two consecutive radii; in a regular polygon of n sides, the central angle equals $\left(\frac{360}{n}\right)^{\circ}$.

Circumference of a Circle. The length of a circle is called its *circumference.* In every circle, however large or small, the ratio of the circumference (C) to its diameter (D) is constant, and equal to 3.1416, or approximately 3⅐ .Thus ratio is called π (pi). Thus any circumference is about 3⅐ times as long as its diameter; that is

$$\frac{C}{D}=\pi=3.1416, \text{ or } C=\pi D.$$

Since $D=2R$, then we also have

$$C=2\pi R.$$

Example 1: Find the circumference of a circle with radius 14″.

Solution:

$$C=2\pi R$$
$$C=2\times\frac{22}{7}\times 14$$
$$=2\times\frac{22}{7}\times 14=88'', \textit{Ans.}$$

Exampll 2: If the circumference of a circle is 110 ft., find its diameter.

Solution:

$$C=\pi D, \text{ or } D=\frac{C}{\pi}$$
$$D=110\div\frac{22}{7}=110\times\frac{7}{22}=35 \text{ ft., } \textit{Ans.}$$

For ordinary rough computations, the value $\pi=3\frac{1}{7}$ may be used; for more exact work, use $\pi=3.14$; and for very accurate work, use $\pi=3.1416$.

Length of an Arc. The linear measure of the length of an arc is found by the proportion:

$$\frac{\text{length of arc}}{\text{circumference}}=\frac{\text{central angle}}{360^{\circ}},$$
$$\text{or, arc}=\frac{\text{central angle}}{360^{\circ}}\times 2\pi R.$$

Example: Find the length of an arc of 80° in a circle with a diameter of 12″.

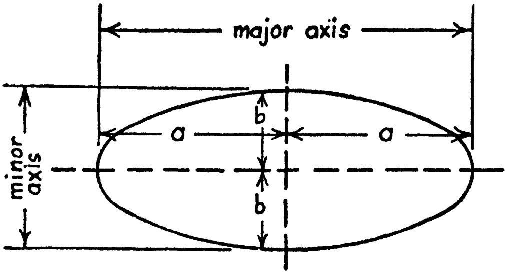
Solution:

$$\text{arc}=\frac{80^{\circ}}{360^{\circ}}\times 2\times\frac{22}{7}\times 6,$$
$$\text{or, arc}=\frac{176}{21}=8.38'', \textit{Ans.}$$

The Ellipse. An ellipse is a closed curve with a *major* and a *minor* diameter. It is perfectly symmetrical with diameter, or axis, and is not to be confused with an oval, which is egg-shaped, or narrower at one end than at the other. If the semi-major diameter $= a$, and the semi-minor diameter $= b$, then the length of the perimeter equals

$$P=\pi(a+b)K,$$

K being a constant depending upon the value of m, where $m=\frac{a-b}{a+b}$. The table shows the value of K for certain values of m:



m	K
0.1	1.002
0.2	1.010
0.3	1.023
0.4	1.040
0.5	1.064
0.6	1.092
0.7	1.127
0.8	1.168
0.9	1.216

EXAMPLE: Find the perimeter of an ellipse in which $a=5''$ and $b=3''$.

SOLUTION:

$$m=\frac{a-b}{a+b}=\frac{5-3}{5+3}=\frac{2}{8}=.25;$$

hence K=midway in value between 1.010 and 1.023, or 1.017 (by interpolation).

$$\begin{aligned}\text{Therefore, perimeter}&=\pi(a+b)K\\&=\frac{22}{7}(5+3)\ (1.017)\\&=25.6'',\ \textit{Ans.}\end{aligned}$$

If $a=b$, then $m=0$ and $K=1$, and the formula becomes $P=\pi(2a)$, or $P=\pi D$, which is what it must be, since if $a=b$, the ellipse has become a circle.

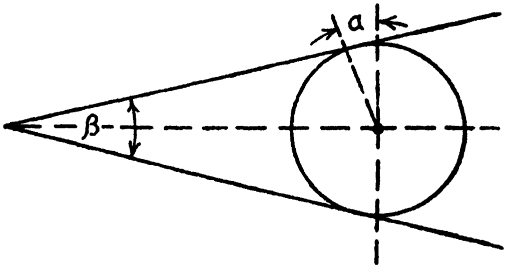
An alternative method for finding the perimeter of an ellipse involves the use of the formula $P=\pi\sqrt{2(a^2+b^2)}$, which also gives an approximate value only, although close enough for most practical purposes. Using this formula for the same problem above, we obtain:

$$\begin{aligned}P&=\pi\sqrt{2(25+9)}=\pi\sqrt{68}\\&=\frac{22}{7}\times8.25=25.9'',\ \textit{Ans.}\end{aligned}$$

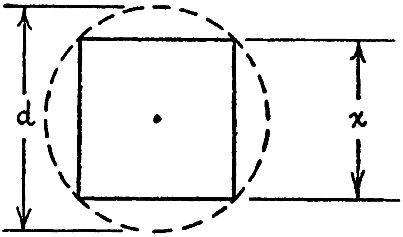
Exercise 60.

1. Find the length of a tangent drawn to a circle with a diameter of 8″ from a point 16″ from the center.

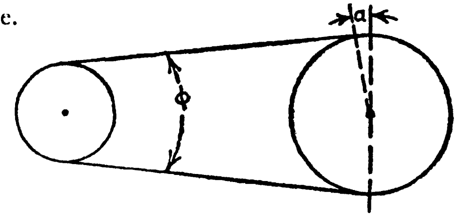
2. Find $\angle a$ if $\angle \beta = 34°20'$; write a formula for $\angle a$ in terms of β. Assume that the radius to the point of tangency is perpendicular to the tangent.



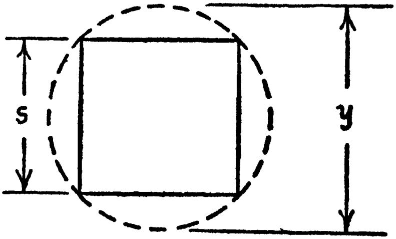
3. Find the side of a square inscribed in a 14-inch circle.
4. What is the value of x, if $d = 6.48''$? Write a formula for x in terms of d.



5. An electric motor is revolving at 1400 revolutions per minute. Through how many ft. does a point on the rim move in one second if the rim has a diameter of 36″?
6. An automobile wheel with tire has an overall diameter of 3½ feet. When traveling 20 miles per hour, how many revolutions is the wheel making per minute?
7. A square measures 6.4″ on a side. Find the diameter of a circle that will circumscribe this square.

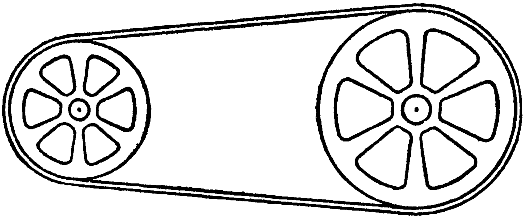


8. If $\angle a = 28°45'$, find $\angle \phi$; write a formula for ϕ in terms of a.
9. A square 10″ on a side is inscribed in a circle; find (a) the diameter, and (b) the circumference of the circle.
10. How many times must an automobile wheel with a 35-inch tire revolve in going half a mile?

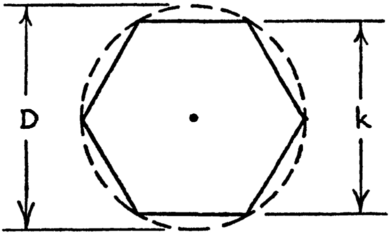


11. Write a formula for finding y when S is known. What is the value of y when $S = .866''$?
12. The rear wheel of a carriage is twice as large in diameter as the front wheel. How much longer is its circumference?
13. A circle 8″ in diameter is circumscribed about a square; find the distance across the flats of the square.

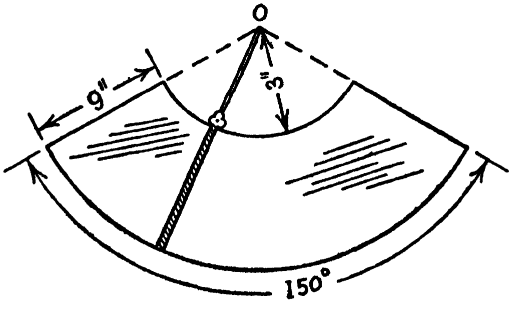
14. An arc of a circle measures 6.6″; if the diameter of the circle is 14″, find the central angle subtended by this arc.
15. A regular hexagon is inscribed in a circle with a diameter of 12″; find the distance across the flats of the hexagon.
16. Two pulleys in a machine shop are connected by a belt. One has a diameter of 18″ and the other a diameter of 12″. For each revolution of the large pulley, how many will the small pulley make?



17. Write a formula for finding k when D is known. What is the value of k when $D=3.12''$?
18. Through how many feet does the hub of a wheel travel during one revolution if the radius of the wheel is 8 ft.?
19. A squarehead nut is milled from round stock 1½″ in diameter; find the distance across the flats of the nut.

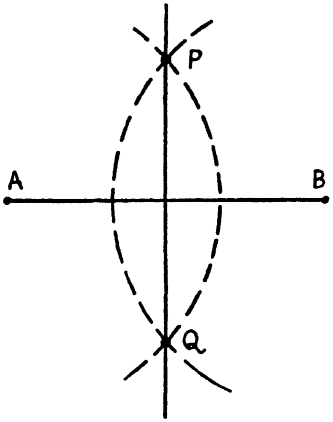


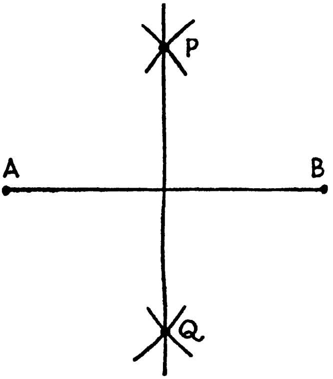
20. A flywheel 6 ft. in diameter turns at the rate of 120 revolutions per minute. What is the speed per minute of a point on the rim?
21. What diameter of round stock must be used to mill an hexagonal nut 1½″ across the flats?
22. A circular disc has a radius of 4.82″. How many inches in length is an arc on the edge of this disc if its subtended angle contains 156°?
23. A metal die is in the shape of an ellipse. If the major and minor axes are 6″ and 5″, respectively, find the perimeter of the ellipse.
24. How much further will one end of the windshield wiper travel than the other in swinging from left to right?
25. A thin cable is wrapped 14 times around the drum of a winch. If the diameter of the drum is 2½ ft., how many feet of cable are wound around the drum?



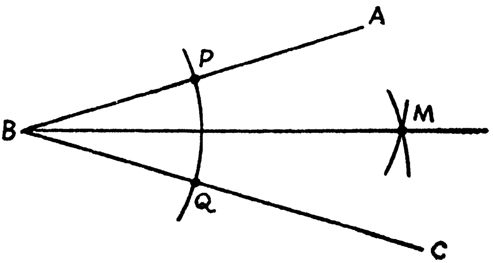
16. GEOMETRIC CONSTRUCTIONS

Bisecting a Given Line. By the term *geometric construction* is meant a drawing or layout made by using only a pair of compasses and a straight edge. A line may be bisected by swinging equal arcs from each end, and then connecting the points of intersection of the arcs by a straight line. Thus the ends of the line AB are used as centers to draw two arcs which intersect at P and Q; the line then drawn through points P and Q will bisect the line AB; it will also be perpendicular to it. Note that the radius used to swing the arcs must be greater than half the length of AB (why?); also, that it is not necessary to draw the arcs in full, but just enough to locate the points of intersection.

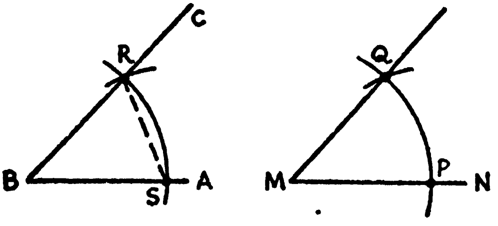




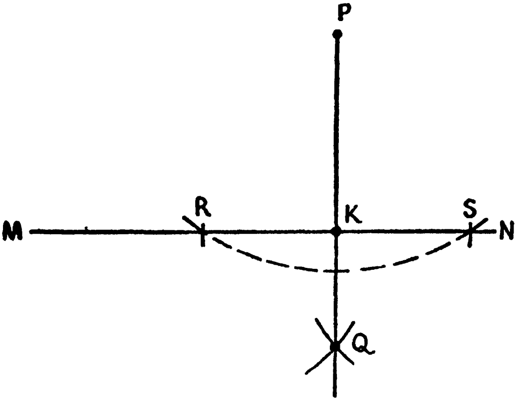
Bisecting a Given Angle. If it is desired to divide a given angle ABC into two equal parts, the vertex of the angle, B, is used as a center, and an arc is swung intersecting the sides of the angle in P and Q. Then using the points P and Q as centers, with any convenient radius, swing two equal arcs which intersect at M; draw a straight line connecting B with M. This line bisects $\angle ABC$.



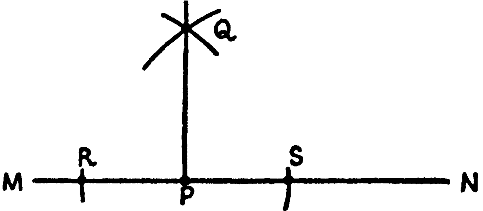
Constructing an Angle Equal to a Given Angle. If it is desired to transfer a given angle, i.e., construct an angle equal to a given angle, the procedure is as follows. Let ABC be the given angle which is to be transferred to the line MN. On ∠ABC, with B as a center, strike any convenient arc, intersecting the sides in R and S. With the same radius, and a center at M, strike another arc, intersecting MN in P. Then measure the *chord* RS; with this distance RS as a radius, and with P as center, strike another arc intersecting the larger arc in Q. Join M with Q. The angle NMQ equals the given angle ABC.



Erecting a Perpendicular from a Given Point outside a Given Line. If from the given point P it is required to construct a line perpendicular to the given line MN, take P as a center and a radius long enough to intersect MN in R and S. Then, with R and S as centers, respectively, strike two arcs with any convenient radius, intersecting in Q. Join P with Q. The line PQ is the perpendicular bisector of the segment RS (why?); hence PK is perpendicular to the given line MN, and, of course, passes through P, as required. (Note that PK does not necessarily bisect MN; also, that MN does not necessarily bisect PQ.)

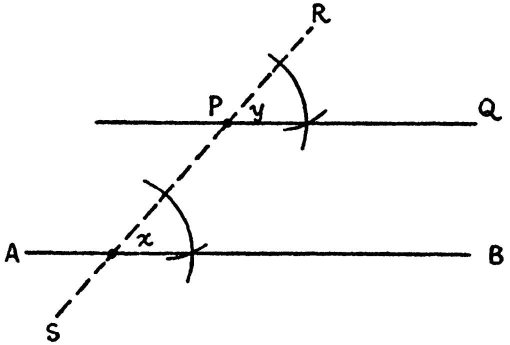


Erecting a Perpendicular at a Given Point on a Given Line. Should it be required to construct a line perpendicular to a given line MN at some particular point P *on* the line MN, the procedure is virtually the same. With P as center, and any convenient radius, strike two arcs intersecting MN in R and S. Then, with R and S as centers, and any convenient radius, strike arcs intersecting in Q;

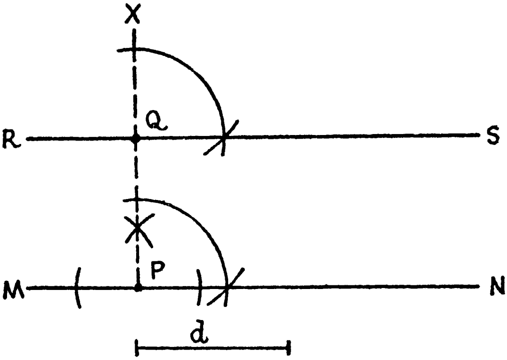


join P with Q. The line PQ is the required perpendicular. If the given point should happen to be at, or very near, either end of the given line, it would first be necessary to extend the line somewhat; then proceed as before.

Constructing a Line Parallel to a Given Line through a Given Point. It is desired to draw a line through P, and parallel to the line AB. Draw RS through P at any convenient angle to AB. Then construct $\angle y = \angle x$, using PR as one side of $\angle y$. The line PQ will be parallel to AB, and, of course, passes through P, as required.

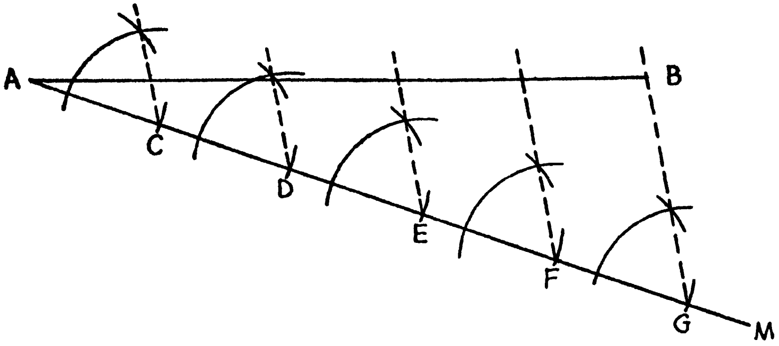


Constructing a Line Parallel to a Given Line at a Given Distance from the Line. It is desired to construct a line which will be parallel to the given line MN, and also at a given distance, *d*, from MN. The procedure follows. At any convenient point P on MN, erect a perpendicular PX, as described above. Then on PX, beginning at P, measure off a length PQ equal to the given distance *d*. Now through Q construct the line RS parallel to MN (by constructing a right angle at Q, using QX as one side). The line RS is the required line, for it is not only parallel to MN, but is also at the given distance *d* from MN.



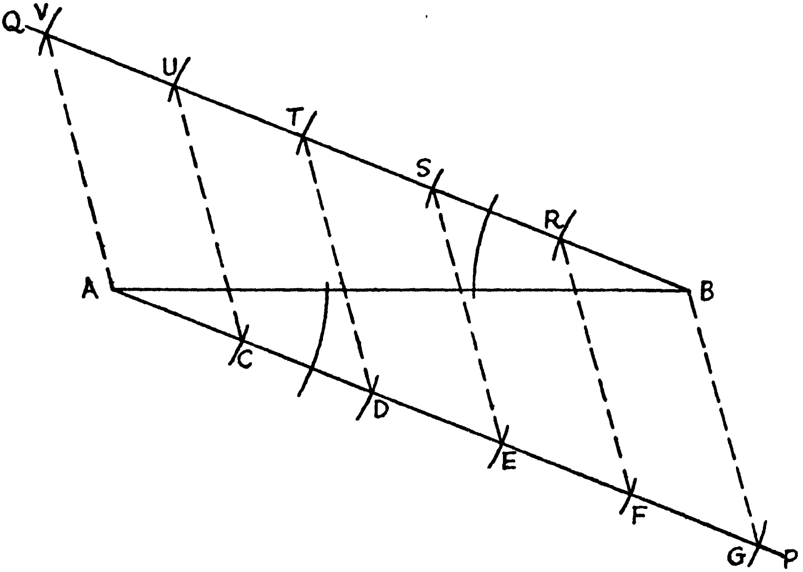
Dividing a Given Line into Any Number of Equal Parts. If it is desired to divide a line of given length into any number of equal parts, either of the following methods may be used.

(a) Suppose the line AB is to be divided into five equal segments; draw line AM at any angle to AB, and long enough to lay out the required number of divisions conveniently. Step off five equal segments on line AM,



starting at A; connect the last point (G) with point B. Then draw lines through C, D, E and F parallel to BG. The intersections of these lines with AB will divide it into the required number of equal parts.

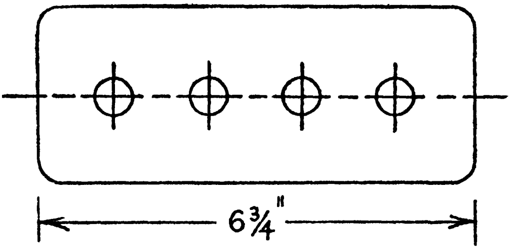
(b) The alternative method consists of drawing two lines, AP and BQ, parallel to each other, through A and B respectively, at any convenient

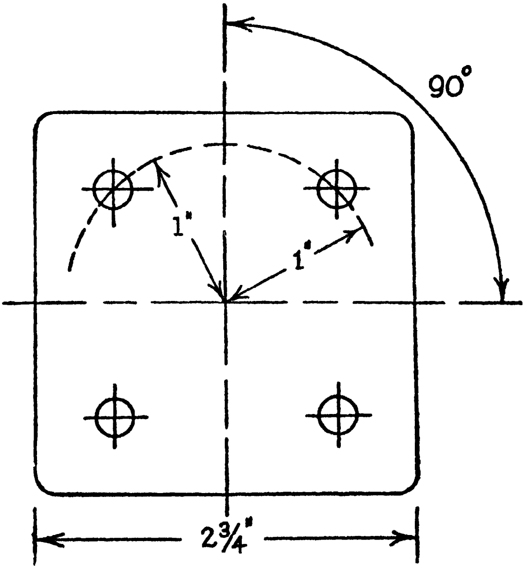


angle to AB. Then lay off the same number of equal segments on AP and BQ, starting in each case from A and B, respectively. Connect AV, CU, DT, ES, FR, and GB. The intersections of these lines with the line AB will divide it into the required number of equal parts.

Exercise 61.

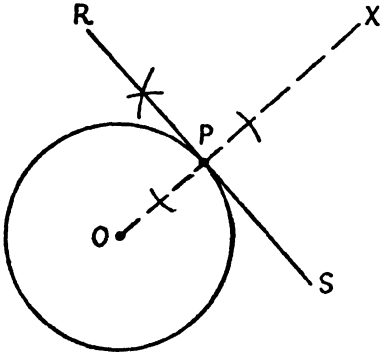
1. Draw a horizontal line 4⅝″ long, and bisect it geometrically; do the same for a vertical line 5³⁄₁₆″ long.
2. Lay out an angle of 70° with a protractor; then bisect the angle, showing all construction lines.
3. With a protractor, draw an angle of 110°; bisect this angle geometrically.
4. Lay out an angle of 27° with a protractor and bisect this angle.
5. Erect a perpendicular to a line 3¾″ long at a point 1½″ from either end of the line.
6. Erect a perpendicular to a line 3⅞″ long at either end of the line. (*Hint:* Extend the line first.)
7. Draw an oblique line 4¼″ long; then mark a point, anywhere, approximately 2″ from either side of the line. Now construct a perpendicular to the line from that point.
8. Draw a line 5⅜″ long and divide it into 7 equal parts.
9. Four equally spaced holes are to be drilled in the plate shown; the end holes are to be as far from the edge as the distance between the holes. Locate the centers by means of a geometric construction.
10. Construct an angle of 90°. Then construct an angle of 45° by bisecting the right angle. Construct an angle of 22½° by bisecting the 45° angle.
11. Lay out (with the protractor) an angle of 60°. Now construct, by bisection, an angle of 30°; of 15°; of 7½°.





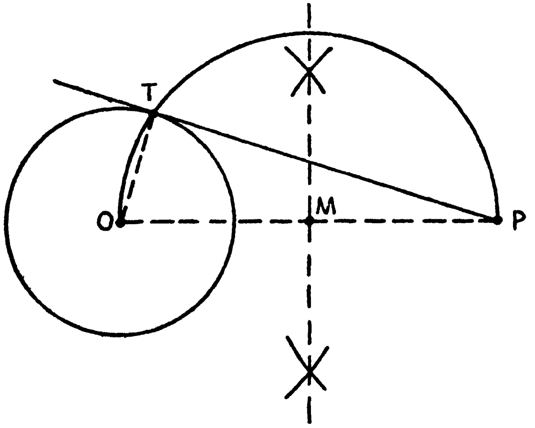
12. Draw an oblique line. Construct a line parallel to this line and at a distance of 1⅞″ from it.
13. Construct a square, 2¾″ on a side; locate its center. Then locate the centers of the four holes as indicated.
14. Draw any acute triangle. Construct (a) an altitude to the longest side; (b) a medium to the shortest side.
15. Construct a right triangle whose short sides are 1½″ and 2½″. Construct (a) the perpendicular bisectors of all three sides; (b) the altitude upon the hypotenuse.

Constructing a Tangent to a Circle at a Given Point on the Circle.

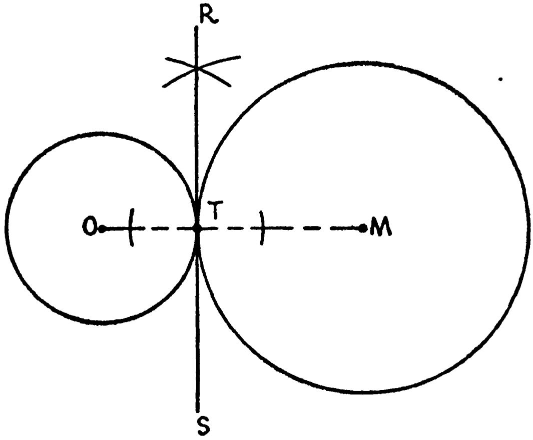


To the given circle O, it is required to construct a tangent touching the circle at the given point P. *Construction:* Draw the radius OP, and extend OP a convenient distance beyond the circle to X. Construct the line RS perpendicular to OX at P. The required tangent to circle O at point P is the line RS.

Constructing a Tangent to a Given Circle from a Given Point outside the Circle. *Construction:* Join the center O of the given circle with the given point P. Determine the midpoint M of OP by constructing the perpendicular bisector of OP. With M as center and MO as a radius, construct a semicircle on OP as diameter, intersecting the given circle in point T. Join T with P, and extend PT, which is the required tangent to the circle from the given point P. (why?) (*Hint:* What can you say of ∠OTP?) The other tangent to the circle from P could of course be constructed in the same way by completing the circle on OP as diameter.



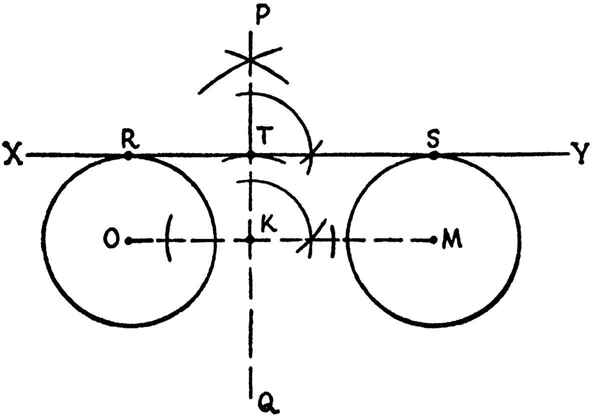
Constructing a Common Tangent to Two Externally Tangent Circles.



Construction: Join the centers O and M of the two given tangent circles. This line of centers will pass through the point of tangency, T. At T, construct a perpendicular to OM. The line RS is the required common tangent to the two circles.

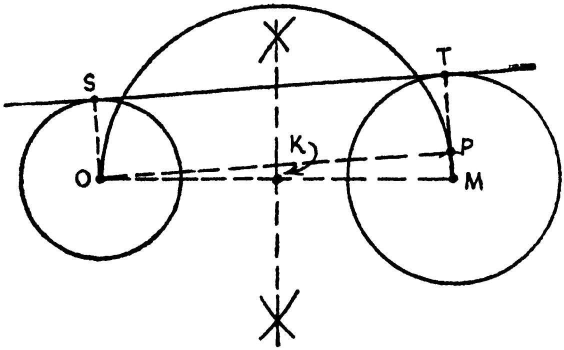
Constructing a Common External Tangent to Two Given Equal Circles.

Construction: Draw the line of centers, OM. At any convenient point on OM, say K, erect a perpendicular to OM, say PQ. On PQ, beginning at K, lay off a distance KT equal to the radius of either of the two equal circles. Through T construct a line XY parallel to OM (or perpendicular to PQ). The line XY will be tangent to the circles at R and S respectively, as required.



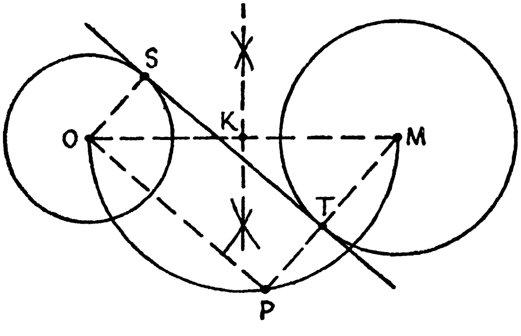
Constructing a Common External Tangent to Two Given Unequal Circles.

Construction: Draw the line of centers OM; bisect OM in K.



With K as center and OK as radius, construct a semicircle on OM as diameter. Then with M as center, and a radius equal to the *difference between the lengths of the radii of the given circles* (*R*—*r*), strike an arc intersecting the semicircle in P. Join M with P and extend to T, the intersection with the circle. Draw OP. Now construct a line through T parallel to OP and extend it; it will touch the other circle at S, and ST is the required external tangent. An alternative procedure would be, instead of constructing TS parallel to OP, to erect a perpendicular to OP at O, and extend this perpendicular to the circle at S; join S with T.

Constructing a Common Internal Tangent to Two Given Unequal Circles. *Construction:* Draw the line of centers OM; bisect OM in K. With K as center and OK as radius, construct a semicircle on OM as diameter. Then with M as center, and a radius equal to *the sum of the lengths of the radii of the given circles*(*R*+*r*), strike an arc intersecting the semicircle in P. Join M with P, and let MP intersect the given circle in T. Draw OP. Now construct a line through T parallel to OP and extend it; it will touch the other circle in S, and ST is the required internal tangent.



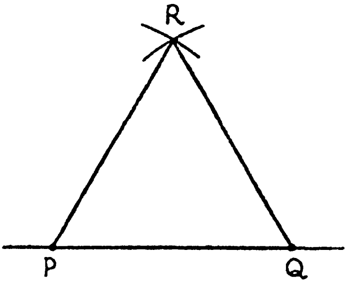
An alternative procedure would be, instead of constructing TS parallel to OP, to erect a perpendicular to OP at O, and extend this perpendicular to the circle at S; join S with T.

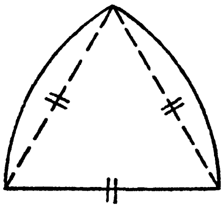
Exercise 62.

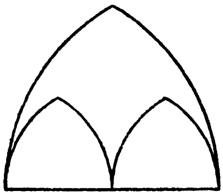
1. Draw a circle having a radius of 2½″; select any point on the circle and construct a tangent to the circle at that point.

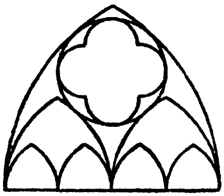
2. Draw a circle with a radius of 1⅞″ and extend any radius to a point P such that P is 3¼″ from the center. Construct a tangent to the circle from P.
3. Draw two externally tangent circles, one with a radius of 1½″, the other with a radius of 2¼″.
4. Construct two circles, each with a radius of ¾″, having their centers 3″ apart. Construct a common external tangent to the two circles.
5. Show by construction how two pulleys each with a 3″ diameter are connected (a) by an open belt; (b) by a crossed belt.
6. Show by construction how a 2″ pulley and a 3½″ pulley are connected by (a) an open belt; (b) a crossed belt.
7. Draw any obtuse triangle. Construct (a) the perpendicular bisectors of the three sides; (b) the circumscribed circle of the triangle.

Constructing an Equilateral Triangle. If it is desired to construct an equilateral triangle with a given length as its side, all that needs be done is to lay off a segment PQ equal to the required side; then, with P and Q as centers, respectively, and with PQ as a radius, strike off two arcs intersecting in R. Join R with P and with Q; the triangle PQR is the required equilateral triangle. This construction is the basis of the well-known Gothic arch of medieval times, commonly used in cathedrals and other public buildings.



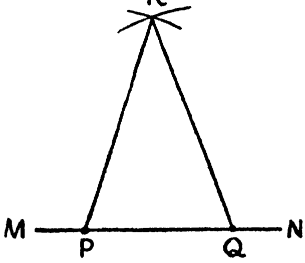




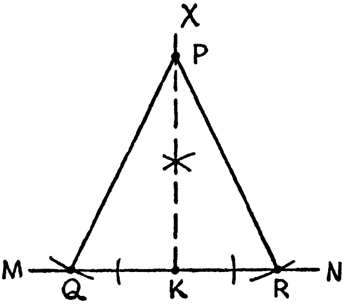


Constructing an Isosceles Triangle. Several possibilities may arise.

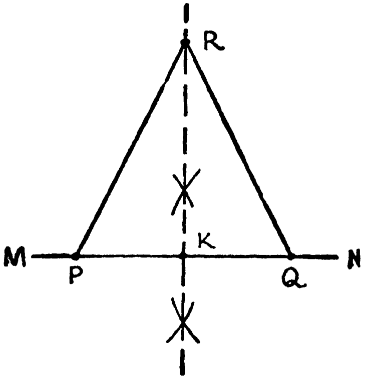
(1) *Given the base and the length of each of the two equal sides.* On any line MN lay off the required base, PQ. With P and Q as centers, respectively, and a radius equal to the given equal sides, strike two arcs intersecting in R. Triangle PQR is the required isosceles triangle.



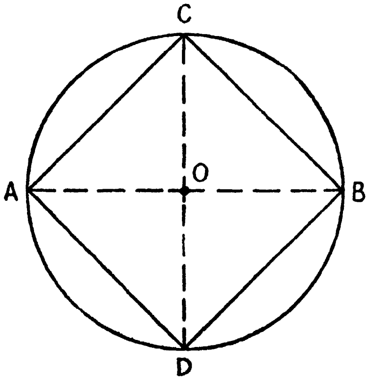
(2) *Given the altitude and the two equal sides.* At any convenient point K on any line MN, erect a perpendicular to MN and extend it to X. On KX, beginning at K, lay off a distance KP equal to the required altitude. With P as a center and a radius equal to the length of the required equal sides, strike two arcs intersecting MN in Q and R. Triangle PQR is the required isosceles triangle.



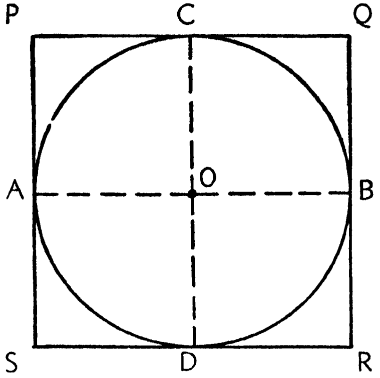
(3) *Given the base and the altitude.* On any line MN lay off the required base, PQ. Bisect PQ in K; at K erect a perpendicular to MN. Beginning at K, lay off a distance KR equal to the required altitude. Join R with P and with Q. Triangle PQR is the required isosceles triangle.



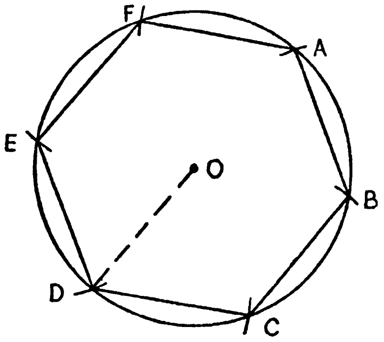
Inscribing a Square in a Given Circle. Draw any convenient diameter AB; then construct another diameter, CD, perpendicular to AB. Join points A, C, B, and D; the resulting figure is the required inscribed square.



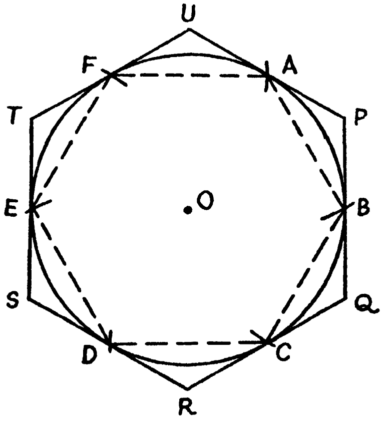
Circumscribing a Square about a Given Circle. Construct two perpendicular diameters, AB and CD. At each of the four extremities A, B, C, and D construct a tangent to the circle. The figure PQRS is the required circumscribed square.



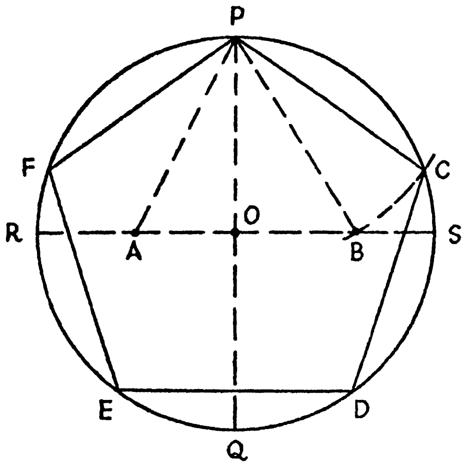
Inscribing a Regular Hexagon in a Given Circle. Starting with any point A on the given circle, and using a radius equal to the radius OA of the given circle, strike off successive arcs at B, C, D, E and F. Join the six points A, B, C, D, E and F; the resulting figure is the required inscribed hexagon. Each side of this hexagon is equal in length to the radius of the circle in which it is inscribed.



Circumscribing a Regular Hexagon about a Given Circle. Divide the given circle, as above, into six equal arcs, the points of division being A, B, C, D, E and F. At each of these points of division construct a tangent to the circle. The figure PQRSTU is the required circumscribed hexagon.



Inscribing a Regular Pentagon in a Given Circle. Construct any two perpendicular diameters, such as RS and PQ. Now bisect RO, and join the midpoint A with point P. With a radius equal to PA, lay off AB=PA; join P and B. Then with a radius equal to PB, and with P as center, strike an arc BC, cutting the circle in C. The chord PC is the side of the required pentagon; step off arc PC five times, beginning at P, and the figure PCDEF will be the desired regular inscribed pentagon.



Exercise 63.

1. Construct an equilateral triangle 1½″ on a side.
2. Construct an angle of 60°; by bisection, construct an angle of 30°; 15°.
3. Show how to construct (with a protractor) an angle of 120°; 75°; 105°; 67½°; 135°; 150°; 165°.
4. Construct an isosceles triangle having a base 2½″ and its equal sides each 3½″ long.
5. Construct an isosceles triangle with a base of 3″ and an altitude of 1¼″.
6. Construct an isosceles triangle with an altitude of ¾″ and equal sides of 1½″ each.
7. Construct a Gothic arch on a base line of 1½″.
8. Construct an equilateral triangle 1⅜″ on a side; then construct the inscribed and circumscribed circles.
9. Construct an isosceles triangle with a base 2¼″ long and base angles of 30° each.
10. Construct an isosceles triangle with equal sides of 1″ each and a vertex angle of 120°.
11. Construct a 30°—60°—90° triangle with an hypotenuse of 2⅞″.
12. Construct an isosceles right triangle 1¼″ on each of the two equal sides.
13. Construct the largest possible square in a 1¾″ circle.
14. Show by construction the largest square which can be milled from a piece of 1½″ round stock.
15. Show by construction the largest hexagon that can be milled on a 2″ round shaft.

16. Draw a circle with a radius of 1½″. Inscribe a rectangle in this circle, having a width of 1″.
17. Construct a 30°—60°—90° triangle such that its shortest side is 1½″ long; now construct its inscribed and circumscribed circles.

17. MEASUREMENT OF AREAS

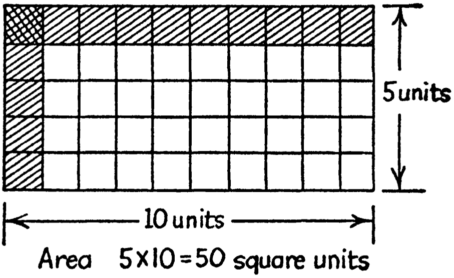
Area of Squares and Rectangles. Surface measure, or the measure of area, is expressed in square units, such as the square inch, the square foot, or the square centimeter. Thus the area of a plane geometric figure may be described as the number of units of surface measure contained in the figure in question. A square inch is defined as a square, each of whose sides measures one linear inch; similarly for a square foot, etc. It is readily seen that the area of a rectangle may be found simply by multiplying the length of the rectangle by its width; or, expressed as a formula:

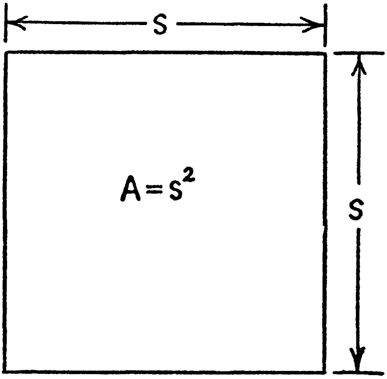
$$A=lw.$$

For a square, this becomes

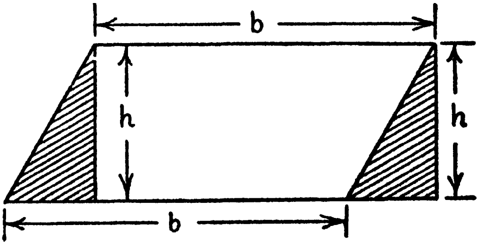
$$A=s\times s,$$

or $A=s^2$.

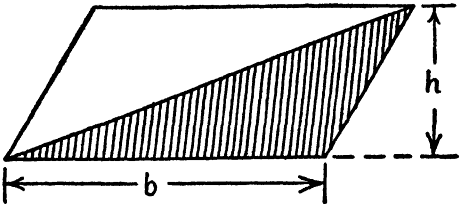




Area of a Parallelogram. In the case of a parallelogram, the area is found by multiplying the base by the altitude, since from the diagram it is clear that the shaded triangles are equivalent in area; also, the parallelogram is equivalent in area to the rectangle, which has the same base and altitude, respectively, as the parallelogram. Hence the area of a parallelogram is $A=bh$.



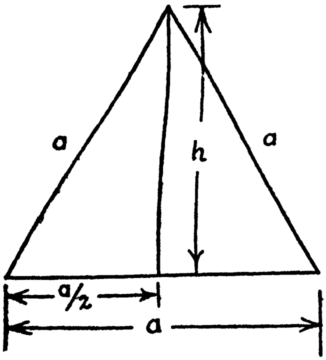
Area of a Triangle. When it is remembered that a parallelogram is divided into two equivalent triangles by either of its diagonals, one method of computing the area of a triangle is readily suggested. The base of the shaded triangle is equal to the base of the parallelogram, and the altitude of this triangle is equal to that of the parallelogram; therefore, since the area of the triangle is half that of the parallelogram, the area of the triangle is half of the base times the altitude; or, $A=\frac{1}{2}bh$.



To use this formula it is necessary to know the length of the altitude drawn to one of the three sides, as well as the length of that side. This may not always be known or conveniently measured. If, for example, we know only the lengths of each of the three sides, the following formula may be used instead:

$A=\sqrt{(s)\ (s-a)\ (s-b)\ (s-c)}$, where a, b, and c represent the lengths of the three sides, respectively, and s equals the *semiperimeter*, or $\frac{1}{2}(a+b+c)$. Thus, if the sides of a triangular sheet of metal measure 8″, 12″, and 16″, the area of the sheet equals $A=\sqrt{(18)\ (18-8)\ (18-12)\ (18-16)}=\sqrt{(18)\ (10)\ (6)\ (2)}=\sqrt{2160}=$ 46.4 sq. in. As a special case, the area of an equilateral triangle yields another formula; since in this case $a=b=c$, the above formula becomes:

$$A=\sqrt{\left(\frac{3a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}=\frac{a^2}{4}\sqrt{3},$$

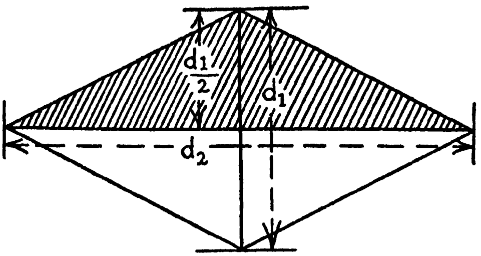


where a represents any one of the three equal sides. The same result can also be obtained by using the altitude and base directly, as follows:

$$h=\frac{a}{2}\sqrt{3};$$

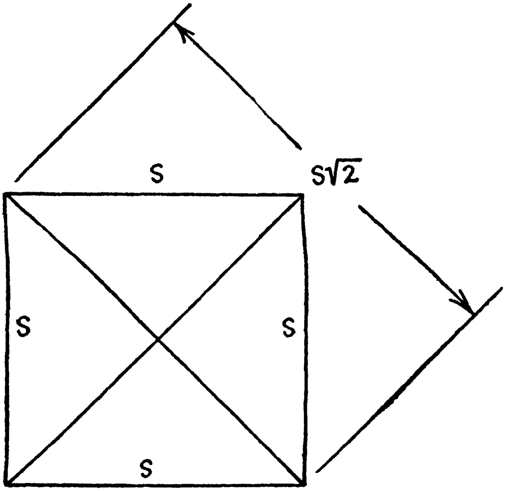
but $A=\frac{1}{2}(\text{base})\times(\text{altitude})$; therefore $A=\frac{1}{2}(a)\left(\frac{a}{2}\sqrt{3}\right)$, or $A=\frac{a^2}{4}\sqrt{3}$, as before. Thus, if the side of an equilateral triangle is 6″, its area equals $\frac{36}{4}\times\sqrt{3}=9\times1.73=15.6$ sq. in.

Area of a Rhombus in Terms of the Diagonals. Since the diagonals of a rhombus not only bisect each other, as in all parallelograms, but inter-

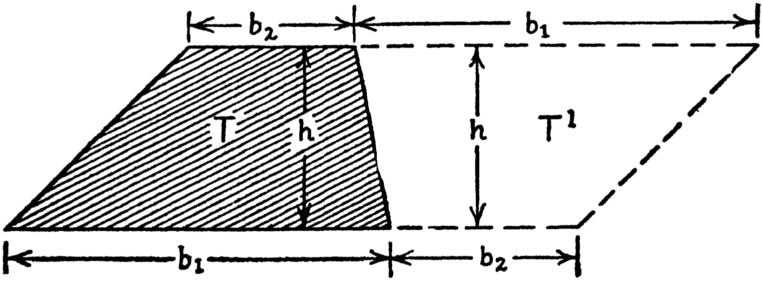


sect at right angles as well (which is true only in a rhombus or a square, i.e., an equilateral parallelogram), then the base of the shaded triangle $=d_2$ and its altitude$=\frac{d_1}{2}$; hence the area of the shaded triangle is $\frac{1}{2}(d_2)\left(\frac{d_1}{2}\right)$, or $\frac{1}{4}d_1d_2$. Therefore the area of the entire rhombus (being twice as large as the shaded triangle) is given by: $A=2(\frac{1}{4}d_1d_2)$, or $A=\frac{1}{2}d_1d_2$.

An interesting illustration of this relation is found by applying it to a square, where each diagonal equals the other, and is given (as we have already seen) by $d=s\sqrt{2}$, where s represents the side of the square. Applying the above formula, then, we obtain: A (of a square)$=\frac{1}{2}(s\sqrt{2})\times(s\sqrt{2})$, or $A=s^2$, as before.



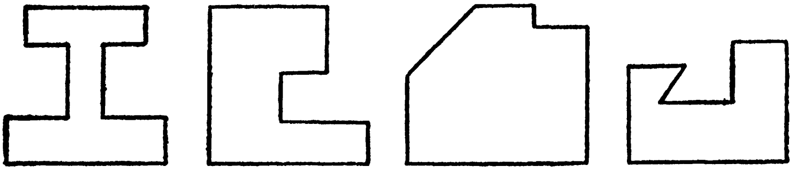
Area of a Trapezoid. The area of the entire parallelogram is evidently $h(b_1+b_2)$; since either trapezoid is half the area of this parallelogram,

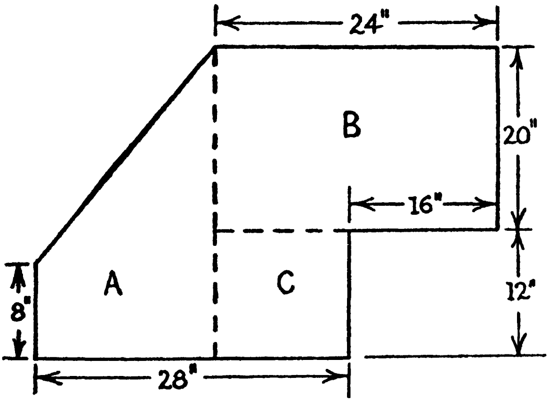


then the area of the trapezoid must be given by $A=\frac{1}{2}h(b_1+b_2)$. *Since the* median of a trapezoid equals one half the sum of its bases, the area of a trapezoid is also equal to the altitude times the median; or,

$$A=hm, \text{ where } m=\tfrac{1}{2}(b_1+b_2).$$

Irregular Structural Shapes. Metal plates and other structural parts are often in the shape of irregular polygons as here shown. If there are no rounded corners or other curved lines in the figure (which is frequently the case, however) the area may be computed by decomposing the figure into convenient rectangles, triangles, trapezoids, etc., taking the appropriate measurements, finding the area of each part, and then simply adding them together.



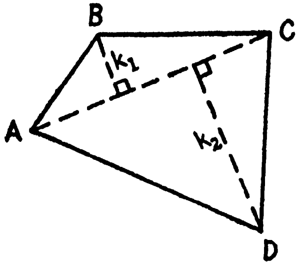


Example: Find the area of the iron plate shown, with dimensions as given.

Solution: Break up the original figure into trapezoid A and rectangles B and C.

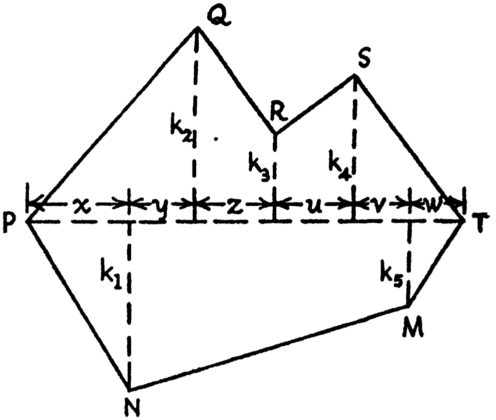
Trapezoid	$A=\frac{1}{2}h(b_1+b_2)=\frac{1}{2}(20)(8+32)$	$=400$
Rectangle	$B=\ 24\times20$	$=480$
Rectangle	$C=\ 12\times\ 8$	$=\ 96$
	Total figure	$=976$ sq. in., *Ans.*

A somewhat more general method, particularly for irregularly shaped figures, consists of drawing a convenient diagonal, dropping and measuring the perpendiculars to this diagonal from each remaining vertex, and measuring the projections of these perpendiculars upon the diagonal as well; with this data it is a simple matter to find the area of each of the separate triangles and trapezoids into which the figure has been decomposed, as suggested below.



Area of ABCD

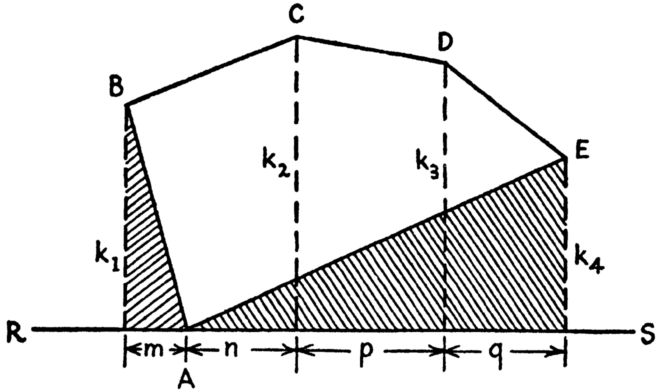
$$=\frac{1}{2}(AC)k_1+\frac{1}{2}(AC)k_2$$
$$=\frac{1}{2}(AC)(k_1+k_2)$$



Area of MNPQRST

$$=\frac{1}{2}(x+y)k_2+\frac{1}{2}(z)(k_2+k_3)+\frac{1}{2}(u)(k_3+k_4)+\text{etc.}$$

Usually the modified method of using a base line (RS) instead of the diagonal is slightly more convenient. Thus the area of ABCDE is given by the sum of the areas of the three trapezoids diminished by the sum of the areas of the two triangles; or,

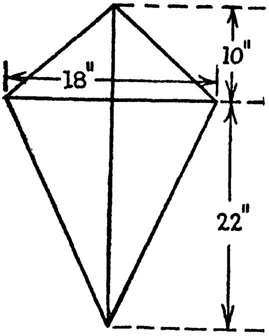


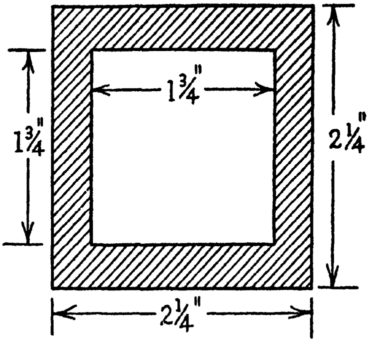
$$A=\frac{1}{2}(m+n)(k_1+k_2)+\frac{1}{2}(p)(k_2+k_3)$$
$$+\frac{1}{2}(q)(k_3+k_4)-\frac{1}{2}mk_1$$
$$-\frac{1}{2}(n+p+q)(k_4).$$

Exercise 64.

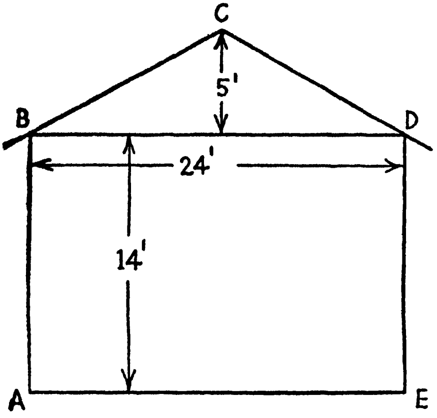
1. Find the area of a 30°—60°—90° triangle whose hypotenuse is 14″.
2. Find the area of an isosceles triangle whose base is 6″ and whose equal sides are each 8″ long.

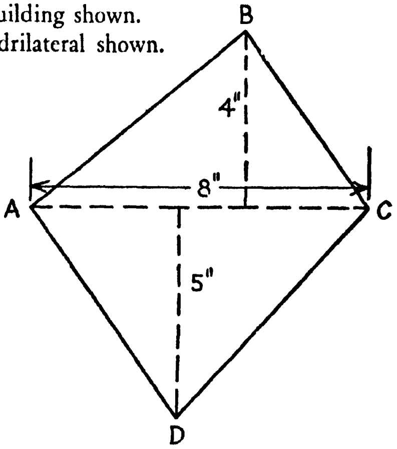
3. The sides of a parallelogram are 6″ and 12″, and its diagonal is 16″. What is its area?



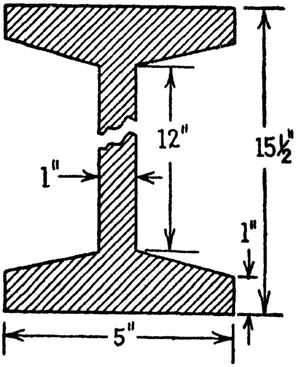


4. How many square inches of paper are needed to cover this kite frame?
5. What is the area of a fiber gasket cut in the dimensions shown?
6. Find the area of the front of the building shown.
7. Find the area of the irregular quadrilateral shown.



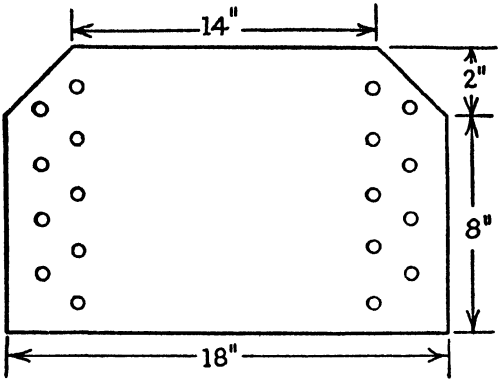


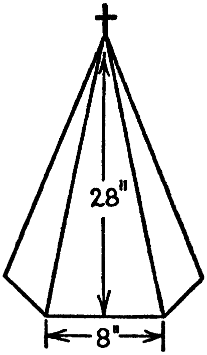
8. What is the cross-sectional area of the beam shown in the sketch?



9. Find the total lateral surface of this 8-sided church steeple.

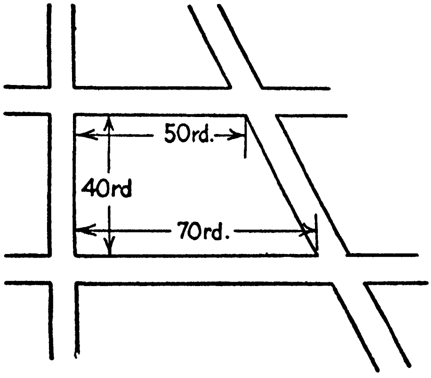
10. A steel plate riveted to a girder has the shape and dimensions shown. Find its area.

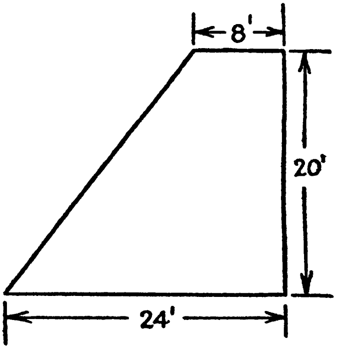




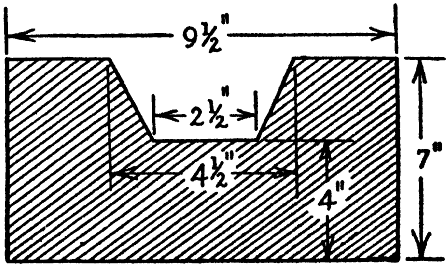
11. A plot of land is bounded by intersecting roads as shown. (a) How many square rods does the plot contain? (b) How many acres is this? (One acre=160 sq. rd.)

12. The cross section of the dam of a reservoir has the dimensions shown. What is the surface area of the end of the dam?

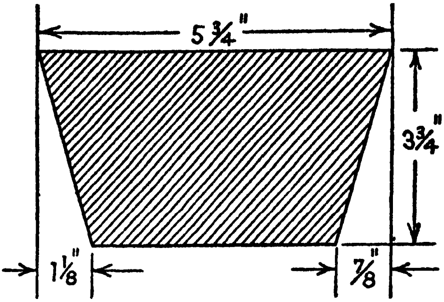


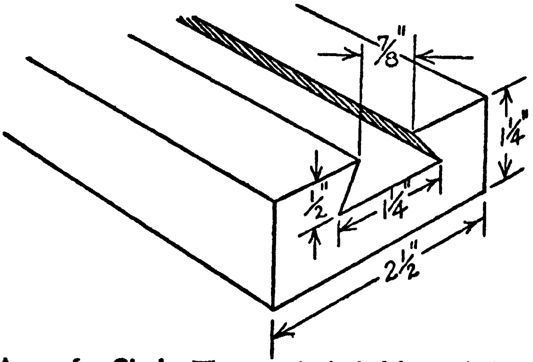


13. Find the area of the cross section of the steel channel beam shown.



14. The cross section of a fixture on a lathe is a trapezoid with the dimensions shown; what is the area of its face?

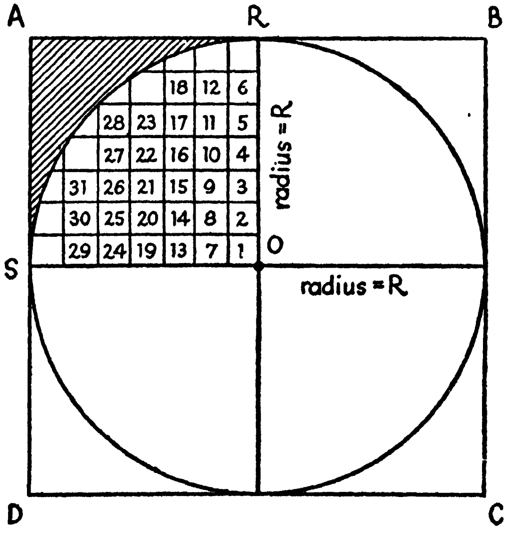




15. Find the cross-sectional area of the wooden dovetail shown in the figure.

Area of a Circle. The area included by a circle is found by squaring the radius and multiplying by 3⅐; i.e.,

$$A=\pi R^2\text{, or, since } R=\frac{D}{2},\ A=\frac{\pi D^2}{4}=.7854D^2.$$



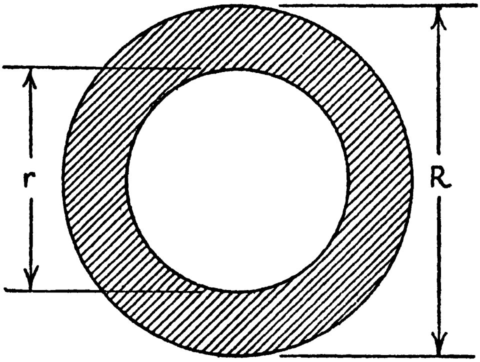
Each side of the square is 14 units long; so is the diameter. The radius is 7 units in length. Now count the number of small squares in the quadrant ORS. There are 31 whole squares. Try to estimate the equivalent number of whole squares in the remaining imperfect squares. If you are careful, you will probably estimate 7 or 8 more; if you are very skillful, you will obtain approximately 7½ more, or about 38½ in all. Now, since a quadrant is ¼ of a circle, the entire circle contains 4×38½, or about 154 square units. Let's call the area of the circle A. Since R=7, then $R^2=49$. By counting the small squares, we found that A=154. Now divide 154 by 49:

$$\frac{A}{R^2}={}^{154}\!/_{49}=3\tfrac{1}{7}=\pi, \text{ showing}$$

$$\text{that } \frac{A}{R^2}=\pi, \text{ or } A=\pi R^2, \text{ as above.}$$

Area of a Ring. The area included between two concentric circles is called a *ring,* and is obviously found by deducting the area of the smaller circle from that of the larger, i.e.,

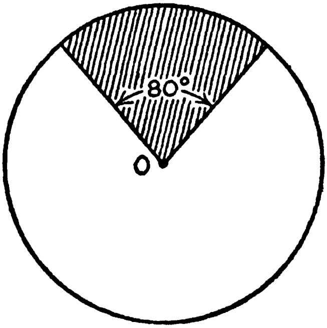
$$A=\pi R^2-\pi r^2, \text{ or}$$
$$A=\pi(R^2-r^2).$$



Area of a Sector. A sector of a circle is a figure bounded by a central angle and its subtended arc. The area of a sector (like the length of an arc) may easily be found by proportion; for

$$\frac{\text{Area of Sector}}{\text{Area of Circle}}=\frac{\text{Central Angle}}{360^\circ};$$

$$\text{or, Sector}=\frac{\text{Central Angle}}{360^\circ}\pi R^2.$$

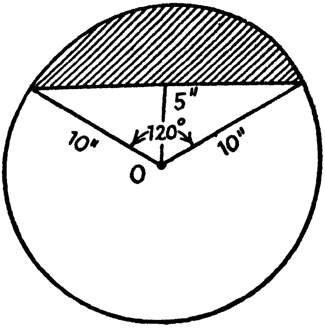


EXAMPLE: Find the area of an 80°-sector in a circle with a diameter of 7 inches.

SOLUTION: $R=3\tfrac{1}{2}$

$$\text{Sector}=\left({}^{80}\!/_{360}\right)\left({}^{22}\!/_{7}\right)\left({}^{7}\!/_{2}\right)\left({}^{7}\!/_{2}\right)=$$
8.56 sq. in., *Ans.*

Area of a Segment. A segment of a circle is that portion bounded by a chord and its subtended arc. Hence its *area may be found by deducting the area of the triangular portion from the entire* area of the corresponding sector.

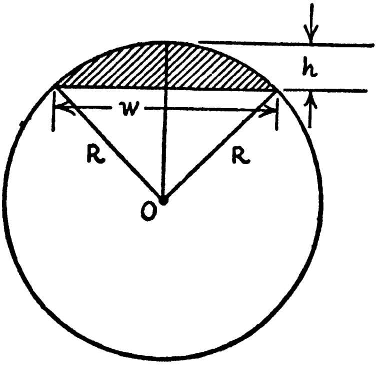


Example: In a circle with a 10″ radius, find the area of a segment whose arc is 120°.

Solution:

$$\text{Sector} = \left(\tfrac{120}{360}\right)(3.14)(100) = 104.7$$
$$\text{Triangle} = \tfrac{100}{4}\sqrt{3} = 43.3$$
$$\text{Segment} = \text{Sector} - \text{Triangle} = 61.4 \text{ sq. in.}, \textit{ Ans.}$$

If the span w only is given, as well as the radius, but not the central angle, the procedure is as follows: First the value of the height h is determined from either of the following formulas, whichever is more convenient:



(1) $h = R - \sqrt{R^2 - (\tfrac{1}{2}w)^2}$

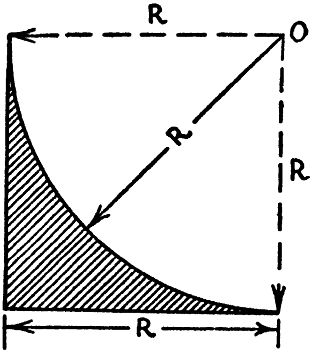
(2) $h = \tfrac{1}{2}(D - \sqrt{D^2 - w^2})$

The area may then be found by using the formula:

$$A = \frac{h(\tfrac{4}{3}w^2 + h^2)}{2w}$$

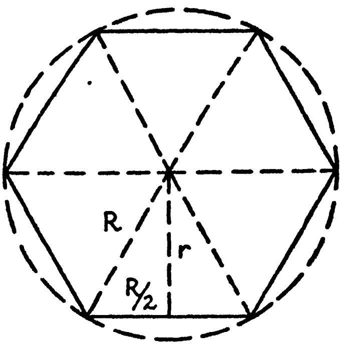
A *fillet* is a structural piece, the cross section of which is bounded by two adjacent sides of a square and the quadrant whose center is the opposite vertex of the square, as shown in the accompanying figure. The area of such a cross section is simply obtained by subtracting the area of the sector, which is one-fourth that of the circle, from the area of the square; thus

$$A = R^2 - \tfrac{1}{4}\pi R^2.$$

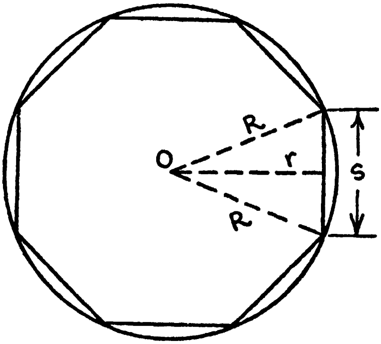


Area of Regular Polygons. The area of the square and the equilateral triangle have already been discussed. The only other regular polygon frequently encountered in the machine shop is the regular hexagon. Its area is simply six times that of one of the six congruent equilateral triangles of which it is made up, each side of the regular hexagon being equal to the radius of the circumscribed circle; thus

$$A=\frac{3R^2\sqrt{3}}{2}$$



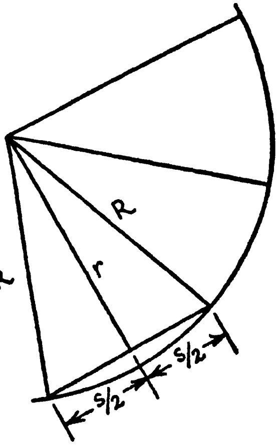
A general way of expressing the area of any regular polygon is in terms of its sides and apothem: $A=\frac{1}{2}(ns)r=Pr$,



where n=number of sides, s=length of each side, and r=the apothem. Of course, if any two of the measurements R (radius), r (apothem) and s (length of side) are known, the third could always be found by the right-triangle rule. This is not always too convenient, however, and trigonometric methods are more practical, as will be seen later. Thus, since

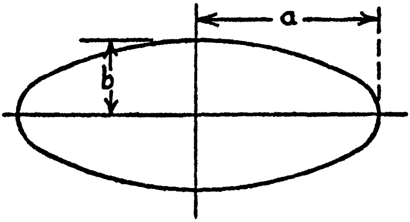
$$r=\sqrt{R^2-\frac{s^2}{4}},\text{ then}$$

$$A=\frac{1}{2}(ns)\sqrt{R^2-\frac{s^2}{4}}.$$

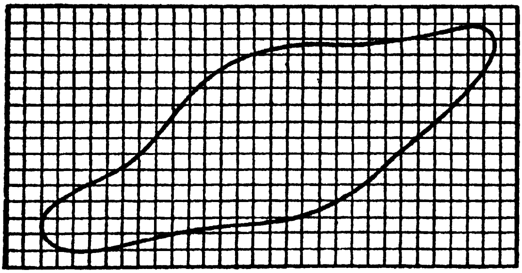


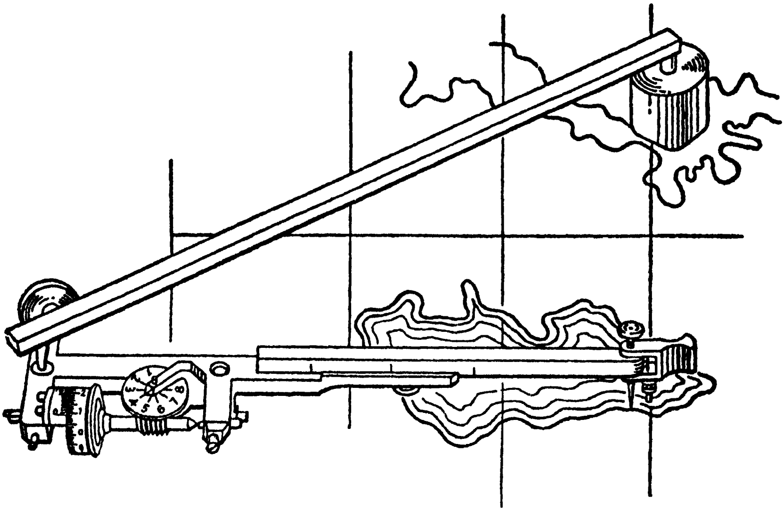
$$R^2=r^2+\left(\frac{s}{2}\right)^2$$

Area of an Ellipse. The area enclosed by an ellipse is found by multiplying the product of the major axis by the minor axis by π; or, $A = \pi ab$. If $a = b$, the ellipse becomes a circle, and $A = \pi aa = \pi a^2$, as already seen.

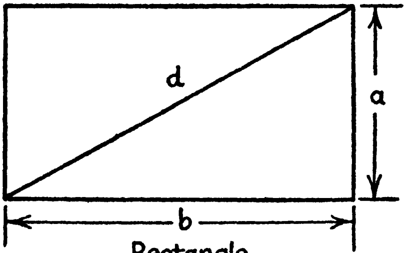


Irregular Areas. Not infrequently it is desired to obtain the area of an irregularly shaped figure, such as a plot of ground, the cross section of a specimen of material, a steam engine indicator, etc. Two methods are available in such cases: (1) an approximate value of the area may be found by drawing the figure to scale against a ruled area, and then counting and estimating the number of square units included within the area. If a more accurate value of the area is required, a *planimeter* may be used. This is a mechanical device which computes the area automatically as the needle of the instrument is allowed to trace the perimeter of the figure in question.





SUMMARY OF FORMULAS

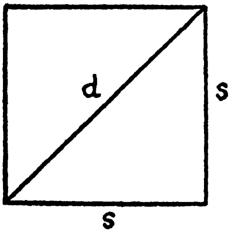


Rectangle

$P=2(a+b)$

$A=ab$

$d=\sqrt{a^2+b^2}$



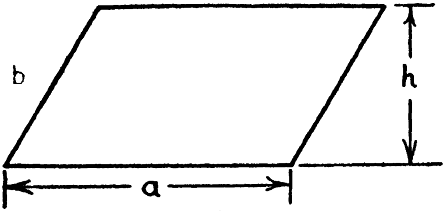
Square

$P=4s$

$A=s^2$

$d=s\sqrt{2}$

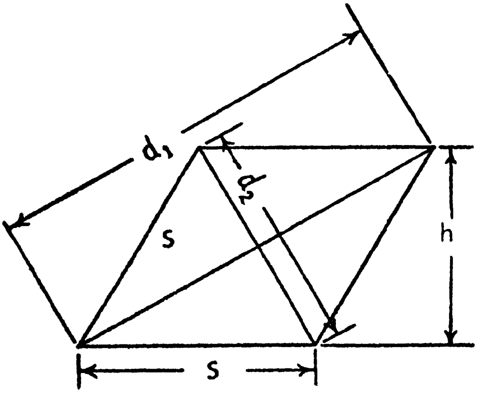
$s=\frac{1}{2}d\sqrt{2}$



Parallelogram

$P=2(a+b)$

$A=ah$

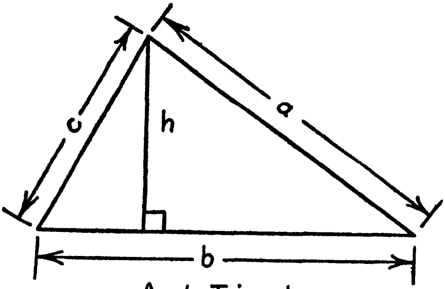


Rhombus

$P=4s$

$A=sh$

$A=\frac{1}{2}d_1d_2$



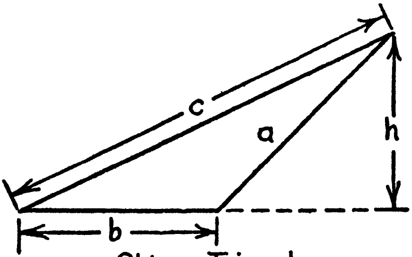
Acute Triangle

$P=a+b+c$

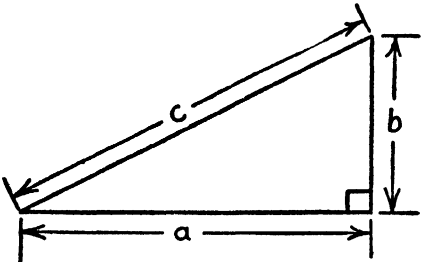
$A=\frac{1}{2}bh$

$A=\sqrt{s(s-a)(s-b)(s-c)}$

where s=semiperimeter



Obtuse Triangle



Right Triangle

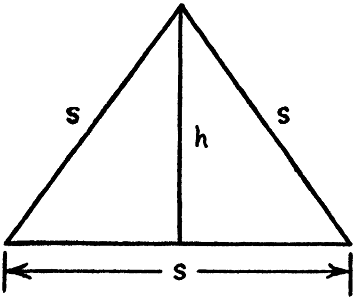
$P=a+b+c$

$A=\frac{1}{2}ab$

$c=\sqrt{a^2+b^2}$

$a=\sqrt{c^2-b^2}$

$b=\sqrt{c^2-a^2}$

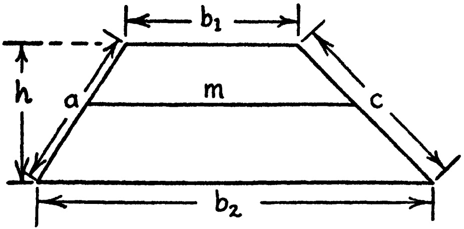


Equilateral Triangle

$P=3s$

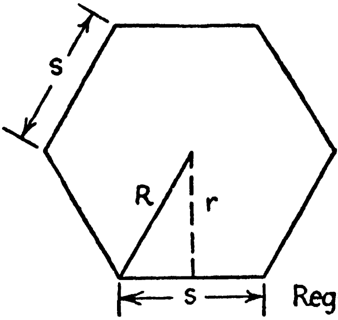
$A=\frac{s^2}{4}\sqrt{3}$

$h=\frac{s}{2}\sqrt{3}$



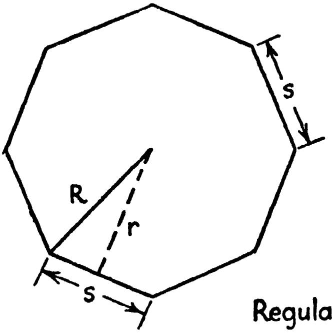
Trapezoid

$$P=a+c+b_1+b_2$$
$$A=\tfrac{1}{2}h(b_1+b_2)$$
$$m=\tfrac{1}{2}(b_1+b_2)$$
$$A=\tfrac{1}{2}mh$$



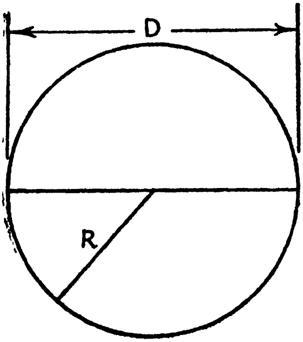
Regular Hexagon

$$P=6s$$
$$R=s$$
$$r=\frac{s}{2}\sqrt{3}$$
$$A=\frac{3s^2}{2}\sqrt{3}$$



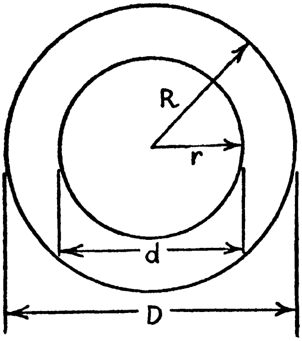
Regular n-gon

$$P=ns$$
$$A=\tfrac{1}{2}Pr=\tfrac{1}{2}nsr$$
$$A=\tfrac{1}{2}(ns)\sqrt{R^2-\frac{s^2}{4}}$$
$$r=\sqrt{R^2-\frac{s^2}{4}}$$



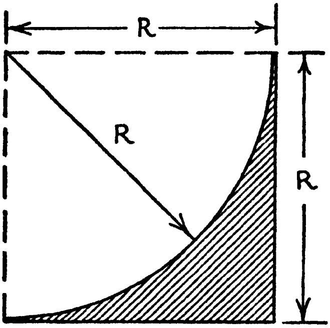
Circle

$$C=2\pi R=\pi D$$
$$A=\pi R^2=\frac{\pi D^2}{4}=.7854D^2$$



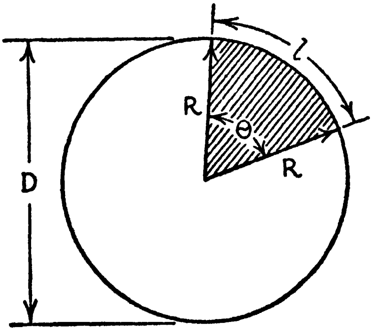
Ring

$$A=\pi(R^2-r^2)$$
$$A=\pi(R+r)\ (R-r)$$
$$A=\frac{\pi}{4}(D+d)\ (D-d)$$



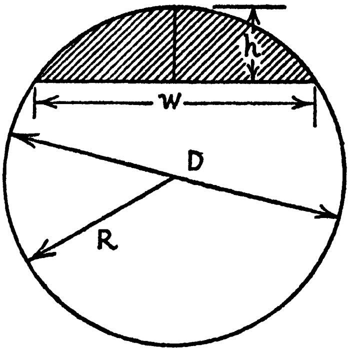
Fillet

$$P=2R+\frac{\pi R}{2}$$
$$A=R^2-\frac{\pi R^2}{4}$$



Sector

$$l\ (\text{arc})=\frac{\pi R\theta}{180}=\frac{\pi D\theta}{360}$$
$$A=\frac{\theta}{360}(\pi R^2)$$
$$A=\frac{\theta}{360}\left(\frac{\pi D^2}{4}\right)$$

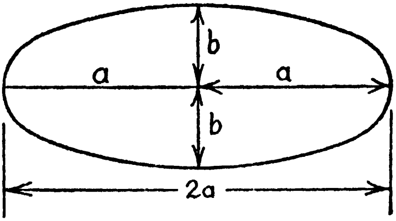


$$h = R - \sqrt{R^2 - (\tfrac{1}{2}w)^2}$$

$$h = \tfrac{1}{2}(D - \sqrt{D^2 - w^2})$$

$$A = \frac{h(\tfrac{4}{3}w^2 + h^2)}{2w}$$

Segment



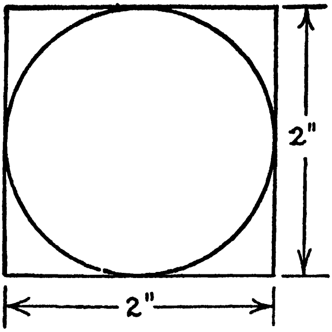
$P = \pi\sqrt{2(a^2 + b^2)}$ (approximately)

$A = \pi ab$

Ellipse

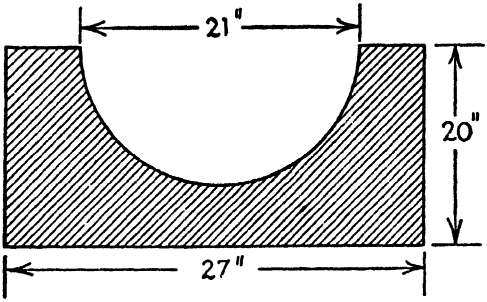
Exercise 65.

1. A circular-shaped rod is to be milled from a piece of square stock 2″ on a side. Find the area of the cross section of the finished rod.



2. Find the side of a square equivalent to a circle one foot in diameter.

3. Find the cross-sectional area of the semicircular wooden trough shown in the diagram.



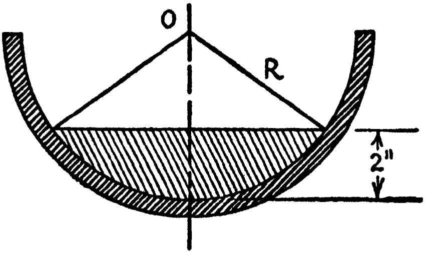
4. If the diameter of a circle is 40″, what is the area of a segment whose arc is 120°?

5. The diameters of two circles are 4″ and 6″, respectively. Find (a) the ratio of their circumferences; (b) the ratio of their areas.

6. What per cent of any circle is wasted if the largest possible hexagon is cut out of it?

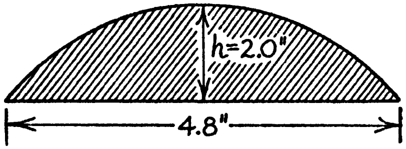
7. Find the cross-sectional area of the metal in a hollow shaft measuring 3½″ outside diameter and 2″ inside diameter.

8. The inside diameter of a horizontal pipe is 10½″. Water is standing to a depth of 2″. Find the cross-sectional area of water in the pipe.

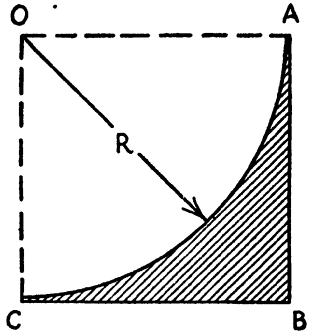


9. A wooden cylinder with a circular cross section is to be planed down to have the largest possible square cross section. If the diameter of the original rod is 3½″, what per cent is wasted in shavings?

10. A metal stamping has the shape of a circular segment with a span of 4.8 in. and a height of 2.0″. Find its area.



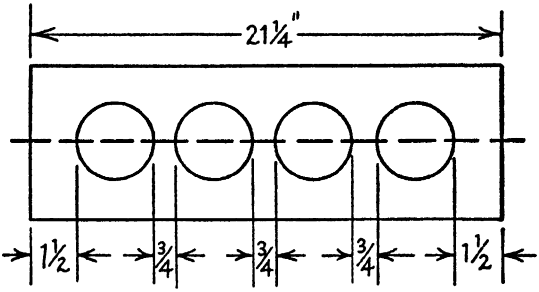
11. Find the area of the shaded fillet shown, where the radius of the fillet is 4″, and OABC is a square.



12. Find the area of a segment having for its chord a side of a regular inscribed hexagon, if the radius of the circle is 10½″.

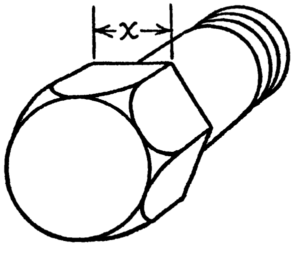
13. The end section of a gasoline tank is an ellipse with a major axis of 28″ and a minor axis of 22″. Find the area of this cross section of the tank.

14. The gasket of an auto engine is as shown; find the area of the metal wasted when the holes are punched out.



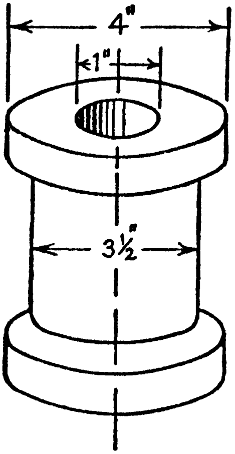
15. In a steam engine having a piston 18″ in diameter, the pressure upon the piston is 84 lb. per sq. in. Find the total pressure on the piston.

16. If $x=1\frac{1}{2}''$, what is the diameter of the round stock needed to make the hexagonal head of this machine stud?



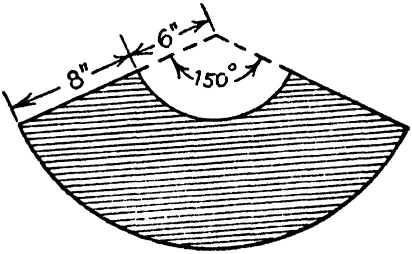
17. If a 3½" circular piece of rubber is cut from a sheet that is 4" square, what per cent of the material is left?

18. An 8"-pipe supplies only ⅔ the amount of water required in a given time. Assuming that the flow is proportional to the cross-sectional area, find the diameter of the smallest pipe that will supply the required amount, if the pipe sizes come only in diameters of whole numbers of inches.



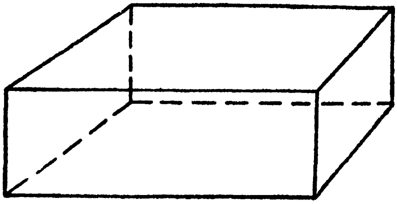
19. Find the area of the end of this spool before the hole is drilled; after the hole is drilled.

20. A parchment lampshade is to be made from a piece of material cut according to the pattern shown; find the area of the parchment required.

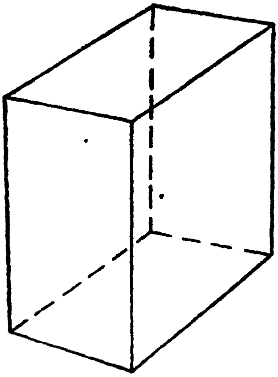


18. MEASUREMENT OF SOLID FIGURES

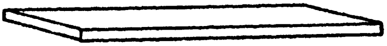
Rectangular Solids. A *rectangular solid* is a solid figure with 6 *faces* and 12 *edges;* all of the faces are rectangles, being, in fact, three pairs of congruent rectangles, and all the edges are perpendicular to the faces which they meet. The six faces are usually referred to as the "top and bottom," or the *bases;* as the "front and back,"

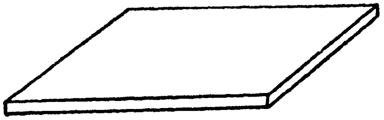


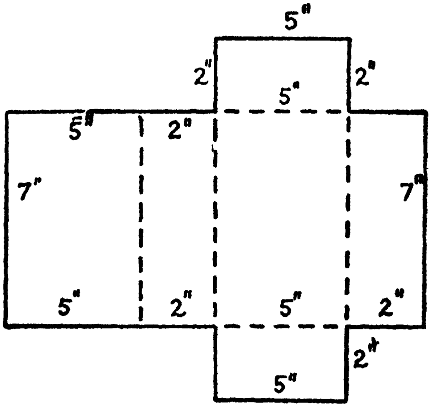
and as the "two ends," the last four constituting the *lateral* faces. Such figures are also known as rectangular ***prisms,*** but there are other types of prisms, too, as will be seen shortly. The three different edges are often known as the ***length, width*** and ***height*** (or thickness), although it is sometimes better to refer to the perpendicular distance between the bases as the *altitude* instead of the height or thickness.



Rectangular-solid shapes are met in shop practice in various proportions. Thus metal bars, planks, and joists are usually rectangular solids with one of the three "dimensions" much greater than the other two, as suggested in the diagram; thus we speak of a "two by four" which is 8 ft. long, meaning a joist 2 in.×4 in.×8 ft. Or again, in machine-shop work, frequent use is made of flat, rectangular ***plates*** and ***sheets*** of metal; if rectangular "pieces" are cut out, they are still rectangular solids, no matter how thin. A rectangular solid may be "developed" according to the following pattern, which will help to understand its geometric properties. In other words, by cutting along the solid lines, and folding along the dotted lines, a rectangular solid would be formed.







Area and Volume of a Rectangular Solid. The *lateral area* of a rectangular solid with dimensions l, w, and h is equal to $2lh+2wh$,

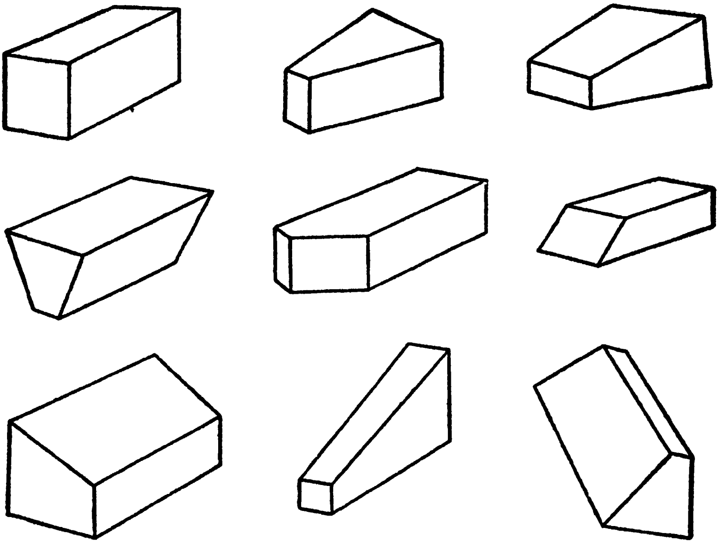
or, $L.A.=2h(l+w)$,

or, $L.A.=ph$, where $p=$ the perimeter of the base.

The ***total area*** is the lateral area plus the area of the two bases, or

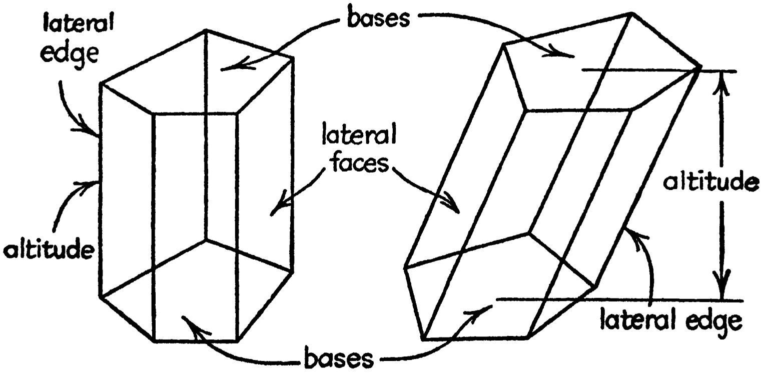
$$T.A.=2lh+2wh+2wl.$$

The *volume* of a rectangular solid, which has already been discussed, equals the product of its three dimensions; thus $V=lwh$,
or $V=Ah$, where A is the area of either base.

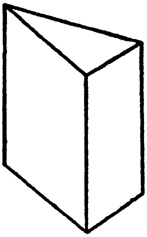


TYPICAL SHAPES OF FIRECLAY AND SILICA BRICK
(for high temperature work in the construction of ovens, furnaces and kilns.)

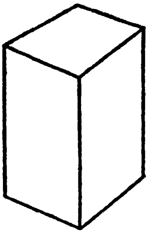
Prisms. Solids bounded partly by a series of flat surfaces (faces) whose intersections are parallel lines, and partly by a pair of additional parallel surfaces (bases) each of which intersects the series of lateral faces are called *prisms.* Such solids may be *right prisms* or *oblique prisms,* according as their *lateral edges* are perpendicular to the *bases* or not. The lateral



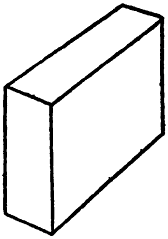
faces of a prism are always parallelograms; they may or may not be congruent, and they may or may not be rectangles. But their lines of intersection are always parallel, whether oblique or perpendicular to the bases. The bases are congruent polygons, parallel to one another, and each base intersects all the lateral faces. These bases may be triangles, squares,



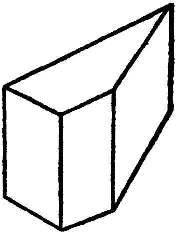
Triangular Prism



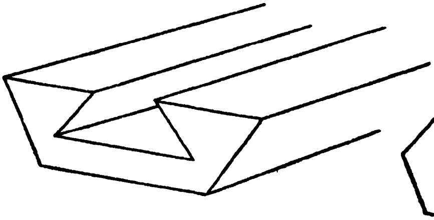
Square Prism



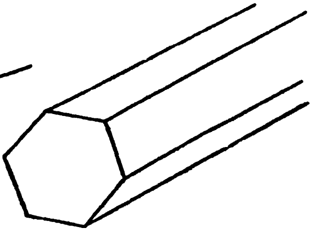
Rectangular Prism



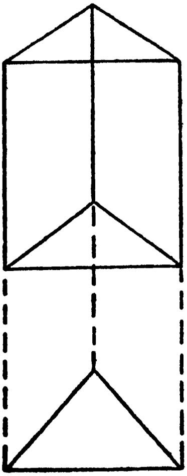
Trapezoidal Prism

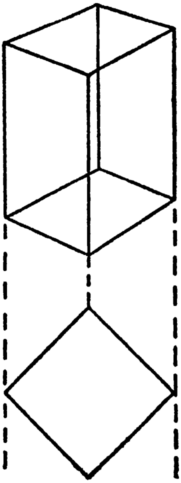


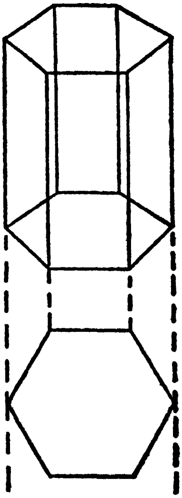
Grooved Prism



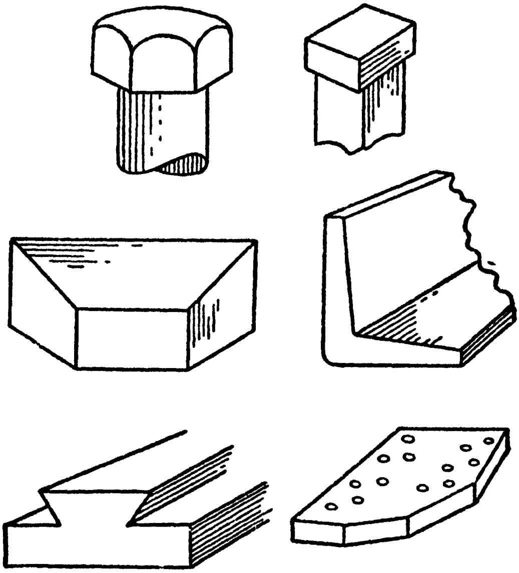
Hexagonal Prism





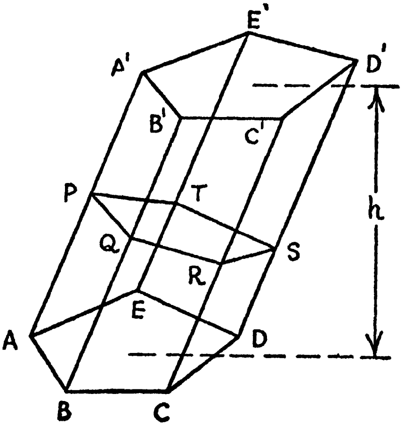


hexagons, etc. The *altitude* of any prism is the perpendicular distance between the two bases. Thus in a right prism the altitude is equal in length to any lateral edge, since all the lateral edges are perpendicular to the bases; in an oblique prism the altitude is less than the length of a lateral edge. In both types of prisms, however, all the lateral edges are equal in length.

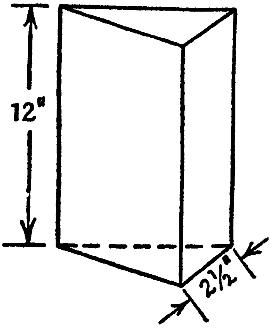


A *regular prism* is any right prism whose bases are regular polygons. In such prisms every non-lateral edge, i.e., base edge, equals every other non-lateral edge. In connection with mechanical work, many parts, such as strips, plates, beams, bars, sheets, angle irons, and the like are in the shape of regular prisms, or partly so, at any rate.

The *lateral area* of any regular prism equals the perimeter of the base times the altitude; the *total area* then equals the lateral area plus twice the area of the base. The *volume* of any regular prism equals the area of the base multiplied by the altitude. For specific types of prisms see the diagram above, or the summary of formulas near the end of this section. If, on the other hand, the figure is an oblique prism, its lateral area can be found by computing the area of each face separately, or by multiplying the perimeter of a right section by the length of a lateral edge; or *L.A.*= $(PQ+QR+RST+\cdots)\times AA'$. The volume is still equal to the area of $(ABCDE)\times h$.



Example 1: Find the volume of a right prism whose base is an equilateral triangle 2½″ on a side and whose altitude is 12″.



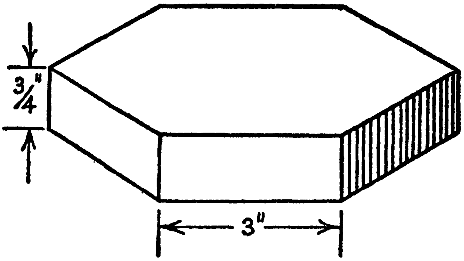
Solution:

$$V=Bh;\ B=\frac{s^2}{4}\sqrt{3}$$

$$B=\left(\frac{1}{4}\right)\left(\frac{25}{4}\right)\sqrt{3}=\frac{25}{16}(1.73)=2.70$$

$$V=Bh=(2.7)(12)=32.4 \text{ cu. in.}, \textit{Ans.}$$

Example 2: What is the lateral area of a regular hexagonal prism, ¾" on a lateral edge, and 3" on each edge of the base?



Solution:

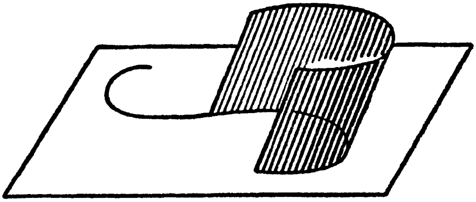
$$L.A.=(6)(3)\left(\frac{3}{4}\right)=13.50$$

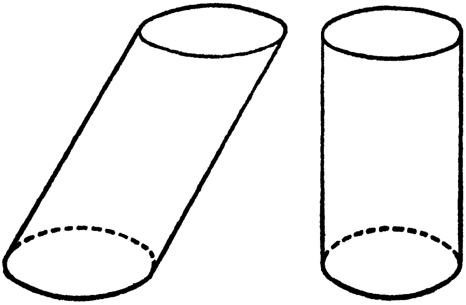
Area of base=

$$\left(6\right)\left(\frac{3^2}{4}\right)\sqrt{3}=23.36$$

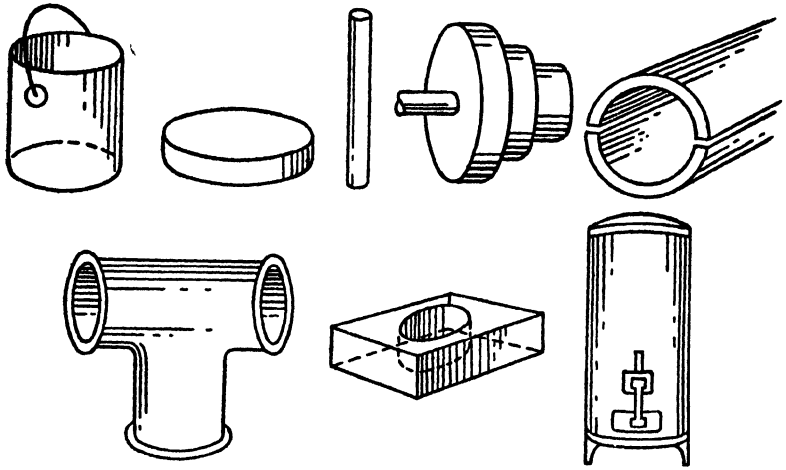
$$T.A.=13.50+(2)(23.36)$$
$$=60.2 \text{ sq. in.}, \textit{Ans.}$$

Cylinders. A *cylindrical surface* is one that has been generated by a straight line moving in such a way that it follows a fixed curve in a plane, while maintaining a constant angle to the plane. If the fixed curve (directrix) whose path is followed is a circle, the surface generated is a circularly cylindrical surface; if in addition the "generating line" remains perpendicular to the plane, it is a *right* circular cylindrical surface. A solid formed by such a surface *and* two parallel planes intersecting the cylindrical surface is called an *oblique circular cylinder* or a *right circular cylinder,* respectively. An approximate illustration of an oblique elliptical cylinder would be the shape of some steamship funnels.

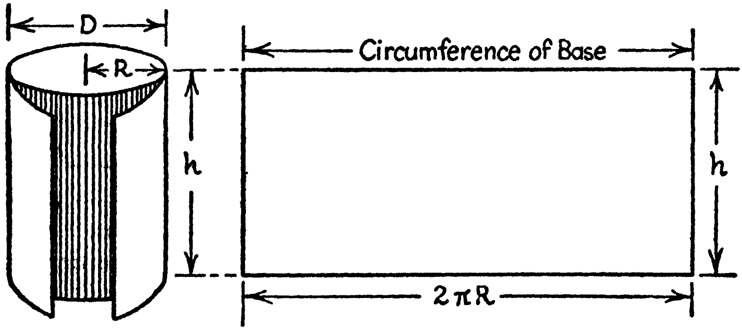




Right Circular Cylinders. Many common objects are in the form of a right circular cylinder: spools, drums, tubes, pipes, discs, pulleys, wheels, containers, cans, etc. In a right circular cylinder, the bases are equal circles, there are no "lateral edges," and the altitude is the perpendicular distance between the two bases. If the altitude of a right circular cylinder is relatively small compared to its diameter, it is usually called a ***disc;*** if the altitude is considerably longer than its diameter, it is called a ***rod*** or a ***tube,*** according to whether it is solid or hollow.



The lateral area of a right circular cylinder will be seen from the following diagram to be given by



$$L.A. = 2\pi Rh = \pi Dh;$$

and the total area is

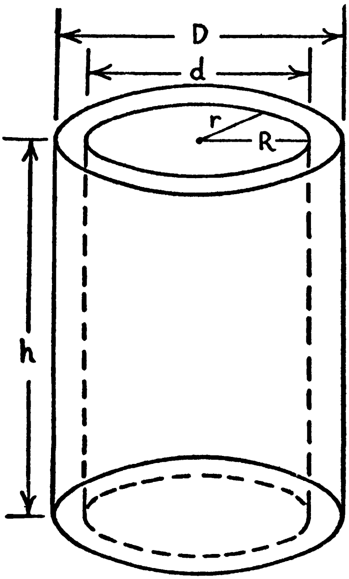
$$T.A. = 2\pi Rh + 2\pi R^2$$

$$\text{or, } T.A. = 2\pi R(h+R)$$

$$= \pi D\left(h + \frac{D}{2}\right)$$

The volume of a right circular cylinder, like that of a right prism, equals the area of the base multiplied by the altitude; or

$$V = \pi R^2 h = \frac{\pi D^2}{4} h.$$



Hollow Cylinders. For such figures the following formulas will be found useful:

$$L.A.\ (inside) = 2\pi r h$$
$$L.A.\ (outside) = 2\pi R h$$
$$End\ area\ (one) = \pi(R^2 - r^2)$$
$$V = \pi h(R^2 - r^2)$$
$$= \frac{\pi h}{4}(D^2 - d^2)$$

Fillet. The volume of a fillet is given by the product of its cross-sectional area multiplied by its length (or height).

EXAMPLE 1: Find the contents in gallons of a cylindrical tank 7 ft. in diameter and 14 ft. high, if filled to a level 9½ ft. above the bottom.

SOLUTION: $V = \pi R^2 h$

$$= \left(\frac{22}{7}\right)\left(\frac{49}{4}\right)\left(\frac{19}{2}\right) = 365.75 \text{ cu. ft.}$$

1 cu. ft.=7½ gal.

365.75×7½=2743 gal., *Ans.*

EXAMPLE 2: What is the area of the inside surface of an open cylindrical tank measuring 3½ ft. in height and 1½ ft. in diameter?

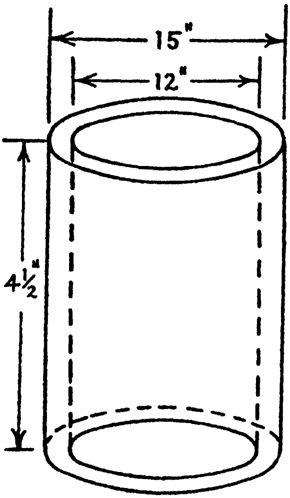
SOLUTION: $L.A. = 2\pi R h$

$$= 2\left(\frac{22}{7}\right)\left(\frac{3}{4}\right)\left(\frac{7}{2}\right) = 16.5 \text{ sq. ft.}$$

$$B = \pi R^2 h$$

$$= \left(\frac{22}{7}\right)\left(\frac{9}{16}\right)\left(\frac{7}{2}\right) = 6.2 \text{ sq. ft.}$$

Total area=22.7 sq. ft., *Ans.*



Example 3: Find the weight of a section of cylindrical metal casting 4½ ft. long, with an inside diameter of 12" and an outside diameter of 15",.if the material weighs 0.24 lb. per cu. in.

$$V=\frac{\pi h}{4}(D^2-d^2)$$

$$V=\left(\frac{22}{7}\right)\left(\frac{9}{2}\right)\left(\frac{12}{4}\right)(225-144)$$

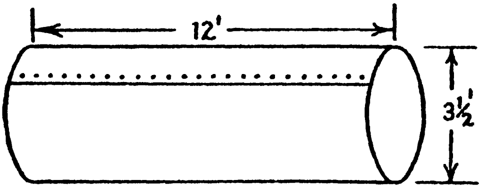
$V=3435$ cu. in.

Weight$=3435\times0.24=824$ lb., *Ans.*

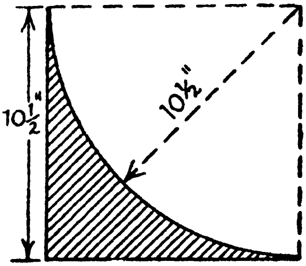
Exercise 66.

(*For reference:* 1 gal.=231 cu. in.; 1 cu. ft.= 7½ gal.; 1 cu. ft. water=62½ lb.)

1. What is the weight of a rectangular steel bar 48"×4"×1½", if the metal weighs .25 lb. per cu. in.?
2. Allowing 4" for overlapping at the seam along its entire length, how many sq. ft. of galvanized iron are required for this section of metal pipe?

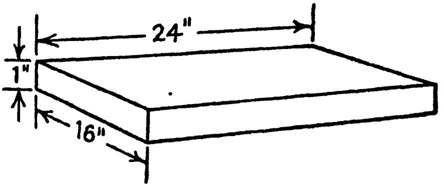


3. A fillet with the cross section shown is 14 ft. in length. How many cu. ft. of material are there in 20 such pieces?

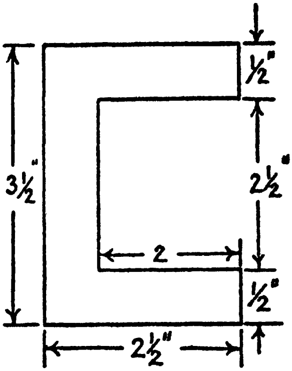


4. Brass discs ½" thick are to be machined from 4" diameter stock. (a) How many cu. in. of metal are there in each disc? (b) What is the weight of each disc if brass weighs .31 lb. per cu. in.?
5. An irregular piece of metal is placed in a cylindrical vessel 3 in. in diameter and partly full of water. If the water rises 3½ in., what is the volume of the piece of metal?

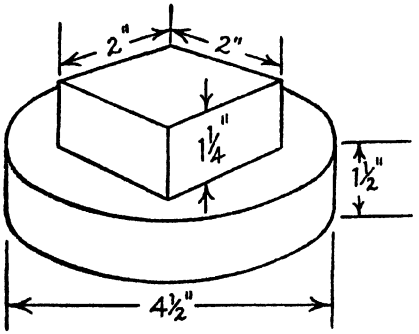
6. At 40¢ per lb., what is the cost of copper required to make 24 open pans with the dimensions shown, if sheet copper weighs .8 lb. to the square foot, and 10% is allowed for lapping and waste?

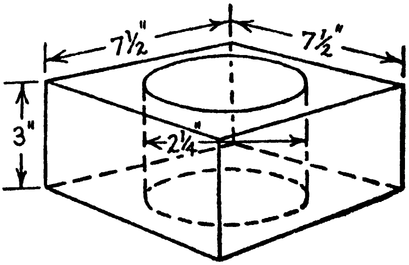


7. Find the weight of a cast-iron cylinder 2.80 in. in diameter and 2 ft. long, if cast iron weighs 448 lb. per cu. ft.
8. Hollow metal bars in the shape of a regular hexagon in 12 ft. lengths are to be plated with a special chromium finish. If the cross section of the bar is a regular hexagon ¾″ on a side, find the total surface to be plated on 100 such bars.

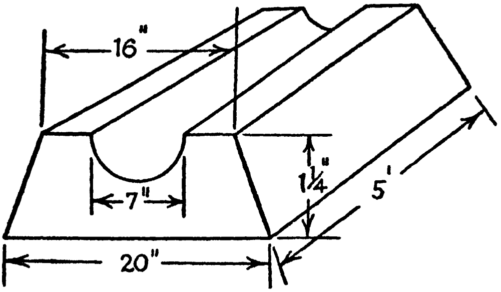


9. Laminations for an electrical machine are made of pieces of sheet iron with the shape and dimensions shown. What is the weight of 1000 such pieces, if the iron used weighs .48 lb. per sq. ft.?
10. A cylindrical hot water boiler has a diameter of 14″ and stands 4′ 8″ high. How many gallons will it hold when full?
11. A rectangular block of gold metal measures 10″×8″×2″. If it is hammered and rolled into sheets 8″ square and .02″ thick, how many such sheets will be obtained?
12. Find the volume of a cylindrical bronze ring having an inside diameter of 4⅜″ and an outside diameter of 6¼″, if it is ¾″ thick.
13. Find the volume of the solid iron plug with dimensions as shown.
14. A circular hole is drilled through the metal block as shown; what is the volume of the piece after the hole has been drilled?



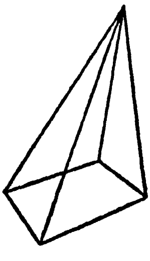


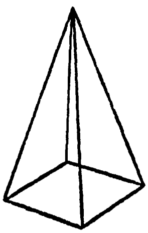
15. A rod of copper 14 ft. long and 2 sq. in. in cross section is melted and formed into a wire ⅛ in. in diameter. Find the length of the wire.
16. Concrete blocks like the one shown in the sketch are made in molds. If the sections are 5 ft. long and 7¼ in. high, how many cu. yd. of concrete are required for 48 such sections?

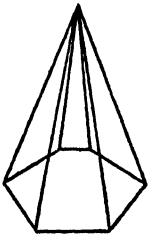


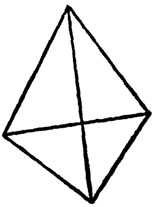
17. If 2 gallons of water are poured into a cylindrical jar 7 inches in diameter, how high will the level of the water rise?
18. Sections of reinforced concrete water main are hollow cylinders 12 ft. long and 4″ thick. If the inside diameter is 48″, and the concrete weighs 105 lb. per cubic foot, what is the weight of one such section?
19. A cylindrical paint can 14 in. high holds exactly 2 gallons. What is the diameter of the can?
20. In a building there are 2100 ft. of steam piping with a 12″ outside diameter. How many sq. ft. of pipe surface does this represent?
21. If a bar of metal 2″ in diameter weighs 10.2 lb. per foot of length, what is the weight per running foot of a bar 3″ square of the same material?
22. A tunnel whose cross section is a semicircle 21 ft. high is ¼ mile long. How many cu. yd. of earth were removed when it was excavated?

Pyramids. Any solid figure, one of whose faces is a polygon and whose other faces are all triangles having a common vertex, is a *pyramid*.

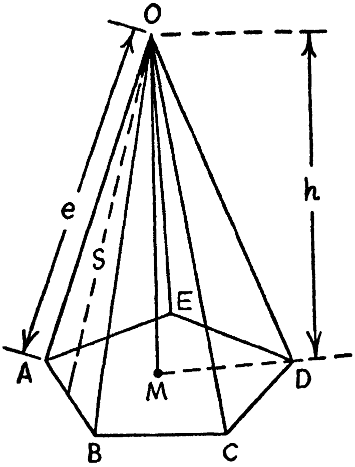

Oblique Pyramid


Regular Square Pyramid


Regular Hexagonal Pyramid


Regular Tetrahedron

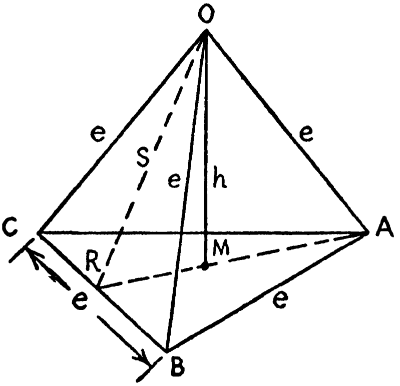
The *altitude* of any pyramid is the perpendicular distance from the



vertex (O) to the plane of the base. If the base is a regular polygon, and the altitude of the pyramid passes through the center of the base, the solid is a *regular pyramid.* In such a regular pyramid, all the lateral faces are obviously equal isosceles triangles; the altitude of any lateral face is known as the *slant height* of the pyramid, and is always greater than the altitude of the pyramid.

The lateral surface of a regular pyramid is given by $L.A. = \frac{1}{2}Ps$, where P=perimeter of the base and s=slant height. The total area, of course, is the lateral area plus the area of the base. The volume of any pyramid, whether regular or not, equals ⅓ the area of its base multiplied by its altitude.

Regular Tetrahedron. A special case of a regular pyramid is the case of a regular triangular pyramid in which not only the base, but each of the three lateral faces, are all equilateral triangles. Such a figure is called a *regular tetrahedron,* since all four faces are identical. If each of the six equal edges is denoted by e, then careful study of the figure will show that the following relations hold:



(1) $OR = s = \frac{e}{2}\sqrt{3}$

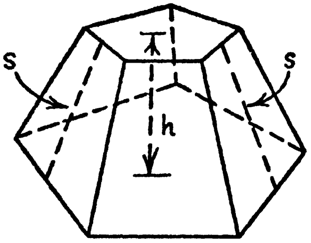
(2) $AR = OR = \frac{e}{2}\sqrt{3}$

(3) $AM = \frac{2}{3}(AR) = \frac{e}{3}\sqrt{3}$

(4) $OM = h = \sqrt{(OA)^2 - (AM)^2}$

$$= \sqrt{e^2 - \left(\frac{e\sqrt{3}}{3}\right)^2} = \frac{e}{3}\sqrt{6}$$

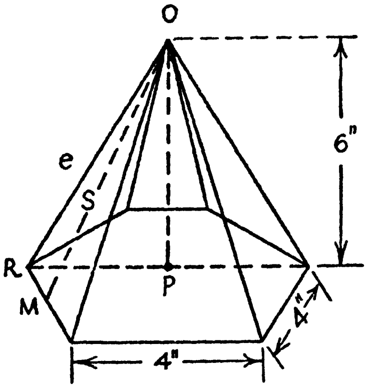
Frustum of a Pyramid. If a portion of a pyramid including the vertex is cut off by a plane parallel to the base, the part left is called the *frustum* of the pyramid. Thus in the frustum of a regular pyramid each lateral face is an isosceles trapezoid; all these lateral faces are equal to one another, and the slant height of the frustum is the altitude of the trapezoid. The altitude of the frustum is the perpendicular distance between the two bases, which are similar regular polygons.



The lateral area of the frustum of a regular pyramid equals half the sum of the perimeters of the bases multiplied by the slant height, or

$$L.A. = \tfrac{1}{2}(P+p)s.$$

The volume of a regular frustrum is given by $V = \tfrac{1}{3}h(B+b+\sqrt{Bb})$, where b and B are the areas of the upper and lower bases, respectively.



EXAMPLE 1: Find the lateral area and volume of a regular hexagonal pyramid whose altitude is 6″, if the sides of the base are 4″ each.

SOLUTION:

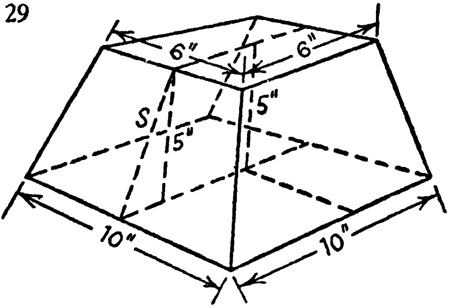
$$(OR)^2 = (OP)^2 + (PR)^2$$
$$e^2 = 4^2 + 6^2 = 16 + 36 = 52$$
$$(OM)^2 = (RM)^2 + (OR)^2$$
$$s^2 = 4 + 52 = 56$$
$$s = \sqrt{56} = 7.5$$

$$L.A. = \tfrac{1}{2}ps = \tfrac{1}{2}(24)(7.5) = 90.0+ \text{ sq. in., } Ans.$$
$$V = \tfrac{1}{3}Bh = \tfrac{1}{3}[(6)(4\sqrt{3})(6)] = 83.0+ \text{cu. in., } Ans.$$

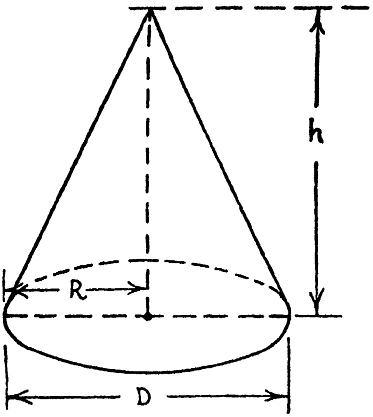
EXAMPLE 2: Find the total area and volume of the frustum of a square pyramid with dimensions as shown.

SOLUTION: $s^2 = 5^2 + 2^2 = 25 + 4 = 29$

$$s = \sqrt{29} = 5.4$$
$$L.A. = \tfrac{1}{2}s(p_1 + p_2)$$
$$= \tfrac{1}{2}(5.4)(24+40)$$
$$= 172.8 \text{ sq. in.}$$
$$T.A. = 172.8 + 36 + 100$$
$$= 308.8 \text{ sq. in., } Ans.$$
$$V = \tfrac{1}{3}h(B_1 + B_2 + \sqrt{B_1B_2})$$
$$= \tfrac{1}{3}(5)(36+100+60)$$
$$= 326.7 \text{ cu. in., } Ans.$$



Right Circular Cones. The properties of right circular cones are analogous to those of regular pyramids, of which, indeed, the cone can be considered as the limiting case as the number of sides is increased indefinitely. Hence we have:

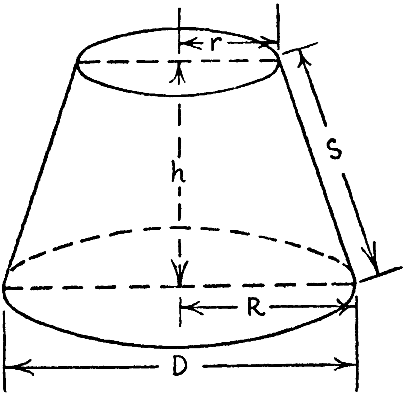


$$L.A.=\tfrac{1}{2}\,Cs=\tfrac{1}{2}(2\pi R)s=\pi Rs$$

$$T.A.=\pi R^2+\pi Rs=\pi R(R+s)$$

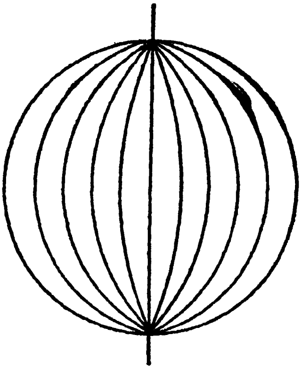
$$V=\tfrac{1}{3}Bh=\tfrac{1}{3}\pi R^2h=\frac{\pi D^2h}{12}$$

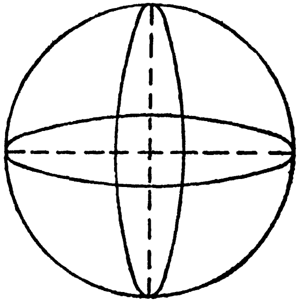
Similarly for the frustum of a regular cone:

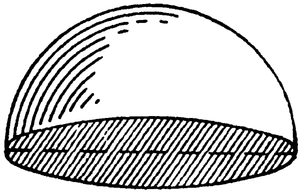


$$\begin{aligned}L.A.&=\tfrac{1}{2}(C+c)s\\&=\tfrac{1}{2}(2\pi R+2\pi r)s\\&=\pi s(R+r).\\T.A.&=\pi s(R+r)+\pi R^2+\pi r^2\\&=\pi[(R+r)s+R^2+r^2]\\V&=\tfrac{1}{3}h[\pi R^2+\pi r^2+\sqrt{\pi R^2+\pi r^2}]\\&=\tfrac{1}{3}\pi h[R^2+r^2+Rr]\end{aligned}$$

Sphere. In practical use, the sphere or ball finds its commonest application where friction between moving parts is to be a minimum, as in ball bearings, or where the symmetry and "perfect beauty" of a sphere is attractive, as in ornamental parts. The sphere is a "closed" curved surface, every point on which is equally distant from a point within called the center. A sphere may also be thought of as the surface generated by revolving a semicircle about its diameter as an axis. A plane passed through the center of a sphere divides it into two equal *hemispheres.* The base of each hemisphere is a circle whose diameter is the same as that of the sphere; they are known as *great circles.*







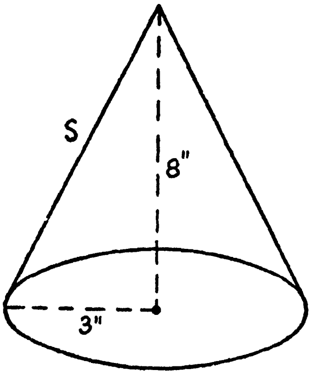
The surface of a sphere is found to be four times that of a great circle of the sphere, or

$$A=4\pi R^2=\pi D^2;$$

and the volume is given by

$$V=\tfrac{4}{3}\pi R^3=\frac{\pi D^3}{6}.$$

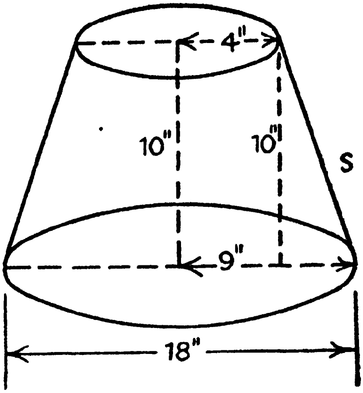
Example 1: Find the total area and volume of a right circular cone if the altitude is 8″ and the diameter of the base is 6″.



Solution:

$$s=\sqrt{8^2+3^2}=\sqrt{73}=8.6$$
$$L.A.=\pi Rs$$
$$=(\tfrac{22}{7})(3)(8.6)=81.1 \text{ sq. in.}$$
$$T.A.=L.A.+(\tfrac{22}{7})(9)$$
$$=81.1+28.3=109.4 \text{ sq. in., } \textit{Ans.}$$
$$V=\pi R^2h=(\tfrac{22}{7})(9)(8)=$$
$$226.4 \text{ cu. in., } \textit{Ans.}$$

Example 2: Find the lateral area and volume of the frustum of a right circular cone whose altitude is 10″, and the diameters of whose bases are 8″ and 18″, respectively.



SOLUTION:

$s^2 = 10^2 + 5^2$

$s = \sqrt{100+25} = \sqrt{125} = 11.2$

$L.A. = \pi s(R+r)$

$= (\frac{22}{7})(11.2)(9+4) =$

44.3 sq. in., *Ans.*

$V = \frac{1}{3}\pi h[R^2 + r^2 + Rr]$

$= (\frac{1}{3})(\frac{22}{7})(10)[81+16+36]$

$= 1393+$ cu. in., *Ans.*

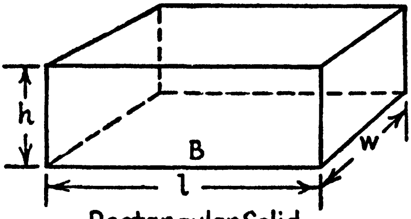
EXAMPLE 3: A hollow metal container is in the shape of a hemisphere whose diameter is 42 cm. What is its capacity?

SOLUTION: $V = \frac{4}{3}\pi R^3(\frac{1}{2})$

$= (\frac{4}{3})(\frac{22}{7})(21^3)(\frac{1}{2}) = 19{,}404$ cu. cm., *Ans.*

SUMMARY OF FORMULAS

Areas and Volumes of Solid Figures

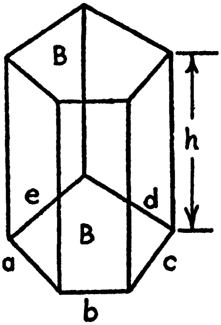


Rectangular Solid

$L.A. = ph = 2h(l+w)$

$T.A. = 2(lw+wh+wl)$

$V = lwh = Bh$

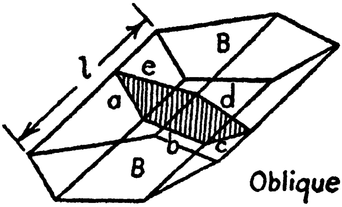


Right Prism

$L.A. = ph = h(a+b+c+\cdots)$

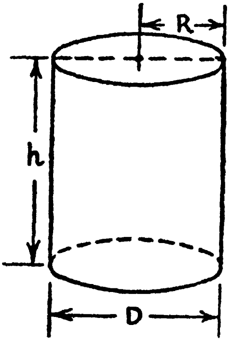
$T.A. = ph + 2B$

$V = Bh$



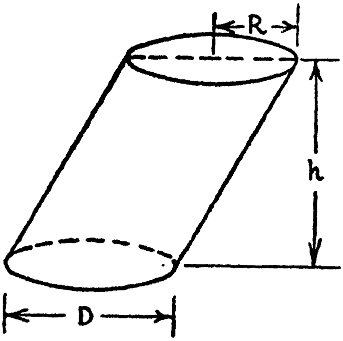
$L.A. = pl = h(a+b+c+\cdots)$
$T.A. = pl + 2B$
$V = Bh$

Oblique Prism



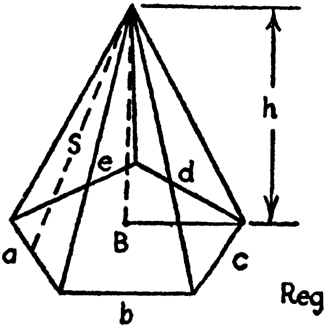
$L.A. = 2\pi Rh$
$T.A. = 2\pi Rh + 2\pi R^2$
$= 2\pi R(h+R)$
$V = \pi R^2 h$
$= \frac{\pi h}{4} D^2$

Right Circular Cylinder



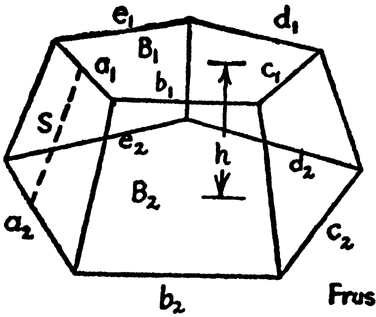
$V = \pi R^2 h$

Oblique Circular Cylinder



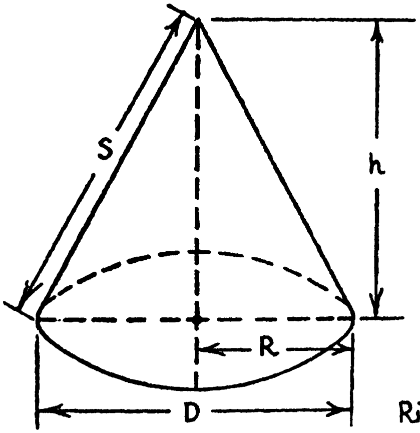
$L.A. = \frac{1}{2}ps = \frac{1}{2}s(a+b+c+\cdots)$
$T.A. = \frac{1}{2}ps + B$
$V = \frac{1}{3}Bh$

Regular Pyramid



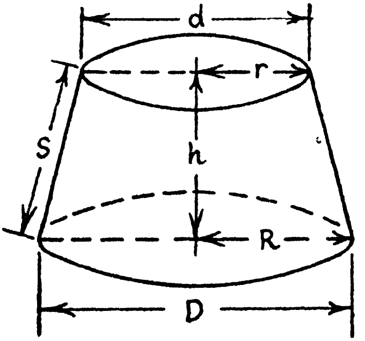
$$
\begin{aligned}
L.A. &= \tfrac{1}{2}s(p_1+p_2)\\
&= \tfrac{1}{2}s[(a_1+b_1+c_1+\cdots)\\
&\qquad +(a_2+b_2+c_2+\ldots)]\\
T.A. &= \tfrac{1}{2}s(p_1+p_2)+B_1+B_2\\
V &= \tfrac{1}{3}h[B_1+B_2+\sqrt{B_1+B_2}]
\end{aligned}
$$

Frustum of Regular Pyramid



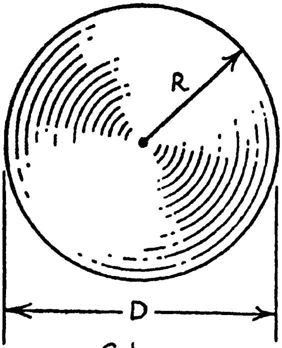
$$
\begin{aligned}
L.A. &= \pi Rs\\
T.A. &= \pi Rs+\pi R^2\\
&= \pi R(s+R)\\
V &= \tfrac{1}{3}\pi R^2 h\\
&= \frac{\pi D^2 h}{12}\\
&= .2618D^2h
\end{aligned}
$$

Right Circular Cone



$$
\begin{aligned}
L.A. &= \tfrac{1}{2}s(C+c)\\
&= \tfrac{1}{2}s(2\pi R+2\pi r)\\
&= \pi s(R+r)\\
T.A. &= \pi s(R+r)+\pi R^2+\pi r^2\\
&= \pi(R^2+r^2+Rs+rs)\\
V &= \tfrac{1}{3}\pi h(R^2+r^2+Rr)\\
&= .2618h(D^2+d^2+Dd)
\end{aligned}
$$

Frustum of Right Circular Cone

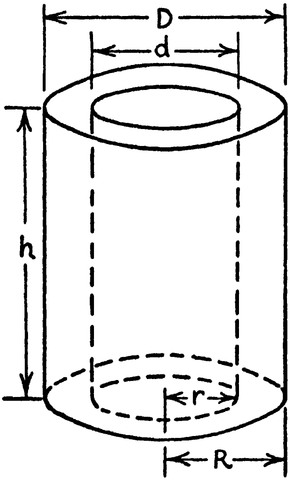


Sphere

$$A = 4\pi R^2 = 12.566R^2 = \pi D^2$$

$$V = \tfrac{4}{3}\pi R^3 = 4.1888R^3$$

$$= \frac{\pi D^3}{6} = .5236D^3$$

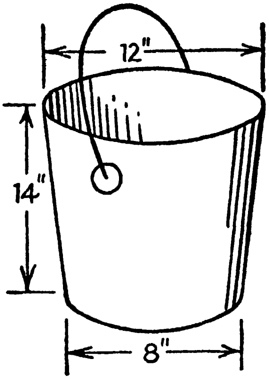


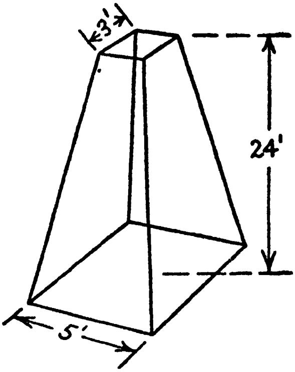
Hollow Cylinder

$$V = \pi h(R^2 - r^2)$$

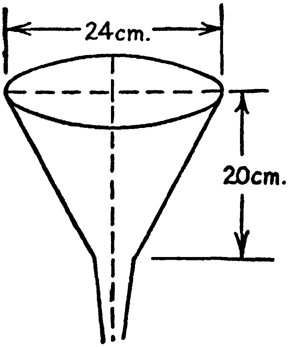
$$= \frac{\pi h}{4}(D^2 - d^2)$$

Exercise 67.

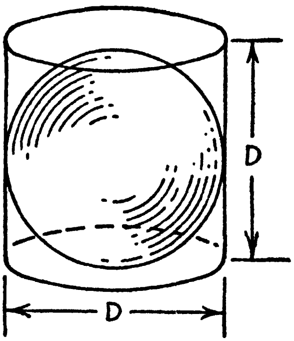




1. How many gallons can this bucket hold, if it has the form of a frustum of a right circular cone with dimensions as shown?
2. A concrete foundation pillar 24 ft. high is a frustum of a square pyramid whose bases are 3 ft. and 5 ft. on a side. How many cu. yd. of concrete are required for the pillar?



3. A funnel measures 24 cm. across the top, and 20 cm. from top to bottom, excluding the stem. Approximately how many cu. cm. of liquid will it hold when full?

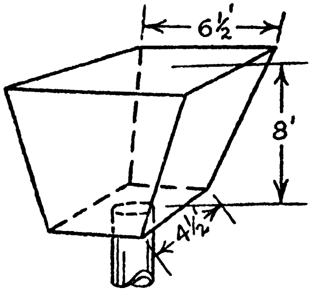


4. A sphere when immersed in a cylinder full of water just touches the sides and bottom, and is flush with the surface of the water remaining. What per cent of the water in the cylinder originally will the overflow be?

5. How many sq. in. of sheet metal will be needed to make a pail 12″ in diameter at the bottom and 14″ across at the top, with a slant height of 18″?

6. The altitude of a regular hexagonal pyramid is 17½″; the hexagonal base measures 4″ across the flats. Find the volume of the pyramid.

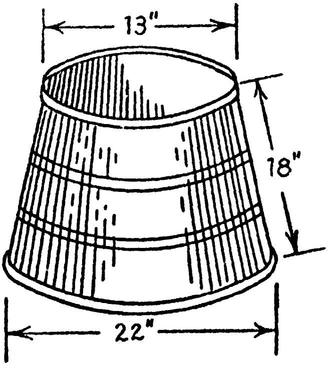
7. A feed hopper is in the form of the frustum of a square pyramid with dimensions shown. Find its cubic contents when filled to its maximum height of 8 ft.



8. A conical pile of sand measures 66 ft. around the circumference of its base. If the slope of the pile is 45°, how many cu. yd. of sand are there in the pile?

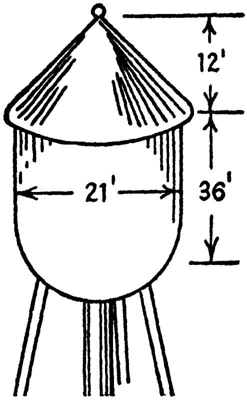
9. A parchment lamp shade has the shape of the frustum of a circular cone with the dimensions shown; find the amount of material required to make it up.

10. A cylindrical "capsule" with hemispherical ends has a diameter of 18″ and is 4½ feet in total length. Find its volume.

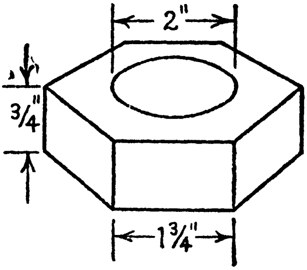


11. A regular triangular prism of glass measures 16.2 cm. in length; the side of each base is 1.8 cm. long. Find the lateral surface of the prism.
12. Find the volume of metal in a hollow sphere if the metal is 1½″ thick and the inside diameter equals 6″.
13. A cone pulley is a frustum of a cone with an altitude of 10½″; the diameters of the bases are 4″ and 6″, respectively. If a 1″ hole is drilled through the pulley, find the volume of the pulley.
14. Find the ratio between the surfaces of two spheres whose radii are 3″ and 6″.

15. A water tank in the shape of a cylinder with a hemispherical bottom and a conical top has the dimensions shown. What is its maximum capacity in gallons?

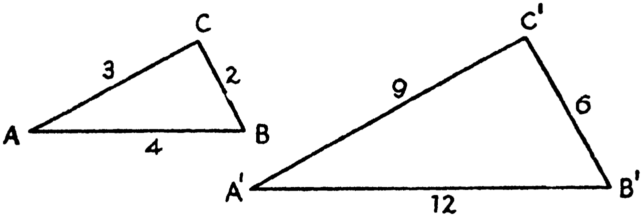


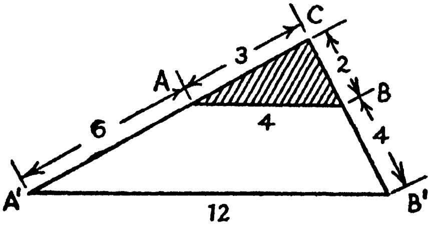
16. A 2″ hole is drilled through a hexagonal piece of stock ¾″ thick and 1¾″ across the flats. Find the volume of metal in the finished piece.



19. SIMILAR FIGURES

Similar Triangles. It was previously pointed out that any two (or more) triangles having the same shape, although differing in size, were said to be *similar triangles*. More specifically, having the "same shape" means





that the corresponding angles of the two triangles are respectively equal, and that their corresponding sides are in proportion. Thus: $\angle A = \angle A'$, $\angle B = \angle B'$, and $\angle C = \angle C'$; also, $\frac{AC}{A'C'} = \frac{AB}{A'B'} = \frac{BC}{B'C'}$.

In other words, in this particular instance, any pair of corresponding sides of the two triangles has the ratio 3:1, (or 1:3); thus:

$$\frac{AC}{A'C'} = 3/9 = 1/3;\ \frac{AB}{A'B'} = 4/12 = 1/3;$$
$$\frac{B'C'}{BC} = 6/2 = 3:1;\ \frac{A'C'}{AC} = 9/3 = 3:1;\ \text{etc.}$$

As a matter of fact, *congruent* triangles may be regarded as a "special case" of similar triangles whose corresponding sides are in the ratio of 1:1.

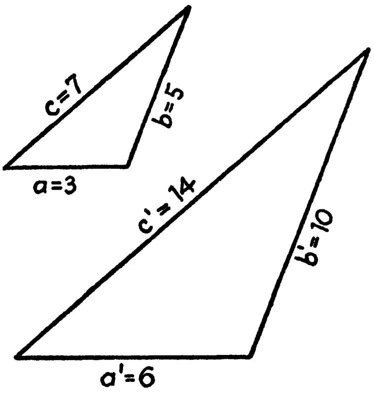
Conditions for Similarity. Triangles are similar under any one of the following conditions:

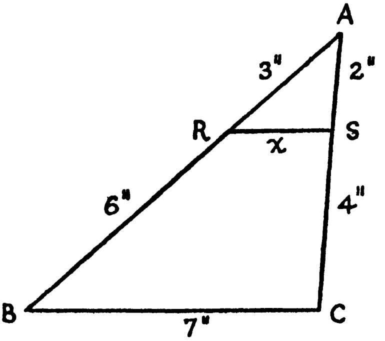
(1) if all three pairs of corresponding angles are equal;
(2) if any two pairs of corresponding angles are equal;
(3) if one pair of corresponding angles is equal, and the sides including them are in proportion;
(4) if all three pairs of corresponding sides are in proportion;
(5) if they are right triangles having one pair of acute angles equal.

Proportional Parts. As has just been seen, if two triangles are similar, their corresponding sides are in proportion; or

$$\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$$
$$3/6 = 5/10 = 7/14 = (1/2).$$

Or again; $\frac{a}{b} = \frac{a'}{b'};\ \frac{a}{c} = \frac{a'}{c'};\ \frac{b}{c} = \frac{b'}{c'}$
$$3/5 = 6/10;\ 3/7 = 6/14;\ 5/7 = 10/14.$$





Moreover, if a line (RS) is drawn parallel to any side (BC) of a triangle, it divides the other two sides into proportional parts, since the triangle cut off by the line is similar to the original triangle. Thus if RS divides AB in the ratio of 3:6 (or 1:2), it will also divide AC in the ratio of 1:2 (2:4). In other words,

$$\frac{AR}{RB}=\frac{AS}{SC}, \text{ or } 3/6=2/4.$$

Also, the length of RS can be determined if BC is known, for:

$$\frac{AR}{AB}=\frac{RS}{BC}, \text{ or } 3/9=\frac{x}{7}; \ x=2\tfrac{1}{3}.$$

$$\text{Or again, } \frac{RS}{BC}=\frac{AS}{AC}, \text{ or } \frac{x}{7}=2/6; \ x=2\tfrac{1}{3}.$$

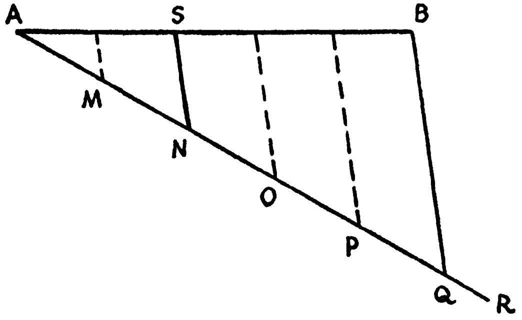
Furthermore, any segment is to the entire side as the corresponding segment is to the other side; i.e.,

$$\frac{AR}{AB}=\frac{AS}{SC}, \text{ or } \frac{3}{3+6}=\frac{2}{2+4};$$

$$\text{and } \frac{RB}{AB}=\frac{SC}{AC}, \text{ or } \frac{6}{3+6}=\frac{4}{2+4}.$$

Dividing a Line into a Given Ratio. A simple method for dividing a given line into any number of parts having any desired ratio is the following construction, based on similar triangles.

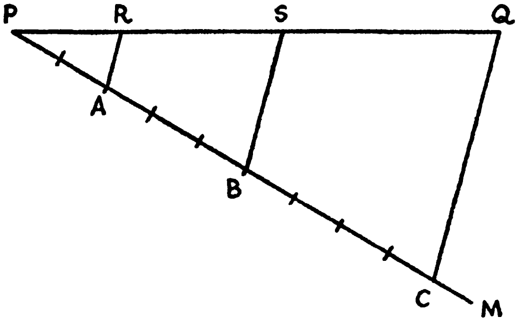
(a) Suppose that the line AB is to be divided into two parts having the ratio 2:3. At any convenient angle, draw an indefinite line through A. On AR step off five convenient equal segments, AM, MN, NO, OP, and PQ. Join Q with B, and through N construct NS || QB. The two required segments are AS and SB.



For, by the construction, △ANS is similar to (∼) △AQB; therefore

$$\frac{AN}{NQ}=\frac{AS}{SB}=\frac{2}{3}.$$

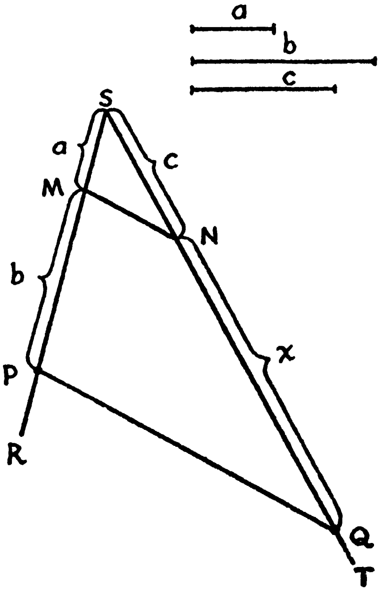
(b) Suppose that a given line PQ is to be divided into three parts having the ratio 2:3:4; a similar procedure is followed. An indefinite line PM is drawn at any convenient angle to PQ; 9 equal segments are then stepped off on PM, beginning at P and ending at C. Point C is joined with Q, and lines constructed through A and B, respectively, parallel to QC, and intersecting PQ in R and S. The required segments are PR, RS and SQ; for



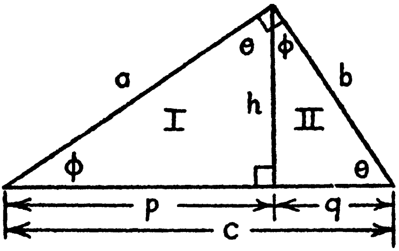
$$PA:AB:BC=PR:RS:SQ=2:3:4$$

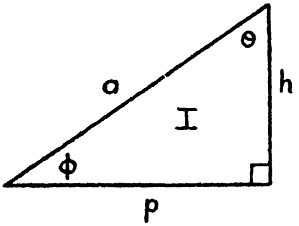
Constructing a Fourth Proportional. The same principle may be used to construct a fourth proportional to three given lines; for example, find the fourth proportional (x) to the given segments a, b and c. On any convenient angle RST with indefinite sides, lay off the given segments a, b and c as shown. Join the ends M and N of segments a and c; through P, the end of segment b, construct a line parallel to MN, intersecting ST in Q. Then NQ is the required fourth proportional x; for

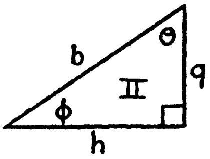
$$\frac{SM}{MP}=\frac{SN}{NQ}, \text{ or } \frac{a}{b}=\frac{c}{x}.$$



The Altitude upon the Hypotenuse. A little consideration will show that if the altitude of a right triangle is drawn to the hypotenuse, the two triangles thus formed are not only similar to each other, but each is similar to the original triangle as well. As a result, the following relations hold; they are occasionally useful in computations and in deriving other properties of such figures:





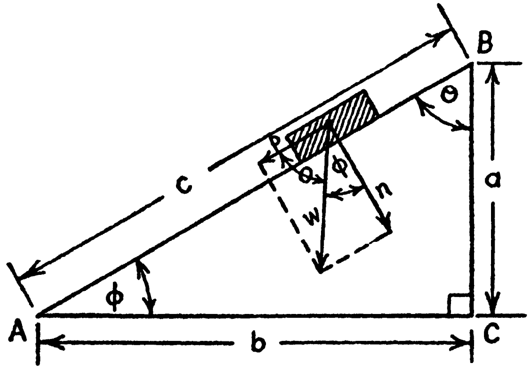


(1) $\frac{h}{p}=\frac{q}{h}$, or $h^2=pq$, $h=\sqrt{pq}$.

(2) $\frac{a}{p}=\frac{p+q}{a}$, or $a^2=p\ (p+q)$;
$a^2=pc$, or $a=\sqrt{pc}$.

(3) $\frac{b}{q}=\frac{p+q}{b}$, or $b^2=q\ (p+q)$;
$b^2=qc$, or $b=\sqrt{qc}$.

Practical Problem. A useful application of similar triangles occurs in connection with a study of forces acting on a body on an inclined plane. Thus the triangle of which *w* and *p* (the downward pull of the weight, and the pull along the inclined plane, respectively)are two sides of a triangle is similar to △ ABC; so is the triangle having *w* and *n* for two of its sides, where *n* is the "normal" (i.e., perpendicular) push upon the plane. Hence:



$\frac{p}{w}=\frac{a}{c}$, or $p=\left(\frac{a}{c}\right)w$; $\frac{n}{w}=\frac{b}{c}$, or $n=\left(\frac{b}{c}\right)w$; $\frac{p}{n}=\frac{a}{b}$.

In other words, if $a{:}b$, for example, equals 1:2, then the pull down the plane is half as great as the pressure exerted perpendicularly against the plane; and so on.

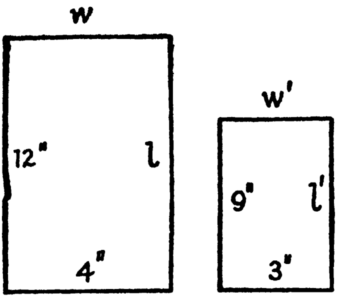
Similar Rectangles. Triangles, of course, are not the only geometric figures which may be similar in shape. Thus if the corresponding sides of two rectangles are in proportion, the rectangles are said to be similar. Here

$$\frac{w}{l}=\frac{w'}{l'}, \text{ or } l\frac{w}{w'}=\frac{l}{l'}.$$

Moreover, the perimeters of two similar rectangles are in the same ratio as any two corresponding sides; i.e.,

$$\frac{P}{P'}=\frac{l}{l'}=\frac{w}{w'}, \text{ or}$$

$$\tfrac{32}{24}=\tfrac{12}{9}=\tfrac{4}{3}.$$

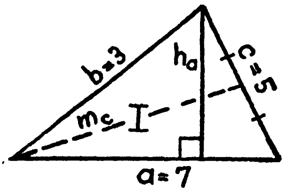


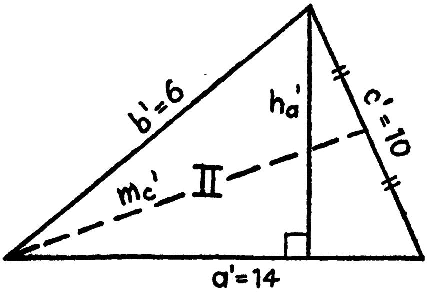
It should also be noted that any two squares are "automatically" similar, and that therefore their perimeters are also in the same ratio as the corresponding sides of the squares.

General Properties of Similar Triangles. This property of the perimeters of similar rectangles holds true of similar triangles also; in fact, in two similar triangles, *any two corresponding "parts"*—altitudes, medians, angle-bisectors—have the same ratio as a pair of corresponding sides; thus in similar triangles I and II we have:

$$\frac{P}{P'}=\frac{ha}{ha'}=\frac{mc}{mc'}=\frac{a}{a'}=\frac{b}{b'}=\frac{c}{c'}=\tfrac{1}{2};$$

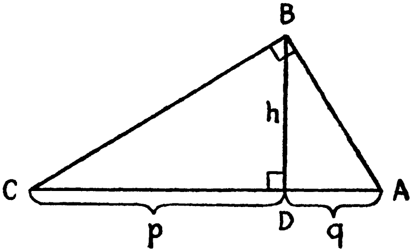
the same ratio (1:2) would likewise hold for any other pair of corresponding altitudes or corresponding medians.





Exercise 68.

1. Two similar rectangles have respective bases of 6″ and 15″. If the altitude of the first is 10″, what is the perimeter of the second rectangle?
2. The base of a triangle is 6 ft. and its altitude is 4 ft. If the corresponding base of a similar triangle is 18 ft., what is the area of the second triangle?
3. If AD=8 and AC=18, find the length of AB; the length of BD.
4. If $h=12$ and $q=9$, find CA and CB.

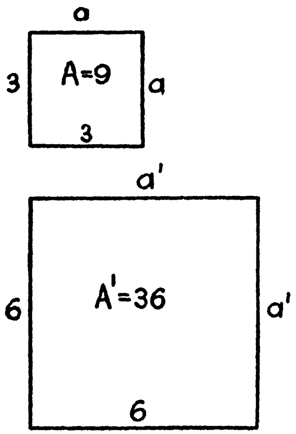


5. A tree casts a shadow 90 ft. long at the same time that a nearby post 6 ft. high casts a shadow 4 ft. long. How high is the tree?
6. The base of a triangle is 20″, and the other sides are 10″ and 16″. A line parallel to the base cuts off 2″ from the lower end of the shorter side. Find (a) the segments of the other side, and (b) the length of the parallel line.
7. The sides of a right triangle are 9″, 12″ and 15″. Find (a) the altitude to the hypotenuse, and (b) the segments of the hypotenuse into which this altitude divides it.
8. The bases of a trapezoid are 6″ and 8″, and the other two sides are 4″ and 5″ respectively. How far must each of these latter sides be produced before they meet in a point?

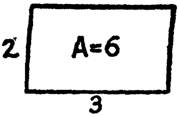
Areas of Similar Figures. It will be noted that in the two squares here shown, the ratio of the sides is 1:2, while the ratio of their *areas* is 1:4; or, doubling the side multiplies the area by *four,* and *not* by two. Similarly, tripling the sides of a rectangle multiplies the area by 9 instead of by 3. In short, the *areas of any two similar figures* are to each other as the *squares* of their respective sides, or the squares of any of their corresponding dimensions. Thus,

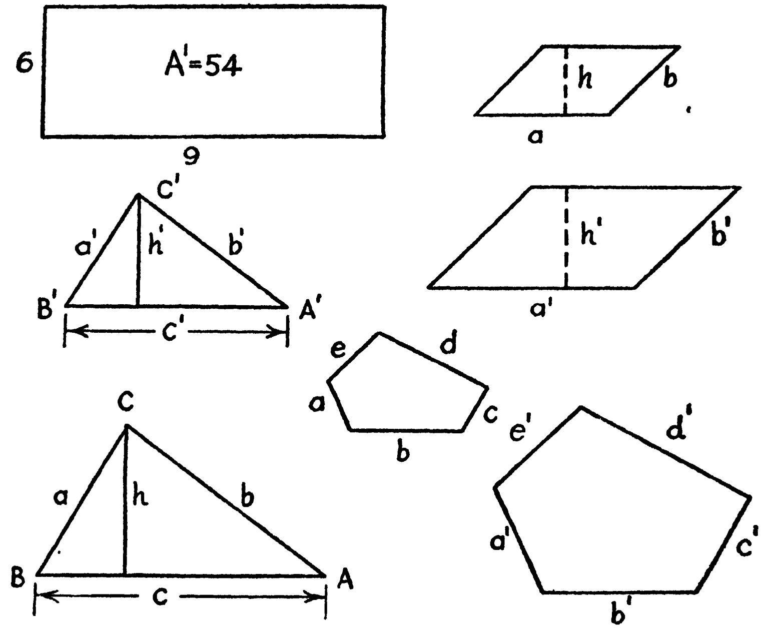
$$\frac{\text{Area}\triangle\ \text{ABC}}{\text{Area}\triangle\text{A}'\text{B}'\text{C}'}=\frac{a^2}{a'^2}=\frac{b^2}{b'^2}=\frac{c^2}{c'^2}=\frac{h^2}{h'^2}=\frac{P^2}{P'^2},$$

where P and P' represent their perimeters. So also in the case of all similar polygons, i.e., polygons having their respective angles



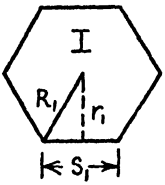
equal and their corresponding sides proportional: the areas of such polygons are to each other as the squares of their corresponding sides, altitudes, etc.

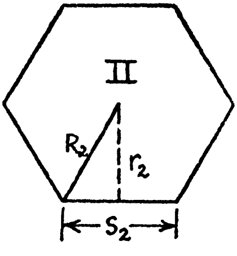




Regular Polygons; Circles. It is easily seen that if any two *regular* polygons have the same number of sides they must be similar. The *perimeters* of any two such regular polygons are to each other as any two corresponding sides; so also is the ratio of their corresponding radii and apothems. Expressed symbolically, we have:

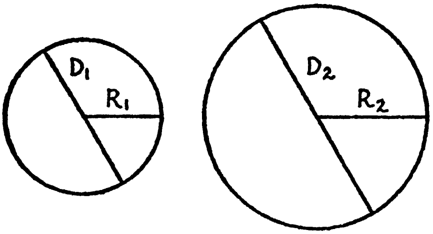
$\frac{P_I}{P_{II}}=\frac{s_1}{s_2}=\frac{R_1}{R_2}=\frac{r_1}{r_2}$, where P_I and P_{II} denote their perimeters, R_1 and R_2 their radii, r_1 and r_2 their apothems, and s_1 and s_2 their sides, respectively. The *areas* of two regular polygons having the same number of sides are in the same ratio as the *squares* of their corresponding parts: thus





$$\frac{A_I}{A_{II}}=\frac{s_1^2}{s_2^2}=\frac{R_1^2}{R_2^2}=\frac{r_1^2}{r_2^2}=\frac{P_I^2}{P_{II}^2}.$$

In the case of circles the relations are very much the same; in a sense, any two circles may be regarded as "similar figures." The perimeters (i.e., circumferences) of any two circles are to each other as their respective radii or diameters; their areas, however, are to each other as the squares of their corresponding radii, diameters, or circumferences; or, $\frac{C_1}{C_2}=\frac{R_1}{R_2}=\frac{D_1}{D_2}$, and $\frac{A_1}{A_2}=\frac{R_1^2}{R_2^2}=\frac{D_1^2}{D_2^2}=\frac{C_1^2}{C_2^2}$.



EXAMPLE 1: The distances across the flats of two hexagonal plates are 1″ and 1½″, respectively. What is the ratio of their cross-sectional areas?

SOLUTION: $\frac{A_1}{A_2}=\frac{(1)^2}{(1\frac{1}{2})^2}=\frac{1/9}{4}=4/9$, *Ans.*

EXAMPLE 2: Two circular discs are 3″ and 5″ in diameter, respectively. How many times larger than the first disc is the second?

SOLUTION: $\frac{A_2}{A_1}=\frac{5^2}{3^2}=\frac{25}{9}=2\frac{7}{9}$ times as large, *Ans.*

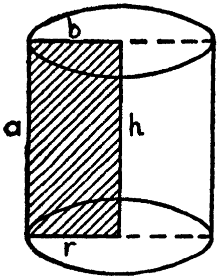
EXAMPLE 3: The area of one circular plate is twice that of another. Find the ratio of their circumferences.

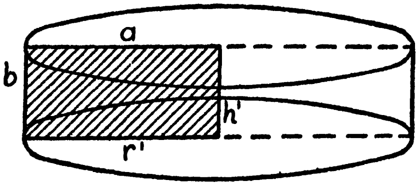
SOLUTION: $\frac{A_1}{A_2}=2/1=\frac{R_1^2}{R_2^2}=\left(\frac{R_1}{R_2}\right)^2$

$\left(\frac{R_1}{R_2}\right)=\sqrt{2/1}=\sqrt{2}=1.41$

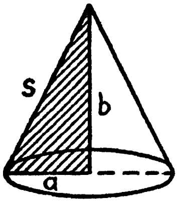
$\frac{C_1}{C_2}=\frac{R_1}{R_2}=1.41$, *Ans.*

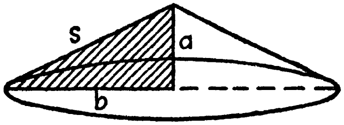
Similar Solids of Revolution. A right circular cylinder may be regarded as having been formed by revolving a rectangle about either of its sides as an axis of rotation; the side of the rectangle about which it is revolved becomes the altitude of the cylinder, and the other side becomes the radius of the base. In the same manner, a right circular cone can be regarded as the solid that is formed by revolving a right triangle about

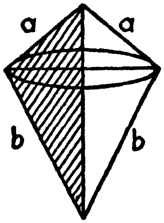




either of its sides as an axis; the side around which it is revolved becomes the altitude, the other side becomes the radius of the base, and the hypotenuse becomes the slant height. If a right triangle is revolved around

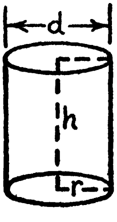




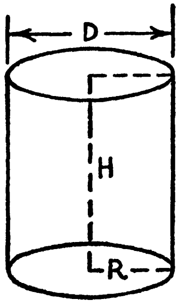


the hypotenuse as an axis, two circular cones are formed (one inverted), having a common base, and a *combined altitude* equal to the hypotenuse.

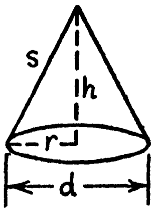
If now we consider cylinders and cones of revolution formed from similar rectangles and similar right triangles, respectively, we have ***similar solids of revolution;*** their ***areas,*** lateral and total, are to each other as the ***squares*** of their corresponding dimensions, and their ***volumes*** are to each other as the ***cubes*** of their corresponding dimensions.



I



II



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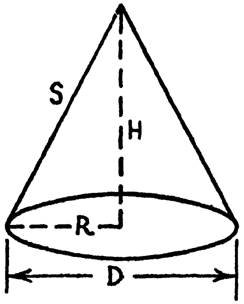


II

Thus for similar cylinders of revolution:

$$\frac{\text{L.A.}_{\text{I}}}{\text{L.A.}_{\text{II}}}=\frac{\text{T.A.}_{\text{I}}}{\text{T.A.}_{\text{II}}}=\frac{h^2}{H^2}=\frac{r^2}{R^2}=\frac{d^2}{D^2};$$

$$\frac{\text{V}_{\text{I}}}{\text{V}_{\text{II}}}=\frac{h^3}{H^3}=\frac{r^3}{R^3}=\frac{d^3}{D^3}.$$

And for similar cones of revolution:

$$\frac{\text{L.A.}_{\text{I}}}{\text{L.A.}_{\text{II}}}=\frac{\text{T.A.}_{\text{I}}}{\text{T.A.}_{\text{II}}}=\frac{h^2}{H^2}=\frac{r^2}{R^2}=\frac{d^2}{D^2}=\frac{s^2}{S^2};$$

$$\frac{\text{V}_{\text{I}}}{\text{V}_{\text{II}}}=\frac{h^3}{H^3}=\frac{r^3}{R^3}=\frac{d^3}{D^3}=\frac{s^3}{S^3}.$$

Spheres. Just as any two circles may be regarded as similar, so any two spheres may be regarded as similar. Therefore we have

$$\frac{\text{A}_{\text{I}}}{\text{A}_{\text{II}}}=\frac{r^2}{R^2}=\frac{d^2}{D^2}, \text{ and } \frac{\text{V}_{\text{I}}}{\text{V}_{\text{II}}}=\frac{r^3}{R^3}=\frac{d^3}{D^3}.$$

EXAMPLE 1: Two cylindrical tin cans are similar, i.e., their heights and diameters are in the proportion of 2:3. Find (a) the ratio of their lateral surfaces; (b) the ratio of their volumes.

SOLUTION: $\frac{\text{L.A.}_{\text{I}}}{\text{L.A.}_{\text{II}}}=\frac{2^2}{3^2}=4/9$, *Ans.*

$\frac{\text{V}_{\text{I}}}{\text{V}_{\text{II}}}=\frac{2^3}{3^3}=8/27$, *Ans.*

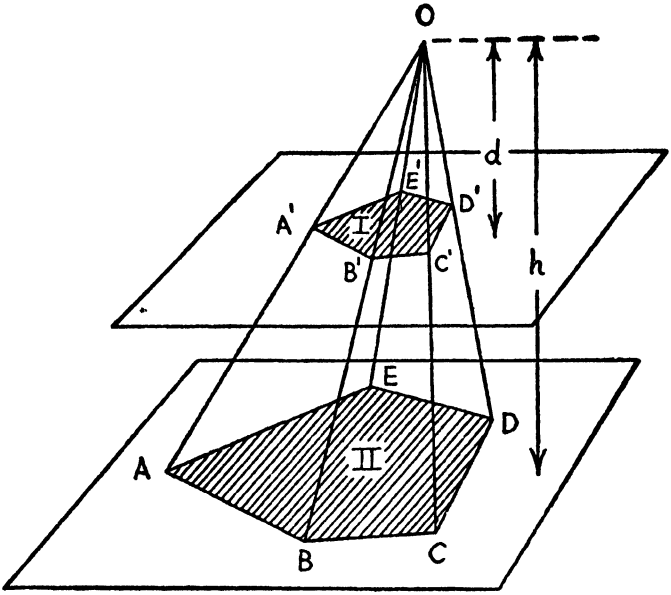
EXAMPLE 2: A right circular cone has a diameter of 4″ and an altitude of 5″. Another cone, similar to the first, has an altitude of 10″. Find (a) the diameter of the second cone; (b) the ratio of their total areas; (c) how many times larger in volume the second cone is than the first.

SOLUTION: (a) $4/5=\frac{x}{10}$; $5x=40$; $x=8$, *Ans.*

(b) $\frac{\text{T.A.}_{\text{I}}}{\text{T.A.}_{\text{II}}}=\frac{4^2}{5^2}=16/25$, *Ans.*

(c) $\frac{\text{V}_{\text{II}}}{\text{V}_{\text{I}}}=\frac{5^3}{4^3}=125/64=1\,61/64$ times as large, *Ans.*

Section of Pyramid Parallel to the Base. If in any pyramid, whether a regular pyramid or an oblique pyramid, a plane section is passed parallel to the base, the polygon formed is similar to the base, and the pyramid



"cut off" is similar to the original pyramid. Furthermore, the lateral edges will be divided in the ratio of $d:(h-d)$, where d is the distance of the cutting plane below the vertex, and h is the original altitude. Or:

$$\frac{OA'}{OA}=\frac{OB'}{OB}=\frac{OC'}{OC}\text{(etc.)}=\frac{d}{h};$$

also, $$\frac{A'B'}{AB}=\frac{B'C'}{BC}=\frac{C'D'}{CD}=\text{(etc.)}=\frac{d}{h};\text{ and}$$

$$\frac{\text{area I}}{\text{area II}}=\frac{(OA')^2}{(OA)^2}=\frac{(A'B')^2}{(AB)^2}=\frac{d^2}{h^2}.$$

Exercise 69.

1. If the radius of a circle is cut in half, how is its circumference changed? its area?
2. The areas of two circles are as 4:9; if the radius of the larger circle is 12 inches, what is the circumference of the smaller circle?
3. If the edge of a cube is doubled, how is its area changed? its volume?
4. The diameters of two circles are 6" and 8" respectively. Find (a) the ratio of their circumference, and (b) the ratio of their areas.
5. The circumference of a circle is 20". What is the circumference of a circle having twice the area of the given circle?

6. If the diameter of a cylindrical shaft is diminished by 10%, how is its circumference changed? the area of its cross section?
7. A water storage tank is fed by two pipe lines, one 8″ and one 6″ in diameter. If these are to be replaced by a single pipe line having the same capacity as the two combined, what diameter pipe should be used? (*Hint:* $\pi R^2 + \pi r^2 = \pi x^2$.)
8. A rectangular zinc cut for photoengraving is reduced "three-fourths"; what is the ratio of the areas involved?
9. If the diameter of a water main is made half again as large, what is the per cent of increase in the capacity of the pipe? (*Hint:* compare the cross-sectional areas.)
10. Find the ratio of the volume of two spheres if their areas are in the ratio of 3:1.
11. Find the lateral area and the volume of a cone of revolution if the radius of the base is 8″ and the slant height forms an angle of 60° with the plane of the base.
12. A cylindrical container 8″ high has a diameter of 4″. If each dimension is increased by 25%, what is the ratio of the new total surface to the original total surface? What is the per cent of increase in total surface?
13. An equilateral triangle whose side is 6″ is revolved about one of its sides as an axis. Find the total surface and volume of the solid generated.
14. The area of the base of a circular cone is 108 sq. in., and its altitude is 6″. If a section is passed parallel to the base and 4″ above it, what is the area of the base of the cone cut off?
15. If a cylindrical metal drum used for shipping chemicals has each dimension increased by 20%, what is the per cent of increase in its capacity?
16. If the inside diameter of a pipe is increased 50%, how much more water will flow through it at the same rate in the same amount of time?
17. If the height of a cylinder is cut in half, and the diameter is doubled, what is the change in volume?
18. A micro-photograph was enlarged "20 diameters." How many times larger than the original is the area of the photograph?
19. One of two circular water pipes is 4″ in diameter, and the other is 8″. (a) How many times larger is the cross-sectional area of the second? (b) How much more water will flow through the second pipe?
20. Because of cloudy weather a photographer enlarged the diaphragm (circular opening) of his camera from a diameter of 0.4 cm. to 0.6 cm. By what per cent did this increase the area of the opening?

TABLE OF TANGENTS, COSINES, AND SINES

Angle	Tangent $\left(\frac{opp.}{adj.}\right)$	Cosine $\left(\frac{adj.}{hyp.}\right)$	Sine $\left(\frac{opp.}{hyp.}\right)$	Angle	Tangent $\left(\frac{opp.}{adj.}\right)$	Cosine $\left(\frac{adj.}{hyp.}\right)$	Sine $\left(\frac{opp.}{hyp.}\right)$
0°	.0000	1.0000	.0000	**45°**	1.0000	.7071	.7071
1°	.0175	.9998	.0175	**46°**	1.0355	.6947	.7193
2°	.0349	.9994	.0349	**47°**	1.0724	.6820	.7314
3°	.0524	.9986	.0523	**48°**	1.1106	.6691	.7431
4°	.0699	.9976	.0698	**49°**	1.1504	.6561	.7547
5°	.0875	.9962	.0872	**50°**	1.1918	.6428	.7660
6°	.1051	.9945	.1045	**51°**	1.2349	.6293	.7771
7°	.1228	.9925	.1219	**52°**	1.2799	.6157	.7880
8°	.1405	.9903	.1392	**53°**	1.3270	.6018	.7986
9°	.1584	.9877	.1564	**54°**	1.3764	.5878	.8090
10°	.1763	.9848	.1736	**55°**	1.4281	.5736	.8192
11°	.1944	.9816	.1908	**56°**	1.4826	.5592	.8290
12°	.2126	.9781	.2079	**57°**	1.5399	.5446	.8387
13°	.2309	.9744	.2250	**58°**	1.6003	.5299	.8480
14°	.2493	.9703	.2419	**59°**	1.6643	.5150	.8572
15°	.2679	.9659	.2588	**60°**	1.7321	.5000	.8660
16°	.2867	.9613	.2756	**61°**	1.8040	.4848	.8746
17°	.3057	.9563	.2924	**62°**	1.8807	.4695	.8829
18°	.3249	.9511	.3090	**63°**	1.9626	.4540	.8910
19°	.3443	.9455	.3256	**64°**	2.0503	.4384	.8988
20°	.3640	.9397	.3420	**65°**	2.1445	.4226	.9063
21°	.3839	.9336	.3584	**66°**	2.2460	.4067	.9135
22°	.4040	.9272	.3746	**67°**	2.3559	.3907	.9205
23°	.4245	.9205	.3907	**68°**	2.4751	.3746	.9272
24°	.4452	.9135	.4067	**69°**	2.6051	.3584	.9336
25°	.4663	.9063	.4226	**70°**	2.7475	.3420	.9397
26°	.4877	.8988	.4384	**71°**	2.9042	.3256	.9455
27°	.5095	.8910	.4540	**72°**	3.0777	.3090	.9511
28°	.5317	.8829	.4695	**73°**	3.2709	.2924	.9563
29°	.5543	.8746	.4848	**74°**	3.4874	.2756	.9613
30°	.5774	.8660	.5000	**75°**	3.7321	.2588	.9659
31°	.6009	.8572	.5150	**76°**	4.0108	.2419	.9703
32°	.6249	.8480	.5299	**77°**	4.3315	.2250	.9744
33°	.6494	.8387	.5446	**78°**	4.7046	.2079	.9781
34°	.6745	.8290	.5592	**79°**	5.1446	.1908	.9816
35°	.7002	.8192	.5736	**80°**	5.6713	.1736	.9848
36°	.7265	.8090	.5878	**81°**	6.3138	.1564	.9877
37°	.7536	.7986	.6018	**82°**	7.1154	.1392	.9903
38°	.7813	.7880	.6157	**83°**	8.1443	.1219	.9925
39°	.8098	.7771	.6293	**84°**	9.5144	.1045	.9945
40°	.8391	.7660	.6428	**85°**	11.4301	.0872	.9962
41°	.8693	.7547	.6561	**86°**	14.3007	.0698	.9976
42°	.9004	.7431	.6691	**87°**	19.0811	.0523	.9986
43°	.9325	.7314	.6820	**88°**	28.6363	.0349	.9994
44°	.9657	.7193	.6947	**89°**	57.2900	.0175	.9998
45°	1.0000	.7071	.7071	**90°**		.0000	1.0000

CHAPTER IV

SHOP TRIGONOMETRY

William L. Schaaf

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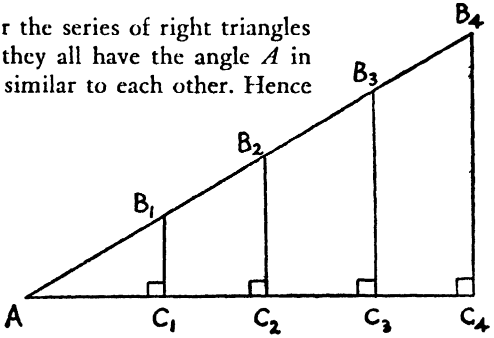
20. USING TRIGONOMETRIC FUNCTIONS

Similar Triangles. Consider the series of right triangles AB_1C_1, AB_2C_2, etc. Since they all have the angle A in common, the triangles are similar to each other. Hence the ratios of the corresponding pairs of sides are equal; or

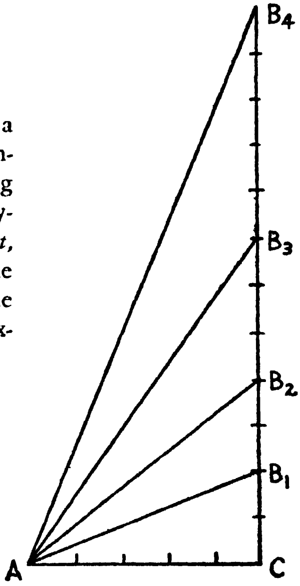
$$\frac{B_1C_1}{AC_1}=\frac{B_2C_2}{AC_2}=\frac{B_3C_3}{AC_3}=\frac{B_4C_4}{AC_4}=k,$$

where k represents some numerically constant value, independent of the units of measure used. In other words, the value of k could be used as a *measure of angle A.* Furthermore, if we consider another series of right triangles having one side fixed, but with the angle at A varying, then the series of ratios is *not constant,* but varies with the size of the angle *A;* if the angle increases, the ratio increases, and if the angle decreases, the ratio decreases. For example:



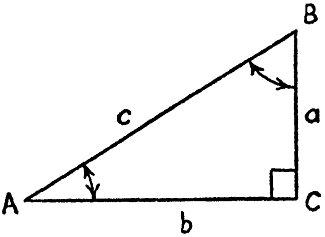
$$\frac{B_1C}{AC}=\tfrac{2}{5}=.4;\quad \frac{B_3C}{AC}=\tfrac{7}{5}=1.4;$$

$$\frac{B_2C}{AC}=\tfrac{4}{5}=.8;\quad \frac{B_4C}{AC}=\tfrac{12}{5}=2.4;\text{ etc.}$$



In any right triangle, the ratio of the side opposite an acute angle to the side adjacent to it is called *the tangent of that angle;* this is written:

$$\tan A = \frac{BC}{AC} = \frac{a}{b} = \frac{\text{opposite side}}{\text{adjacent side}}.$$



Similarly:

$$\tan B = \frac{AC}{BC} = \frac{b}{a}.$$

Trigonometric Ratios. In the same way, any other pair of sides of the right triangle might be used to form a ratio which can be regarded as a measure of the angle. Such ratios are called *trigonometric functions;* their names and definitions are given below:

$$\sin A = \frac{\text{opposite side}}{\text{hypotenuse}} = \frac{a}{c} \qquad \sec A = \frac{\text{hypotenuse}}{\text{adjacent side}} = \frac{c}{b}$$

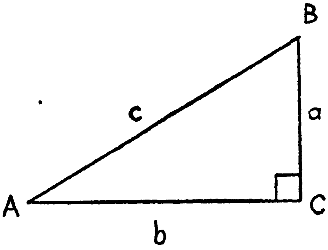
$$\cos A = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{b}{c} \qquad \csc A = \frac{\text{hypotenuse}}{\text{opposite side}} = \frac{c}{a}$$

$$\tan A = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{a}{b} \qquad \cot A = \frac{\text{adjacent side}}{\text{opposite side}} = \frac{b}{a}$$

It will be sufficient for most practical purposes to use the three functions in the first column, viz., *sine, cosine* and *tangent;* these should by all means be memorized. The other three, the *secant, cosecant,* and *cotangent,* are perhaps not so important for ordinary use.

Table of Natural Functions. The numerical values of these ratios for various angles from 1° to 90° have been carefully worked out to several decimal places; part of such a table is given above for the three most commonly used functions—the sine, the cosine, and the tangent. How such a table of the values of the various functions is used will be shown as we go along.

Co-functions. By studying the diagram and referring to the definitions once more, the following relations will be seen to hold:



$$\sin A=\frac{a}{c}=\cos B,$$

$$\text{and } \cos A=\frac{b}{c}=\sin B.$$

In other words, since $\angle A+\angle B=90°$, the sine of any angle equals the cosine of its complement, and the cosine of an angle equals the sine of its complement. This explains the meaning of the word cosine, i.e., *co-sine,* or "complement's sine." Thus if sin 32° = .5299, then cos 58° also equals .5299; etc. Similarly,

$$\tan A=\frac{a}{b}=\cot B;$$

$$\cot A=\frac{b}{a}=\tan B;$$

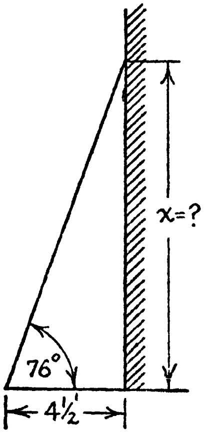
and also:

$$\sec A=\frac{c}{b}=\csc B;$$

$$\csc A=\frac{c}{a}=\sec B.$$

Using the Tangent. Several common types of problems arise where the use of the tangent is very convenient.

Example 1: A ladder leaning against the wall of a building makes an angle of 76° with the ground. If the foot of the ladder is 4½ ft. from the base of the wall, how high above the ground is the point on the wall where the top of the ladder touches it?



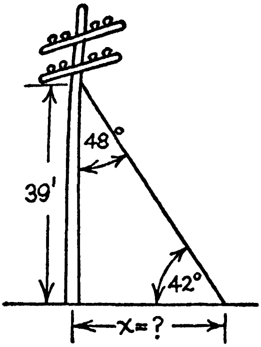
Solution:

$$\tan 76°=\frac{x}{4.5} \text{ (from the figure)}$$

$$\tan 76°=4.0108 \text{ (from the table)}$$

$$\text{hence} \frac{x}{4.5}=4.0108$$

$$\text{or } x=(4.5)(4.0108)=18.05 \text{ ft., } \textit{Ans.}$$

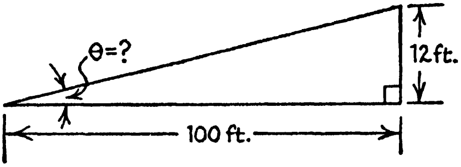


Example 2: The guy wire supporting a telegraph pole is fastened to the pole at a point 39 ft. above the ground, and makes an angle of 42° with the ground. Find how far from the base of the pole must the stake be driven to fasten the guy wire.

Solution:

$\frac{x}{39}=\tan 48°=1.1106$

$x=(39)(1.1106)=43.3$ ft., *Ans.*



Example 3: A ramp rises 12 ft. in a horizontal distance of 100 ft. Find the angle (θ) of inclination.

Solution:

$\tan \theta=12/100=.1200$

From the table: $\tan 7°=.1228$

$\theta=7°$ (approx.), *Ans.*

Note: More exact results can be found by interpolation similar to that used in finding logarithms, or by using a more complete table of trigonometric functions showing values for degrees and minutes. Such a table is given at the back of this book. Thus, by using the complete table, the angle in Ex. 3, expressed to the nearest minute, is 6° 51′.

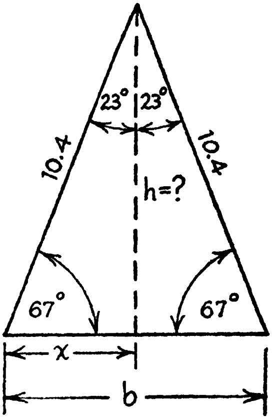
Exercise 70.

In the following problems, use the table of values at the back of the book when finding angles, obtaining your result to the nearest minute.

1. Find the side of an equilateral triangle whose altitude is 22.8″.
2. The slope of a roof is 4 inches in each horizontal foot. What angle does it make with the horizontal?
3. The altitude of an isosceles triangle is 12 and the base is 4. Find the length of the equal sides and the angles of the triangle.
4. A railroad track makes an angle of 10° with the horizontal. How many feet does it rise in 1000 ft. along the horizontal?
5. Find the apothem of a regular octagon whose sides are 4″ each in length.
6. A rectangle is 36″×48″. Find the angle between the diagonal and the longer side.

Using Sines and Cosines. How these functions are similarly utilized in practical problems involving right triangles will now be illustrated.

EXAMPLE 1: Find the altitude of an isosceles triangle whose vertex angle is 46° and whose equal sides are each 10.4 inches. What is the length of its base?



SOLUTION: $\frac{h}{10.4}=\sin 67°=.9205$

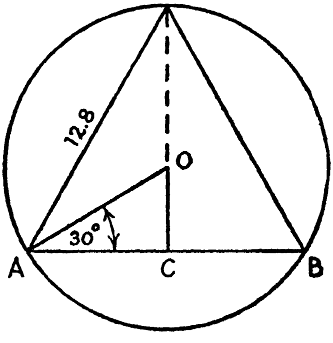
$h=(10.4)(.9205)=9.57$, *Ans.*

$\frac{x}{10.4}=\cos 67°=.3907$

$x=(10.4)(.3907)=4.06$

$b=(2)(4.06)=8.12$, *Ans.*

EXAMPLE 2: Find the radius of the circle circumscribed around an equilateral triangle 12.8″ on a side.



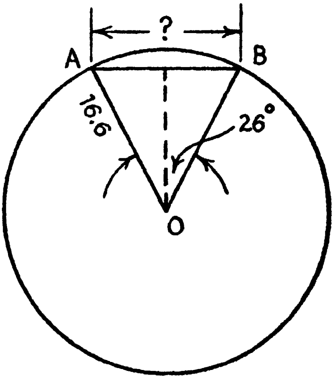
SOLUTION:

$AC=\frac{1}{2}(12.8)=6.4$

$\frac{AC}{r}=\cos 30°$

$r=\frac{AC}{\cos 30°}=\frac{6.4}{.8660}=7.39$, *Ans.*

EXAMPLE 3: How long is a chord of a circle that subtends an angle of 26° at the center, if the radius of the circle is 16.6 inches? What is the distance of the chord from the center?



SOLUTION: $\frac{AM}{16.6}=\sin 13°=.2250$

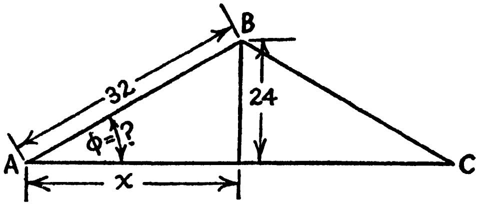
$AM=(16.6)(.2250)=3.74$

$\text{chord}=2\times(AM)=7.48$, *Ans.*

$\frac{OM}{16.6}=\cos 13°=.9744$

$OM=(16.6)(.9744)=16.18$, *Ans.*

Example 4: The equal sides of an isosceles triangle are each 32″ and the altitude is 24″. Find the base and the angles.



Solution:

$$\sin \phi = \frac{24}{32} = .7500$$

$$\phi = 48°35', \textit{ Ans.}$$

$$\frac{x}{32} = \cos 48°35' = .6615$$

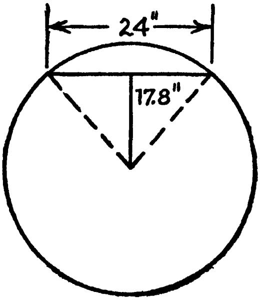
$$x = (32)(.6615) = 21.17$$

$$AC = 2x = (2)(21.17) = 42.34, \textit{ Ans.}$$

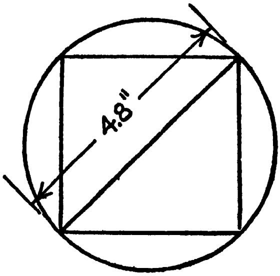
Exercise 71.

In the following problems, use the table in the back of the book when finding angles, obtaining the answer to the nearest minute.

1. Find the radius of a circle if a 12-inch chord subtends an angle of 28° at the center.

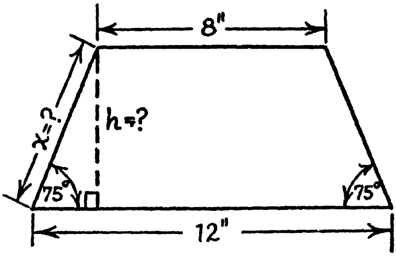


2. The equal sides of an isosceles triangle are 22″ long and the altitude is 9.2″. Find the base and the angles.
3. A chord 24 inches long is 17.8 inches distant from the center. Find (a) the radius of the circle, and (b) the subtended angle of the chord.



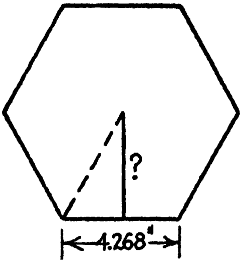
4. What is the largest square that can be milled from a circular disc 4.8 inches in diameter?
5. An isosceles triangle has sides of 16″, 16″ and 10″. Find the angles and the altitude.

6. The bases of an isosceles trapezoid are 8″ and 12″. If the base angles are 75°, find the equal sides and the altitude.

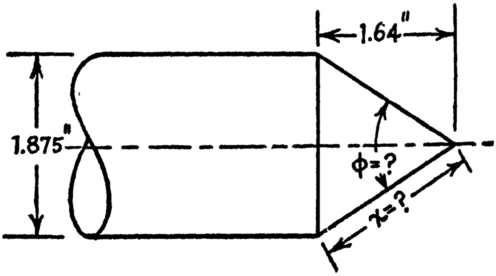


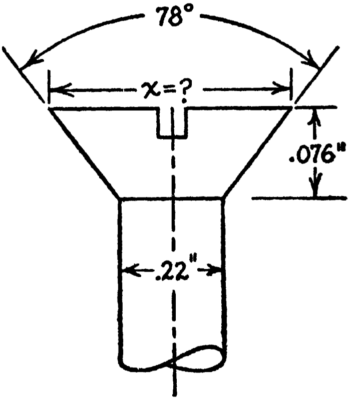
7. If a chord is 18½″ long and the radius of the circle is 14.2″, find the distance of the chord from the center and the central angle.

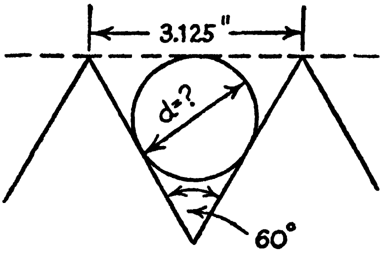
8. Find the perpendicular distance from the center to the sides of a regular hexagon whose sides are each 4.268″ long?

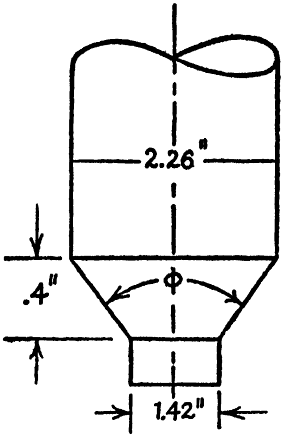


9. Find the radius of a circle inscribed in an equilateral triangle whose perimeter is 45 inches.
10. The side of a regular pentagon is 18.6 inches. Find the radius of the inscribed and circumscribed circles, respectively.
11. Gable rafters 19 ft. long project 1½ ft. beyond the walls of a house and are set with a pitch (angle with the horizontal) of 38°. Find the height of the ridgepole and the width of the house.
12. Find the value of ϕ and x from the dimensions given.
13. Find x in the flat-head screw as shown.





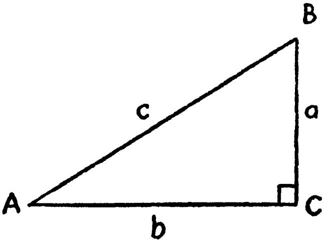




14. Find the diameter *d* of the wire laid in the groove of the screw thread as shown.
15. Find the angle ϕ in the spindle as shown.

21. PRACTICAL APPLICATIONS OF RIGHT TRIANGLES

Solution of Right Triangles. From the foregoing discussion it should be clear that a right triangle is completely determined if a side and any other part are known. In other words, given the length of a side and any other part, all the other parts can then be found; this is called solving the right triangle. By way of summary, the solution of a right triangle may be effected by the use of one or more of the following fundamental relationships:



(1) $a^2+b^2=c^2$

(2) $\sin A=\frac{a}{c}=\cos B$

(3) $\cos A=\frac{b}{c}=\sin B$

(4) $\tan A=\frac{a}{b}=\cot B$

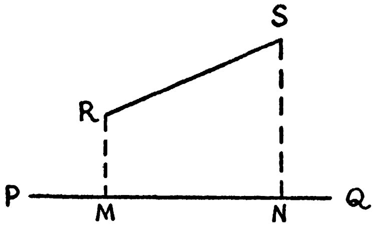
(5) $A+B=90°$

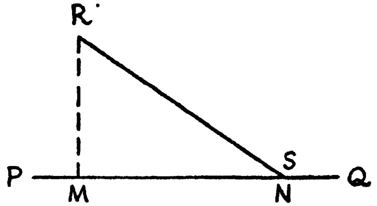
The rest of the present section will deal with practical uses of right triangles. Standard notation, i.e., side "*a*" opposite $\angle A$, side "*b*" opposite $\angle B$, etc., $\angle C$=right angle, will be used consistently throughout.

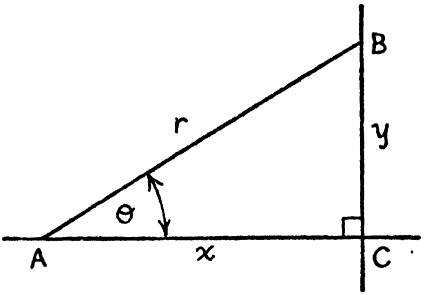
Projections. In physics and mechanics frequent use is made of horizontal and vertical projections. As we saw in Chapter III, section 13, the projection of one line segment upon another was the segment on the second line between the feet of the perpendiculars drawn to the second line from the ends of the first line. Thus, the projection of RS upon PQ in each case here shown is the segment MN. If now, we project an oblique line, such as AB or r, upon each of two mutually perpendicular axes, respectively, then the projection of AB upon the horizontal axis is AC, or x; and the projection of AB upon the vertical axis is BC, or y. These projections, AC and BC, can be expressed as follows:

$$AC = x = r \cdot \cos \theta,$$

$$\text{and } BC = y = r \cdot \sin \theta.$$





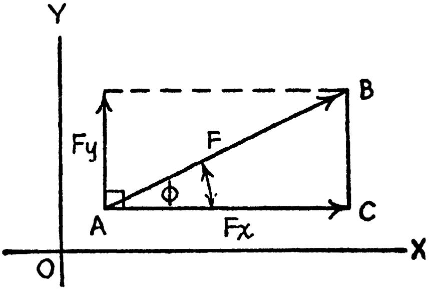


This leads to two simple but important rules concerning projections:

Rule 1: The horizontal projection of any line segment equals the length of the segment multiplied by the cosine of the angle of inclination, or the angle with the horizontal.

Rule 2: The vertical projection of any line segment equals the length of the segment multiplied by the sine of the angle of inclination.

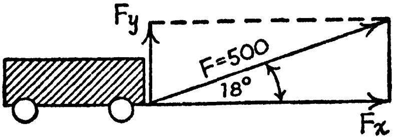
Component Forces and Velocities. These principles may be applied to the component parts of forces or velocities acting obliquely to the horizontal and vertical directions. Thus if AB represents a force F acting at an angle ϕ to the horizontal, the two perpendicular component forces F_x and F_y are equivalent, when considered jointly, to the single original force F. Furthermore, from the preceding paragraph, we now see that



$$F_x = F \cos \phi,$$
$$\text{and } F_y = F \sin \phi.$$

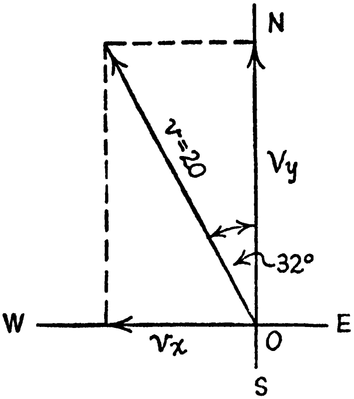
It is also clear that $F_x^2 + F_y^2 = F^2$. From these relations, many simple problems concerning forces and velocities can be solved, as will be shown below.

Example 1: A pull of 500 lb. is applied to a cart at an angle of 18° to the horizontal. What is the effective horizontal pull? How much is the force that tends to lift the cart vertically upward?



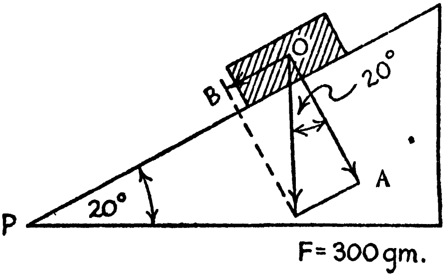
Solution: $F_x = 500(\cos 18°) = 500(.9511) = 476.6$ lb., *Ans.*
$F_y = 500(\sin 18°) = 500(.3090) = 154.5$ lb., *Ans.*

Example 2: A ship steering a course 32° west of north is moving at 20 miles an hour. Where will it be one hour after it leaves point O?



Solution: $v_x = 20(\cos 58°) = 20(.5299) = 10.6$ mi. west of point O, *Ans.*
$v_y = 20(\sin 58°) = 20(.8480) = 17.0$ mi. north of point O, *Ans.*

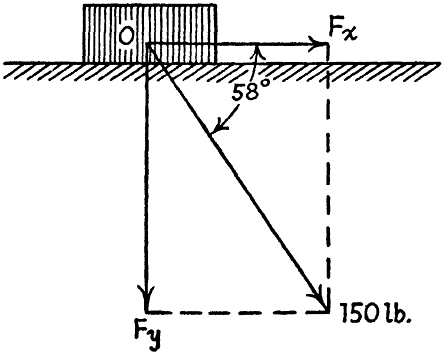
Example 3: A block rests upon an incline of 20°. If the block weighs 300 gm., find (a) the pressure exerted perpendicularly upon the inclined plane; and (b) the pull parallel to the plane, tending to cause it to slide down the plane.



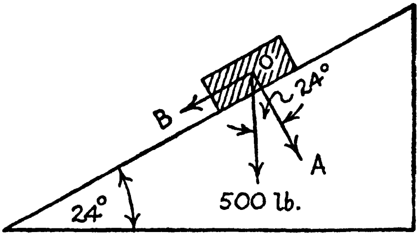
Solution: If the angle at $P=20°$, then $\angle AOF$ also equals 20°. Hence,
$OA=300(\cos 20°)=300(.9397)=281.9$ gm., *Ans.*
$OB=300(\sin 20°)=300(.3420)=102.6$ gm., *Ans.*

Exercise 72.

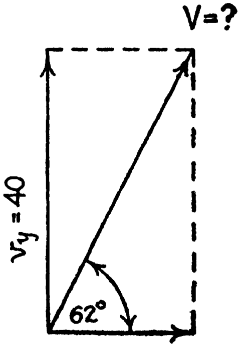
1. An object is moving with a velocity of 200 ft. per minute along a line making an angle of 33° with the horizontal. Find the horizontal component of this velocity.
2. An airplane is flying northeast at the rate of 300 miles per hour. At what rate is it moving eastward? at what rate northward?



3. A force of 150 lb. is applied to a block resting on the horizontal. If the force makes an angle of 58° with the horizontal, what force tends to draw the block horizontally? What force tends to pull it vertically downward?
4. A force of 600 lb. acting in a direction inclined 50° from the vertical is applied to a heavy object. Find (a) the force which tends to move the block horizontally, and (b) the force which tends to lift it vertically.
5. A block resting on an incline of 24° weighs 500 lb. Find (a) the perpendicular pressure (OA) exerted by the block against the inclined plane, and (b) the pull (OB) along the plane.

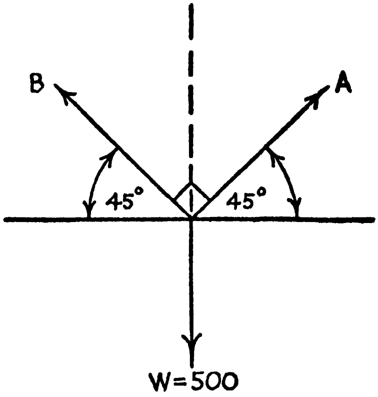


6. Find the velocity of a body moving at an angle of 62° with the horizontal, if the vertical component of its velocity is 40 ft. per second. How fast is it moving horizontally?

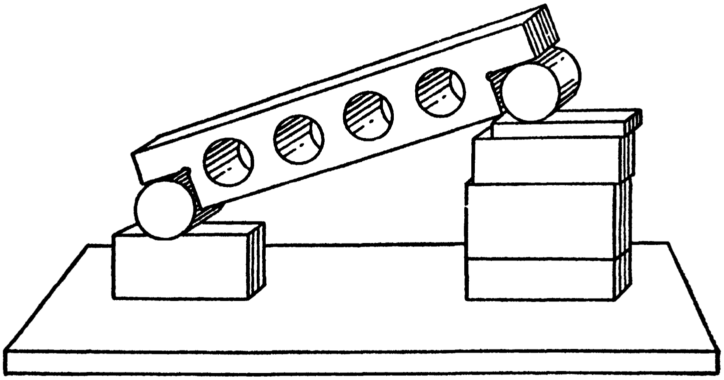


7. The horizontal and vertical components of a force acting on an object are 800 lb. and 600 lb., respectively. Find the original force and its direction of action.

8. A weight *W* of 500 lb. is suspended from point O by two stout chains, OA and OB. If each chain bears half the share of the total weight *W*, find the upward vertical pull exerted by each chain.



The Sine Bar. The sine bar is a device commonly used to facilitate precision angle-measurements. It consists of a very accurately made, heat-treated alloy-steel straight edge, to which are attached two hardened cylinders. All of the surfaces of this straight edge are parallel to each other and to the center line between the two rolls or plugs. These rolls are extremely accurately spaced, and their cylindrical surfaces made square to the measuring surfaces of the bar. The distance between studs on the 5-inch bar is controlled to within ±.0002 inch, and between ±.00025 inch on the 10-inch bar. These are the two standard sizes of the sine bar. The accompanying illustration shows how the sine bar is used in conjunction with stacks of gage blocks. A 1-inch block is generally used under the lower roll; the stack of blocks under the higher end is adjusted to create the desired angle.

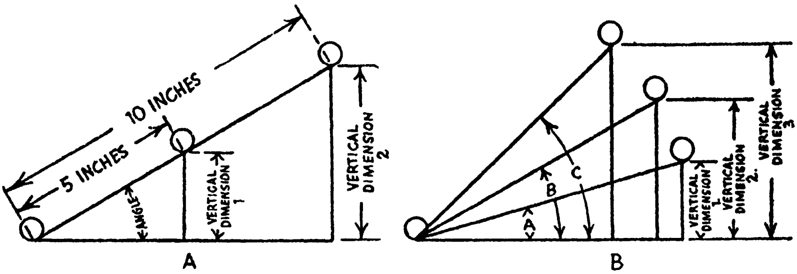


Use of Sine Bar in conjunction with Gage Blocks to obtain precision angular measurement.

Theory of the Sine Bar. The sine bar makes application of the known relation between the sides of a right triangle and its angles.

The right triangle does not exist physically as a solid triangle, but is partly imaginary. The bar itself is the only part of the triangle that actually exists. The base of the triangle is an imaginary line on the surface plate if the lower button is resting on the plate. The vertical leg of the triangle is an imaginary line from the bottom of the sine bar button to the surface plate. It may take the form of a stack of precision gage blocks if the bar is supported by the blocks, but if the bar is clamped to an angle plate, it will probably be just an imaginary line.

The fundamental relation that holds for all angles in a right triangle is that the ratio of the side opposite the angle to the length of the longest side varies with the size of the triangle.

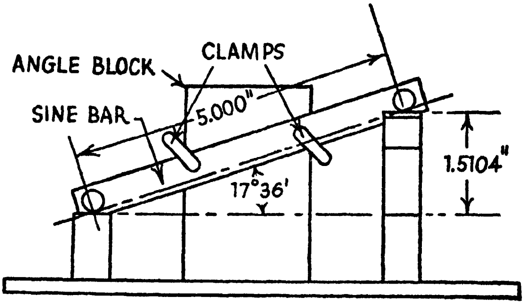


PRINCIPLE OF THE SINE BAR

Now, if the side opposite the right angle is kept constant at some figure, such as 5 inches, then the angle varies directly as the length of the side opposite, and a table of angles and corresponding lengths of opposite sides can be developed. With the aid of such a table, it is necessary to know only the length of the opposite side to determine the angle, and vice versa.

The tables of natural sines found in almost any handbook are developed on the basis of a 1-inch side opposite the right angle, or a 1-inch sine bar. For example, the table of natural sines gives a value of .56184 for the sine of 34° 11′. This means that if the side opposite the right angle is one inch, then the side opposite the angle is .56184 of an inch; and from what has been developed above, it also means that if the side opposite the right angle is 5 inches, then the side opposite the angle is 5 times .56184, or 2.8092 inches. To set up a 5-inch sine bar to an angle of 34° 11′, a stack of gage blocks 2.8092 inches high would be required under one end, with the other resting on the surface plate.

Setting Up an Angle on the Sine Bar. Let it be assumed that a part master has an angle of 17° 36′ to be checked with a sine bar. The sine bar is set up to 17° 36′ by noting the value in any sine bar table and building up a stack of blocks so that the opposite side has this value. If the lower end of the bar rests on a block one inch high, then one inch also must be added to the stack supporting the upper end; because it is the difference between the upper and the lower end which determines the opposite side of the triangle.



SETTING UP A GIVEN ANGLE

Any sine bar table would show that a difference of 1.5104 is necessary to produce an angle of 17° 36′. This means that the high end must be set at 2.5104 off the surface plate. With the bar resting on the blocks, it is clamped to the angle iron.

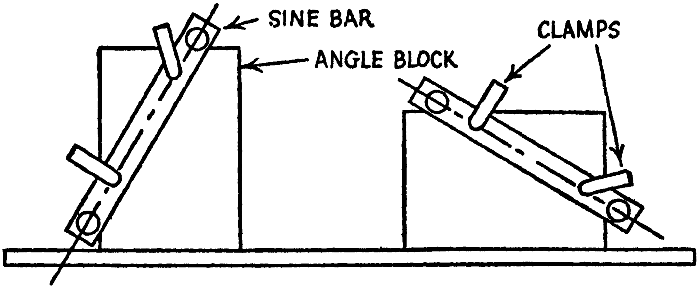
The work is then clamped to the bar so that the 17° 36′ angle on the piece is level with the surface of the plate. The surface is then indicated with a height gage and sensitive lathe indicator to verify the correctness of the angle.

Setting Up to Measure an Unknown Angle. This procedure is similar to setting up for checking a known angle. The angle is measured roughly with a bevel protractor, and the stacks of blocks necessary to produce this angle are made up from the figure read in the table.

The work is clamped to the sine bar after it is set up, and indicated with a height gage and indicator. The error noted is corrected by increasing or decreasing the height of the stack of blocks under the high end. When the indicator shows the surface to be level, the length of the stack is noted. From this, the stack at the lower end is subtracted, and the net value is the opposite side. Referring to a sine bar table, the value nearest this dimension represents the angle measured to the nearest minute. Although it might be possible to compute the number of seconds, for all practical purposes "to the nearest minute" is considered satisfactory.

Applications of the Sine Bar. The application of the sine bar generally is reserved for measurements which require that an angle be determined closer than five minutes, which is the limit of accuracy of the bevel protractor. Of course, with the sine bar it is a simple matter to determine an angle within one minute.

The sine bar is most accurate when measuring small angles, since a small change in the angle at or near the horizontal produces a greater change in the vertical dimension measured by the blocks. As the angle approaches 90 degrees, it is easy to see that the change in height corresponding to a given change in angle is much smaller. For this reason a sine bar should not be set up to measure an angle over 60 degrees, but the setup should be changed so that the complement of the angle can be measured. The complement of the angle is the difference between the angle and 90 degrees, and is obtained by tipping over the angle block to which the sine bar is set up.



MEASURING THE COMPLEMENT

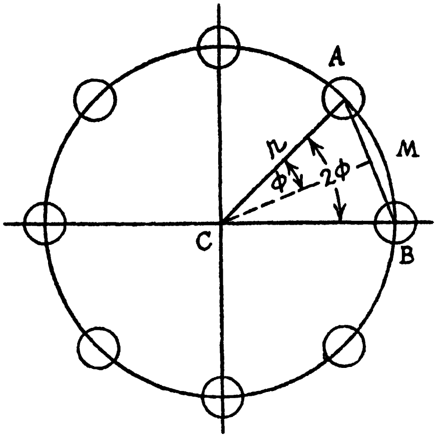
The reason for tipping the bar over and measuring the complement of a large angle instead of the angle directly is illustrated by the following example. An angle of 80 degrees has a sine of .98481, and the opposite side, using a 5-inch bar, would be 4.92405. An angle of 80 degrees, 1 minute would have an opposite side of 4.92430. This means that one minute is represented by a difference of .00025 of an inch in the height of the blocks.

If this angle is turned over on its side it becomes an angle of 10 degrees. A table of sine bar values quickly shows .86825 as the height of the stack of blocks necessary to produce an angle of 10 degrees. An angle of 10 degrees, 1 minute is produced by a stack .86965 of an inch high. One minute is represented by a difference of .0014, which means that at 10 degrees, eliminating the error in the tools themselves, a measurement is more than five times as accurate as one made at 80 degrees. This is the reason why sine bar tables usually stop at 60 degrees, and the inspector or toolmaker using a sine bar will always try to set up an angle over 60 degrees so that he checks the complement instead of the angle itself.

Spacing Holes on a Circle. In order to find the distance from center to center between holes spaced on a circle, it is only necessary to find the length of the chord (AB) joining the centers A and B. If the angle of arc AB is 2ϕ, then clearly,

$$\sin\phi = \frac{AM}{r},$$

or $AM = r\sin\phi$; since $AB = 2(AM)$, then $AB = 2r\sin\phi$; or, in terms of the diameter, the distance between adjacent holes equals $d\sin\phi$. In practice, bolt holes are usually spaced at equal distances from one another; i.e., so that their arcs have equal central angles. The circle drawn through the centers of the holes is called the ***bolt circle,*** and its diameter is, of course, less than that of the rim or outer edge of the disc, wheel, etc.



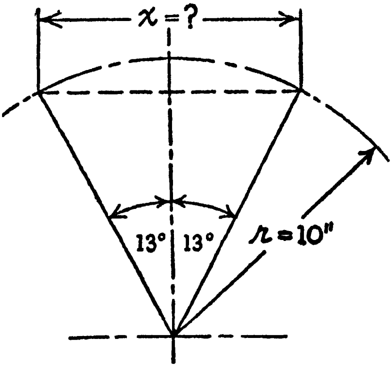
Example: Find the distance between 12 equally spaced holes on a 20″ circle.

Solution: $360° \div 12 = 30°$,
$\frac{1}{2} \times 30° = 15°$.

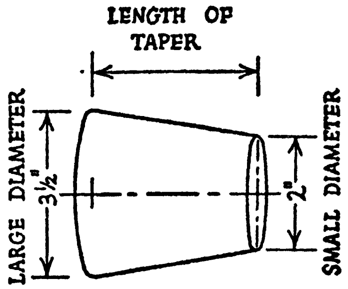
Hence distance between centers $= d\sin 15°$
$= 20(.2588) = 5.176''$, *Ans.*

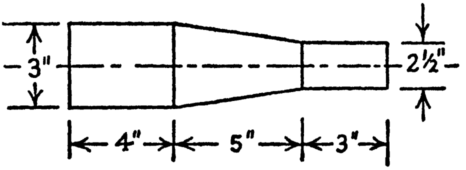
Exercise 73

1. If 8 equally spaced holes are to be drilled on a 14-in. circle, find the distance between the centers of any two adjacent holes.
2. Three holes are to be drilled 120° apart on a 12-in. bolt circle. What is the center to center distance between any two holes?
3. Find the center to center distance x between the two holes shown, if they are spaced 26° apart.
4. A bronze casting 3′ 8″ in diameter is to have 6 holes drilled in it. If the bolt circle on which the holes are to be placed is 1½″ away from the outer edge of the casting, how far apart must the holes be placed?



Tapers. In designing a conical piece of work such as here shown, the amount of the slope of the sides is called the *taper*. Such a tapered piece is actually the frustum of a cone. The amount of taper is defined as the difference in diameter per unit length of the tapered part. Frequently only part of an entire piece is tapered, the rest of it being cylindrical. If D=large diameter, d=small diameter, L=length of tapered part only, T. T.=total taper, and T= the taper per inch, then, in the second diagram:

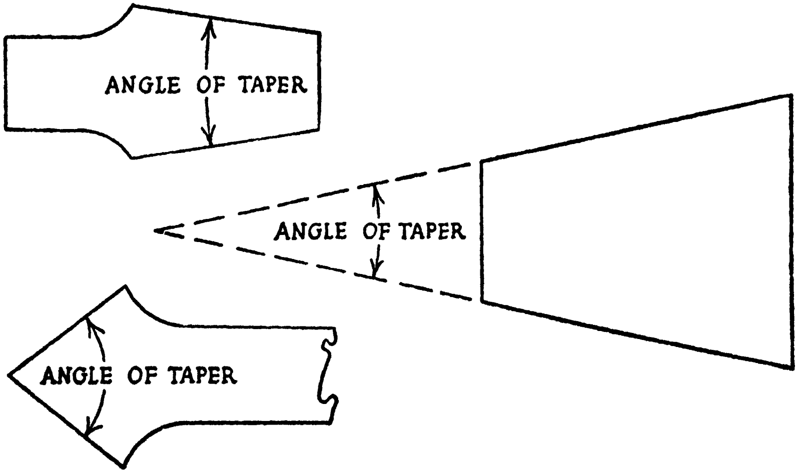




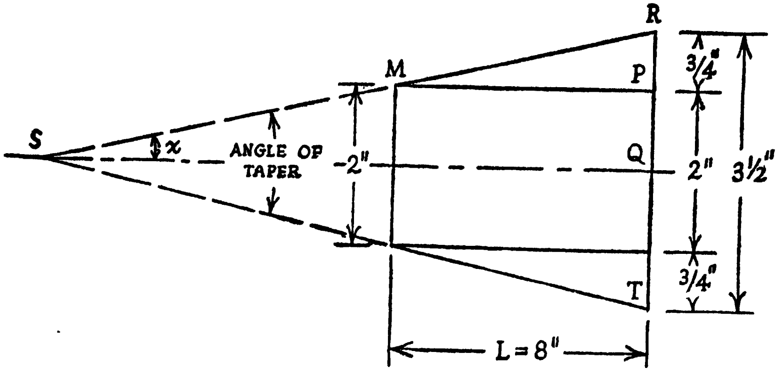
$T.\,T. = D - d = 3'' - 2\tfrac{1}{2}'' = \tfrac{1}{2}''$;

and $T = \frac{D-d}{L} = \frac{\tfrac{1}{2}}{5} = 0.1''$ per inch.

Angle of Taper. The taper on a piece of work may be expressed as so many inches per inch, or as many inches per foot. It may also be described in terms of the angle included between the sides (or the prolongation of the sides) of the piece of work.



Note carefully that the angle between a sloping side (or its extension) and the center line represents only *half* the taper angle. Computation of tapers and taper angles involves the tangent of an angle, very simply.



It is clearly seen that △ RMP is similar to △ RSQ, and that ∠RMP= ∠RSQ=½ (taper angle). Hence:

$$\tan x = \frac{\text{RP}}{\text{MP}} = \frac{\frac{1}{2}(D-d)}{L} = \frac{\frac{1}{2}(3\frac{1}{2}-2)}{8} = \frac{\frac{3}{4}}{8} = .0938, \text{ or } x = 5°21\frac{1}{2}'.$$

taper angle=2×5°21½′=10°43′.

Example 1: What is the angle of taper in a piece of work having a taper of ¾″ per foot?

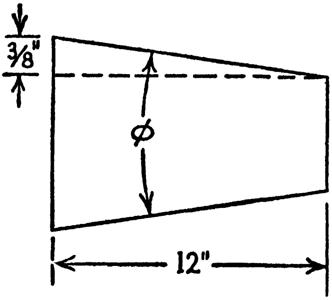
Solution:

$$\tan(\tfrac{1}{2}\text{ taper angle}) = \frac{\frac{1}{2}(\text{taper})}{\text{length in inches}}$$

$$\tan\frac{\phi}{2} = \frac{\frac{1}{2}\times\frac{3}{4}}{12} = \tfrac{1}{2}\times\tfrac{3}{4}\times\tfrac{1}{12} = .0313.$$

$$\frac{\phi}{2} = 1°48'$$

$\phi = 2\times 1°48' = 3°36'$, *Ans.*



Example 2: What is the taper per foot of a piece of work if the angle of taper is 8°30′?

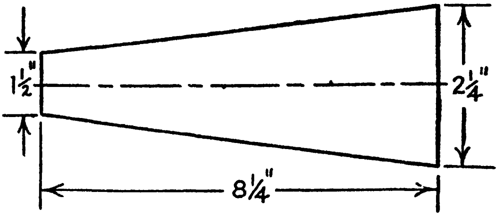
Solution: ½(taper angle)=½×8°30′=4°15′.
tan 4°15′=.0743.

Hence taper per inch=2×.0743=.1486″,
and taper per foot=12×.1486=1.7832″, *Ans.*

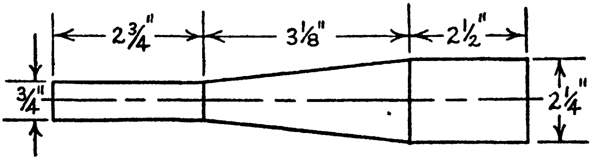
In other words, to find the taper per inch when the taper angle is given, we find the tangent of half the taper angle; this gives the taper from the center line in inches per inch. To find the total taper per inch, we multiply this result by 2. To find the total taper per foot, multiply the total taper per inch by 12.

Exercise 74.

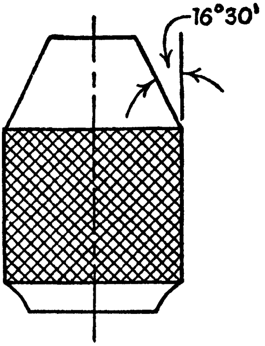
1. Find the angle of taper of the piece shown in the accompanying diagram.



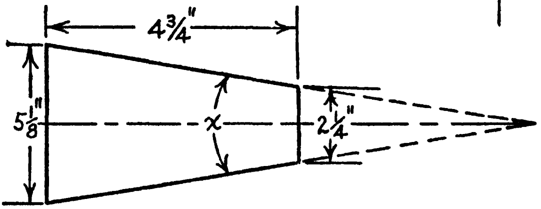
2. Find the taper angle in the following piece of work.



3. In the tapered drill chuck shown, find (a) the angle of taper, and (b) the taper per inch.

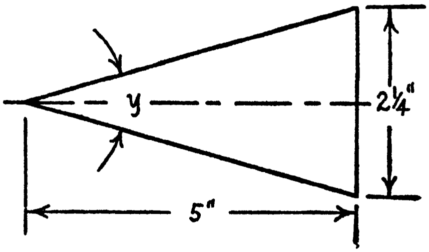


4. Determine angle x in the tapered piece shown.



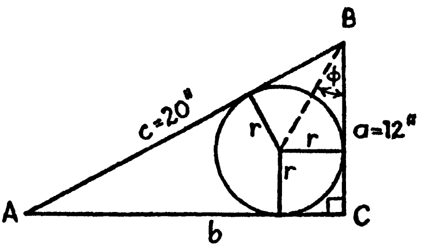
5. What is the taper per foot of a piece whose taper angle is 12°10′?

6. Find $\angle y$; if the length which is now 5″ were doubled, what would $\angle y$ then be equal to?



Miscellaneous Applications. Many problems of design in the machine shop are readily solved by making use of trigonometric relationships. Study the following and you will see how helpful trigonometry can be.

Example 1: Find the diameter of the circle inscribed in a right triangle, one of whose sides is 12″ and whose hypotenuse is 20″; also, find ϕ.



Solution: It can be shown that, when a circle is inscribed in any right triangle, the following relation holds:

hypotenuse+diameter=sum of the sides;

$$c+2r=a+b$$

or, diameter$=a+b-c$

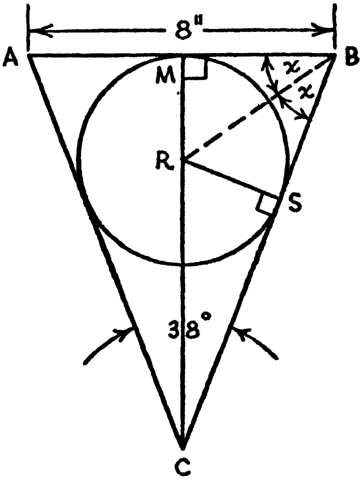
Hence, $b=\sqrt{(20)^2-(12)^2}=16''$

and diameter$=12+16-20=8''$, *Ans.*

$\tan B={}^{16}\!/_{12}=1.3333$

$B=53^\circ 8'$

$\phi=\frac{1}{2}(B)=26^\circ 34'$, *Ans.*



Example 2: Given a circle inscribed in an isosceles triangle ABC. Find the diameter of the circle if AB $=8''$ and $\angle ABC=38^\circ$.

Solution:

$\angle MCB=19^\circ$; $MB=4''$;

$\angle x=\frac{1}{2}(90^\circ-19^\circ)=35^\circ 30'$

$MR=(MB)(\tan x)$

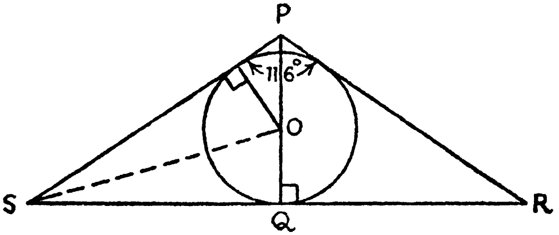
$=(4)(\tan 35^\circ 30')$

$=(4)(.7133)=2.8532$

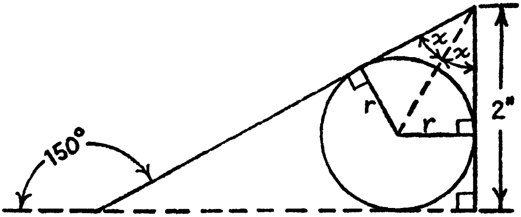
diameter$=2\times 2.8532=5.706''$, *Ans.*

Exercise 75.

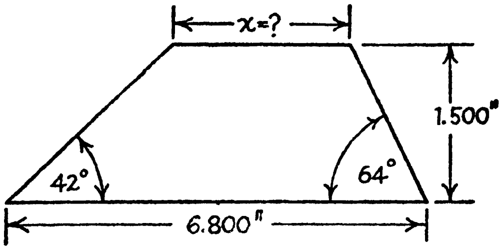
1. **Find the diameter of a circle inscribed in a right triangle whose hypotenuse is 18″ and whose longer side is 10″.**
2. **What is the diameter of a circle inscribed in a right triangle whose shorter side is 6″, if the angle adjacent to that side is 42°?**
3. If PQ=12″, and ∠SPR=116°, find the radius of the inscribed circle.

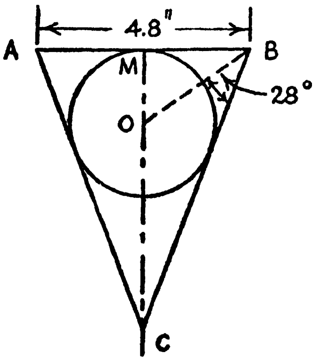


4. Find the diameter of the inscribed circle in the diagram shown.



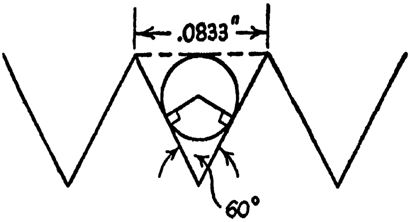
5. Find the shorter base of the trapezoid in the given diagram.



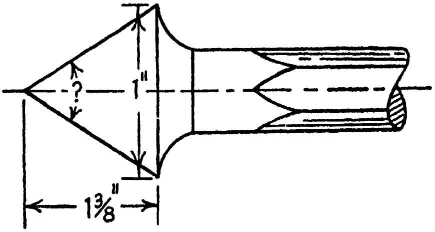


6. If ∠OBC=28°, find (a) the diameter of the inscribed circle, and (b) the depth MC.

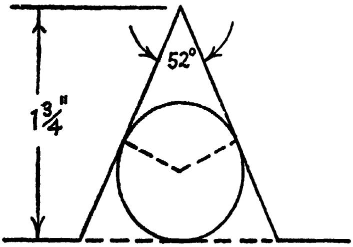
7. Find the diameter of the wire flush with the top of the 60°-screw thread shown, if the distance between two peaks is .0833″.

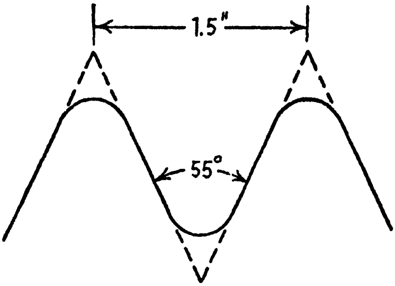


8. Find the included angle in the point of the countersink tool shown.

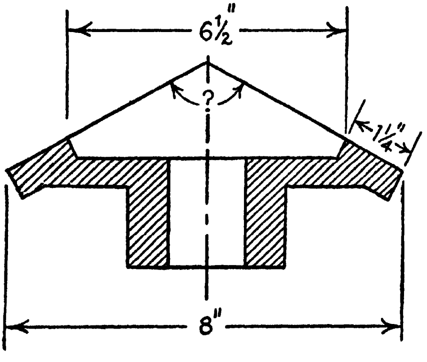


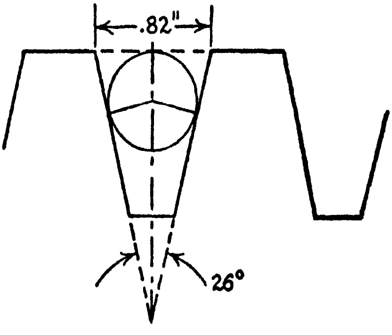
9. Find the diameter of the plug required to fit the 52° angle shown.





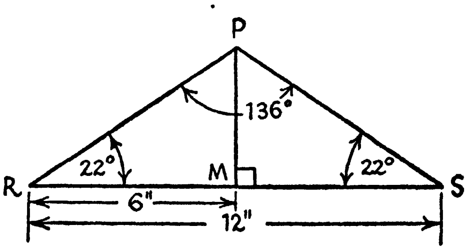
10. The special 55°-thread screw has a depth equal to ⅔ of the depth of the triangle formed when the sides of the thread are extended. Find the depth of the thread when the distance from top to top is 1.5″.
11. Find the included angle of the bevel gear blank shown below.
12. Find the diameter of the wire inserted in the worm thread with the 26°-included angle as shown.





22. SOLUTION OF OBLIQUE TRIANGLES

Oblique Triangles. By an oblique triangle is meant a triangle none of whose angles is a right angle. To solve an oblique triangle means to find the remaining sides and angles when some of these parts are known. This can sometimes be done by breaking the triangle down into right triangles; for example, if we wish to find the sides of an isosceles triangle whose base is 12″ and whose vertex angle in 136°, we draw PM perpendicular to RS, which divides the given triangle into two right triangles from which we can readily find PR. Thus:



$$\frac{RM}{RP} = \sin 68^\circ$$

$$RP = \frac{RM}{\sin 68^\circ} = \frac{6}{.9272} = 6.5, \textit{ Ans.}$$

However, it is not always convenient or possible to do this with oblique triangles, and so other methods must be used. We shall continue to use the standard notation for the sides and angles of a triangle, exactly as was done in the case of right triangles.

Functions of an Obtuse Angle. Before introducing these new methods, however, it is necessary to show how to find the functions of an obtuse angle. We already know how to find the functions of $(90^\circ - a)$; thus

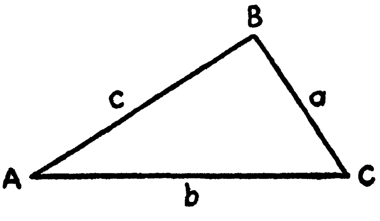
$$\sin (90^\circ - a) = \cos a$$
$$\cos (90^\circ - a) = \sin a$$
$$\tan (90^\circ - a) = \cot a$$

It can be proved, although we shall not stop to do it here, that the transformations for finding functions of $(90^\circ + a)$ and $(180^\circ - a)$ are as follows:

$$\sin (90^\circ + a) = \cos a \qquad \sin (180^\circ - a) = \sin a$$
$$\cos (90^\circ + a) = -\sin a \qquad \cos (180^\circ - a) = -\cos a$$
$$\tan (90^\circ + a) = -\cot a \qquad \tan (180^\circ - a) = -\tan a$$

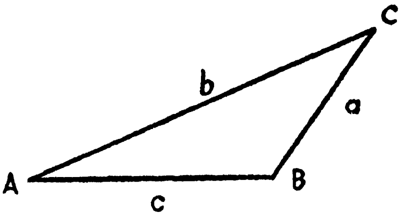
Law of Sines. A convenient trigonometrical relationship which may be utilized in solving oblique triangles is the ***law of sines,*** which may be stated as follows:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$



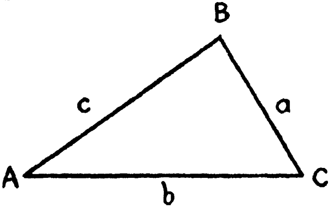
Expressed in words: *any side of a triangle is to the sine of the angle opposite that side as any other side is to the sine of its opposite angle.* Or, it may be written in another form, viz.:

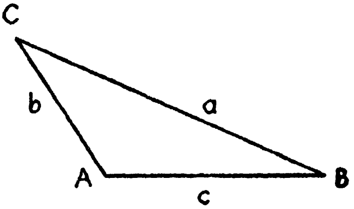
$$\frac{a}{b}=\frac{\sin A}{\sin B};\quad \frac{a}{c}=\frac{\sin A}{\sin C};\quad \frac{b}{c}=\frac{\sin B}{\sin C}.$$



That is, the ratio of any two sides of a triangle is equal to the ratio of the sines of the angles opposite them, respectively.

Law of Cosines. This law states that in any triangle, the square of any side equals the sum of the squares of the other two sides diminished by the product of those two sides and the cosine of their included angle.





$$a^2=b^2+c^2-2bc\cos A$$
$$b^2=a^2+c^2-2ac\cos B$$
$$c^2=a^2+b^2-2ab\cos C.$$

Or, solving for the angles, we get:

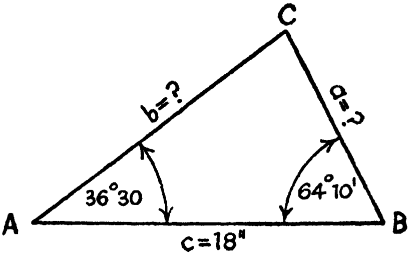
$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$
$$\cos B=\frac{a^2+c^2-b^2}{2ac}$$
$$\cos C=\frac{a^2+b^2-c^2}{2ab}.$$

Types of Problems in Solving Oblique Triangles. It is convenient to consider four types of problems, or sets of given conditions, when dealing with oblique triangles, viz.:

I. Given one side and any two angles.
II. Given two sides and the included angle.
III. Given two sides and an angle opposite one of them.
IV. Given the three sides only.

We shall explain the method used to solve each of these four types; in the illustrative problems we shall use the tables in the back of the book.

TYPE I: Given One Side and Any Two Angles. In this case the two given angles may be adjacent to the given side (a.s.a.) or they may not be (s.a.a.); it makes no difference, however, since if any two angles of a triangle are known, the third may be found immediately by subtracting their sum from 180°.



EXAMPLE 1: Given: $C=18''$, $A=36°30'$, and $B=64°10'$. Find the remaining sides.

SOLUTION: $C=180°-(36°30'+64°10')=79°20'$

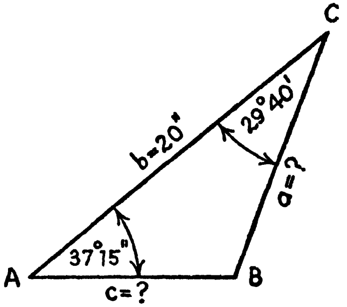
$$\frac{b}{c}=\frac{\sin B}{\sin C} \qquad \frac{a}{c}=\frac{\sin A}{\sin C}$$

$$\frac{b}{18}=\frac{\sin 64°10'}{\sin 79°20'} \qquad \frac{a}{18}=\frac{\sin 36°30'}{\sin 79°20'}$$

$$\frac{b}{18}=\frac{.9001}{.9827} \qquad \frac{a}{18}=\frac{.5948}{.9827}$$

$$b=\frac{(18)(.9001)}{.9827} \qquad a=\frac{(18)(.5948)}{.9827}$$

$b=16.49''$, *Ans.* $a=10.89''$, *Ans.*



EXAMPLE 2: Given: $b=20''$, $A=37°15'$, and $C=29°40'$. Solve the triangle.

SOLUTION: $B=180°-(37°15'+29°40')=113°5'$

$\sin B=\sin 113°5'=\sin(90°+23°5')=\cos 23°5'$

$$\frac{c}{b}=\frac{\sin C}{\sin B} \qquad \frac{a}{b}=\frac{\sin A}{\sin B}$$

$$\frac{c}{20}=\frac{\sin 29°40'}{\cos 23°5'} \qquad \frac{a}{20}=\frac{\sin 37°15'}{\cos 23°5'}$$

$$\frac{c}{20}=\frac{.4950}{.9199}$$

$$c=\frac{(20)(.4950)}{.9199}$$

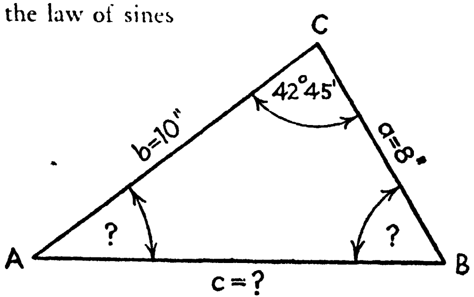
$$c=10.76'', \textit{ Ans.}$$

$$\frac{a}{20}=\frac{.6053}{.9199}$$

$$a=\frac{(20)(.6053)}{.9199}$$

$$a=13.16'', \textit{ Ans.}$$

TYPE II: Given Two Sides and the Included Angle. In this case we use the law of cosines, as well as the law of sines



EXAMPLE 1: Given: $a=8''$, $b=10''$, and $C=42°45'$. Solve the triangle.

SOLUTION: $c^2=a^2+b^2-2ab\cos C$

$$c^2=8^2+10^2-2(8)(10)(\cos 42°45')$$

$$c^2=64+100-(160)(.7343)=46.512$$

$$c=\sqrt{45.87}=6.82'', \textit{ Ans.}$$

Using the sine law to find A, we have:

$$\frac{a}{\sin A}=\frac{c}{\sin C}$$

$$\sin A=\frac{a\sin C}{c}=\frac{(8)(.6788)}{6.82}=.7962$$

$$A=52°46', \textit{ Ans.}$$

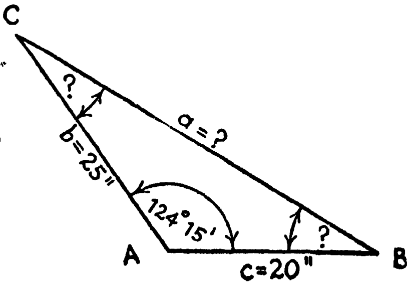
Angle B may be found by difference:

$$B=180°-(42°45'+52°46')=84°29', \textit{ Ans.}$$

As a check, use the law of sines:

$$\sin B=\frac{b\sin C}{c}=\frac{(10)(.6788)}{6.82}=.9953$$

from table, $\sin 84°29'=.9954$, Check.



EXAMPLE 2: Given: $b=25''$, $c=20''$, and $A=124°15'$. Solve the triangle.

Solution: $a^2=b^2+c^2-2bc\cos A$

$a^2=(25)^2+(20)^2-2(25)(20)(\cos 124°15')$.

But, $\cos 124°15'=\cos(90°+34°15')=-\sin 34°15'$

Hence, $a^2=625+400-(1000)(\sin 34°15')$

$a^2=625+400-(1000)(.5628)=1587.8$

$a=\sqrt{1587.8}=39.847''$, *Ans.*

$$\frac{c}{\sin C}=\frac{a}{\sin A}$$

$$\sin C=\frac{c\sin A}{a}=\frac{(20)(\sin 124°15')}{39.847}.$$

But, $\sin 124°15'=\sin(90°+34°15')=\cos 34°15'=.8266$

$$\sin C=\frac{(20)(.8266)}{39.847}=.4149$$

$C=24°31'$, *Ans.*

$B=180°-(124°15'+24°31')=31°12'$, *Ans.*

Check: $\sin B=\dfrac{b\sin A}{a}=\dfrac{(25)(.8266)}{39.847}=.5186$

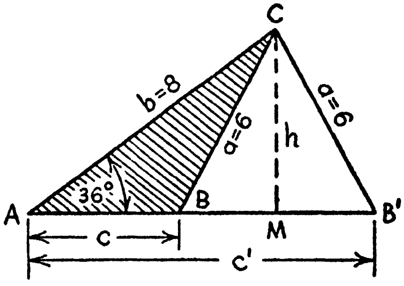
from the table, $\sin 31°12'=.5180$, Check.

Exercise 76.

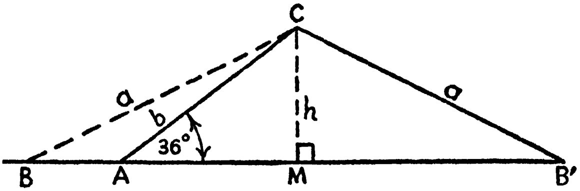
Solve the following oblique triangle, given the parts specified:

1. $A=42°30'$, $B=54°40'$, $c=20''$
2. $A=39°10'$, $C=112°$, $b=24''$
3. $A=38°45'$, $B=62°12'$, $a=18.2''$
4. $a=16''$, $c=10''$, $B=47°20'$
5. $a=20''$, $b=12''$, $C=135°$

TYPE III: Given Two Sides and an Angle Opposite One of Them. This case is a bit different from the others in that there is the possibility of two solutions, one solution, or no solution, depending upon the conditions of the problem. Consider the following diagram; given $a=6$, $b=8$, and $A=36°$. By studying the diagram it will be realized that not only the shaded triangle ABC, but also the large triangle AB′C (where CB=CB′) will satisfy the given conditions. In other words, if $a>h$ (or $b\sin A$) but $<b$, then *two* solutions are possible, viz.,

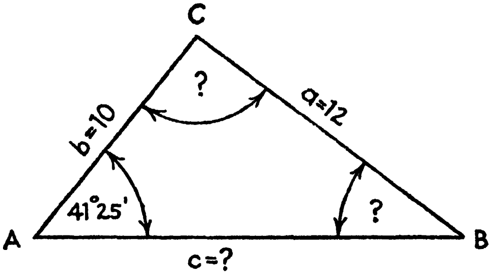


$\triangle$ABC and $\triangle$AB'C. If $a=h$ (or $b \sin A$), then only *one* solution is possible, viz., right triangle AMC. If $a<h$, *no* solution is possible, for no triangle exists in that event. Finally, if $a>h$ and also $a>b$, then again only *one* solution is possible, viz., $\triangle$ACB', since $\triangle$ACB, while it contains the given parts a and b, does *not* contain $\angle A$ (36°) as one of its interior angles, and hence does not represent a required solution. This will now be further illustrated.



Example 1: Given: $a=12$, $b=10$, and $A=41°25'$. Solve the triangle.

Solution: By constructing the triangle approximately, it is obvious that only one solution is possible.



$$\sin B=\frac{b \sin A}{a}=\frac{(10)(\sin 41°25')}{12}$$

$$\sin B=\frac{(10)(.6615)}{12}=.5513$$

$B=33°27'$, *Ans.*

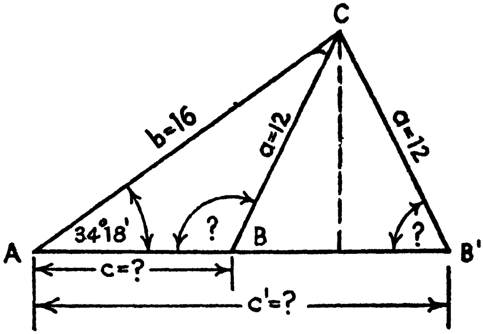
$C=180°-(41°25'+33°27')=105°8'$, *Ans.*

$$c=\frac{a \sin C}{\sin A}=\frac{(12)(\sin 105°8')}{\sin 41°25'}=\frac{(12)(\sin 74°52')}{\sin 41°25'}$$

$$c=\frac{(12)(.9653)}{.6615}=17.511,\ \textit{Ans.}$$

Example 2: Given: $a=12$, $b=16$, and $A=34°18'$. Solve the triangle.

Solution: By constructing the triangle approximately, it is seen that two triangles are possible—either △ABC or △AB′C. We shall solve △AB′C first.



$$\sin B' = \frac{b \sin A}{a} = \frac{(16)(.5635)}{12} = .7180$$

$B' = 45°54'$, *Ans.*

$C = 180° - (A+B) = 99°48'$, *Ans.*

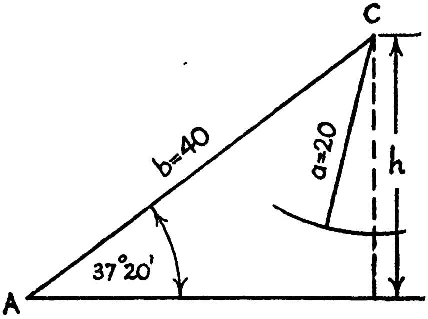
$$c' = \frac{a \sin C}{\sin A} = \frac{(12)(\sin 99°48')}{\sin 34°18'} = \frac{(12)(.9854)}{.5635}$$

$c' = 20.984$, *Ans.*

Now, to find c in the ***second*** solution, we note that ∠CBA=180° —45°54′=134°6′, since △ CBB′ is isosceles and ∠CBB′=∠CB′B= 45°54′. Also note that ∠ABC therefore equals 11°36′.

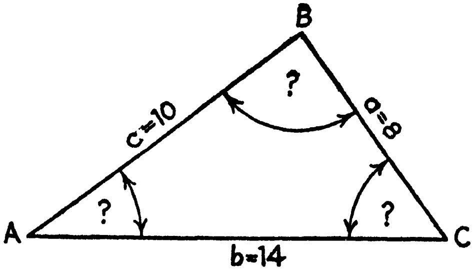
$$\text{Thus, } c = \frac{b \sin C}{\sin B} = \frac{(16)(\sin 11°36')}{\sin 134°6'};\ c = \frac{(16)(.2011)}{.7181} = 4.481, \textit{ Ans.}$$

Example 3: Given: $a=20$, $b=40$, and $A=37°20'$. Solve the triangle.



Solution: In attempting to construct the triangle approximately, it will be seen that, since $h=b\ (\sin 37°20') = (40)(.6065) = 24.26$, the length of a is $<h$ and so the triangle cannot be constructed with the parts given; thus no solution is possible.

TYPE IV: Given the Three Sides Only. When only the three sides and no angles are given, the law of cosines is used once more. In this case a triangle is always possible so long as the sum of any two of the given sides exceeds the third.



Example: Given: $a=8$, $b=14$, $c=10$. Solve the triangle.

Solution:

$$\cos A=\frac{b^2+c^2-a^2}{2bc}=\frac{(14)^2+(10)^2-(8)^2}{2(14)\ (10)}=\frac{232}{280}=.8286$$

$A=34°3'$, *Ans.*

$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{(8)^2+(14)^2-(10)^2}{2(8)(14)}=\frac{160}{224}=.7143$$

$C=44°25'$, *Ans.*

$B=180°-(A+C)=101°32'$, *Ans.*

Check:

$$\sin B=\frac{b\sin A}{a}=\frac{(14)(.5599)}{8}=.9798$$

$\sin 101°32'=\cos 11°32'=.9798$

Exercise 77.

Solve the triangles, given the following parts:

1. $A=66°$, $a=28''$, $b=16''$
2. $a=10''$, $b=12''$, $c=18''$
3. $b=5.6''$, $c=2.4''$, $B=110°$
4. $a=16.5''$, $b=12.8''$, $c=20''$
5. $A=27°30'$, $a=10''$, $c=18''$

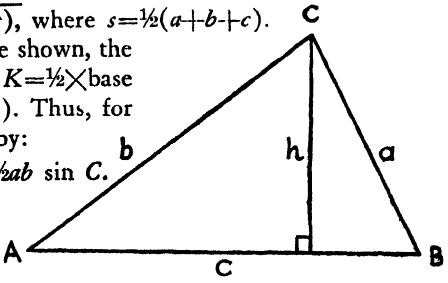
Area of Triangles. We have already seen, in the previous chapter, that the area of a triangle (K) is given by the following expressions:

(1) $K=\frac{1}{2}$ (base) (altitude);

(2) $K=\sqrt{s(s-a)(s-b)(s-c)}$, where $s=\frac{1}{2}(a+b+c)$.

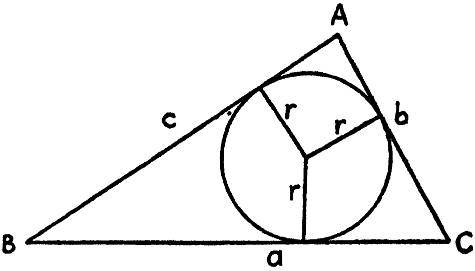
By considering the diagram here shown, the altitude $h=b\sin A$; hence area $K=\frac{1}{2}\times$base $\times$ altitude, or $K=\frac{1}{2}c(b\sin A)$. Thus, for any triangle, the area is given by:

$K=\frac{1}{2}bc\sin A=\frac{1}{2}ac\sin B=\frac{1}{2}ab\sin C.$



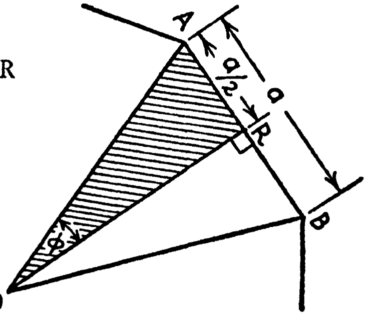
From (2) above, without actually proving it here, it can also be shown that the area of a triangle in terms of its sides and the radius of the inscribed circle is given by:

$K = s \times r$, where r = radius of the inscribed circle, and $s = \frac{1}{2}(a+b+c)$.



Area of Regular Polygons. If the side of a regular polygon is a, and there are n such sides, then $\phi = \frac{360°}{2n} = \frac{180°}{n}$, and OR = (AR) (cot ϕ) $= \frac{a}{2}$ (cot ϕ). Hence the area of $\triangle$OAR $= \frac{1}{2}(\text{AR})(\text{OR}) = \frac{1}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)(\cot\phi) = \frac{a^2}{8}\cot\phi$. But as there are $2n$ such triangles in the entire regular polygon, the area of the polygon thus equals $2n \times \frac{a^2}{8}\cot\phi$, or

$$K = \frac{1}{4}na^2 \cot\frac{180°}{n}.$$



In a similar way it can be shown that the following relations also hold for regular polygons having n sides:

(1) Area of a regular polygon circumscribed about a circle whose radius is R:

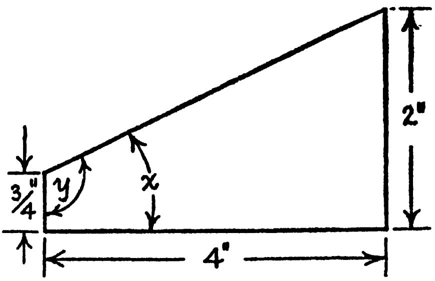
$$K = nR^2 \tan\frac{180°}{n}.$$

(2) Area of a regular polygon inscribed in a circle whose radius is r:

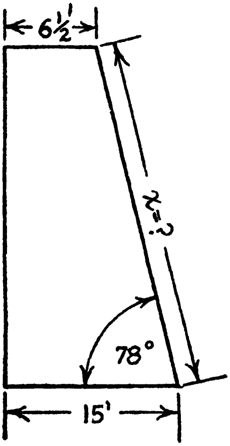
$$K = \frac{1}{2}nr^2 \sin\frac{360°}{n}.$$

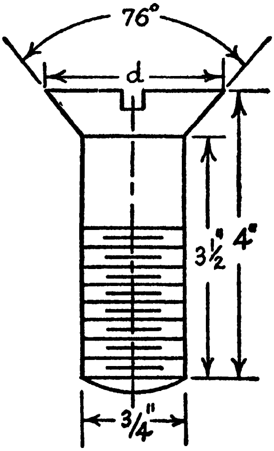
Exercise 78.

1. Find the area of an oblique triangle in which $A=34°30'$, $c=18.2''$, and $b=22.5''$.
2. Find the area of a parallelogram whose sides are 20″ and 32″, and one of whose angles is 42°20′.
3. Find the area of a regular polygon of 10 sides inscribed in a circle whose diameter is 40 cm.
4. Find the radius of a circle inscribed in a triangle whose sides are 12″, 18″ and 22″.
5. What is the area of a regular polygon of 12 sides, if each side of the polygon is 6″ long?
6. Find the number of degrees and minutes in angles x and y in the accompanying figure.
7. A parallelogram has two sides equal to 6″ and 9″, respectively, and the included angle is 39°. Find the shorter diagonal.



8. What is the length of the sloping side of the cross section of a concrete embankment if it makes an angle of 78° with the base, which is 15 ft. wide, and the top of the embankment is 6½ ft. wide?





9. Find the diameter d of the bolt with dimensions as shown.
10. The sides of a parallelogram are 10″ and 16″ long, respectively. If the longer diagonal is 20″, what are the angles of the parallelogram?

SELECTED REFERENCES FOR FURTHER STUDY

Trade Mathematics

AXELROD, A. Machine Shop Mathematics. McGraw-Hill Book Co.

BURNHAM, R. W. Mathematics for Machinists. J. Wiley & Sons.

CUSHMAN, F. Mathematics and the Machinist's Job. The Practical Mathematics of the Machinist's Trade. J. Wiley & Sons.

DOOLEY, W. H. & KRIEGEL, D. New Vocational Mathematics for Boys. D. C. Heath & Co.

FELKER, C. A. Shop Mathematics. Bruce Publishing Co.

HARPER, H. D. General Shop Mathematics. D. Van Nostrand Co.

JOHNSON, J. F. Applied Mathematics. Bruce Publishing Co.

MOYER, J. A. & SAMPSON, C. H. Practical Trade Mathematics. J. Wiley & Sons.

RAY, H. B. & DOUB, A. V. Preparatory Mathematics for Use in Technical Schools. J. Wiley & Sons.

SLADE, S. & MARGOLIS, L. Mathematics for Technical and Vocational Schools. J. Wiley & Sons.

Unit Course in Mathematics for Beginners in Machine Shop Practice. The Greenwood Co., Albany, N. Y.

VAN LEUVEN, E. P. General Trade Mathematics. McGraw-Hill Book Co.

WOLFE, J. H. & PHELPS, E. R. Practical Shop Mathematics: Vol. I, Elementary. McGraw-Hill Book Co.

NATURAL SINES AND COSINES

′	0°		1°		2°		3°		4°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	0000	1.000	0175	9998	0349	9994	0523	9986	0698	9976	60
1	0003	1.000	0177	9998	0352	9994	0526	9986	0700	9975	59
2	0006	1.000	0180	9998	0355	9994	0529	9986	0703	9975	58
3	0009	1.000	0183	9998	0358	9994	0532	9986	0706	9975	57
4	0012	1.000	0186	9998	0361	9993	0535	9986	0709	9975	56
5	0015	1.000	0189	9998	0364	9993	0538	9986	0712	9975	55
6	0017	1.000	0192	9998	0366	9993	0541	9985	0715	9974	54
7	0020	1.000	0195	9998	0369	9993	0544	9985	0718	9974	53
8	0023	1.000	0198	9998	0372	9993	0547	9985	0721	9974	52
9	0026	1.000	0201	9998	0375	9993	0550	9985	0724	9974	51
10	0029	1.000	0204	9998	0378	9993	0552	9985	0727	9974	50
11	0032	1.000	0207	9998	0381	9993	0555	9985	0729	9973	49
12	0035	1.000	0209	9998	0384	9993	0558	9984	0732	9973	48
13	0038	1.000	0212	9998	0387	9993	0561	9984	0735	9973	47
14	0041	1.000	0215	9998	0390	9992	0564	9984	0738	9973	46
15	0044	1.000	0218	9998	0393	9992	0567	9984	0741	9973	45
16	0047	1.000	0221	9998	0396	9992	0570	9984	0744	9972	44
17	0049	1.000	0224	9997	0398	9992	0573	9984	0747	9972	43
18	0052	1.000	0227	9997	0401	9992	0576	9983	0750	9972	42
19	0055	1.000	0230	9997	0404	9992	0579	9983	0753	9972	41
20	0058	1.000	0233	9997	0407	9992	0581	9983	0756	9971	40
21	0061	1.000	0236	9997	0410	9992	0584	9983	0758	9971	39
22	0064	1.000	0239	9997	0413	9991	0587	9983	0761	9971	38
23	0067	1.000	0241	9997	0416	9991	0590	9983	0764	9971	37
24	0070	1.000	0244	9997	0419	9991	0593	9982	0767	9971	36
25	0073	1.000	0247	9997	0422	9991	0596	9982	0770	9970	35
26	0076	1.000	0250	9997	0425	9991	0599	9982	0773	9970	34
27	0079	1.000	0253	9997	0427	9991	0602	9982	0776	9970	33
28	0081	1.000	0256	9997	0430	9991	0605	9982	0779	9970	32
29	0084	1.000	0259	9997	0433	9991	0608	9982	0782	9969	31
30	0087	1.000	0262	9997	0436	9990	0610	9981	0785	9969	30
31	0090	1.000	0265	9996	0439	9990	0613	9981	0787	9969	29
32	0093	1.000	0268	9996	0442	9990	0616	9981	0790	9969	28
33	0096	1.000	0270	9996	0445	9990	0619	9981	0793	9968	27
34	0099	1.000	0273	9996	0448	9990	0622	9981	0796	9968	26
35	0102	9999	0276	9996	0451	9990	0625	9980	0799	9968	25
36	0105	9999	0279	9996	0454	9990	0628	9980	0802	9968	24
37	0108	9999	0282	9996	0457	9990	0631	9980	0805	9968	23
38	0111	9999	0285	9996	0459	9989	0634	9980	0808	9967	22
39	0113	9999	0288	9996	0462	9989	0637	9980	0811	9967	21
40	0116	9999	0291	9996	0465	9989	0640	9980	0814	9967	20
41	0119	9999	0294	9996	0468	9989	0642	9979	0816	9967	19
42	0122	9999	0297	9996	0471	9989	0645	9979	0819	9966	18
43	0125	9999	0300	9996	0474	9989	0648	9979	0822	9966	17
44	0128	9999	0302	9995	0477	9989	0651	9979	0825	9966	16
45	0131	9999	0305	9995	0480	9988	0654	9979	0828	9966	15
46	0134	9999	0308	9995	0483	9988	0657	9978	0831	9965	14
47	0137	9999	0311	9995	0486	9988	0660	9978	0834	9965	13
48	0140	9999	0314	9995	0488	9988	0663	9978	0837	9965	12
49	0143	9999	0317	9995	0491	9988	0666	9978	0840	9965	11
50	0145	9999	0320	9995	0494	9988	0669	9978	0843	9964	10
51	0148	9999	0323	9995	0497	9988	0671	9977	0845	9964	9
52	0151	9999	0326	9995	0500	9987	0674	9977	0848	9964	8
53	0154	9999	0329	9995	0503	9987	0677	9977	0851	9964	7
54	0157	9999	0332	9995	0506	9987	0680	9977	0854	9963	6
55	0160	9999	0334	9994	0509	9987	0683	9977	0857	9963	5
56	0163	9999	0337	9994	0512	9987	0686	9976	0860	9963	4
57	0166	9999	0340	9994	0515	9987	0689	9976	0863	9963	3
58	0169	9999	0343	9994	0518	9987	0692	9976	0866	9962	2
59	0172	9999	0346	9994	0520	9986	0695	9976	0869	9962	1
60	0175	9999	0349	9994	0523	9986	0698	9976	0872	9962	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	89°		88°		87°		86°		85°		′

NATURAL SINES AND COSINES

′	5°		6°		7°		8°		9°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
●	0872	9962	1045	9945	1219	9925	1392	9903	1564	9877	60
1	0874	9962	1048	9945	1222	9925	1395	9902	1567	9876	59
2	0877	9961	1051	9945	1224	9925	1397	9902	1570	9876	58
3	0880	9961	1054	9944	1227	9924	1400	9901	1573	9876	57
4	0883	9961	1057	9944	1230	9924	1403	9901	1576	9875	56
5	0886	9961	1060	9944	1233	9924	1406	9901	1579	9875	55
6	0889	9960	1063	9943	1236	9923	1409	9900	1582	9874	54
7	0892	9960	1066	9943	1239	9923	1412	9900	1584	9874	53
8	0895	9960	1068	9943	1241	9923	1415	9899	1587	9873	52
9	0898	9960	1071	9942	1245	9922	1418	9899	1590	9873	51
10	0901	9959	1074	9942	1248	9922	1421	9899	1593	9872	50
11	0903	9959	1077	9942	1250	9922	1423	9898	1596	9872	49
12	0906	9959	1080	9942	1253	9921	1426	9898	1599	9871	48
13	0909	9959	1083	9941	1256	9921	1429	9897	1602	9871	47
14	0912	9958	1086	9941	1259	9920	1432	9897	1605	9870	46
15	0915	9958	1089	9941	1262	9920	1435	9897	1607	9870	45
16	0918	9958	1092	9940	1265	9920	1438	9896	1610	9869	44
17	0921	9958	1094	9940	1268	9919	1441	9896	1613	9869	43
18	0924	9957	1097	9940	1271	9919	1444	9895	1616	9869	42
19	0927	9957	1100	9939	1274	9919	1446	9895	1619	9868	41
20	0929	9957	1103	9939	1276	9918	1449	9894	1622	9868	40
21	0932	9956	1106	9939	1279	9918	1452	9894	1625	9867	39
22	0935	9956	1109	9938	1282	9917	1455	9894	1628	9867	38
23	0938	9956	1112	9938	1285	9917	1458	9893	1630	9866	37
24	0941	9956	1115	9938	1288	9917	1461	9893	1633	9866	36
25	0944	9955	1118	9937	1291	9916	1464	9802	1636	9865	35
26	0947	9955	1120	9937	1294	9916	1467	9892	1639	9865	34
27	0950	9955	1123	9937	1297	9916	1469	9891	1642	9864	33
28	0953	9955	1126	9936	1299	9915	1472	9891	1645	9864	32
29	0956	9954	1129	9936	1302	9915	1475	9891	1648	9863	31
30	0958	9954	1132	9936	1305	9914	1478	9890	1650	9863	30
31	0961	9954	1135	9935	1308	9914	1481	9890	1653	9862	29
32	0964	9953	1138	9935	1311	9914	1484	9889	1656	986	28
33	0967	9953	1141	9935	1314	9913	1487	9889	1659	986.	27
34	0970	9953	1144	9934	1317	9913	1490	9888	1662	9861	26
35	0973	9953	1146	9934	1320	9913	1492	9888	1665	9860	25
36	0976	9952	1149	9934	1323	9912	1495	9888	1668	9860	24
37	0979	9952	1152	9933	1325	9912	1498	9887	1671	9859	23
38	0982	9952	1155	9933	1328	9911	1501	9887	1673	9859	22
39	0985	9951	1158	9933	1331	9911	1504	9886	1676	9859	21
40	0987	9951	1161	9932	1334	9911	1507	9886	1679	9858	20
41	0990	9951	1164	9932	1337	9910	1510	9885	1682	9858	19
42	0993	9951	1167	9932	1340	9910	1513	9885	1685	9857	18
43	0996	9950	1170	9931	1343	9909	1515	9884	1688	9857	17
44	0999	9950	1172	9931	1346	9909	1518	9884	1691	9856	16
45	1002	9950	1175	9931	1349	9909	1521	9884	1693	9856	15
46	1005	9949	1178	9930	1351	9908	1524	9883	1696	9855	14
47	1008	9949	1181	9930	1354	9908	1527	9883	1699	9855	13
48	1011	9949	1184	9930	1357	9907	1530	9882	1702	9854	12
49	1013	9949	1187	9929	1360	9907	1533	9882	1705	9854	11
50	1016	9948	1190	9929	1363	9907	1536	9881	1708	9853	10
51	1019	9948	1193	9929	1366	9906	1538	9881	1711	9853	9
52	1022	9948	1196	9928	1369	9906	1541	9880	1714	9852	8
53	1025	9947	1198	9928	1372	9905	1544	9880	1716	9852	7
54	1028	9947	1201	9928	1374	9905	1547	9880	1719	9851	6
55	1031	9947	1204	9927	1377	9905	1550	9879	1722	9851	5
56	1034	9946	1207	9927	1380	9904	1553	9879	1725	9850	4
57	1037	9946	1210	9927	1383	9904	1556	9878	1728	9850	3
58	1039	9946	1213	9926	1386	9903	1559	9878	1731	9849	2
59	1042	9946	1216	9926	1389	9903	1561	9877	1734	9849	1
60	1045	9945	1219	9925	1392	9903	1564	9877	1736	9848	●
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	84°		83°		82°		81°		80°		′

NATURAL SINES AND COSINES

′	10°		11°		12°		13°		14°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	1736	9848	1908	9816	2079	9781	2250	9744	2419	9703	60
1	1739	9848	1911	9816	2082	9781	2252	9743	2422	9702	59
2	1742	9847	1914	9815	2085	9780	2255	9742	2425	9702	58
3	1745	9847	1917	9815	2088	9780	2258	9742	2428	9701	57
4	1748	9846	1920	9814	2090	9779	2261	9741	2431	9700	56
5	1751	9846	1922	9813	2093	9778	2264	9740	2433	9699	55
6	1754	9845	1925	9813	2096	9778	2267	9740	2436	9699	54
7	1757	9845	1928	9812	2099	9777	2269	9739	2439	9698	53
8	1759	9844	1931	9812	2102	9777	2272	9738	2442	9697	52
9	1762	9843	1934	9811	2105	9776	2275	9738	2445	9697	51
10	1765	9843	1937	9811	2108	9775	2278	9737	2447	9696	50
11	1768	9842	1939	9810	2110	9775	2281	9736	2450	9695	49
12	1771	9842	1942	9810	2113	9774	2284	9736	2453	9694	48
13	1774	9841	1945	9809	2116	9774	2286	9735	2456	9694	47
14	1777	9841	1948	9808	2119	9773	2289	9734	2459	9693	46
15	1779	9840	1951	9808	2122	9772	2292	9734	2462	9692	45
16	1782	9840	1954	9807	2125	9772	2295	9733	2464	9692	44
17	1785	9839	1957	9807	2127	9771	2298	9732	2467	9691	43
18	1788	9839	1959	9806	2130	9770	2300	9732	2470	9690	42
19	1791	9838	1962	9806	2133	9770	2303	9731	2473	9689	41
20	1794	9838	1965	9805	2136	9769	2306	9730	2476	9689	40
21	1797	9837	1968	9804	2139	9769	2309	9730	2478	9688	39
22	1799	9837	1971	9804	2142	9768	2312	9729	2481	9687	38
23	1802	9836	1974	9803	2145	9767	2315	9728	2484	9687	37
24	1805	9836	1977	9803	2147	9767	2317	9728	2487	9686	36
25	1808	9835	1979	9802	2150	9766	2320	9727	2490	9685	35
26	1811	9835	1982	9802	2153	9765	2323	9726	2493	9684	34
27	1814	9834	1985	9801	2156	9765	2326	9726	2495	9684	33
28	1817	9834	1988	9800	2159	9764	2329	9725	2498	9683	32
29	1819	9833	1991	9800	2162	9764	2332	9724	2501	9682	31
30	1822	9833	1994	9799	2164	9763	2334	9724	2504	9681	30
31	1825	9832	1997	9799	2167	9762	2337	9723	2507	9681	29
32	1828	9831	1999	9798	2170	9762	2340	9722	2509	9680	28
33	1831	9831	2002	9798	2173	9761	2343	9722	2512	9679	27
34	1834	9830	2005	9797	2176	9760	2346	9721	2515	9679	26
35	1837	9830	2008	9796	2179	9760	2349	9720	2518	9678	25
36	1840	9829	2011	9796	2181	9759	2351	9720	2521	9677	24
37	1842	9829	2014	9795	2184	9759	2354	9719	2524	9676	23
38	1845	9828	2016	9795	2187	9758	2357	9718	2526	9676	22
39	1848	9828	2019	9794	2190	9757	2360	9718	2529	9675	21
40	1851	9827	2022	9793	2193	9757	2363	9717	2532	9674	20
41	1854	9827	2025	9793	2196	9756	2366	9716	2535	9673	19
42	1857	9826	2028	9792	2198	9755	2368	9715	2538	9673	18
43	1860	9826	2031	9792	2201	9755	2371	9715	2540	9672	17
44	1862	9825	2034	9791	2204	9754	2374	9714	2543	9671	16
45	1865	9825	2036	9790	2207	9753	2377	9713	2546	9670	15
46	1868	9824	2039	9790	2210	9753	2380	9713	2549	9670	14
47	1871	9823	2042	9789	2213	9752	2383	9712	2552	9669	13
48	1874	9823	2045	9789	2215	9751	2385	9711	2554	9668	12
49	1877	9822	2048	9788	2218	9751	2388	9711	2557	9667	11
50	1880	9822	2051	9787	2221	9750	2391	9710	2560	9667	10
51	1882	9821	2054	9787	2224	9750	2394	9709	2563	9666	9
52	1885	9821	2056	9786	2227	9749	2397	9709	2566	9665	8
53	1888	9820	2059	9786	2230	9748	2399	9708	2569	9665	7
54	1891	9820	2062	9785	2233	9748	2402	9707	2571	9664	6
55	1894	9819	2065	9784	2235	9747	2405	9706	2574	9663	5
56	1897	9818	2068	9784	2238	9746	2408	9706	2577	9662	4
57	1900	9818	2071	9783	2241	9746	2411	9705	2580	9662	3
58	1902	9817	2073	9783	2244	9745	2414	9704	2583	9661	2
59	1905	9817	2076	9782	2247	9744	2416	9704	2585	9660	1
60	1908	9816	2079	9781	2250	9744	2419	9703	2588	9659	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	79°		78°		77°		76°		75°		′

NATURAL SINES AND COSINES

′	15°		16°		17°		18°		19°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	2588	9659	2756	9613	2924	9563	3090	9511	3256	9455	60
1	2591	9659	2759	9612	2926	9562	3093	9510	3258	9454	59
2	2594	9658	2762	9611	2929	9561	3096	9509	3261	9453	58
3	2597	9657	2765	9610	2932	9560	3098	9508	3264	9452	57
4	2599	9656	2768	9609	2935	9560	3101	9507	3267	9451	56
5	2602	9655	2770	9609	2938	9559	3104	9506	3269	9450	55
6	2605	9655	2773	9608	2940	9558	3107	9505	3272	9449	54
7	2608	9654	2776	9607	2943	9557	3110	9504	3275	9449	53
8	2611	9653	2779	9606	2946	9556	3112	9503	3278	9448	52
9	2613	9652	2782	9605	2949	9555	3115	9502	3280	9447	51
10	2616	9652	2784	9605	2952	9555	3118	9502	3283	9446	50
11	2619	9651	2787	9604	2954	9554	3121	9501	3286	9445	49
12	2622	9650	2790	9603	2957	9553	3123	9500	3289	9444	48
13	2625	9649	2793	9602	2960	9552	3126	9499	3291	9443	47
14	2628	9649	2795	9601	2963	9551	3129	9498	3294	9442	46
15	2630	9648	2798	9600	2965	9550	3132	9497	3297	9441	45
16	2633	9647	2801	9600	2968	9549	3134	9496	3300	9440	44
17	2636	9646	2804	9599	2971	9548	3137	9495	3302	9439	43
18	2639	9646	2807	9598	2974	9548	3140	9494	3305	9438	42
19	2642	9645	2809	9597	2977	9547	3143	9493	3308	9437	41
20	2644	9644	2812	9596	2979	9546	3145	9492	3311	9436	40
21	2647	9643	2815	9596	2982	9545	3148	9492	3313	9435	39
22	2650	9642	2818	9595	2985	9544	3151	9491	3316	9434	38
23	2653	9642	2821	9594	2988	9543	3154	9490	3319	9433	37
24	2656	9641	2823	9593	2990	9542	3156	9489	3322	9432	36
25	2658	9640	2826	9592	2993	9542	3159	9488	3324	9431	35
26	2661	9639	2829	9591	2996	9541	3162	9487	3327	9430	34
27	2664	9639	2832	9591	2999	9540	3165	9486	3330	9429	33
28	2667	9638	2835	9590	3002	9539	3168	9485	3333	9428	32
29	2670	9637	2837	9589	3004	9538	3170	9484	3335	9427	31
30	2672	9636	2840	9588	3007	9537	3173	9483	3338	9426	30
31	2675	9636	2843	9587	3010	9536	3176	9482	3341	9425	29
32	2678	9635	2846	9587	3013	9535	3179	9481	3344	9424	28
33	2681	9634	2849	9586	3015	9535	3181	9480	3346	9423	27
34	2684	9633	2851	9585	3018	9534	3184	9480	3349	9423	26
35	2686	9632	2854	9584	3021	9533	3187	9479	3352	9422	25
36	2689	9632	2857	9583	3024	9532	3190	9478	3355	9421	24
37	2692	9631	2860	9582	3026	9531	3192	9477	3357	9420	23
38	2695	9630	2862	9582	3029	9530	3195	9476	3360	9419	22
39	2698	9629	2865	9581	3032	9529	3198	9475	3363	9418	21
40	2700	9628	2868	9580	3035	9528	3201	9474	3365	9417	20
41	2703	9628	2871	9579	3038	9527	3203	9473	3368	9416	19
42	2706	9627	2874	9578	3040	9527	3206	9472	3371	9415	18
43	2709	9626	2876	9577	3043	9526	3209	9471	3374	9414	17
44	2712	9625	2879	9577	3046	9525	3212	9470	3376	9413	16
45	2714	9625	2882	9576	3049	9524	3214	9469	3379	9412	15
46	2717	9624	2885	9575	3051	9523	3217	9468	3382	9411	14
47	2720	9623	2888	9574	3054	9522	3220	9467	3385	9410	13
48	2723	9622	2890	9573	3057	9521	3223	9466	3387	9409	12
49	2726	9621	2893	9572	3060	9520	3225	9466	3390	9408	11
50	2728	9621	2896	9572	3062	9520	3228	9465	3393	9407	10
51	2731	9620	2899	9571	3065	9519	3231	9464	3396	9406	9
52	2734	9619	2901	9570	3068	9518	3234	9463	3398	9405	8
53	2737	9618	2904	9569	3071	9517	3236	9462	3401	9404	7
54	2740	9617	2907	9568	3074	9516	3239	9461	3404	9403	6
55	2742	9617	2910	9567	3076	9515	3242	9460	3407	9402	5
56	2745	9616	2913	9566	3079	9514	3245	9459	3409	9401	4
57	2748	9615	2915	9566	3082	9513	3247	9458	3412	9400	3
58	2751	9614	2918	9565	3085	9512	3250	9457	3415	9399	2
59	2754	9613	2921	9564	3087	9511	3253	9456	3417	9398	1
60	2756	9613	2924	9563	3090	9511	3256	9455	3420	9397	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	74°		73°		72°		71°		70°		′

′	20°		21°		22°		23°		24°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	3420	9397	3584	9336	3746	9272	3907	9205	4067	9135	60
1	3423	9396	3586	9335	3749	9271	3910	9204	4070	9134	59
2	3426	9395	3589	9334	3751	9270	3913	9203	4073	9133	58
3	3428	9394	3592	9333	3754	9269	3915	9202	4075	9132	57
4	3431	9393	3595	9332	3757	9267	3918	9200	4078	9131	56
5	3434	9392	3597	9331	3760	9266	3921	9199	4081	9130	55
6	3437	9391	3600	9330	3762	9265	3923	9198	4083	9128	54
7	3439	9390	3603	9328	3765	9264	3926	9197	4086	9127	53
8	3442	9389	3605	9327	3768	9263	3929	9196	4089	9126	52
9	3445	9388	3608	9326	3770	9262	3931	9195	4091	9125	51
10	3448	9387	3611	9325	3773	9261	3934	9194	4094	9124	50
11	3450	9386	3614	9324	3776	9260	3937	9192	4097	9122	49
12	3453	9385	3616	9323	3778	9259	3939	9191	4099	9121	48
13	3456	9384	3619	9322	3781	9258	3942	9190	4102	9120	47
14	3458	9383	3622	9321	3784	9257	3945	9189	4105	9119	46
15	3461	9382	3624	9320	3786	9255	3947	9188	4107	9118	45
16	3464	9381	3627	9319	3789	9254	3950	9187	4110	9116	44
17	3467	9380	3630	9318	3792	9253	3953	9186	4112	9115	43
18	3469	9379	3633	9317	3795	9252	3955	9184	4115	9114	42
19	3472	9378	3635	9316	3797	9251	3958	9183	4118	9113	41
20	3475	9377	3638	9315	3800	9250	3961	9182	4120	9112	40
21	3478	9376	3641	9314	3803	9249	3963	9181	4123	9110	39
22	3480	9375	3643	9313	3805	9248	3966	9180	4126	9109	38
23	3483	9374	3646	9312	3808	9247	3969	9179	4128	9108	37
24	3486	9373	3649	9311	3811	9245	3971	9178	4131	9107	36
25	3488	9372	3651	9309	3813	9244	3974	9176	4134	9106	35
26	3491	9371	3654	9308	3816	9243	3977	9175	4136	9104	34
27	3494	9370	3657	9307	3819	9242	3979	9174	4139	9103	33
28	3497	9369	3660	9306	3821	9241	3982	9173	4142	9102	32
29	3499	9368	3662	9305	3824	9240	3985	9172	4144	9101	31
30	3502	9367	3665	9304	3827	9239	3987	9171	4147	9100	30
31	3505	9366	3668	9303	3830	9238	3990	9169	4150	9098	29
32	3508	9365	3670	9302	3832	9237	3993	9168	4152	9097	28
33	3510	9364	3673	9301	3835	9235	3995	9167	4155	9096	27
34	3513	9363	3676	9300	3838	9234	3998	9166	4158	9095	26
35	3516	9362	3679	9299	3840	9233	4001	9165	4160	9094	25
36	3518	9361	3681	9298	3843	9232	4003	9164	4163	9092	24
37	3521	9360	3684	9297	3846	9231	4006	9162	4165	9091	23
38	3524	9359	3687	9296	3848	9230	4009	9161	4168	9090	22
39	3527	9358	3689	9295	3851	9229	4011	9160	4171	9089	21
40	3529	9356	3692	9293	3854	9228	4014	9159	4173	9088	20
41	3532	9355	3695	9292	3856	9227	4017	9158	4176	9086	19
42	3535	9354	3697	9291	3859	9225	4019	9157	4179	9085	18
43	3537	9353	3700	9290	3862	9224	4022	9155	4181	9084	17
44	3540	9352	3703	9289	3864	9223	4025	9154	4184	9083	16
45	3543	9351	3706	9288	3867	9222	4027	9153	4187	9081	15
46	3546	9350	3708	9287	3870	9221	4030	9152	4189	9080	14
47	3548	9349	3711	9286	3872	9220	4033	9151	4192	9079	13
48	3551	9348	3714	9285	3875	9219	4035	9150	4195	9078	12
49	3554	9347	3716	9284	3878	9218	4038	9148	4197	9077	11
50	3557	9346	3719	9283	3881	9216	4041	9147	4200	9075	10
51	3559	9345	3722	9282	3883	9215	4043	9146	4202	9074	9
52	3562	9344	3724	9281	3886	9214	4046	9145	4205	9073	8
53	3565	9343	3727	9279	3889	9213	4049	9144	4208	9072	7
54	3567	9342	3730	9278	3891	9212	4051	9143	4210	9070	6
55	3570	9341	3733	9277	3894	9211	4054	9141	4213	9069	5
56	3573	9340	3735	9276	3897	9210	4057	9140	4216	9068	4
57	3576	9339	3738	9275	3899	9208	4059	9139	4218	9067	3
58	3578	9338	3741	9274	3902	9207	4062	9138	4221	9066	2
59	3581	9337	3743	9273	3905	9206	4065	9137	4224	9064	1
60	3584	9336	3746	9272	3907	9205	4067	9135	4226	9063	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	69°		68°		67°		66°		65°		′

′	25°		26°		27°		28°		29°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	4226	9063	4384	8988	4540	8910	4695	8829	4848	8746	60
1	4229	9062	4386	8987	4542	8909	4697	8828	4851	8745	59
2	4231	9061	4389	8985	4545	8907	4700	8827	4853	8743	58
3	4234	9059	4392	8984	4548	8906	4702	8825	4856	8742	57
4	4237	9058	4394	8983	4550	8905	4705	8824	4858	8741	56
5	4239	9057	4397	8982	4553	8903	4708	8823	4861	8739	55
6	4242	9056	4399	8980	4555	8902	4710	8821	4863	8738	54
7	4245	9054	4402	8979	4558	8901	4713	8820	4866	8736	53
8	4247	9053	4405	8978	4561	8899	4715	8819	4868	8735	52
9	4250	9052	4407	8976	4563	8898	4718	8817	4871	8733	51
10	4253	9051	4410	8975	4566	8897	4720	8816	4874	8732	50
11	4255	9050	4412	8974	4568	8895	4723	8814	4876	8731	49
12	4258	9048	4415	8973	4571	8894	4726	8813	4879	8729	48
13	4260	9047	4418	8971	4574	8893	4728	8812	4881	8728	47
14	4263	9046	4420	8970	4576	8892	4731	8810	4884	8726	46
15	4266	9045	4423	8969	4579	8890	4733	8809	4886	8725	45
16	4268	9043	4425	8967	4581	8889	4736	8808	4889	8724	44
17	4271	9042	4428	8966	4584	8888	4738	8806	4891	8722	43
18	4274	9041	4431	8965	4586	8886	4741	8805	4894	8721	42
19	4276	9040	4433	8964	4589	8885	4743	8803	4896	8719	41
20	4279	9038	4436	8962	4592	8884	4746	8802	4899	8718	40
21	4281	9037	4439	8961	4594	8882	4749	8801	4901	8716	39
22	4284	9036	4441	8960	4597	8881	4751	8799	4904	8715	38
23	4287	9035	4444	8958	4599	8879	4754	8798	4907	8714	37
24	4289	9033	4446	8957	4602	8878	4756	8796	4909	8712	36
25	4292	9032	4449	8956	4605	8877	4759	8795	4912	8711	35
26	4295	9031	4452	8955	4607	8875	4761	8794	4914	8709	34
27	4297	9030	4454	8953	4610	8874	4764	8792	4917	8708	33
28	4300	9028	4457	8952	4612	8873	4766	8791	4919	8706	32
29	4302	9027	4459	8951	4615	8871	4769	8790	4922	8705	31
30	4305	9026	4462	8949	4617	8870	4772	8788	4924	8704	30
31	4308	9025	4465	8948	4620	8869	4774	8787	4927	8702	29
32	4310	9023	4467	8947	4623	8867	4777	8785	4929	8701	28
33	4313	9022	4470	8945	4625	8866	4779	8784	4932	8699	27
34	4316	9021	4472	8944	4628	8865	4782	8783	4934	8698	26
35	4318	9020	4475	8943	4630	8863	4784	8781	4937	8696	25
36	4321	9018	4478	8942	4633	8862	4787	8780	4939	8695	24
37	4323	9017	4480	8940	4636	8861	4789	8778	4942	8694	23
38	4326	9016	4483	8939	4638	8859	4792	8777	4944	8692	22
39	4329	9015	4485	8938	4641	8858	4795	8776	4947	8691	21
40	4331	9013	4488	8936	4643	8857	4797	8774	4950	8689	20
41	4334	9012	4491	8935	4646	8855	4800	8773	4952	8688	19
42	4337	9011	4493	8934	4648	8854	4802	8771	4955	8686	18
43	4339	9010	4496	8932	4651	8853	4805	8770	4957	8685	17
44	4342	9008	4498	8931	4654	8851	4807	8769	4960	8683	16
45	4344	9007	4501	8930	4656	8850	4810	8767	4962	8682	15
46	4347	9006	4504	8928	4659	8849	4812	8766	4965	8681	14
47	4350	9004	4506	8927	4661	8847	4815	8764	4967	8679	13
48	4352	9003	4509	8926	4664	8846	4818	8763	4970	8678	12
49	4355	9002	4511	8925	4666	8844	4820	8762	4972	8676	11
50	4358	9001	4514	8923	4669	8843	4823	8760	4975	8675	10
51	4360	8999	4517	8922	4672	8842	4825	8759	4977	8673	9
52	4363	8998	4519	8921	4674	8840	4828	8757	4980	8672	8
53	4365	8997	4522	8919	4677	8839	4830	8756	4982	8670	7
54	4368	8996	4524	8918	4679	8838	4833	8755	4985	8669	6
55	4371	8994	4527	8917	4682	8836	4835	8753	4987	8668	5
56	4373	8993	4530	8915	4684	8835	4838	8752	4990	8666	4
57	4376	8992	4532	8914	4687	8834	4840	8750	4992	8665	3
58	4378	8990	4535	8913	4690	8832	4843	8749	4995	8663	2
59	4381	8989	4537	8911	4692	8831	4846	8748	4997	8662	1
60	4384	8988	4540	8910	4695	8829	4848	8746	5000	8660	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	64°		63°		62°		61°		60°		′

NATURAL SINES AND COSINES

′	30°		31°		32°		33°		34°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	5000	8660	5150	8572	5299	8480	5446	8387	5592	8290	60
1	5003	8659	5153	8570	5302	8479	5449	8385	5594	8289	59
2	5005	8657	5155	8569	5304	8477	5451	8384	5597	8287	58
3	5008	8656	5158	8567	5307	8476	5454	8382	5599	8285	57
4	5010	8654	5160	8566	5309	8474	5456	8380	5602	8284	56
5	5013	8653	5163	8564	5312	8473	5459	8379	5604	8282	55
6	5015	8652	5165	8563	5314	8471	5461	8377	5606	8281	54
7	5018	8650	5168	8561	5316	8470	5463	8376	5609	8279	53
8	5020	8649	5170	8560	5319	8468	5466	8374	5611	8277	52
9	5023	8647	5173	8558	5321	8467	5468	8372	5614	8276	51
10	5025	8646	5175	8557	5324	8465	5471	8371	5616	8274	50
11	5028	8644	5178	8555	5326	8463	5473	8369	5618	8272	49
12	5030	8643	5180	8554	5329	8462	5476	8368	5621	8271	48
13	5033	8641	5183	8552	5331	8460	5478	8366	5623	8269	47
14	5035	8640	5185	8551	5334	8459	5480	8364	5626	8268	46
15	5038	8638	5188	8549	5336	8457	5483	8363	5628	8266	45
16	5040	8637	5190	8548	5339	8456	5485	8361	5630	8264	44
17	5043	8635	5193	8546	5341	8454	5488	8360	5633	8263	43
18	5045	8634	5195	8545	5344	8453	5490	8358	5635	8261	42
19	5048	8632	5198	8543	5346	8451	5493	8356	5638	8259	41
20	5050	8631	5200	8542	5348	8450	5495	8355	5640	8258	40
21	5053	8630	5203	8540	5351	8448	5498	8353	5642	8256	39
22	5055	8628	5205	8539	5353	8446	5500	8352	5645	8254	38
23	5058	8627	5208	8537	5356	8445	5502	8350	5647	8253	37
24	5060	8625	5210	8536	5358	8443	5505	8348	5650	8251	36
25	5063	8624	5213	8534	5361	8442	5507	8347	5652	8249	35
26	5065	8622	5215	8532	5363	8440	5510	8345	5654	8248	34
27	5068	8621	5218	8531	5366	8439	5512	8344	5657	8246	33
28	5070	8619	5220	8529	5368	8437	5515	8342	5659	8245	32
29	5073	8618	5223	8528	5371	8435	5517	8340	5662	8243	31
30	5075	8616	5225	8526	5373	8434	5519	8339	5664	8241	30
31	5078	8615	5227	8525	5375	8432	5522	8337	5666	8240	29
32	5080	8613	5230	8523	5378	8431	5524	8336	5669	8238	28
33	5083	8612	5232	8522	5380	8429	5527	8334	5671	8236	27
34	5085	8610	5235	8520	5383	8428	5529	8332	5674	8235	26
35	5088	8609	5237	8519	5385	8426	5531	8331	5676	8233	25
36	5090	8607	5240	8517	5388	8425	5534	8329	5678	8231	24
37	5093	8606	5242	8516	5390	8423	5536	8328	5681	8230	23
38	5095	8604	5245	8514	5393	8421	5539	8326	5683	8228	22
39	5098	8603	5247	8513	5395	8420	5541	8324	5686	8226	21
40	5100	8601	5250	8511	5398	8418	5544	8323	5688	8225	20
41	5103	8600	5252	8510	5400	8417	5546	8321	5690	8223	19
42	5105	8599	5255	8508	5402	8415	5548	8320	5693	8221	18
43	5108	8597	5257	8507	5405	8414	5551	8318	5695	8220	17
44	5110	8596	5260	8505	5407	8412	5553	8316	5698	8218	16
45	5113	8594	5262	8504	5410	8410	5556	8315	5700	8216	15
46	5115	8593	5265	8502	5412	8409	5558	8313	5702	8215	14
47	5118	8591	5267	8500	5415	8407	5561	8311	5705	8213	13
48	5120	8590	5270	8499	5417	8406	5563	8310	5707	8211	12
49	5123	8588	5272	8497	5420	8404	5565	8308	5710	8210	11
50	5125	8587	5275	8496	5422	8403	5568	8307	5712	8208	10
51	5128	8585	5277	8494	5424	8401	5570	8305	5714	8207	9
52	5130	8584	5279	8493	5427	8399	5573	8303	5717	8205	8
53	5133	8582	5282	8491	5429	8398	5575	8302	5719	8203	7
54	5135	8581	5284	8490	5432	8396	5577	8300	5721	8202	6
55	5138	8579	5287	8488	5434	8395	5580	8299	5724	8200	5
56	5140	8578	5289	8487	5437	8393	5582	8297	5726	8198	4
57	5143	8576	5292	8485	5439	8391	5585	8295	5729	8197	3
58	5145	8575	5294	8484	5442	8390	5587	8294	5731	8195	2
59	5148	8573	5297	8482	5444	8388	5590	8292	5733	8193	1
60	5150	8572	5299	8480	5446	8387	5592	8290	5736	8192	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	59°		58°		57°		56°		55°		′

NATURAL SINES AND COSINES

′	35°		36°		37°		38°		39°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	5736	8192	5878	8090	6018	7986	6157	7880	6293	7771	60
1	5738	8190	5880	8088	6020	7985	6159	7878	6295	7770	59
2	5741	8188	5883	8087	6023	7983	6161	7877	6298	7768	58
3	5743	8187	5885	8085	6025	7981	6163	7875	6300	7766	57
4	5745	8185	5887	8083	6027	7979	6166	7873	6302	7764	56
5	5748	8183	5890	8082	6030	7978	6168	7871	6305	7762	55
6	5750	8181	5892	8080	6032	7976	6170	7869	6307	7760	54
7	5752	8180	5894	8078	6034	7974	6173	7868	6309	7759	53
8	5755	8178	5897	8076	6037	7972	6175	7866	6311	7757	52
9	5757	8176	5899	8075	6039	7971	6177	7864	6314	7755	51
10	5760	8175	5901	8073	6041	7969	6180	7862	6316	7753	50
11	5762	8173	5904	8071	6044	7967	6182	7860	6318	7751	49
12	5764	8171	5906	8070	6046	7965	6184	7859	6320	7749	48
13	5767	8170	5908	8068	6048	7964	6186	7857	6323	7748	47
14	5769	8168	5911	8066	6051	7962	6189	7855	6325	7746	46
15	5771	8166	5913	8064	6053	7960	6191	7853	6327	7744	45
16	5774	8165	5915	8063	6055	7958	6193	7851	6329	7742	44
17	5776	8163	5918	8061	6058	7956	6196	7850	6332	7740	43
18	5779	8161	5920	8059	6060	7955	6198	7848	6334	7738	42
19	5781	8160	5922	8058	6062	7953	6200	7846	6336	7737	41
20	5783	8158	5925	8056	6065	7951	6202	7844	6338	7735	40
21	5786	8156	5927	8054	6067	7950	6205	7842	6341	7733	39
22	5788	8155	5930	8052	6069	7948	6207	7841	6343	7731	38
23	5790	8153	5932	8051	6071	7946	6209	7839	6345	7729	37
24	5793	8151	5934	8049	6074	7944	6211	7837	6347	7727	36
25	5795	8150	5937	8047	6076	7942	6214	7835	6350	7725	35
26	5798	8148	5939	8045	6078	7941	6216	7833	6352	7724	34
27	5800	8146	5941	8044	6081	7939	6218	7832	6354	7722	33
28	5802	8145	5944	8042	6083	7937	6221	7830	6356	7720	32
29	5805	8143	5946	8040	6085	7935	6223	7828	6359	7718	31
30	5807	8141	5948	8039	6088	7934	6225	7826	6361	7716	30
31	5809	8139	5951	8037	6090	7932	6227	7824	6363	7714	29
32	5812	8138	5953	8035	6092	7930	6230	7822	6365	7713	28
33	5814	8136	5955	8033	6095	7928	6232	7821	6368	7711	27
34	5816	8134	5958	8032	6097	7926	6234	7819	6370	7709	26
35	5819	8133	5960	8030	6099	7925	6237	7817	6372	7707	25
36	5821	8131	5962	8028	6101	7923	6239	7815	6374	7705	24
37	5824	8129	5965	8026	6104	7921	6241	7813	6376	7703	23
38	5826	8128	5967	8025	6106	7919	6243	7812	6379	7701	22
39	5828	8126	5969	8023	6108	7918	6246	7810	6381	7700	21
40	5831	8124	5972	8021	6111	7916	6248	7808	6383	7698	20
41	5833	8123	5974	8020	6113	7914	6250	7806	6385	7696	19
42	5835	8121	5976	8018	6115	7912	6252	7804	6388	7694	18
43	5838	8119	5979	8016	6118	7910	6255	7802	6390	7692	17
44	5840	8117	5981	8014	6120	7909	6257	7801	6392	7690	16
45	5842	8116	5983	8013	6122	7907	6259	7799	6394	7688	15
46	5845	8114	5986	8011	6124	7905	6262	7797	6397	7687	14
47	5847	8112	5988	8009	6127	7903	6264	7795	6399	7685	13
48	5850	8111	5990	8007	6129	7902	6266	7793	6401	7683	12
49	5852	8109	5993	8006	6131	7900	6268	7792	6403	7681	11
50	5854	8107	5995	8004	6134	7898	6271	7790	6406	7679	10
51	5857	8106	5997	8002	6136	7896	6273	7788	6408	7677	9
52	5859	8104	6000	8000	6138	7894	6275	7786	6410	7675	8
53	5861	8102	6002	7999	6141	7893	6277	7784	6412	7674	7
54	5864	8100	6004	7997	6143	7891	6280	7782	6414	7672	6
55	5866	8099	6007	7995	6145	7889	6282	7781	6417	7670	5
56	5868	8097	6009	7993	6147	7887	6284	7779	6419	7668	4
57	5871	8095	6011	7992	6150	7885	6286	7777	6421	7666	3
58	5873	8094	6014	7990	6152	7884	6289	7775	6423	7664	2
59	5875	8092	6016	7988	6154	7882	6291	7773	6426	7662	1
60	5878	8090	6018	7986	6157	7880	6293	7771	6428	7660	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	54°		53°		52°		51°		50°		′

′	40°		41°		42°		43°		44°		′
	sin	cos	sin	cos	sin	cos	sin	cos	sin	cos	
0	6428	7660	6561	7547	6691	7431	6820	7314	6947	7193	60
1	6430	7659	6563	7545	6693	7430	6822	7312	6949	7191	59
2	6432	7657	6565	7543	6696	7428	6824	7310	6951	7189	58
3	6435	7655	6567	7541	6698	7426	6826	7308	6953	7187	57
4	6437	7653	6569	7539	6700	7424	6828	7306	6955	7185	56
5	6439	7651	6572	7538	6702	7422	6831	7304	6957	7183	55
6	6441	7649	6574	7536	6704	7420	6833	7302	6959	7181	54
7	6443	7647	6576	7534	6706	7418	6835	7300	6961	7179	53
8	6446	7645	6578	7532	6709	7416	6837	7298	6963	7177	52
9	6448	7644	6580	7530	6711	7414	6839	7296	6965	7175	51
10	6450	7642	6583	7528	6713	7412	6841	7294	6967	7173	50
11	6452	7640	6585	7526	6715	7410	6843	7292	6970	7171	49
12	6455	7638	6587	7524	6717	7408	6845	7290	6972	7169	48
13	6457	7636	6589	7522	6719	7406	6848	7288	6974	7167	47
14	6459	7634	6591	7520	6722	7404	6850	7286	6976	7165	46
15	6461	7632	6593	7518	6724	7402	6852	7284	6978	7163	45
16	6463	7630	6596	7516	6726	7400	6854	7282	6980	7161	44
17	6466	7629	6598	7515	6728	7398	6856	7280	6982	7159	43
18	6468	7627	6600	7513	6730	7396	6858	7278	6984	7157	42
19	6470	7625	6602	7511	6732	7394	6860	7276	6986	7155	41
20	6472	7623	6604	7509	6734	7392	6862	7274	6988	7153	40
21	6475	7621	6607	7507	6737	7390	6865	7272	6990	7151	39
22	6477	7619	6609	7505	6739	7388	6867	7270	6992	7149	38
23	6479	7617	6611	7503	6741	7387	6869	7268	6995	7147	37
24	6481	7615	6613	7501	6743	7385	6871	7266	6997	7145	36
25	6483	7613	6615	7499	6745	7383	6873	7264	6999	7143	35
26	6486	7612	6617	7497	6747	7381	6875	7262	7001	7141	34
27	6488	7610	6620	7495	6749	7379	6877	7260	7003	7139	33
28	6490	7608	6622	7493	6752	7377	6879	7258	7005	7137	32
29	6492	7606	6624	7491	6754	7375	6881	7256	7007	7135	31
30	6494	7604	6626	7490	6756	7373	6884	7254	7009	7133	30
31	6497	7602	6628	7488	6758	7371	6886	7252	7011	7130	29
32	6499	7600	6631	7486	6760	7369	6888	7250	7013	7128	28
33	6501	7598	6633	7484	6762	7367	6890	7248	7015	7126	27
34	6503	7596	6635	7482	6764	7365	6892	7246	7017	7124	26
35	6506	7595	6637	7480	6767	7363	6894	7244	7019	7122	25
36	6508	7593	6639	7478	6769	7361	6896	7242	7022	7120	24
37	6510	7591	6641	7476	6771	7359	6898	7240	7024	7118	23
38	6512	7589	6644	7474	6773	7357	6900	7238	7026	7116	22
39	6514	7587	6646	7472	6775	7355	6903	7236	7028	7114	21
40	6517	7585	6648	7470	6777	7353	6905	7234	7030	7112	20
41	6519	7583	6650	7468	6779	7351	6907	7232	7032	7110	19
42	6521	7581	6652	7466	6782	7349	6909	7230	7034	7108	18
43	6523	7579	6654	7464	6784	7347	6911	7228	7036	7106	17
44	6525	7578	6657	7463	6786	7345	6913	7226	7038	7104	16
45	6528	7576	6659	7461	6788	7343	6915	7224	7040	7102	15
46	6530	7574	6661	7459	6790	7341	6917	7222	7042	7100	14
47	6532	7572	6663	7457	6792	7339	6919	7220	7044	7098	13
48	6534	7570	6665	7455	6794	7337	6921	7218	7046	7096	12
49	6536	7568	6667	7453	6797	7335	6924	7216	7048	7094	11
50	6539	7566	6670	7451	6799	7333	6926	7214	7050	7092	10
51	6541	7564	6672	7449	6801	7331	6928	7212	7053	7090	9
52	6543	7562	6674	7447	6803	7329	6930	7210	7055	7088	8
53	6545	7560	6676	7445	6805	7327	6932	7208	7057	7085	7
54	6547	7559	6678	7443	6807	7325	6934	7206	7059	7083	6
55	6550	7557	6680	7441	6809	7323	6936	7203	7061	7081	5
56	6552	7555	6683	7439	6811	7321	6938	7201	7063	7079	4
57	6554	7553	6685	7437	6814	7319	6940	7199	7065	7077	3
58	6556	7551	6687	7435	6816	7318	6942	7197	7067	7075	2
59	6558	7549	6689	7433	6818	7316	6944	7195	7069	7073	1
60	6561	7547	6691	7431	6820	7314	6947	7193	7071	7071	0
	cos	sin	cos	sin	cos	sin	cos	sin	cos	sin	
′	49°		48°		47°		46°		45°		′

NATURAL TANGENTS AND COTANGENTS

′	0°		1°		2°		3°		4°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	0000	Infinite	0175	57.2900	0349	28.6363	0524	19.0811	0699	14.3007	60
1	0003	3437.75	0177	56.3506	0352	28.3994	0527	18.9755	0702	14.2411	59
2	0006	1718.87	0180	55.4415	0355	28.1664	0530	18.8711	0705	14.1821	58
3	0009	1145.92	0183	54.5613	0358	27.9372	0533	18.7678	0708	14.1235	57
4	0012	859.436	0186	53.7086	0361	27.7117	0536	18.6656	0711	14.0655	56
5	0015	687.549	0189	52.8821	0364	27.4899	0539	18.5645	0714	14.0079	55
6	0017	572.957	0192	52.0807	0367	27.2715	0542	18.4645	0717	13.9507	54
7	0020	491.106	0195	51.3032	0370	27.0566	0544	18.3655	0720	13.8940	53
8	0023	429.718	0198	50.5485	0373	26.8450	0547	18.2677	0723	13.8378	52
9	0026	381.971	0201	49.8157	0375	26.6367	0550	18.1708	0726	13.7821	51
10	0029	343.774	0204	49.1039	0378	26.4316	0553	18.0750	0729	13.7267	50
11	0032	312.521	0207	48.4121	0381	26.2296	0556	17.9802	0731	13.6719	49
12	0035	286.478	0209	47.7395	0384	26.0307	0559	17.8863	0734	13.6174	48
13	0038	264.441	0212	47.0853	0387	25.8348	0562	17.7934	0737	13.5634	47
14	0041	245.552	0215	46.4489	0390	25.6418	0565	17.7015	0740	13.5098	46
15	0044	229.182	0218	45.8294	0393	25.4517	0568	17.6106	0743	13.4566	45
16	0047	214.858	0221	45.2261	0396	25.2644	0571	17.5205	0746	13.4039	44
17	0049	202.219	0224	44.6386	0399	25.0798	0574	17.4314	0749	13.3515	43
18	0052	190.984	0227	44.0661	0402	24.8978	0577	17.3432	0752	13.2996	42
19	0055	180.932	0230	43.5081	0405	24.7185	0580	17.2558	0755	13.2480	41
20	0058	171.885	0233	42.9641	0407	24.5418	0582	17.1693	0758	13.1969	40
21	0061	163.700	0236	42.4335	0410	24.3675	0585	17.0837	0761	13.1461	39
22	0064	156.259	0239	41.9158	0413	24.1957	0588	16.9990	0764	13.0958	38
23	0067	149.465	0241	41.4106	0416	24.0263	0591	16.9150	0767	13.0458	37
24	0070	143.237	0244	40.9174	0419	23.8593	0594	16.8319	0769	12.9962	36
25	0073	137.507	0247	40.4358	0422	23.6945	0597	16.7496	0772	12.9469	35
26	0076	132.219	0250	39.9655	0425	23.5321	0600	16.6681	0775	12.8981	34
27	0079	127.321	0253	39.5059	0428	23.3718	0603	16.5874	0778	12.8496	33
28	0081	122.774	0256	39.0568	0431	23.2137	0606	16.5075	0781	12.8014	32
29	0084	118.540	0259	38.6177	0434	23.0577	0609	16.4283	0784	12.7536	31
30	0087	114.589	0262	38.1885	0437	22.9038	0612	16.3499	0787	12.7062	30
31	0090	110.892	0265	37.7686	0440	22.7519	0615	16.2722	0790	12.6591	29
32	0093	107.426	0268	37.3579	0442	22.6020	0617	16.1952	0793	12.6124	28
33	0096	104.171	0271	36.9560	0445	22.4541	0620	16.1190	0796	12.5660	27
34	0099	101.107	0274	36.5627	0448	22.3081	0623	16.0435	0799	12.5199	26
35	0102	98.2179	0276	36.1776	0451	22.1640	0626	15.9687	0802	12.4742	25
36	0105	95.4895	0279	35.8006	0454	22.0217	0629	15.8945	0805	12.4288	24
37	0108	92.9085	0282	35.4313	0457	21.8813	0632	15.8211	0808	12.3838	23
38	0111	90.4633	0285	35.0695	0460	21.7426	0635	15.7483	0810	12.3390	22
39	0113	88.1436	0288	34.7151	0463	21.6056	0638	15.6762	0813	12.2946	21
40	0116	85.9398	0291	34.3678	0466	21.4704	0641	15.6048	0816	12.2505	20
41	0119	83.8435	0294	34.0273	0469	21.3369	0644	15.5340	0819	12.2067	19
42	0122	81.8470	0297	33.6935	0472	21.2049	0647	15.4638	0822	12.1632	18
43	0125	79.9434	0300	33.3662	0475	21.0747	0650	15.3943	0825	12.1201	17
44	0128	78.1263	0303	33.0452	0477	20.9460	0653	15.3254	0828	12.0772	16
45	0131	76.3900	0306	32.7303	0480	20.8188	0655	15.2571	0831	12.0346	15
46	0134	74.7292	0308	32.4213	0483	20.6932	0658	15.1893	0834	11.9923	14
47	0137	73.1390	0311	32.1181	0486	20.5691	0661	15.1222	0837	11.9504	13
48	0140	71.6151	0314	31.8205	0489	20.4465	0664	15.0557	0840	11.9087	12
49	0143	70.1533	0317	31.5284	0492	20.3253	0667	14.9898	0843	11.8673	11
50	0146	68.7501	0320	31.2416	0495	20.2056	0670	14.9244	0846	11.8262	10
51	0148	67.4019	0323	30.9599	0498	20.0872	0673	14.8596	0849	11.7853	9
52	0151	66.1055	0326	30.6833	0501	19.9702	0676	14.7954	0851	11.7448	8
53	0154	64.8580	0329	30.4116	0504	19.8546	0679	14.7317	0854	11.7045	7
54	0157	63.6567	0332	30.1446	0507	19.7403	0682	14.6685	0857	11.6645	6
55	0160	62.4992	0335	29.8823	0509	19.6273	0685	14.6059	0860	11.6248	5
56	0163	61.3829	0338	29.6245	0512	19.5156	0688	14.5438	0863	11.5853	4
57	0166	60.3058	0340	29.3711	0515	19.4051	0690	14.4823	0866	11.5461	3
58	0169	59.2659	0343	29.1220	0518	19.2959	0693	14.4212	0869	11.5072	2
59	0172	58.2612	0346	28.8771	0521	19.1879	0696	14.3607	0872	11.4685	1
60	0175	57.2900	0349	28.6363	0524	19.0811	0699	14.3007	0875	11.4301	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	89°		88°		87°		86°		85°		′

′	5°		6°		7°		8°		9°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	0875	11.4301	1051	9.5144	1228	8.1443	1405	7.1154	1584	6.3138	60
1	0878	11.3919	1054	9.4878	1231	8.1248	1408	7.1004	1587	6.3019	59
2	0881	11.3540	1057	9.4614	1234	8.1054	1411	7.0855	1590	6.2901	58
3	0884	11.3163	1060	9.4352	1237	8.0860	1414	7.0706	1593	6.2783	57
4	0887	11.2789	1063	9.4090	1240	8.0667	1417	7.0558	1596	6.2666	56
5	0890	11.2417	1066	9.3831	1243	8.0476	1420	7.0410	1599	6.2549	55
6	0892	11.2048	1069	9.3572	1246	8.0285	1423	7.0264	1602	6.2432	54
7	0895	11.1681	1072	9.3315	1249	8.0095	1426	7.0117	1605	6.2316	53
8	0898	11.1316	1075	9.3060	1251	7.9906	1429	6.9972	1608	6.2200	52
9	0901	11.0954	1078	9.2806	1254	7.9718	1432	6.9827	1611	6.2085	51
10	0904	11.0594	1080	9.2553	1257	7.9530	1435	6.9682	1614	6.1970	50
11	0907	11.0237	1083	9.2302	1260	7.9344	1438	6.9538	1617	6.1856	49
12	0910	10.9882	1086	9.2052	1263	7.9158	1441	6.9395	1620	6.1742	48
13	0913	10.9529	1089	9.1803	1266	7.8973	1444	6.9252	1623	6.1628	47
14	0916	10.9178	1092	9.1555	1269	7.8789	1447	6.9110	1626	6.1515	46
15	0919	10.8829	1095	9.1309	1272	7.8606	1450	6.8969	1629	6.1402	45
16	0922	10.8483	1098	9.1065	1275	7.8424	1453	6.8828	1632	6.1290	44
17	0925	10.8139	1101	9.0821	1278	7.8243	1456	6.8687	1635	6.1178	43
18	0928	10.7797	1104	9.0579	1281	7.8062	1459	6.8548	1638	6.1066	42
19	0931	10.7457	1107	9.0338	1284	7.7883	1462	6.8408	1641	6.0955	41
20	0934	10.7119	1110	9.0098	1287	7.7704	1465	6.8269	1644	6.0844	40
21	0936	10.6783	1113	8.9860	1290	7.7525	1468	6.8131	1647	6.0734	39
22	0939	10.6450	1116	8.9623	1293	7.7348	1471	6.7994	1650	6.0624	38
23	0942	10.6118	1119	8.9387	1296	7.7171	1474	6.7856	1653	6.0514	37
24	0945	10.5789	1122	8.9152	1299	7.6996	1477	6.7720	1655	6.0405	36
25	0948	10.5462	1125	8.8919	1302	7.6821	1480	6.7584	1658	6.0296	35
26	0951	10.5136	1128	8.8686	1305	7.6647	1483	6.7448	1661	6.0188	34
27	0954	10.4813	1131	8.8455	1308	7.6473	1486	6.7313	1664	6.0080	33
28	0957	10.4491	1134	8.8225	1311	7.6301	1489	6.7179	1667	5.9972	32
29	0960	10.4172	1136	8.7996	1314	7.6129	1492	6.7045	1670	5.9865	31
30	0963	10.3854	1139	8.7769	1317	7.5958	1495	6.6912	1673	5.9758	30
31	0966	10.3538	1142	8.7542	1319	7.5787	1497	6.6779	1676	5.9651	29
32	0969	10.3224	1145	8.7317	1322	7.5618	1500	6.6646	1679	5.9545	28
33	0972	10.2913	1148	8.7093	1325	7.5449	1503	6.6514	1682	5.9439	27
34	0975	10.2602	1151	8.6870	1328	7.5281	1506	6.6383	1685	5.9333	26
35	0978	10.2294	1154	8.6648	1331	7.5113	1509	6.6252	1688	5.9228	25
36	0981	10.1988	1157	8.6427	1334	7.4947	1512	6.6122	1691	5.9124	24
37	0983	10.1683	1160	8.6208	1337	7.4781	1515	6.5992	1694	5.9019	23
38	0986	10.1381	1163	8.5989	1340	7.4615	1518	6.5863	1697	5.8915	22
39	0989	10.1080	1166	8.5772	1343	7.4451	1521	6.5734	1700	5.8811	21
40	0992	10.0780	1169	8.5555	1346	7.4287	1524	6.5606	1703	5.8708	20
41	0995	10.0483	1172	8.5340	1349	7.4124	1527	6.5478	1706	5.8605	19
42	0998	10.0187	1175	8.5126	1352	7.3962	1530	6.5350	1709	5.8502	18
43	1001	9.9893	1178	8.4913	1355	7.3800	1533	6.5223	1712	5.8400	17
44	1004	9.9601	1181	8.4701	1358	7.3639	1536	6.5097	1715	5.8298	16
45	1007	9.9310	1184	8.4490	1361	7.3479	1539	6.4971	1718	5.8197	15
46	1010	9.9021	1187	8.4280	1364	7.3319	1542	6.4846	1721	5.8095	14
47	1013	9.8734	1189	8.4071	1367	7.3160	1545	6.4721	1724	5.7994	13
48	1016	9.8448	1192	8.3863	1370	7.3002	1548	6.4596	1727	5.7894	12
49	1019	9.8164	1195	8.3656	1373	7.2844	1551	6.4472	1730	5.7794	12
50	1022	9.7882	1198	8.3450	1376	7.2687	1554	6.4348	1733	5.7694	10
51	1025	9.7601	1201	8.3245	1379	7.2531	1557	6.4225	1736	5.7594	9
52	1028	9.7322	1204	8.3041	1382	7.2375	1560	6.4103	1739	5.7495	8
53	1030	9.7044	1207	8.2838	1385	7.2220	1563	6.3980	1742	5.7396	7
54	1033	9.6768	1210	8.2636	1388	7.2066	1566	6.3859	1745	5.7297	6
55	1036	9.6499	1213	8.2434	1391	7.1912	1569	6.3737	1748	5.7199	5
56	1039	9.6220	1216	8.2234	1394	7.1759	1572	6.3617	1751	5.7101	4
57	1042	9.5949	1219	8.2035	1397	7.1607	1575	6.3496	1754	5.7004	3
58	1045	9.5679	1222	8.1837	1399	7.1455	1578	6.3376	1757	5.6906	2
59	1048	9.5411	1225	8.1640	1402	7.1304	1581	6.3257	1760	5.6809	1
60	1051	9.5144	1228	8.1443	1405	7.1154	1584	6.3138	1763	5.6713	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	84°		83°		82°		81°		80°		′

NATURAL TANGENTS AND COTANGENTS

′	10°		11°		12°		13°		14°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	1763	5.6713	1944	5.1446	2126	4.7046	2309	4.3315	2493	4.0108	60
1	1766	5.6617	1947	5.1366	2129	4.6979	2312	4.3257	2496	4.0058	59
2	1769	5.6521	1950	5.1286	2132	4.6912	2315	4.3200	2499	4.0009	58
3	1772	5.6425	1953	5.1207	2135	4.6845	2318	4.3143	2503	3.9959	57
4	1775	5.6330	1956	5.1128	2138	4.6779	2321	4.3086	2506	3.9910	56
5	1778	5.6234	1959	5.1049	2141	4.6712	2324	4.3029	2509	3.9861	55
6	1781	5.6140	1962	5.0970	2144	4.6646	2327	4.2972	2512	3.9812	54
7	1784	5.6045	1965	5.0892	2147	4.6580	2330	4.2916	2515	3.9763	53
8	1787	5.5951	1968	5.0814	2150	4.6514	2333	4.2859	2518	3.9714	52
9	1790	5.5857	1971	5.0736	2153	4.6448	2336	4.2803	2521	3.9665	51
10	1793	5.5764	1974	5.0658	2156	4.6382	2339	4.2747	2524	3.9617	50
11	1796	5.5671	1977	5.0581	2159	4.6317	2342	4.2691	2527	3.9568	49
12	1799	5.5578	1980	5.0504	2162	4.6252	2345	4.2635	2530	3.9520	48
13	1802	5.5485	1983	5.0427	2165	4.6187	2349	4.2580	2533	3.9471	47
14	1805	5.5393	1986	5.0350	2168	4.6122	2352	4.2524	2537	3.9423	46
15	1808	5.5301	1989	5.0273	2171	4.6057	2355	4.2468	2540	3.9375	45
16	1811	5.5209	1992	5.0197	2174	4.5993	2358	4.2413	2543	3.9327	44
17	1814	5.5118	1995	5.0121	2177	4.5928	2361	4.2358	2546	3.9279	43
18	1817	5.5026	1998	5.0045	2180	4.5864	2364	4.2303	2549	3.9232	42
19	1820	5.4936	2001	4.9969	2183	4.5800	2367	4.2248	2552	3.9184	41
20	1823	5.4845	2004	4.9894	2186	4.5736	2370	4.2193	2555	3.9136	40
21	1826	5.4755	2007	4.9819	2189	4.5673	2373	4.2139	2558	3.9089	39
22	1829	5.4665	2010	4.9744	2193	4.5609	2376	4.2084	2561	3.9042	38
23	1832	5.4575	2013	4.9669	2196	4.5546	2379	4.2030	2564	3.8995	37
24	1835	5.4486	2016	4.9594	2199	4.5483	2382	4.1976	2568	3.8947	36
25	1838	5.4397	2019	4.9520	2202	4.5420	2385	4.1922	2571	3.8900	35
26	1841	5.4308	2022	4.9446	2205	4.5357	2388	4.1868	2574	3.8854	34
27	1844	5.4219	2025	4.9372	2208	4.5294	2392	4.1814	2577	3.8807	33
28	1847	5.4131	2028	4.9298	2211	4.5232	2395	4.1760	2580	3.8760	32
29	1850	5.4043	2031	4.9225	2214	4.5169	2398	4.1706	2583	3.8714	31
30	1853	5.3955	2035	4.9152	2217	4.5107	2401	4.1653	2586	3.8667	30
31	1856	5.3868	2038	4.9078	2220	4.5045	2404	4.1600	2589	3.8621	29
32	1859	5.3781	2041	4.9006	2223	4.4983	2407	4.1547	2592	3.8575	28
33	1862	5.3694	2044	4.8933	2226	4.4922	2410	4.1493	2595	3.8528	27
34	1865	5.3607	2047	4.8860	2229	4.4860	2413	4.1441	2599	3.8482	26
35	1868	5.3521	2050	4.8788	2232	4.4799	2416	4.1388	2602	3.8436	25
36	1871	5.3435	2053	4.8716	2235	4.4737	2419	4.1335	2605	3.8391	24
37	1874	5.3349	2056	4.8644	2238	4.4676	2422	4.1282	2608	3.8345	23
38	1877	5.3263	2059	4.8573	2241	4.4615	2425	4.1230	2611	3.8299	22
39	1880	5.3178	2062	4.8501	2244	4.4555	2428	4.1178	2614	3.8254	21
40	1883	5.3093	2065	4.8430	2247	4.4494	2432	4.1126	2617	3.8208	20
41	1887	5.3008	2068	4.8359	2251	4.4434	2435	4.1074	2620	3.8163	19
42	1890	5.2924	2071	4.8288	2254	4.4374	2438	4.1022	2623	3.8118	18
43	1893	5.2839	2074	4.8218	2257	4.4313	2441	4.0970	2627	3.8073	17
44	1896	5.2755	2077	4.8147	2260	4.4253	2444	4.0918	2630	3.8028	16
45	1899	5.2672	2080	4.8077	2263	4.4194	2447	4.0867	2633	3.7983	15
46	1902	5.2588	2083	4.8007	2266	4.4134	2450	4.0815	2636	3.7938	14
47	1905	5.2505	2086	4.7937	2269	4.4075	2453	4.0764	2639	3.7893	13
48	1908	5.2422	2089	4.7867	2272	4.4015	2456	4.0713	2642	3.7848	12
49	1911	5.2339	2092	4.7798	2275	4.3956	2459	4.0662	2645	3.7804	11
50	1914	5.2257	2095	4.7729	2278	4.3897	2462	4.0611	2648	3.7760	10
51	1917	5.2174	2098	4.7659	2281	4.3838	2465	4.0560	2651	3.7715	9
52	1920	5.2092	2101	4.7591	2284	4.3779	2469	4.0509	2655	3.7671	8
53	1923	5.2011	2104	4.7522	2287	4.3721	2472	4.0459	2658	3.7627	7
54	1926	5.1929	2107	4.7453	2290	4.3662	2475	4.0408	2661	3.7583	6
55	1929	5.1848	2110	4.7385	2293	4.3604	2478	4.0358	2664	3.7539	5
56	1932	5.1767	2113	4.7317	2296	4.3546	2481	4.0308	2667	3.7495	4
57	1935	5.1686	2116	4.7249	2299	4.3488	2484	4.0257	2670	3.7451	3
58	1938	5.1606	2119	4.7181	2303	4.3430	2487	4.0207	2673	3.7408	2
59	1941	5.1526	2123	4.7114	2306	4.3372	2490	4.0158	2676	3.7364	1
60	1944	5.1446	2126	4.7046	2309	4.3315	2493	4.0108	2679	3.7321	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	79°		78°		77°		76°		75°		′

NATURAL TANGENTS AND COTANGENTS

′	15°		16°		17°		18°		19°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	2679	3.7321	2867	3.4874	3057	3.2709	3249	3.0777	3443	2.9042	60
1	2683	3.7277	2871	3.4836	3060	3.2675	3252	3.0746	3447	2.9015	59
2	2686	3.7234	2874	3.4798	3064	3.2641	3256	3.0716	3450	2.8987	58
3	2689	3.7191	2877	3.4760	3067	3.2607	3259	3.0686	3453	2.8960	57
4	2692	3.7148	2880	3.4722	3070	3.2573	3262	3.0655	3456	2.8933	56
5	2695	3.7105	2883	3.4684	3073	3.2539	3265	3.0625	3460	2.8905	55
6	2698	3.7062	2886	3.4646	3076	3.2506	3269	3.0595	3463	2.8878	54
7	2701	3.7019	2890	3.4608	3080	3.2472	3272	3.0565	3466	2.8851	53
8	2704	3.6976	2893	3.4570	3083	3.2438	3275	3.0535	3469	2.8824	52
9	2708	3.6933	2896	3.4533	3086	3.2405	3278	3.0505	3473	2.8797	51
10	2711	3.6891	2899	3.4495	3089	3.2371	3281	3.0475	3476	2.8770	50
11	2714	3.6848	2902	3.4458	3092	3.2338	3285	3.0445	3479	2.8743	49
12	2717	3.6806	2905	3.4420	3096	3.2305	3288	3.0415	3482	2.8716	48
13	2720	3.6764	2908	3.4383	3099	3.2272	3291	3.0385	3486	2.8689	47
14	2723	3.6722	2912	3.4346	3102	3.2238	3294	3.0356	3489	2.8662	46
15	2726	3.6680	2915	3.4308	3105	3.2205	3298	3.0326	3492	2.8636	45
16	2729	3.6638	2918	3.4271	3108	3.2172	3301	3.0296	3495	2.8609	44
17	2733	3.6596	2921	3.4234	3111	3.2139	3304	3.0267	3499	2.8582	43
18	2736	3.6554	2924	3.4197	3115	3.2106	3307	3.0237	3502	2.8556	42
19	2739	3.6512	2927	3.4160	3118	3.2073	3310	3.0208	3505	2.8529	41
20	2742	3.6470	2931	3.4124	3121	3.2041	3314	3.0178	3508	2.8502	40
21	2745	3.6429	2934	3.4087	3124	3.2008	3317	3.0149	3512	2.8476	39
22	2748	3.6387	2937	3.4050	3127	3.1975	3320	3.0120	3515	2.8449	38
23	2751	3.6346	2940	3.4014	3131	3.1943	3323	3.0090	3518	2.8423	37
24	2754	3.6305	2943	3.3977	3134	3.1910	3327	3.0061	3522	2.8397	36
25	2758	3.6264	2946	3.3941	3137	3.1878	3330	3.0032	3525	2.8370	35
26	2761	3.6222	2949	3.3904	3140	3.1845	3333	3.0003	3528	2.8344	34
27	2764	3.6181	2953	3.3868	3143	3.1813	3336	2.9974	3531	2.8318	33
28	2767	3.6140	2956	3.3832	3147	3.1780	3339	2.9945	3535	2.8291	32
29	2770	3.6100	2959	3.3796	3150	3.1748	3343	2.9916	3538	2.8265	31
30	2773	3.6059	2962	3.3759	3153	3.1716	3346	2.9887	3541	2.8239	30
31	2776	3.6018	2965	3.3723	3156	3.1684	3349	2.9858	3544	2.8213	29
32	2780	3.5978	2968	3.3687	3159	3.1652	3352	2.9829	3548	2.8187	28
33	2783	3.5937	2972	3.3652	3163	3.1620	3356	2.9800	3551	2.8161	27
34	2786	3.5897	2975	3.3616	3166	3.1588	3359	2.9772	3554	2.8135	26
35	2789	3.5856	2978	3.3580	3169	3.1556	3362	2.9743	3558	2.8109	25
36	2792	3.5816	2981	3.3544	3172	3.1524	3365	2.9714	3561	2.8083	24
37	2795	3.5776	2984	3.3509	3175	3.1492	3369	2.9686	3564	2.8057	23
38	2798	3.5736	2987	3.3473	3179	3.1460	3372	2.9657	3567	2.8032	22
39	2801	3.5696	2991	3.3438	3182	3.1429	3375	2.9629	3571	2.8006	21
40	2805	3.5656	2994	3.3402	3185	3.1397	3378	2.9600	3574	2.7980	20
41	2808	3.5616	2997	3.3367	3188	3.1366	3382	2.9572	3577	2.7955	19
42	2811	3.5576	3000	3.3332	3191	3.1334	3385	2.9544	3581	2.7929	18
43	2814	3.5536	3003	3.3297	3195	3.1303	3388	2.9515	3584	2.7903	17
44	2817	3.5497	3006	3.3261	3198	3.1271	3391	2.9487	3587	2.7878	16
45	2820	3.5457	3010	3.3226	3201	3.1240	3395	2.9459	3590	2.7852	15
46	2823	3.5418	3013	3.3191	3204	3.1209	3398	2.9431	3594	2.7827	14
47	2827	3.5379	3016	3.3156	3207	3.1178	3401	2.9403	3597	2.7801	13
48	2830	3.5339	3019	3.3122	3211	3.1146	3404	2.9375	3600	2.7776	12
49	2833	3.5300	3022	3.3087	3214	3.1115	3408	2.9347	3604	2.7751	11
50	2836	3.5261	3026	3.3052	3217	3.1084	3411	2.9319	3607	2.7725	10
51	2839	3.5222	3029	3.3017	3220	3.1053	3414	2.9291	3610	2.7700	9
52	2842	3.5183	3032	3.2983	3223	3.1022	3417	2.9263	3613	2.7675	8
53	2845	3.5144	3035	3.2948	3227	3.0991	3421	2.9235	3617	2.7650	7
54	2849	3.5105	3038	3.2914	3230	3.0961	3424	2.9208	3620	2.7625	6
55	2852	3.5067	3041	3.2880	3233	3.0930	3427	2.9180	3623	2.7600	5
56	2855	3.5028	3045	3.2845	3236	3.0899	3430	2.9152	3627	2.7575	4
57	2858	3.4989	3048	3.2811	3240	3.0868	3434	2.9125	3630	2.7550	3
58	2861	3.4951	3051	3.2777	3243	3.0838	3437	2.9097	3633	2.7525	2
59	2864	3.4912	3054	3.2743	3246	3.0807	3440	2.9070	3636	2.7500	1
60	2867	3.4874	3057	3.2709	3249	3.0777	3443	2.9042	3640	2.7475	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	74°		73°		72°		71°		70°		′

NATURAL TANGENTS AND COTANGENTS

′	20°		21°		22°		23°		24°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	3640	2.7475	3839	2.6051	4040	2.4751	4245	2.3559	4452	2.2460	60
1	3643	2.7450	3842	2.6028	4044	2.4730	4248	2.3539	4456	2.2443	59
2	3646	2.7425	3845	2.6006	4047	2.4709	4252	2.3520	4459	2.2425	58
3	3650	2.7400	3849	2.5983	4050	2.4689	4255	2.3501	4463	2.2408	57
4	3653	2.7376	3852	2.5961	4054	2.4668	4258	2.3483	4466	2.2390	56
5	3656	2.7351	3855	2.5938	4057	2.4648	4262	2.3464	4470	2.2373	55
6	3659	2.7326	3859	2.5916	4061	2.4627	4265	2.3445	4473	2.2355	54
7	3663	2.7302	3862	2.5893	4064	2.4606	4269	2.3426	4477	2.2338	53
8	3666	2.7277	3865	2.5871	4067	2.4586	4272	2.3407	4480	2.2320	52
9	3669	2.7253	3869	2.5848	4071	2.4566	4276	2.3388	4484	2.2303	51
10	3673	2.7228	3872	2.5826	4074	2.4545	4279	2.3369	4487	2.2286	50
11	3676	2.7204	3875	2.5804	4078	2.4525	4283	2.3351	4491	2.2268	49
12	3679	2.7179	3879	2.5782	4081	2.4504	4286	2.3332	4494	2.2251	48
13	3683	2.7155	3882	2.5759	4084	2.4484	4289	2.3313	4498	2.2234	47
14	3686	2.7130	3885	2.5737	4088	2.4464	4293	2.3294	4501	2.2216	46
15	3689	2.7106	3889	2.5715	4091	2.4443	4296	2.3276	4505	2.2199	45
16	3693	2.7082	3892	2.5693	4095	2.4423	4300	2.3257	4508	2.2182	44
17	3696	2.7058	3895	2.5671	4098	2.4403	4303	2.3238	4512	2.2165	43
18	3699	2.7034	3899	2.5649	4101	2.4383	4307	2.3220	4515	2.2148	42
19	3702	2.7009	3902	2.5627	4105	2.4362	4310	2.3201	4519	2.2130	41
20	3706	2.6985	3906	2.5605	4108	2.4342	4314	2.3183	4522	2.2113	40
21	3709	2.6961	3909	2.5533	4111	2.4322	4317	2.3164	4526	2.2096	39
22	3712	2.6937	3912	2.5561	4115	2.4302	4320	2.3146	4529	2.2079	38
23	3716	2.6913	3916	2.5539	4118	2.4282	4324	2.3127	4533	2.2062	37
24	3719	2.6889	3919	2.5517	4122	2.4262	4327	2.3109	4536	2.2045	36
25	3722	2.6865	3922	2.5495	4125	2.4242	4331	2.3090	4540	2.2028	35
26	3726	2.6841	3926	2.5473	4129	2.4222	4334	2.3072	4543	2.2011	34
27	3729	2.6818	3929	2.5452	4132	2.4202	4338	2.3053	4547	2.1994	33
28	3732	2.6794	3932	2.5430	4135	2.4182	4341	2.3035	4550	2.1977	32
29	3736	2.6770	3936	2.5408	4139	2.4162	4345	2.3017	4554	2.1960	31
30	3739	2.6746	3939	2.5386	4142	2.4142	4348	2.2998	4557	2.1943	30
31	3742	2.6723	3942	2.5365	4146	2.4122	4352	2.2980	4561	2.1926	29
32	3745	2.6699	3946	2.5343	4149	2.4102	4355	2.2962	4564	2.1909	28
33	3749	2.6675	3949	2.5322	4152	2.4083	4359	2.2944	4568	2.1892	27
34	3752	2.6652	3953	2.5300	4156	2.4063	4362	2.2925	4571	2.1876	26
35	3755	2.6628	3956	2.5279	4159	2.4043	4365	2.2907	4575	2.1859	25
36	3759	2.6605	3959	2.5257	4163	2.4023	4369	2.2889	4578	2.1842	24
37	3762	2.6581	3963	2.5236	4166	2.4004	4372	2.2871	4582	2.1825	23
38	3765	2.6558	3966	2.5214	4169	2.3984	4376	2.2853	4585	2.1808	22
39	3769	2.6534	3969	2.5193	4173	2.3964	4379	2.2835	4589	2.1792	21
40	3772	2.6511	3973	2.5172	4176	2.3945	4383	2.2817	4592	2.1775	20
41	3775	2.6488	3976	2.5150	4180	2.3925	4386	2.2799	4596	2.1758	19
42	3779	2.6464	3979	2.5129	4183	2.3906	4390	2.2781	4599	2.1742	18
43	3782	2.6441	3983	2.5108	4187	2.3886	4393	2.2763	4603	2.1725	17
44	3785	2.6418	3986	2.5086	4190	2.3867	4397	2.2745	4607	2.1708	16
45	3789	2.6395	3990	2.5065	4193	2.3847	4400	2.2727	4610	2.1692	15
46	3792	2.6371	3993	2.5044	4197	2.3828	4404	2.2709	4614	2.1675	14
47	3795	2.6348	3996	2.5023	4200	2.3808	4407	2.2691	4617	2.1659	13
48	3799	2.6325	4000	2.5002	4204	2.3789	4411	2.2673	4621	2.1642	12
49	3802	2.6302	4003	2.4981	4207	2.3770	4414	2.2655	4624	2.1625	11
50	3805	2.6279	4006	2.4960	4210	2.3750	4417	2.2637	4628	2.1609	10
51	3809	2.6256	4010	2.4939	4214	2.3731	4421	2.2620	4631	2.1592	9
52	3812	2.6233	4013	2.4918	4217	2.3712	4424	2.2602	4635	2.1576	8
53	3815	2.6210	4017	2.4897	4221	2.3693	4428	2.2584	4638	2.1560	7
54	3819	2.6187	4020	2.4876	4224	2.3673	4431	2.2566	4642	2.1543	6
55	3822	2.6165	4023	2.4855	4228	2.3654	4435	2.2549	4645	2.1527	5
56	3825	2.6142	4027	2.4834	4231	2.3635	4438	2.2531	4649	2.1510	4
57	3829	2.6119	4030	2.4813	4234	2.3616	4442	2.2513	4652	2.1494	3
58	3832	2.6096	4033	2.4792	4238	2.3597	4445	2.2496	4656	2.1478	2
59	3835	2.6074	4037	2.4772	4241	2.3578	4449	2.2478	4660	2.1461	1
60	3839	2.6051	4040	2.4751	4245	2.3559	4452	2.2460	4663	2.1445	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	69°		68°		67°		66°		65°		′

NATURAL TANGENTS AND COTANGENTS

′	25°		26°		27°		28°		29°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	4663	2.1445	4877	2.0503	5095	1.9626	5317	1.8807	5543	1.8040	60
1	4667	2.1429	4881	2.0488	5099	1.9612	5321	1.8794	5547	1.8028	59
2	4670	2.1413	4885	2.0473	5103	1.9598	5325	1.8781	5551	1.8016	58
3	4674	2.1396	4888	2.0458	5106	1.9584	5328	1.8768	5555	1.8003	57
4	4677	2.1380	4892	2.0443	5110	1.9570	5332	1.8755	5558	1.7991	56
5	4681	2.1364	4895	2.0428	5114	1.9556	5336	1.8741	5562	1.7979	55
6	4684	2.1348	4899	2.0413	5117	1.9542	5340	1.8728	5566	1.7966	54
7	4688	2.1332	4903	2.0398	5121	1.9528	5343	1.8715	5570	1.7954	53
8	4691	2.1315	4906	2.0383	5125	1.9514	5347	1.8702	5574	1.7942	52
9	4695	2.1299	4910	2.0368	5128	1.9500	5351	1.8689	5577	1.7930	51
10	4699	2.1283	4913	2.0353	5132	1.9486	5354	1.8676	5581	1.7917	50
11	4702	2.1267	4917	2.0338	5136	1.9472	5358	1.8663	5585	1.7905	49
12	4706	2.1251	4921	2.0323	5139	1.9458	5362	1.8650	5589	1.7893	48
13	4709	2.1235	4924	2.0308	5143	1.9444	5366	1.8637	5593	1.7881	47
14	4713	2.1219	4928	2.0293	5147	1.9430	5369	1.8624	5596	1.7868	46
15	4716	2.1203	4931	2.0278	5150	1.9416	5373	1.8611	5600	1.7856	45
16	4720	2.1187	4935	2.0263	5154	1.9402	5377	1.8598	5604	1.7844	44
17	4723	2.1171	4939	2.0248	5158	1.9388	5381	1.8585	5608	1.7832	43
18	4727	2.1155	4942	2.0233	5161	1.9375	5384	1.8572	5612	1.7820	42
19	4731	2.1139	4946	2.0219	5165	1.9361	5388	1.8559	5616	1.7808	41
20	4734	2.1123	4950	2.0204	5169	1.9347	5392	1.8546	5619	1.7796	40
21	4738	2.1107	4953	2.0189	5172	1.9333	5396	1.8533	5623	1.7783	39
22	4741	2.1092	4957	2.0174	5176	1.9319	5399	1.8520	5627	1.7771	38
23	4745	2.1076	4960	2.0160	5180	1.9306	5403	1.8507	5631	1.7759	37
24	4748	2.1060	4964	2.0145	5184	1.9292	5407	1.8495	5635	1.7747	36
25	4752	2.1044	4968	2.0130	5187	1.9278	5411	1.8482	5639	1.7735	35
26	4755	2.1028	4971	2.0115	5191	1.9265	5415	1.8469	5642	1.7723	34
27	4759	2.1013	4975	2.0101	5195	1.9251	5418	1.8456	5646	1.7711	33
28	4763	2.0997	4979	2.0086	5198	1.9237	5422	1.8443	5650	1.7699	32
29	4766	2.0981	4982	2.0072	5202	1.9223	5426	1.8430	5654	1.7687	31
30	4770	2.0965	4986	2.0057	5206	1.9210	5430	1.8418	5658	1.7675	30
31	4773	2.0950	4989	2.0042	5209	1.9196	5433	1.8405	5662	1.7663	29
32	4777	2.0934	4993	2.0028	5213	1.9183	5437	1.8392	5665	1.7651	28
33	4780	2.0918	4997	2.0013	5217	1.9169	5441	1.8379	5669	1.7639	27
34	4784	2.0903	5000	1.9999	5220	1.9155	5445	1.8367	5673	1.7627	26
35	4788	2.0887	5004	1.9984	5224	1.9142	5448	1.8354	5677	1.7615	25
36	4791	2.0872	5008	1.9970	5228	1.9128	5452	1.8341	5681	1.7603	24
37	4795	2.0856	5011	1.9955	5232	1.9115	5456	1.8329	5685	1.7591	23
38	4798	2.0840	5015	1.9941	5235	1.9101	5460	1.8316	5688	1.7579	22
39	4802	2.0825	5019	1.9926	5239	1.9088	5464	1.8303	5692	1.7567	21
40	4806	2.0809	5022	1.9912	5243	1.9074	5467	1.8291	5696	1.7556	20
41	4809	2.0794	5026	1.9897	5246	1.9061	5471	1.8278	5700	1.7544	19
42	4813	2.0778	5029	1.9883	5250	1.9047	5475	1.8265	5704	1.7532	18
43	4816	2.0763	5033	1.9868	5254	1.9034	5479	1.8253	5708	1.7520	17
44	4820	2.0748	5037	1.9854	5258	1.9020	5482	1.8240	5712	1.7508	16
45	4823	2.0732	5040	1.9840	5261	1.9007	5486	1.8228	5715	1.7496	15
46	4827	2.0717	5044	1.9825	5265	1.8993	5490	1.8215	5719	1.7485	14
47	4831	2.0701	5048	1.9811	5269	1.8980	5494	1.8202	5723	1.7473	13
48	4834	2.0686	5051	1.9797	5272	1.8967	5498	1.8190	5727	1.7461	12
49	4838	2.0671	5055	1.9782	5276	1.8953	5501	1.8177	5731	1.7449	11
50	4841	2.0655	5059	1.9768	5280	1.8940	5505	1.8165	5735	1.7437	10
51	4845	2.0640	5062	1.9754	5284	1.8927	5509	1.8152	5739	1.7426	9
52	4849	2.0625	5066	1.9740	5287	1.8913	5513	1.8140	5743	1.7414	8
53	4852	2.0609	5070	1.9725	5291	1.8900	5517	1.8127	5746	1.7402	7
54	4856	2.0594	5073	1.9711	5295	1.8887	5520	1.8115	5750	1.7391	6
55	4859	2.0579	5077	1.9697	5298	1.8873	5524	1.8103	5754	1.7379	5
56	4863	2.0564	5081	1.9683	5302	1.8860	5528	1.8090	5758	1.7367	4
57	4867	2.0549	5084	1.9669	5306	1.8847	5532	1.8078	5762	1.7355	3
58	4870	2.0533	5088	1.9654	5310	1.8834	5535	1.8065	5766	1.7344	2
59	4874	2.0518	5092	1.9640	5313	1.8820	5539	1.8053	5770	1.7332	1
60	4877	2.0503	5095	1.9626	5317	1.8807	5543	1.8040	5774	1.7321	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	64°		63°		62°		61°		60°		′

NATURAL TANGENTS AND COTANGENTS

′	30°		31°		32°		33°		34°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	5774	1.7321	6009	1.6643	6249	1.6003	6494	1.5399	6745	1.4826	**60**
1	5777	1.7309	6013	1.6632	6253	1.5993	6498	1.5389	6749	1.4816	59
2	5781	1.7297	6017	1.6621	6257	1.5983	6502	1.5379	6754	1.4807	58
3	5785	1.7286	6020	1.6610	6261	1.5972	6506	1.5369	6758	1.4798	57
4	5789	1.7274	6024	1.6599	6265	1.5962	6511	1.5359	6762	1.4788	56
5	5793	1.7262	6028	1.6588	6269	1.5952	6515	1.5350	6766	1.4779	**55**
6	5797	1.7251	6032	1.6577	6273	1.5941	6519	1.5340	6771	1.4770	54
7	5801	1.7239	6036	1.6566	6277	1.5931	6523	1.5330	6775	1.4761	53
8	5805	1.7228	6040	1.6555	6281	1.5921	6527	1.5320	6779	1.4751	52
9	5808	1.7216	6044	1.6545	6285	1.5911	6531	1.5311	6783	1.4742	51
10	5812	1.7205	6048	1.6534	6289	1.5900	6536	1.5301	6787	1.4733	**50**
11	5816	1.7193	6052	1.6523	6293	1.5890	6540	1.5291	6792	1.4724	49
12	5820	1.7182	6056	1.6512	6297	1.5880	6544	1.5282	6796	1.4715	48
13	5824	1.7170	6060	1.6501	6301	1.5869	6548	1.5272	6800	1.4705	47
14	5828	1.7159	6064	1.6490	6305	1.5859	6552	1.5262	6805	1.4696	46
15	5832	1.7147	6068	1.6479	6310	1.5849	6556	1.5253	6809	1.4687	**45**
16	5836	1.7136	6072	1.6469	6314	1.5839	6560	1.5243	6813	1.4678	44
17	5840	1.7124	6076	1.6458	6318	1.5829	6565	1.5233	6817	1.4669	43
18	5844	1.7113	6080	1.6447	6322	1.5818	6569	1.5224	6822	1.4659	42
19	5847	1.7102	6084	1.6436	6326	1.5808	6573	1.5214	6826	1.4650	41
20	5851	1.7090	6088	1.6426	6330	1.5798	6577	1.5204	6830	1.4641	**40**
21	5855	1.7079	6092	1.6415	6334	1.5788	6581	1.5195	6834	1.4632	39
22	5859	1.7067	6096	1.6404	6338	1.5778	6585	1.5185	6839	1.4623	38
23	5863	1.7056	6100	1.6393	6342	1.5768	6590	1.5175	6843	1.4614	37
24	5867	1.7045	6104	1.6383	6346	1.5757	6594	1.5166	6847	1.4605	36
25	5871	1.7033	6108	1.6372	6350	1.5747	6598	1.5156	6851	1.4596	**35**
26	5875	1.7022	6112	1.6361	6354	1.5737	6602	1.5147	6856	1.4586	34
27	5879	1.7011	6116	1.6351	6358	1.5727	6606	1.5137	6860	1.4577	33
28	5883	1.6999	6120	1.6340	6363	1.5717	6610	1.5127	6864	1.4568	32
29	5887	1.6988	6124	1.6329	6367	1.5707	6615	1.5118	6869	1.4559	31
30	5890	1.6977	6128	1.6319	6371	1.5697	6619	1.5108	6873	1.4550	**30**
31	5894	1.6965	6132	1.6308	6375	1.5687	6623	1.5099	6877	1.4541	29
32	5898	1.6954	6136	1.6297	6379	1.5677	6627	1.5089	6881	1.4532	28
33	5902	1.6943	6140	1.6287	6383	1.5667	6631	1.5080	6886	1.4523	27
34	5906	1.6932	6144	1.6276	6387	1.5657	6636	1.5070	6890	1.4514	26
35	5910	1.6920	6148	1.6265	6391	1.5647	6640	1.5061	6894	1.4505	**25**
36	5914	1.6909	6152	1.6255	6395	1.5637	6644	1.5051	6899	1.4496	24
37	5918	1.6898	6156	1.6244	6399	1.5627	6648	1.5042	6903	1.4487	23
38	5922	1.6887	6160	1.6234	6403	1.5617	6652	1.5032	6907	1.4478	22
39	5926	1.6875	6164	1.6223	6408	1.5607	6657	1.5023	6911	1.4469	21
40	5930	1.6864	6168	1.6212	6412	1.5597	6661	1.5013	6916	1.4460	**20**
41	5934	1.6853	6172	1.6202	6416	1.5587	6665	1.5004	6920	1.4451	19
42	5938	1.6842	6176	1.6191	6420	1.5577	6669	1.4994	6924	1.4442	18
43	5942	1.6831	6180	1.6181	6424	1.5567	6673	1.4985	6929	1.4433	17
44	5945	1.6820	6184	1.6170	6428	1.5557	6678	1.4975	6933	1.4424	16
45	5949	1.6808	6188	1.6160	6432	1.5547	6682	1.4966	6937	1.4415	**15**
46	5953	1.6797	6192	1.6149	6436	1.5537	6686	1.4957	6942	1.4406	14
47	5957	1.6786	6196	1.6139	6440	1.5527	6690	1.4947	6946	1.4397	13
48	5961	1.6775	6200	1.6128	6445	1.5517	6694	1.4938	6950	1.4388	12
49	5965	1.6764	6204	1.6118	6449	1.5507	6699	1.4928	6954	1.4379	11
50	5969	1.6753	6208	1.6107	6453	1.5497	6703	1.4919	6959	1.4370	**10**
51	5973	1.6742	6212	1.6097	6457	1.5487	6707	1.4910	6963	1.4361	9
52	5977	1.6731	6216	1.6087	6461	1.5477	6711	1.4900	6967	1.4352	8
53	5981	1.6720	6220	1.6076	6465	1.5468	6716	1.4891	6972	1.4344	7
54	5985	1.6709	6224	1.6066	6469	1.5458	6720	1.4882	6976	1.4335	6
55	5989	1.6698	6228	1.6055	6473	1.5448	6724	1.4872	6980	1.4326	**5**
56	5993	1.6687	6233	1.6045	6478	1.5438	6728	1.4863	6985	1.4317	4
57	5997	1.6676	6237	1.6034	6482	1.5428	6732	1.4854	6989	1.4308	3
58	6001	1.6665	6241	1.6024	6486	1.5418	6737	1.4844	6993	1.4299	2
59	6005	1.6654	6245	1.6014	6490	1.5408	6741	1.4835	6998	1.4290	1
60	6009	1.6643	6249	1.6003	6494	1.5399	6745	1.4826	7002	1.4281	**0**
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	59°		58°		57°		56°		55°		′

NATURAL TANGENTS AND COTANGENTS

′	35°		36°		37°		38°		39°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	7002	1.4281	7265	1.3764	7536	1.3270	7813	1.2799	8098	1.2349	60
1	7006	1.4273	7270	1.3755	7540	1.3262	7818	1.2792	8103	1.2342	59
2	7011	1.4264	7274	1.3747	7545	1.3254	7822	1.2784	8107	1.2334	58
3	7015	1.4255	7279	1.3739	7549	1.3246	7827	1.2776	8112	1.2327	57
4	7019	1.4246	7283	1.3730	7554	1.3238	7832	1.2769	8117	1.2320	56
5	7024	1.4237	7288	1.3722	7558	1.3230	7836	1.2761	8122	1.2312	55
6	7028	1.4229	7292	1.3713	7563	1.3222	7841	1.2753	8127	1.2305	54
7	7032	1.4220	7297	1.3705	7568	1.3214	7846	1.2746	8132	1.2298	53
8	7037	1.4211	7301	1.3697	7572	1.3206	7850	1.2738	8136	1.2290	52
9	7041	1.4202	7306	1.3688	7577	1.3198	7855	1.2731	8141	1.2283	51
10	7046	1.4193	7310	1.3680	7581	1.3190	7860	1.2723	8146	1.2276	50
11	7050	1.4185	7314	1.3672	7586	1.3182	7865	1.2715	8151	1.2268	49
12	7054	1.4176	7319	1.3663	7590	1.3175	7869	1.2708	8156	1.2261	48
13	7059	1.4167	7323	1.3655	7595	1.3167	7874	1.2700	8161	1.2254	47
14	7063	1.4158	7328	1.3647	7600	1.3159	7879	1.2693	8165	1.2247	46
15	7067	1.4150	7332	1.3638	7604	1.3151	7883	1.2685	8170	1.2239	45
16	7072	1.4141	7337	1.3630	7609	1.3143	7888	1.2677	8175	1.2232	44
17	7076	1.4132	7341	1.3622	7613	1.3135	7893	1.2670	8180	1.2225	43
18	7080	1.4124	7346	1.3613	7618	1.3127	7898	1.2662	8185	1.2218	42
19	7085	1.4115	7350	1.3605	7623	1.3119	7902	1.2655	8190	1.2210	41
20	7089	1.4106	7355	1.3597	7627	1.3111	7907	1.2647	8195	1.2203	40
21	7094	1.4097	7359	1.3588	7632	1.3103	7912	1.2640	8199	1.2196	39
22	7098	1.4089	7364	1.3580	7636	1.3095	7916	1.2632	8204	1.2189	38
23	7102	1.4080	7368	1.3572	7641	1.3087	7921	1.2624	8209	1.2181	37
24	7107	1.4071	7373	1.3564	7646	1.3079	7926	1.2617	8214	1.2174	36
25	7111	1.4063	7377	1.3555	7650	1.3072	7931	1.2609	8219	1.2167	35
26	7115	1.4054	7382	1.3547	7655	1.3064	7935	1.2602	8224	1.2160	34
27	7120	1.4045	7386	1.3539	7659	1.3056	7940	1.2594	8229	1.2153	33
28	7124	1.4037	7391	1.3531	7664	1.3048	7945	1.2587	8234	1.2145	32
29	7129	1.4028	7395	1.3522	7669	1.3040	7950	1.2579	8238	1.2138	31
30	7133	1.4019	7400	1.3514	7673	1.3032	7954	1.2572	8243	1.2131	30
31	7137	1.4011	7404	1.3506	7678	1.3024	7959	1.2564	8248	1.2124	29
32	7142	1.4002	7409	1.3498	7683	1.3017	7964	1.2557	8253	1.2117	28
33	7146	1.3994	7413	1.3490	7687	1.3009	7969	1.2549	8258	1.2109	27
34	7151	1.3985	7418	1.3481	7692	1.3001	7973	1.2542	8263	1.2102	26
35	7155	1.3976	7422	1.3473	7696	1.2993	7978	1.2534	8268	1.2095	25
36	7159	1.3968	7427	1.3465	7701	1.2985	7983	1.2527	8273	1.2088	24
37	7164	1.3959	7431	1.3457	7706	1.2977	7988	1.2519	8278	1.2081	23
38	7168	1.3951	7436	1.3449	7710	1.2970	7992	1.2512	8283	1.2074	22
39	7173	1.3942	7440	1.3440	7715	1.2962	7997	1.2504	8287	1.2066	21
40	7177	1.3934	7445	1.3432	7720	1.2954	8002	1.2497	8292	1.2059	20
41	7181	1.3925	7449	1.3424	7724	1.2946	8007	1.2489	8297	1.2052	19
42	7186	1.3916	7454	1.3416	7729	1.2938	8012	1.2482	8302	1.2045	18
43	7190	1.3908	7458	1.3408	7734	1.2931	8016	1.2475	8307	1.2038	17
44	7195	1.3899	7463	1.3400	7738	1.2923	8021	1.2467	8312	1.2031	16
45	7199	1.3891	7467	1.3392	7743	1.2915	8026	1.2460	8317	1.2024	15
46	7203	1.3882	7472	1.3384	7747	1.2907	8031	1.2452	8322	1.2017	14
47	7208	1.3874	7476	1.3375	7752	1.2900	8035	1.2445	8327	1.2009	13
48	7212	1.3865	7481	1.3367	7757	1.2892	8040	1.2437	8332	1.2002	12
49	7217	1.3857	7485	1.3359	7761	1.2884	8045	1.2430	8337	1.1995	11
50	7221	1.3848	7490	1.3351	7766	1.2876	8050	1.2423	8342	1.1988	10
51	7226	1.3840	7495	1.3343	7771	1.2869	8055	1.2415	8346	1.1981	9
52	7230	1.3831	7499	1.3335	7775	1.2861	8059	1.2408	8351	1.1974	8
53	7234	1.3823	7504	1.3327	7780	1.2853	8064	1.2401	8356	1.1967	7
54	7239	1.3814	7508	1.3319	7785	1.2846	8069	1.2393	8361	1.1960	6
55	7243	1.3806	7513	1.3311	7789	1.2838	8074	1.2386	8366	1.1953	5
56	7248	1.3798	7517	1.3303	7794	1.2830	8079	1.2378	8371	1.1946	4
57	7252	1.3789	7522	1.3295	7799	1.2822	8083	1.2371	8376	1.1939	3
58	7257	1.3781	7526	1.3287	7803	1.2815	8088	1.2364	8381	1.1932	2
59	7261	1.3772	7531	1.3278	7808	1.2807	8093	1.2356	8386	1.1925	1
60	7265	1.3764	7536	1.3270	7813	1.2799	8098	1.2349	8391	1.1918	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	54°		53°		52°		51°		50°		′

′	40°		41°		42°		43°		44°		′
	tan	cot	tan	cot	tan	cot	tan	cot	tan	cot	
0	8391	1.1918	8693	1.1504	9004	1.1106	9325	1.0724	9657	1.0355	60
1	8396	1.1910	8698	1.1497	9009	1.1100	9331	1.0717	9663	1.0349	59
2	8401	1.1903	8703	1.1490	9015	1.1093	9336	1.0711	9668	1.0343	58
3	8406	1.1896	8708	1.1483	9020	1.1087	9341	1.0705	9674	1.0337	57
4	8411	1.1889	8713	1.1477	9025	1.1080	9347	1.0699	9679	1.0331	56
5	8416	1.1882	8718	1.1470	9030	1.1074	9352	1.0692	9685	1.0325	55
6	8421	1.1875	8724	1.1463	9036	1.1067	9358	1.0686	9691	1.0319	54
7	8426	1.1868	8729	1.1456	9041	1.1061	9363	1.0680	9696	1.0313	53
8	8431	1.1861	8734	1.1450	9046	1.1054	9369	1.0674	9702	1.0307	52
9	8436	1.1854	8739	1.1443	9052	1.1048	9374	1.0668	9708	1.0301	51
10	8441	1.1847	8744	1.1436	9057	1.1041	9380	1.0661	9713	1.0295	50
11	8446	1.1840	8749	1.1430	9062	1.1035	9385	1.0655	9719	1.0289	49
12	8451	1.1833	8754	1.1423	9067	1.1028	9391	1.0649	9725	1.0283	48
13	8456	1.1826	8759	1.1416	9073	1.1022	9396	1.0643	9730	1.0277	47
14	8461	1.1819	8765	1.1410	9078	1.1016	9402	1.0637	9736	1.0271	46
15	8466	1.1812	8770	1.1403	9083	1.1009	9407	1.0630	9742	1.0265	45
16	8471	1.1806	8775	1.1396	9089	1.1003	9413	1.0624	9747	1.0259	44
17	8476	1.1799	8780	1.1389	9094	1.0996	9418	1.0618	9753	1.0253	43
18	8481	1.1792	8785	1.1383	9099	1.0990	9424	1.0612	9759	1.0247	42
19	8486	1.1785	8790	1.1376	9105	1.0983	9429	1.0606	9764	1.0241	41
20	8491	1.1778	8796	1.1369	9110	1.0977	9435	1.0599	9770	1.0235	40
21	8496	1.1771	8801	1.1363	9115	1.0971	9440	1.0593	9776	1.0230	39
22	8501	1.1764	8806	1.1356	9121	1.0964	9446	1.0587	9781	1.0224	38
23	8506	1.1757	8811	1.1349	9126	1.0958	9451	1.0581	9787	1.0218	37
24	8511	1.1750	8816	1.1343	9131	1.0951	9457	1.0575	9793	1.0212	36
25	8516	1.1743	8821	1.1336	9137	1.0945	9462	1.0569	9798	1.0206	35
26	8521	1.1736	8827	1.1329	9142	1.0939	9468	1.0562	9804	1.0200	34
27	8526	1.1729	8832	1.1323	9147	1.0932	9473	1.0556	9810	1.0194	33
28	8531	1.1722	8837	1.1316	9153	1.0926	9479	1.0550	9816	1.0188	32
29	8536	1.1715	8842	1.1310	9158	1.0919	9484	1.0544	9821	1.0182	31
30	8541	1.1708	8847	1.1303	9163	1.0913	9490	1.0538	9827	1.0176	30
31	8546	1.1702	8852	1.1296	9169	1.0907	9495	1.0532	9833	1.0170	29
32	8551	1.1695	8858	1.1290	9174	1.0900	9501	1.0526	9838	1.0164	28
33	8556	1.1688	8863	1.1283	9179	1.0894	9506	1.0519	9844	1.0158	27
34	8561	1.1681	8868	1.1276	9185	1.0888	9512	1.0513	9850	1.0152	26
35	8566	1.1674	8873	1.1270	9190	1.0881	9517	1.0507	9856	1.0147	25
36	8571	1.1667	8878	1.1263	9195	1.0875	9523	1.0501	9861	1.0141	24
37	8576	1.1660	8884	1.1257	9201	1.0869	9528	1.0495	9867	1.0135	23
38	8581	1.1653	8889	1.1250	9206	1.0862	9534	1.0489	9873	1.0129	22
39	8586	1.1647	8894	1.1243	9212	1.0856	9540	1.0483	9879	1.0123	21
40	8591	1.1640	8899	1.1237	9217	1.0850	9545	1.0477	9884	1.0117	20
41	8596	1.1633	8904	1.1230	9222	1.0843	9551	1.0470	9890	1.0111	19
42	8601	1.1626	8910	1.1224	9228	1.0837	9556	1.0464	9896	1.0105	18
43	8606	1.1619	8915	1.1217	9233	1.0831	9562	1.0458	9902	1.0099	17
44	8611	1.1612	8920	1.1211	9239	1.0824	9567	1.0452	9907	1.0094	16
45	8617	1.1606	8925	1.1204	9244	1.0818	9573	1.0446	9913	1.0088	15
46	8622	1.1599	8931	1.1197	9249	1.0812	9578	1.0440	9919	1.0082	14
47	8627	1.1592	8936	1.1191	9255	1.0805	9584	1.0434	9925	1.0076	13
48	8632	1.1585	8941	1.1184	9260	1.0799	9590	1.0428	9930	1.0070	12
49	8637	1.1578	8946	1.1178	9266	1.0793	9595	1.0422	9936	1.0064	11
50	8642	1.1571	8952	1.1171	9271	1.0786	9601	1.0416	9942	1.0058	10
51	8647	1.1565	8957	1.1165	9276	1.0780	9606	1.0410	9948	1.0052	9
52	8652	1.1558	8962	1.1158	9282	1.0774	9612	1.0404	9954	1.0047	8
53	8657	1.1551	8967	1.1152	9287	1.0768	9618	1.0398	9959	1.0041	7
54	8662	1.1544	8972	1.1145	9293	1.0761	9623	1.0392	9965	1.0035	6
55	8667	1.1538	8978	1.1139	9298	1.0755	9629	1.0385	9971	1.0029	5
56	8672	1.1531	8983	1.1132	9303	1.0749	9634	1.0379	9977	1.0023	4
57	8678	1.1524	8988	1.1126	9309	1.0742	9640	1.0373	9983	1.0017	3
58	8683	1.1517	8994	1.1119	9314	1.0736	9646	1.0367	9988	1.0012	2
59	8688	1.1510	8999	1.1113	9320	1.0730	9651	1.0361	9994	1.0006	1
60	8693	1.1504	9004	1.1106	9325	1.0724	9657	1.0355	1.000	1.0000	0
	cot	tan	cot	tan	cot	tan	cot	tan	cot	tan	
′	49°		48°		47°		46°		45°		′

ANSWERS

Ex. 1:

1. $7\frac{1}{3}$ ft. 2. 43′2″ 3. 23,760 ft. per min.
4. 184.5 sq. ft. 5. 60 mi. per hr. 6. 10 gal.
7. $3.36 8. 20 ft./sec. 9. 69.4 sq. ft.
10. .134 cu. ft.; 4.2 cu. ft. 11. 8.32 lb. 12. 12,480 lb.
13. 40 cu. ft. 14. 84,000 lb. 15. 600 cu. ft./min.

Ex. 2:

1. 8.75 oz. 2. 1.8 in. 3. 37.5 cm. 4. 5.4 kg. 5. 20″
6. 165 lb. 7. 162.5 mm. 8. 226.8 gm. 9. 77.4 kg. 10. 5′11.2″
11. 1900 cc. 12. 10.6 qt. 13. 1.59 pt. 14. 7.6 l. 15. 1300 sq. cm.

Ex. 3:

1. $\frac{24}{64}$″; $\frac{36}{16}$″; $\frac{52}{32}$″; $\frac{652}{64}$″ 2. $4\frac{1}{2}$ eighths; $\frac{1}{16}$″ more; $\frac{3}{16}$″ less
3. $\frac{40}{16}$; $\frac{26}{16}$; $\frac{51}{16}$; $\frac{12}{16}$ 4. 23
5. $\frac{58}{16}$; $\frac{116}{32}$ 6. No; yes; $\frac{3}{4}$, $\frac{7}{8}$, $2\frac{3}{64}$, $2\frac{1}{16}$

Ex. 4:

1. $1\frac{5}{12}$ 2. $1\frac{3}{8}$ 3. $1\frac{3}{4}$ 4. $\frac{15}{16}$ 5. $8\frac{3}{8}$
6. $6\frac{15}{16}$ 7. $8\frac{5}{16}$ 8. $8\frac{17}{32}$ 9. $2\frac{3}{8}$ 10. $2\frac{3}{16}$
11. $\frac{7}{64}$ 12. $1\frac{15}{32}$ 13. $5\frac{29}{32}$″ 14. $29\frac{5}{8}$″ 15. $80\frac{1}{4}$″
16. $1\frac{5}{64}$″ 17. $1\frac{1}{8}$″; $2\frac{1}{16}$″; $\frac{11}{16}$″ 18. $5\frac{5}{8}$″; $\frac{5}{8}$″ 19. $17\frac{1}{16}$″ 20. $10\frac{5}{16}$″

Ex. 5:

1. $3\frac{7}{16}$ 2. 87 3. 78 4. $5\frac{1}{2}$ 5. 8 6. $\frac{5}{8}$
7. 160.3 gal. 8. $5\frac{11}{16}$″ 9. $2\frac{3}{4}$″ 10. 19 11. $282\frac{3}{4}$ lb. 12. 55

Ex. 6:

1. 0.276 2. 0.0154 3. 0.0007 4. 0.00425 5. 0.100
6. Three hundred seventy-nine and two-tenth thousandths.
7. Six-tenth thousandths.
8. Two hundred and two-tenth thousandths.
9. Seventy and one-half thousandths.
10. Two hundred eighty-one and six-tenth thousandths.
11. Ninety-six thousandths.
12. Four hundred forty-four and four-tenth thousandths.
13. Fifteen and eight-tenth thousandths.
14. One-quarter thousandth.

Ex. 7:

1. 2.5175″ 2. 2.3758″ 3. 1.192″ 4. .6175″
5. .694″ 6. 4.3917″ 7. 1.372″ 8. 4.214″; 1.81″
9. 1.503″; 3.678″ 10. .0007″ 11. .239″ 12. 1.444″; 2.786″; .7883″

Ex. 8:

1. 5.95″ 2. 445.09 lb. 3. .682″ 4. 216.5 lb. 5. .108″
6. 7.57 lb. 7. .648″ 8. 139.7 cal. 9. $336.86 10. .10752″

Ex. 9:

1. 73.5 cu. ft. 2. 30.5 cu. ft. 3. 16.61 cm. 4. 36 5. 5.438″ 6. 96

Ex. 10:

1. .125 2. .09375 3. .4375 4. .234375 5. .28125
6. .9375 7. .84375 8. .6875 9. 3.142857 10. .555556
11. .9230769 12. .181818 13. .171875 14. .765625 15. .208333
16. .19531 17. 7/16 18. 13/16 19. 1/16 20. 1/320
21. 13/32 22. 29/32 23. 5/64 24. 59/64 25. 17/32

Ex. 11:

1. (a) 7/32 (b) 27/32 (c) 3/32 2. (a) 31/64 (b) 11/64 (c) 51/64
3. 1.3866 to 1.3946; 6.246 to 6.254; .961 to .969; 1.038 to 1.046
4. .88125 to .93125; 1.13125 to 1.18125; 1.1625 to 1.2125; .5375 to .5875; 2.100 to 2.150

Ex. 12:

1. .908″; .035″ 2. $x = 2\frac{5}{8}''$, $y = 2\frac{1}{4}''$; $x = 2.640''$, $y = 2.2575''$
3. 13/16″, 2 11/16″, 3 5/32″, 1 7/16″ 4. .762″; 2.637″; 3.258″; 1.141″
5. At least .003″, but not more than .007″

Ex. 13:

1. (a) .465 (b) .904 (c) .595 (d) .132 (e) .022 (f) .991
2. (a) .3473 (b) .8839 (c) .4662 (d) .2114 (e) .5776 (f) .6241

Ex. 14:

1. .1004″, .144″, .150″ 2. .1002″, .350″, .103″
3. .1005″, .200″, .126″ 4. .550″, .116″
5. .1002″, .150″; .108″, .040″ 6. .1008″, .200″, .133″
7. 3.000″, .1001″, .700″, .106″ 8. 2.000″, .1002″, .500″, .107″
9. 1.000″, .1009″, .650″, .103″
10. 4.000″, 3.000″, 1.000″, .900″, .500″, .140″, .1002″
11. 4.000″, 3.000″, .144″, .050″, .1004″ 12. 3.000″, .1008″, .800″, .109″

Ex. 15:

1. 57.22 lb. 2. 94.6 H.P. 3. 2769 r.p.m.
4. 90 H.P. 5. 7½ lb. 6. $12.04; $8.74
7. 21 lb.; 40½ lb.; 88½ lb. 8. 27.5 lb. 9. 6 pieces
10. 4.728″ and 4.872″ 11. 217½ lb.; 797½ lb.; 435 lb.

Ex. 16:

1. 16⅔% 2. 15% 3. 12% 4. 25.7%
5. 4.5% 6. 12½% 7. 40% 8. 1.6%
9. .0332% 10. 25% 11. 1.8% 12. 8.6%

Ex. 17:

1. 156; 195 *2.* 22.5″ *3.* 1000 ft. *4.* 88 lb. *5.* $160,000 *6.* $48.50

Ex. 18:

1. $10.80 *2.* $1.60 *3.* 20% *4.* $70 *5.* 12½%
6. $22,400 *7.* 3.6 oz. *8.* 6⅔% *9.* 390 cu. ft. *10.* 3⅛%
11. 368 *12.* 4.2% *13.* $560 *14.* $115.63 *15.* $120

Ex. 19:

1. 2:3 *2.* 1:6 *3.* 2:3
4. 8:19 *5.* 13:17 *6.* 17½″
7. 15:23 *8.* 7 in. *9.* .04 cm.
10. 7 in.

Ex. 20:

1. 5:2; 2:5 *2.* 943:1000 *3.* 17½″, 24½″ *4.* 30°, 60°, 90°
5. 19 liters *6.* 17.32″; 3″ *7.* 1:4; 55 *8.* 200.26 cm.
9. 18 lb. *10.* 300, 700, 1000 *11.* .88 *12.* 460 lb.
13. 1:4 *14.* 2:5; 5:3; 30% *15.* 5:8; 2:5; 33⅓%

Ex. 21:

1. 1″×2⅛″ *2.* 2½″×1¾″
3. 8″; 20″; 15′; 6½″ *4.* 2½′; 1″; ¼″; 1.2″; 40′; 72½′
5. (*a*) 1¾″; 65′ (*b*) 3″; 60″ (*c*) 1½″; 19′ (*d*) 12½″; 11½ mi.
(*e*) 1⅛″; 180 ft.
6. 14″×24½″ *7.* 3¼″×4½″ *8.* 8.17″ *9.* 14′×23′ *10.* 27′×37½′
11. 37½ ft. *12.* 2½″
13. 1″=1′; 1″=10′; 3″; 12′; 10″; 1″=2′ *14.* 3½′×7′
15. L.R., 12′×20′; Dinette, 9½′×10′; B.R., 11½′×17′; Alcove, 8½′×12½′

Ex. 22:

1. 20 *2.* 40 *3.* 21 *4.* 20 *5.* 12 *6.* 27
7. 9 *8.* 42 *9.* 6 *10.* 24 *11.* 6 *12.* 28

Ex. 23:

1. 1.8″ *2.* $56 *3.* 525 pieces *4.* 2380 mi.
5. 525 ohms *6.* 780 lb. *7.* 42 ft.; 250 lb. *8.* 326⅔; 642.9
9. 288 cu. ft.; 125 lb. *10.* .56 mi. nearer

Ex. 24:

1. $C_2=12$; $R_2=60$ *2.* 576 cu. in.; 72 lb. per sq. in. *3.* $w_2=3$ lb.; $d_2=8''$
4. .0024 *5.* 24 r.p.m. *6.* 24″
7. 700 r.p.m. *8.* 13 teeth *9.* 2500 r.p.m.
10. 27″

Ex. 25:

1. Twice one number increased by three times another number.
2. Five times one number diminished by half another number.
3. One number decreased by a second and increased by a third, and the result divided by three.
4. Two-thirds of a number increased by eight.
5. One number decreased by the reciprocal of another number.
6. Five times the product of two numbers.
7. The difference between two numbers divided by twice a third number.
8. Nine-fifths of a number increased by thirty-two.
9. Twice the product of the length by the width added to twice the product of the length and height, added to twice the product of the width and height.
10. The difference of two numbers divided by the difference of two other numbers.

Ex. 26:

1. 11.4 2. 10.33 3. 90
4. 3.586 5. 179.7 6. .0689
7. 211.1 8. 2, 3, 4, 5, 7½ 9. 0, 8, 16, 24, 32
10. 3, 6, 7½, 9, 18 11. 0, ¼, ½, ¾, 1¼ 12. 2⅔, 5⅓, 8, 10⅔, 16
13. ⅔, 2, 3⅓, 4⅔, 6⅔ 14. 0, .32, .64, 3.19 15. 0, 4½, 9, 18, 45

Ex. 27:

1. $+6; +38; 0; -9; -22; -12; +12$ 2. $+23; +13; -6; 0; -2; -36; 0$
3. $-63; +60; -60; +2; +6.2; -72k; +30m; -12a; -48r$
4. $-9; -9; +3; +12; -2; +1; -2; -8/a; -8x$

Ex. 28:

1. $21p; 26r; 9m; 8.6h; -22\frac{1}{2}x$ 2. $4k; -6a; 26xy; -36lw; +2\frac{1}{2}ab$
3. $34x$ 4. $26k$ 5. $4h+4l+4w$ 6. $12e$ 7. $8\frac{1}{4}k$ 8. $9d$

Ex. 29:

1. $32m$; $30k$; $15pq$; $12abh$; $12a^3b$; $66h$; $\frac{1}{3}st$; $5r^2h$; $\frac{4}{3}R^3$; $7.2a^2t^3$; $12a^3b^3$; $5x^7$; m^5; $6x^2y^2$; $\frac{1}{6}a^5$
2. $7m$; 8; 24; $3/x$; $7\frac{1}{2}k$; $7y$; $12a^2/b$; 5; $3r/s^3$; $-8d^2/c$; $2Rh$; $-2a^2b^2$

Ex. 30:

1. $2l+2w$ 2. $ka+kb-kc$ 3. $a+nd-d$
4. $IR+Ir$ 5. $P+PRT$ 6. $S=\frac{na}{2}+\frac{nl}{2}$
7. $\frac{hB}{2}+\frac{hb}{2}$ 8. $\pi R^2-\pi r^2$ 9. Lt_0-Lt_1
10. $2\pi rl+2\pi r^2$ 11. $na+\frac{n^2d}{2}-\frac{nd}{2}$ 12. $V=\frac{hb_1}{6}+\frac{hb_2}{6}+\frac{2}{3}hM$
13. $p\ (1+i)$ 14. $h\ (p-q)$ 15. $I^2\ (R+r)$
16. $2a\ (b-c)$ 17. $P\ (1+rt)$ 18. $\frac{1}{2}h\ (B_1+B_2)$
19. $\pi D\ (l+1)$ 20. $2\pi R\ (l+R)$ 21. 100; 20
22. 21.58 23. .5625 24. 8448

Ex. 31:

1. 64
2. 256
3. 216
4. 1728
5. 1
6. 27
7. 400
8. 343
9. $\frac{1}{4}$
10. $\frac{4}{9}$
11. $\frac{1}{64}$
12. $\frac{1}{100}$
13. $\frac{27}{64}$
14. .0001
15. .000001
16. .000001

Ex. 32:

1. 3,400,000
2. 6,200,000,000
3. 91,000,000,000
4. 430,000,000,000
5. 5,200,000
6. 1,380,000,000,000
7. 862,000,000
8. 24,630,000,000,000
9. .000035
10. .00000023
11. .0000000049
12. .000000000326
13. 37×10^{6}
14. 5.8×10^{6}
15. 12.4×10^{6}
16. 4.9×10^{9}
17. 4×10^{-5}
18. 2.8×10^{-7}
19. 3.92×10^{-4}
20. 7.6×10^{-10}
21. 5,882,500,000,000
22. 58.93×10^{-5}

Ex. 33:

1. $\sqrt[3]{a}$; $\sqrt[4]{x}$; $\sqrt{5a}$; $\sqrt[3]{p^2}$; $\sqrt{A^3}$; $\sqrt{mn}$
2. $p^{1/3}$; $k^{1/4}$; $a^{3/4}$; $(10x)^{1/5}$; $\left(\frac{2s}{g}\right)^{1/2}$; $\left(\frac{A}{\pi}\right)^{1/2}$
3. 12; $1\frac{1}{2}$; 8; 4; $6a$; 12; 216; 4,000,000
4. $\frac{1}{8}$; 8; 9; .2; 16; 2; .3; 8; $\frac{1}{2}$; .01; $\frac{1}{25x^2}$; $\frac{1}{4a^2}$

Ex. 34:

1. 364 in.
2. $37\frac{1}{2}$ in.
3. 456
4. 64.4°
5. $510
6. 2.45
7. 200,000
8. 15.84
9. 59,040
10. 26.4 cal.
11. 153.6 H.P.
12. 128,060 sq. ft.

Ex. 35:

1. 13
2. 32
3. .35
4. 20
5. 12
6. 15
7. 14
8. 320
9. 12
10. 36
11. 16
12. 160

Ex. 36:

1. $l=\frac{A}{w}$
2. $M=DV$
3. $T=\frac{D}{R}$
4. $R=\frac{E}{I}$
5. $R=\frac{C}{2\pi}$
6. $s=\frac{d}{1.4}$
7. $P=\frac{kT}{V}$
8. $T=\frac{I}{PR}$
9. $h=\frac{2A}{b}$
10. $r=\frac{S}{2\pi h}$
11. $w_2=\frac{l_1w_1}{l_2}$
12. $t^2=\frac{2S}{g}$

Ex. 37:

1. 22
2. 14
3. 38
4. 95
5. 14
6. 36
7. $3\frac{1}{2}$
8. 10.7
9. 2.1
10. 9.25
11. 22.5
12. 38.3

Ex. 38:

1. 2 *2.* 4 *3.* $\frac{n-p}{k}$ *4.* $\frac{C-q}{4}$
5. $\frac{P-2l}{2}$ *6.* $\frac{V-E}{r}$ *7.* $\frac{12D-TL}{12}$ *8.* $\frac{V-v_0}{g}$
9. $\frac{2A-bh}{h}$ *10.* $\frac{A-P}{PT}$ *11.* $\frac{l-a}{n-1}$ *12.* $N-2PC$
13. $\frac{DL}{2\pi}$ *14.* $NW+W; \frac{s}{N+1}$ *15.* $\frac{FRg}{v^2}; \frac{Wv^2}{gF}$ *16.* $\frac{Q(t_1-t_0)+wh}{w}$

Ex. 39:

1. 4.2 *2.* 5.5 *3.* 3.5 *4.* 6.4 *5.* 7.7
6. 4.6 *7.* 7.4 *8.* 8.6 *9.* 11.4 *10.* 12.2
11. 13.1 *12.* 14.1 *13.* 9.3 *14.* 7.4 *15.* 2.5
16. 19.5 *17.* 23.8 *18.* 54.2 *19.* 8.3 *20.* 20.7

Ex. 40:

1. 7.4 *2.* 7.6 *3.* 13.3 *4.* 32.0
5. 6.4 *6.* 11.22 *7.* 166.7 *8.* 35.4 mi.
9. 40 sq. ft. *10.* 9.2 in.

Ex. 41:

1. $v=\sqrt{\frac{2E}{m}}$ *2.* $t=\sqrt{\frac{2v}{g}}$ *3.* $h=\frac{2m^2}{3}$
4. $h=\frac{s}{2}\sqrt{3}$ *5.* $s=\sqrt{\frac{4A}{\sqrt{3}}}$ *6.* $R=\sqrt{\frac{3V}{\pi h}}$
7. $C=\frac{1}{4\pi^2 f^2 L}$ *8.* $C=\sqrt{\frac{6I}{\pi ab}}$ *9.* $v=10\sqrt{\frac{11gP}{W}}$
10. $R=\frac{1}{C}\sqrt{E^2-(2\pi nLC)^2}$
11. $h=\frac{.36P^2}{A^2}$ *12.* 38.0 sq. ft.

Ex. 42:

1. 2.7; —1.4 *2.* 12.4; —2.4 *3.* 3.3; —4.3
4. 1.5; —1.3 *5.* 3.8; —.3 *6.* 3.0; —5.0

Ex. 43:

1. Doubled; halved; divided by 10; multiplied by 1½; divided by 8
2. Multiplied by 9; divided by 6; constant; multiplied by 4
3. V decreases; P decreases; P is doubled; V is divided by 3; P is divided by 5
4. Multiplied by 4; divided by 9; multiplied by 25; divided by 100

Ex. 48:

1. 2.9694 *2.* 2.6112 *3.* 9.7024—10 *4.* 1.8554
5. 6.7210—10 *6.* 5.8082 *7.* .9051 *8.* 7.5328—10
9. 9.8408—10 *10.* 6.6532—10 *11.* 3.6817 *12.* 2.3746

Ex. 49:

1. 24.02 *2.* 4.397 *3.* 265.3 *4.* .4116 *5.* 25.06 *6.* .1996
7. 5.788 *8.* .02619 *9.* 276.4 *10.* 52,663 *11.* .008134 *12.* 1112

Ex. 50:

1. 1.333 *2.* .00986 *3.* 1051 *4.* 4185 *5.* .2374 *6.* .004759

Ex. 51:

1. 9.110 *2.* 3.894 *3.* .05604 *4.* 2.116 *5.* 15.78
6. 2.222 *7.* 1.486 *8.* 65,000 *9.* .01985 *10.* 19,620
11. 465.8 *12.* 10.72 *13.* 2.466 *14.* 4.444 *15.* 5.46 in.

Ex. 52:

1. 1.796 *2.* 13.75 *3.* 1.181 *4.* 4.166 *5.* 2.56
6. 1.14 *7.* 1.79 *8.* 47.9 *9.* 18.3 yr. *10.* $678.70; 4.3 yr.

Ex. 53:

1. 950 *2.* 565 *3.* 57.4 *4.* .231 *5.* 123.5 *6.* 116.1 *7.* 710
8. 4.03 *9.* 38.3 *10.* 4.16 *11.* 79.2 *12.* 1.44 *13.* 5.21 *14.* 5.12

Ex. 54:

1. 6.63 *2.* 4.98 *3.* 8.4 *4.* 11.7 *5.* 2.87 *6.* .92
7. .0179 *8.* .253 *9.* 58.7 *10.* .07 *11.* 1.75 *12.* .087

Ex. 55:

1. 51°; 67½°; 45°40′; 22°18′48″
2. 148°; 135°; 90°; 58°; 122°31′; 19°15′12″
3. 85°20′ *4.* 100° *5.* 85° *6.* 56°; 34° *7.* 34° *8.* 50°

Ex. 56:

1. 142°; 68°; 25° *2.* $x=14°$; $y=166°$; $z=76°$
3. 120°; 58°; 360° *4.* 115°; 68°; 130°; 68°; 132°
5. 64°45′ *6.* 133°
7. AM, MC, AB; BC, BM, BM *8.* OR, point O, OS; OM, OB, OP

Ex. 57:

1. 75° 24′ *2.* 16° *3.* 45° 27′ *4.* 28° 35′ 50″ *5.* 64° 38′ *6.* 109° 12′

Ex. 58:

1. 12.04 in. *2.* 20.6 ft. *3.* 11.8 in. *4.* 6.4 in. *5.* 22.4 in.
6. 8.9 in. *7.* 7.2 in.; 10.8 in. *8.* 22′ 9″ *9.* 5.66 in. *10.* 9.798 in.

Ex. 59:

1. 200°
2. 215°
3. 158°
5. $\widehat{AB}=40°$; $\widehat{DC}=40°$; $\widehat{BC}=220°$
6. $\angle P=95°$; $\angle Q=65°$; $\angle R=85°$; $\angle S=115°$

Ex. 60:

1. 15.5 in.
2. 17° 10′; $a=\frac{\beta}{2}$
3. 9.899 in.
4. 4.58 in.; $x=\frac{1}{2}d\sqrt{2}$
5. 220 ft. per sec.
6. 160 r.p.m.
7. 9.05 in.
8. $\phi=57° 30'$; $\phi=2a$
9. 14.14″; 44.44″
10. 288 times
11. 1.225″; $y=s\sqrt{2}$
12. Twice as long
13. 5.66″
14. 54°
15. 6″
16. 1½ revolutions
17. $k=\frac{D}{2}\sqrt{3}$; 2.70″
18. 50.3 ft.
19. 1.06″
20. 2263 ft. per min.
21. 3 in.
22. 13.13 in.
23. 17.34 in.
24. 23.6 in.
25. 110 ft.

Ex. 64:

1. 42.4 sq. in.
2. 22.2 sq. in.
3. 61.2 sq. in.
4. 288 sq. in.
5. 2 sq. in.
6. 396 sq. ft.
7. 36 sq. in.
8. 26.5 sq. in.
9. 896 sq. ft.
10. 176 sq. in.
11. 2400 sq. rd.; 15A.
12. 320 sq. ft.
13. 56 sq. in.
14. 17.8 sq. in.
15. 2.59 sq. in.

Ex. 65:

1. 3.14 sq. in.
2. 10.63 in.
3. 366.7 sq. in.
4. 245.6 sq. in.
5. 2:3; 4:9
6. 17.3%
7. 6.48 sq. in.
8. 22.2 sq. in.
9. 36.5%
10. 7.2 sq. in.
11. 3.4 sq. in.
12. 10.1 sq. in.
13. 484 sq. in.
14. 50.3 sq. in.
15. 21,384 lb.
16. 3 in.
17. 40%
18. 10 in.
19. 12.57 sq. in.; 11.78 sq. in.
20. 209.3 sq. in.

Ex. 66:

1. 72 lb.
2. 136 sq. ft.
3. 46.1 cu. ft.
4. 6.29 cu. in.; 1.95 lb.
5. 24.75 cu. in.
6. $27.20
7. 38.29 lb.
8. 450 sq. ft.
9. 12½ lb.
10. 37.4 gal.
11. 125
12. 11.7 cu. in.
13. 28.9 cu. in.
14. 156.8 cu. in.
15. 2282.8 ft.
16. 6.9 cu. yd.
17. 12 in.
18. 5,715 lb.
19. 6.48 in.
20. 6,594 sq. ft.
21. 29.2 lb.
22. 33,880 cu. yd.

Ex. 67:

1. 4.8 gal.
2. 14.5 cu. yd.
3. 3,014 cu. cm.
4. 33⅓%
5. 847.8 sq. in.
6. 242.5 cu. in.
7. 187.7 cu. ft.
8. 44.9 cu. yd.

9. 990 sq. in.   *10.* 7.07 cu. ft.   *11.* 87.5 sq. cm.   *12.* 268.7 cu. in.
13. 200.8 cu. in.   *14.* 1:4   *15.* 122,033 gal.   *16.* 3.6 cu. in.

Ex. 68:

1. 80 in.   *2.* 108 sq. ft.   *3.* BD=8.94; AB=12.00
4. AC=15; BC=20   *5.* 135 ft.   *6.* (a) 3.2″, 12.8″; (b) 16″
7. (a) 7.2″; (b) 9.6″; 5.4″   *8.* 12″ and 15″, respectively

Ex. 69:

1. Half; one-fourth   *2.* 16π, or 50.24″
3. Area, 4 times; volume, 8 times   *4.* 3:4; 9:16
5. 28.28 in.
6. Circumference, 10% less; area, 19% less
7. 10 in.   *8.* 9:16   *9.* 125%   *10.* 5.19 to 1
11. L.A.=401.9 sq. in.; V=927.9 cu. in.   *12.* 56.25% increase
13. T.A.=195.7 sq. in.; V=169.4 cu. in.   *14.* 12 sq. in.
15. 72.8%   *16.* 125%
17. Twice as large   *18.* 400 times larger
19. 4 times; 4 times   *20.* 125%

Ex. 70:

1. 26.3 in.   *2.* 18° 26′   *3.* 12.16; 80° 32′; 18° 58′
4. 176.3 ft.   *5.* 4.83   *6.* 36° 52′

Ex. 71:

1. 24.8 in.   *2.* 40.0″; 24° 43′; 130° 34′   *3.* 21.47″; 67° 58′
4. 3.39-inch square   *5.* 71° 47′; 36° 26′; 15.2 in.   *6.* 7.46″; 7.73″
7. 10.77″; 81° 18′   *8.* 3.696″   *9.* 4.33″
10. 12.8″; 15.8″   *11.* 10.77′; 27.58′   *12.* 59° 30′; 1.89″
13. .343″   *14.* 1.805″   *15.* 92° 48′

Ex. 72:

1. 167.7 ft. per min.   *2.* 212.13 mi. per hr. N.; same for E.
3. 79.5 lb.; 127.2 lb.   *4.* 459.6 lb.; 385.7 lb.
5. 456.75 lb.; 203.35 lb.   *6.* 45.3 ft. per sec.; 21.3 ft. per sec.
7. 1000 lb.; 36° 52′ with the horizontal   *8.* 176.78 lb.

Ex. 73:

1. 5.358″   *2.* 10.39″   *3.* 4.50″   *4.* 20.5″

Ex. 74:

1. 5° 12′   *2.* 26° 59′   *3.* 33°; .5924″
4. 33° 40′   *5.* 2.5584″   *6.* 25° 22′; 12° 50′

Ex. 75:

1. 6.96″   *2.* 3.33″   *3.* 5.5″   *4.* 1.46″
5. 4.402″   *6.* 2.55″; 3.56″.   *7.* .048″   *8.* 39° 58′
9. 1.06″   *10.* .96″   *11.* 73° 44′   *12.* .65″

Ex. 76:

1. $a=13.6''$; $b=16.4''$; $C=82^\circ\ 50'$
2. $a=31.4''$; $c=46.1''$; $B=28^\circ\ 50'$
3. $b=25.7''$; $c=28.5''$; $C=79^\circ\ 3'$
4. $b=11.79''$; $A=94^\circ\ 5'$; $C=38^\circ\ 35'$
5. $c=29.72''$; $A=28^\circ\ 25'$; $B=16^\circ\ 35'$

Ex. 77:

1. $c=30.4''$; $C=82^\circ\ 32'$; $B=31^\circ\ 28'$
2. $A=31^\circ\ 35'$; $B=38^\circ\ 57'$; $C=109^\circ\ 28'$
3. $a=4.3''$; $A=46^\circ\ 15'$; $C=23^\circ\ 45'$
4. $A=55^\circ\ 17'$; $B=39^\circ\ 37'$; $C=85^\circ\ 6'$
5. $b=21.5''$; $B=96^\circ\ 17'$; $C=56^\circ\ 13'$

Ex. 78:

1. 115.97 sq. in.
2. 430.98 sq. in.
3. 1175.6 sq. cm.
4. 4.26 in.
5. 403.1 sq. in.
6. $x=17^\circ\ 21'$; $y=107^\circ\ 21'$
7. 5.75 in.
8. 40.9 ft.
9. 1.53 in.
10. 82° 6′; 97° 54′

PART TWO

TECHNICAL SUBJECTS

V TRADE AND MECHANICAL DRAWING
VI APPLIED PHYSICS
VII STRENGTH OF MATERIALS
VIII PRACTICAL CHEMISTRY
IX MATERIALS OF TRADE AND INDUSTRY
X MACHINE ELEMENTS

CHAPTER V

TRADE AND MECHANICAL DRAWING

Sidney M. Sheldon

23. TOOLS AND EQUIPMENT; LETTERING

Introduction. In a very real sense, mechanical drawing is a universal language. By this we mean that it is a conventional sign language used by all skilled mechanics in their daily work. A properly executed mechanical drawing tells a complete story: it tells the precise size and shape of the object, with detailed dimensions; it gives the tolerances allowed; it indicates the materials of which it is to be made, and what finish the material is to have; it gives the number of the machine part, or other identification mark; and so on.

While it is not essential that a machinist or mechanic be a good draftsman, it is nevertheless very important that he be able to understand a blueprint and interpret a mechanical drawing. Hence it is extremely useful to master the basic elements of drafting; that is all that this chapter attempts to do for the reader. We shall therefore first become acquainted with the draftsman's common tools and equipment.

The Drawing Board. The drawing board is usually made of soft wood, such as basswood. It generally consists of several small widths jointed

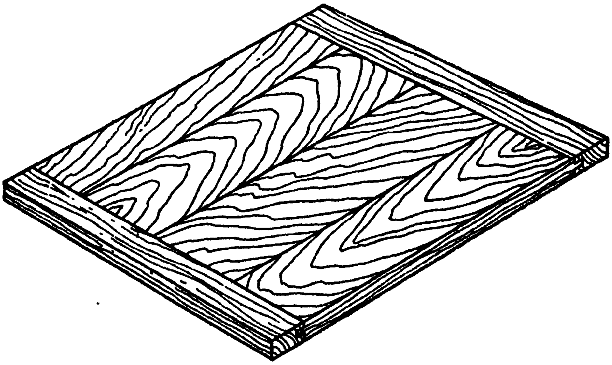


FIG. 1.—The Drawing Board

and glued together, the boards being placed with the grain running in opposite directions. The separate pieces are then fastened together by the side cleats, the edges of which are perfectly true.

The T-square. This instrument is so-called because of its shape. It is used only to draw horizontal lines, and, in conjunction with the triangles, in drawing both vertical and slanting lines. As may be seen from the diagram, the T-square consists of two parts, the blade and the head. The blade is fixed firmly on the head with five small screws and glue. When used, the position of the T-square is always as shown in Fig. 3, with the head always held firmly and flush against the edge of the drawing board.

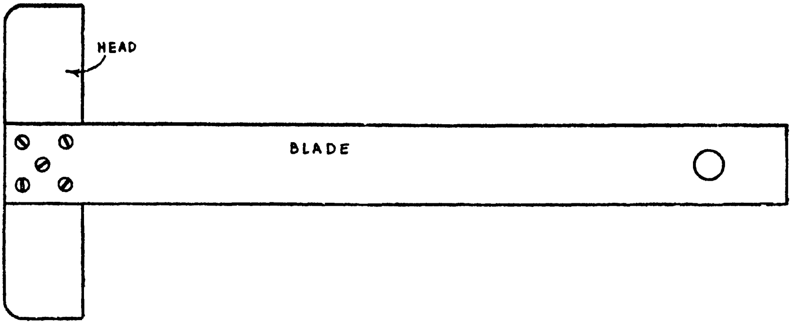


FIG. 2.—The T-square

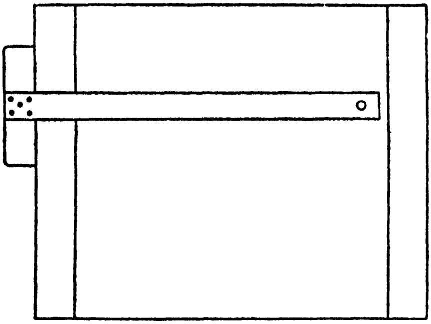


FIG. 3.—Position of T-square in Use

The Triangles. In mechanical drawing we use two different triangles. One is called the 45-degree triangle, because two of its angles are 45° each, while the third is 90°; the other triangle is called the 30–60-degree triangle because one angle is 30°, the second is 60°, and the third is 90°. These triangles are commonly made of wood, hard rubber, celluloid, or plastic material. Their edges must be absolutely straight and true. The

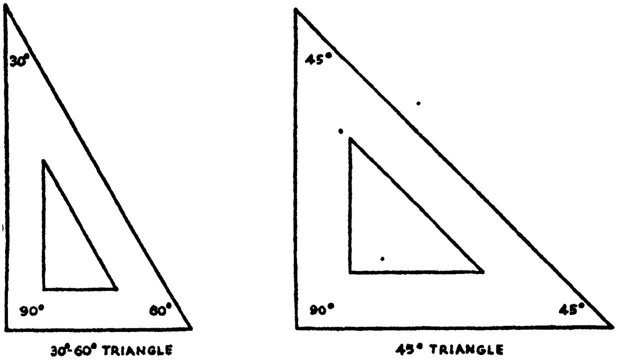


FIG. 4.—Triangles

triangles are used, together with the T-square, to draw vertical and slanted lines, as suggested in Fig. 5.

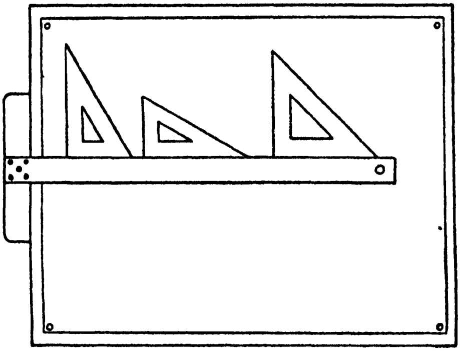


FIG. 5.—Position of Triangles in Use

The Compass. The compass is an instrument used in drawing circles, segments of arcs, or other circular figures. Fig. 6 shows a typical compass, consisting of two legs connected at the top by a yoke. The legs are movable, and may be adjusted so as to draw any desired radius within the limits of their size. One leg has a needle point at its end and the other leg has a pencil inserted at its end.

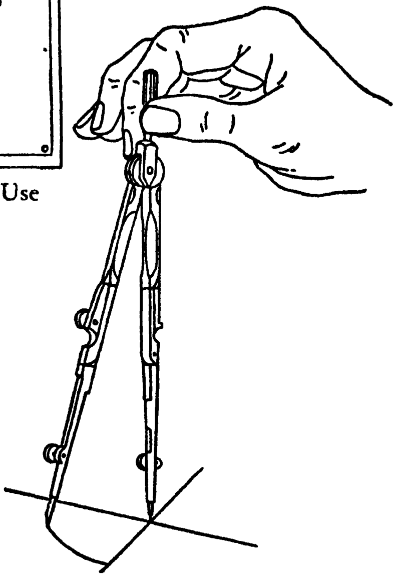


FIG. 6.—The Compass

The Scale. Scales are used in measuring distances on the drawing. The one most frequently used by the draftsman is the triangular scale. This instrument has ten different scales on its surfaces, plus a regulation 12″-rule. The scales are 3⁄32″, 3⁄16″, ⅛″, ¼″, ½″, 1″, ⅜″, ¾″, 1½″, and 3″. These various divisions on the scale enable the draftsman to make a drawing to scale, i.e., in the desired proportion to the actual size of the object. For example: in a full-size drawing, 12″ equals 1′–0″; in half-size drawings, 6″ equals 1′–0″; in quarter-size drawings, 3″ equals 1′–0″; in ⅛-size drawings, 1½″ equals 1′–0″; and in 1⁄16-size drawings, ¾″ equals 1′–0″.

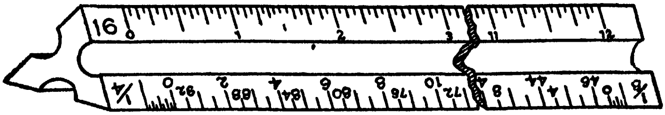


Fig. 7.—The Triangular Scale

French Curves. These irregularly-shaped curves are used for drawing curves that cannot be executed with a compass. In using such a curve a series of points is first located on the drawing; these points are then con-

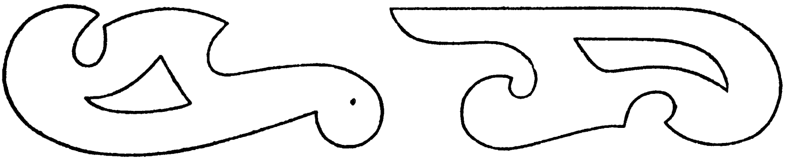


Fig. 8.—French Curves

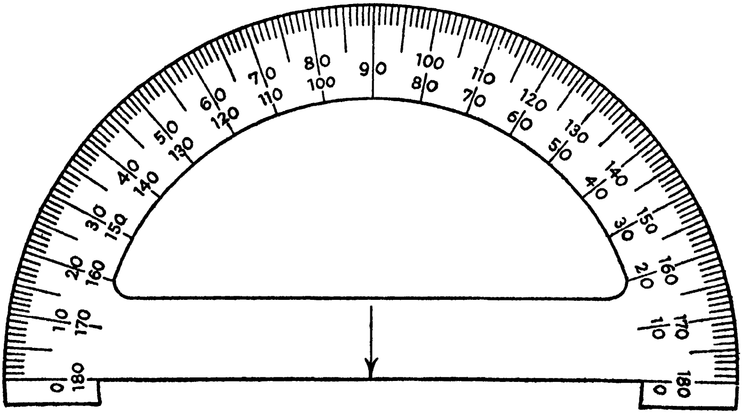


Fig. 9.—The Protractor

nected to each other by placing the French curve in an appropriate position so that the resultant curve will pass through all these points.

The Protractor. The protractor is a device used for measuring angles. Its circular edge is divided into 180 degrees, and each degree is further divided into tenths.

The Pencil. The pencils most commonly used by draftsmen are hard, preferably number "3H".

Lettering. Lettering plays a very important part in mechanical drawing. It is absolutely necessary that the information on the drawing be as clear and legible as possible. When one letters, patience and time are required. One cannot start by lettering quickly and expect to be neat about it. It requires considerable practice to be able to letter a job quickly and neatly. One must also remember that when lettering, guide lines are indispensable. No draftsman, no matter how expert, will ever letter a job without first drawing his guide lines in order to keep all the letters the same height. Another thing should also be remembered: *all lettering should be done free hand.* A good draftsman never uses a stencil when lettering.

Slanted Lettering. There are two types of lettering in common use. First there is the *slanted type,* which is a Gothic letter slanted at an angle of 75°. An easy way of learning how to slant your letters is to draw two

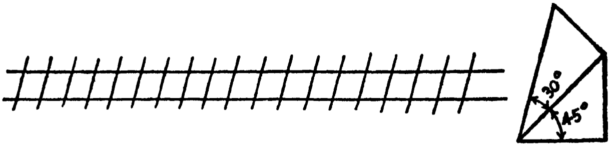


Fig. 10.—75°-Lines

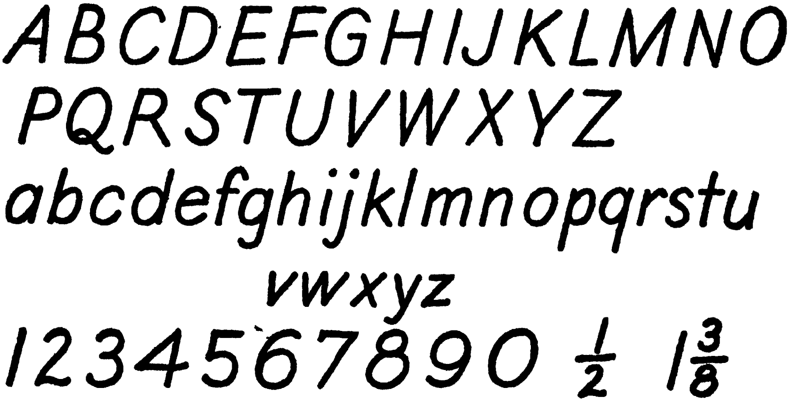


Fig. 11.—Slanted Alphabet

guide lines spaced horizontally; then draw light 75°-lines, a quarter of an inch apart, as indicated in Fig. 10. We are then ready for our lettering. We guide our strokes with the 75°-slanted lines and thus complete our alphabet.

There are two alphabets in the slanted Gothic: the ***upper case*** and the ***lower case,*** as shown in the following illustration:

The lower case, or smaller letter, is approximately two-thirds the height of the upper case, or capital letters.

Upright Gothic. The second type of lettering is the ***upright Gothic*** alphabet. The illustration shows the upper case (capital) and lower case (small) letters. These are lettered exactly as the slanted Gothic, but no 75°, or vertical guide lines are necessary in this case.

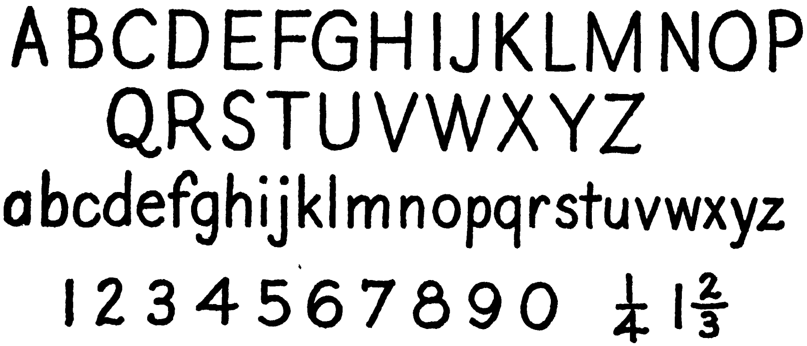


FIG. 12.—Gothic Alphabet

Some Important Rules. It is absolutely essential, for good lettering, to remember and obey the following rules:

1. Never crowd your letters together.
2. Always letter in between your two guide lines. The top and bottom of your letters should always touch the top and bottom guide line, respectively.
3. Unless you are an expert letterer, never hurry your lettering. Take your time and be patient, and your lettering will improve.
4. Never "mix" your letters, that is, never use upper case letters and lower case letters together, or in the same word, except, of course, when capitalizing:

Examples

DRaWinG (*wrong*)

Drawing or DRAWING (*correct*)

5. Never use fancy lettering on mechanical drawing jobs. The job always shows up to better advantage when only simple Gothic letters are used.

24. THE ALPHABET OF LINES; CONVENTIONS; REPRODUCTIONS

Types of Lines and Their Uses. As has already been mentioned, mechanical drawing is very much like a language. And just as other languages have an alphabet, mechanical drawing also has a unique alphabet of its own. It is called the "alphabet of lines." Thus, there are five separate and distinct types of lines used. Each is different from the other both in appearance and in use.

Visible Edges. The first line in our alphabet is the so-called *visible edge.* It is a solid line denoting any edge of an object which can be seen.

EXAMPLE: ————————————————

Thus, in the following drawing of a block, all the edges that are actually seen by the eye have been drawn as visible edges.

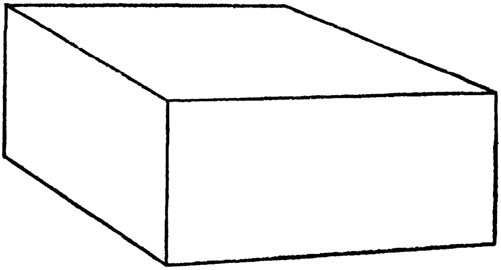


FIG. 13.—Visible Edges of a Block

Invisible Edges. The second line is the *invisible edge,* sometimes also called the hidden edge. This line indicates that while there is an edge on the object, it cannot however be seen by the eye; it is hidden by the material of the object. This line is a "dotted" line, really consisting of dashes; each dash is approximately ⅛″ long, and the space between dashes is 1⁄16″. This line is drawn just as darkly as the visible edge.

EXAMPLE: — — — — — — — — — — — — — —

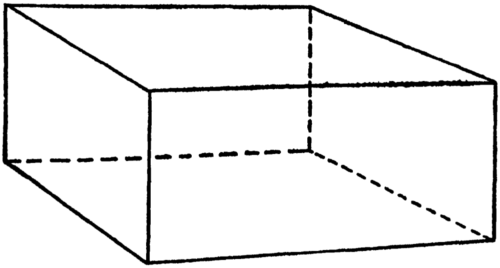


FIG. 14.—Visible and Invisible Edges

The dotted lines in Fig. 14 show the invisible lines or hidden edges; edges, in other words, that cannot be seen because they are hidden.

Projection Lines. The third line in our alphabet of lines is the *projection line*. In constructing a building or when painting a room, the bricklayer or painter erects a temporary scaffold upon which he stands to perform his work. When the work is completed this scaffold is removed; not being a part of the finished job, it is no longer necessary. Similarly, in mechanical drawing, we also use a "scaffold" with which we perform our work. That scaffold takes the form of a projection line; it is also called an extension or construction line. It should be noted that the first two lines described above were called edges, the visible, and the invisible or hidden edge. These edges are actually part of the object being drawn. The projection line and the next two lines to be discussed are not, however, actual parts of the object, but simply lines that aid in understanding the drawing of the object. The projection line is also a dash line, but consists of dashes ½ inch long and spaces 1/16 inch wide.

EXAMPLE: —— —— —— —— —— ——

This line is used to extend or project one view of an object from or to corresponding parts of another view of the same object. These lines are always drawn very lightly. Since they have no other value or use, we do not want to "see" them on our finished drawing, i.e., they should not stand out emphatically; Fig. 15 shows the use of the projection line..

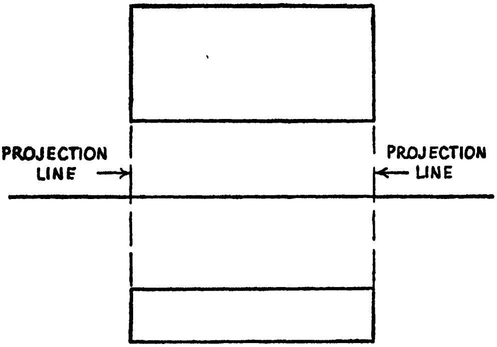


FIG. 15.—Projection Lines

The Center Line. This is the fourth of our five lines in the alphabet. This line is also a dash line, but is not drawn as lightly as the extension line. It consists of a ½-inch dash, followed by a 1/16-inch space, then a 1/16-inch dash, then a 1/16-inch space; this is then repeated, with a ½-inch dash, etc.

EXAMPLE: —— - —— - —— - —— - ——

This line is used for only one purpose, viz., to show the center or middle of an object which is circular in shape, or to show the center of circle.

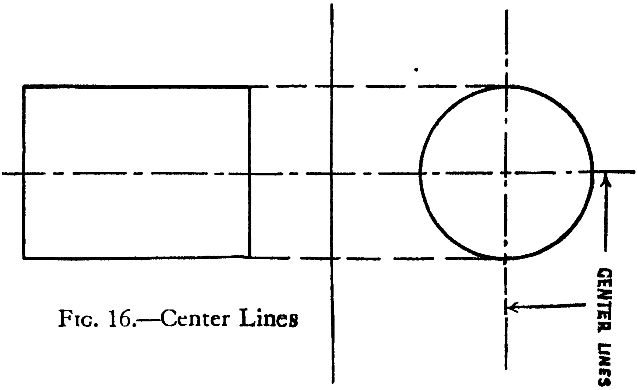


Fig. 16.—Center Lines

Dimension Lines. The last of our lines is the *dimension line*. This is also a light line, at the ends of which are drawn arrows, each about ⅛-inch long. These lines are used to indicate the actual size of the object being drawn. They are placed so·that the length of the object line and the length of the dimension.line are exactly alike. Thus in Fig. 17 we see a two-view drawing of a bearing base with all the necessary dimensions shown.

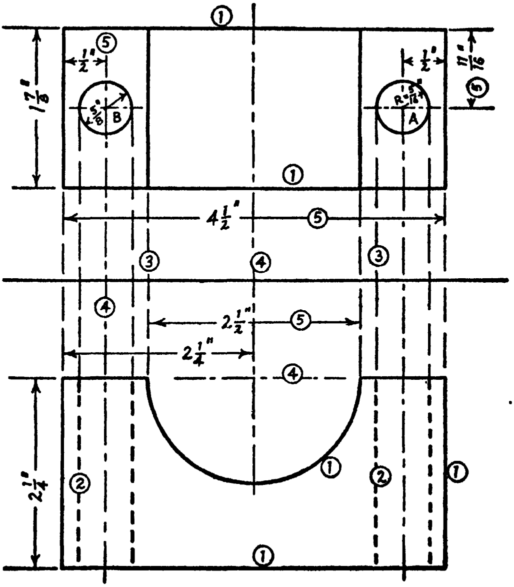
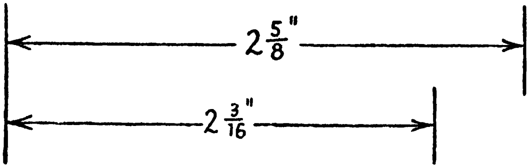


Fig. 17.—Bearing Base

Notice carefully how the radius of a circle is indicated (dimension A). The diameter (B) is indicated as a line going through the center of the circle with the arrow heads at both ends touching the circumference.

All dimensions should be drawn at least ¼ of an inch away from the object lines, and ¼ of an inch away from other dimension lines. Another important fact to remember is that all vertical dimension lines are read from the right side of the drawing. If the dimension is a fraction like ½ or 1¼, the fraction line should be drawn flush with the dimension line.

EXAMPLES:



While dimension lines are usually shown *outside* the drawing itself, sometimes it is desirable, in order to avoid confusion, to show the dimension line within the drawing; this is entirely permissible. It should also be noted that in the case of somewhat elaborate drawings, where the same dimension could be shown many times, it is generally considered good practice to indicate each dimension only once. When working with such drawings, therefore, care must be taken in reading the dimensions lest any of them escape our attention.

By way of review, let us turn back to Fig. 17, where all the lines of the alphabet of lines are illustrated; thus

lines (1) are visible edges,
lines (2) are invisible edges,
lines (3) are projection lines,
lines (4) are center lines,
lines (5) are dimension lines.

Sections. Sometimes the object to be drawn has several hollow parts, or the construction of the inside is somewhat complicated. It would be rather difficult and confusing to draw the various views of this object; there would be so many invisible lines and so many other lines crossing and recrossing one another so that the final drawing might in effect become a mass of meaningless lines. Under such conditions we simply imagine that a portion of the object has been cut away and then proceed to draw the object in that position. Such a device is known as a *cross-section*. The "exposed" surface of the material, where the object has been supposedly cut away, is then drawn with closely spaced lines, called *cross-hatching*. Fig. 18 shows a cross-section of a hand wheel.

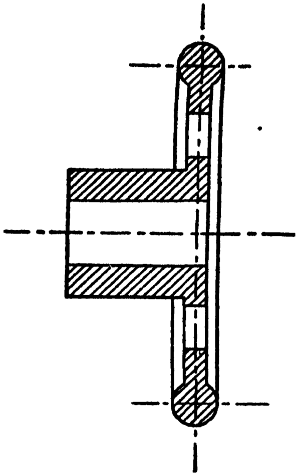


Fig. 18.—Cross-section of a Hand Wheel

Material Sections. In order to aid the mechanic in reading the blueprint, the draftsman employs a different type of cross-hatching for various metals or other materials. These conventional symbols are universally used; the following illustration shows the typical cross-hatchings used for some of the more important materials ordinarily encountered in mechanical drawings for shop work and in trade practice.

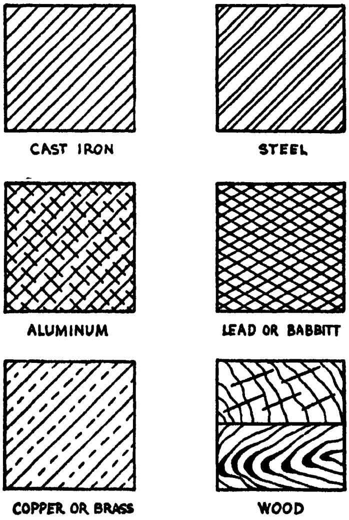


Fig. 19.—Cross-hatching for Materials

As a matter of fact, the methods of indicating materials have not only been highly standardized, but have also become quite elaborate. It is possible, for example, to represent on a drawing any one of a dozen or so different grades of steel. In modern machine drawing, section-lining to represent ferrous metals makes use of faint dull lines, while non-ferrous metals are represented by dotted lines. Such conventions are published by the American Society of Mechanical Engineers, and are often given in standard engineering handbooks or reference books in special fields.

Abbreviations. Certain abbreviations are frequently used on mechanical drawings for convenience and economy. Some of those most commonly used are given below for reference; the reader will do well to familiarize himself with them in order to facilitate working with blueprints and shop drawings.

A.S.A. = *American Standards Association*
B.C. = *bolt circle*
C.I. = *cast iron*
C.L. = *center line*
C to C = *center to center*
D = *diameter*
ff = *file finish*
N C = *National Coarse threads*
N F = *National Fine threads*
N.T.S. = *not to scale*
O.D. = *outside diameter*
Pat. = *pattern*
Pcs. = *pieces*
R = *radius*
Req. = *required*
Std. = *standard*
T. = *teeth*
W.I. = *wrought iron*
# = *number; or, pounds*

Reproductions. In the shop the mechanic usually works from a drawing called a *blueprint.* These blueprints are reproductions of the original drawings, which are made in pencil, and then traced in ink on tracing cloth or tracing paper. The blueprint is made on a sensitized paper; the tracing serves as a negative, and the reproduction is a sort of photograph.

The blueprint paper is coated with a chemical preparation that is affected by the action of light. When the sensitized paper is placed in a blueprint frame or machine, the tracing is placed upon the paper so that the drawing faces upwards and the sensitized paper lies under the tracing. A very strong light is then cast upon these papers. The light passes through the tracing and reacts with the chemical preparation on the sensitized paper, turning it blue. Where the light does not pass through, due to the opaque lines of the drawing, these portions turn white, since no chemical reaction has taken place. The sensitized paper, having been exposed in this manner for a period of time, is then placed in a fixing bath, where all excess chemicals are washed off. The next and final step is the drying of the prints. In modern practice, blueprint machines automatically complete the entire process of printing, washing and drying; the tracing and sensitized paper are fed in one end, and the finished, dry blueprint is the product at the other end.

Photostat prints are seldom used by mechanics. These prints are actual photographs, taken by a camera, of the original drawing. They may be reduced in size, or may be identical in size with the original. They may be had "white on black" (a negative), or "black on white" (a positive). Photostats are somewhat more expensive than blueprints. They are particularly useful, however, when it is necessary to reproduce plans or drawings which have been made on opaque material instead of a translucent tracing material.

25. TWO VIEW DRAWINGS

Purpose. The aim in mechanical drawing is to be able to place the shape and size of an object on paper in such manner as to enable the mechanic to read the print intelligently and accurately. If we show a picture of an object such as the simple block in Fig. 20, we would not be able to tell what the exact size and shape of the object is. Actually, this picture gives us a distorted view of the object. It is necessary to break it up and to show each view of the object as one sees it by looking directly at it.

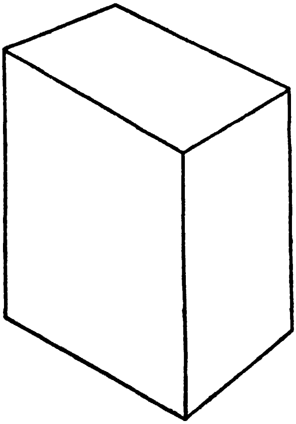


FIG. 20.

In order to do this we may assume that this simple block is suspended in a glass box (Fig. 21).

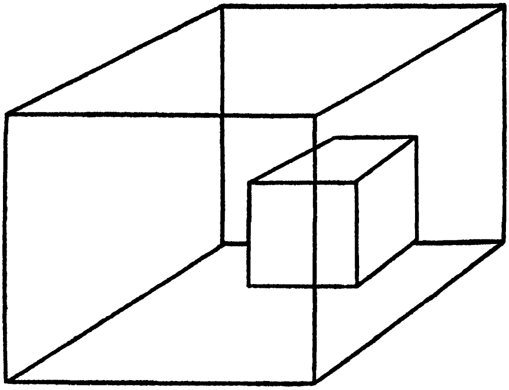


Fig. 21.

Looking down on the top of this box, we see the outline of the top view of the block (Fig. 22). In this figure we see the true length and the true width of the block.

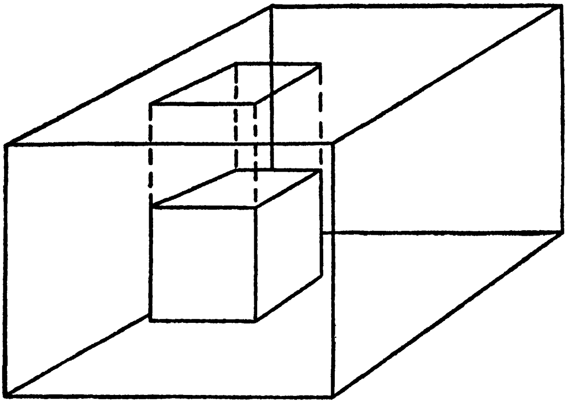


Fig. 22.

Fig. 23 shows us the front of the glass frame, and we see what the front view of the block looks like. The true length and the true height of the block can readily be seen.

Let us assume that the glass in the frame box can retain the pictures of the top and front views projected from the block. Opening the frame, we see the following, as shown in Fig. 24. The top view and front view of

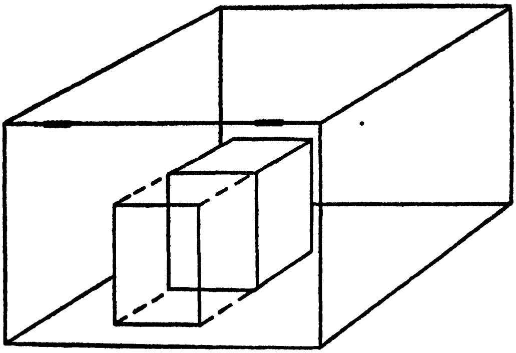


Fig. 23.

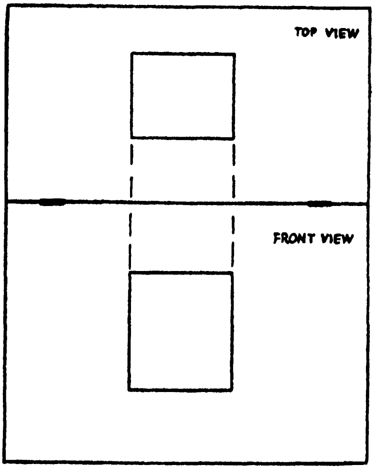


Fig. 24.

the block are clearly shown. Notice that the top view is directly over the front view, and that in the top view the length is exactly the same as the length in the front view. The front and top view are connected by means of projection lines. It is very important that the arrangement of the views is correct. Even though you may draw each view correctly, they must be placed together in the proper position; otherwise the drawing is incorrect and does not give the mechanic a true picture of the object. Always keep your top view and front view at least one inch from the horizontal axis line. When dealing with this type of drawing we must always draw the horizontal axis line between the two views. This line is an imaginary line, drawn visibly, which separates the top view from the front view.

The following plates and instructions will assist the reader in perfecting his drawing skill. In executing these plates it is essential to follow the instructions very carefully in order to get the full benefit of the problem.

Example: (Fig. 25).

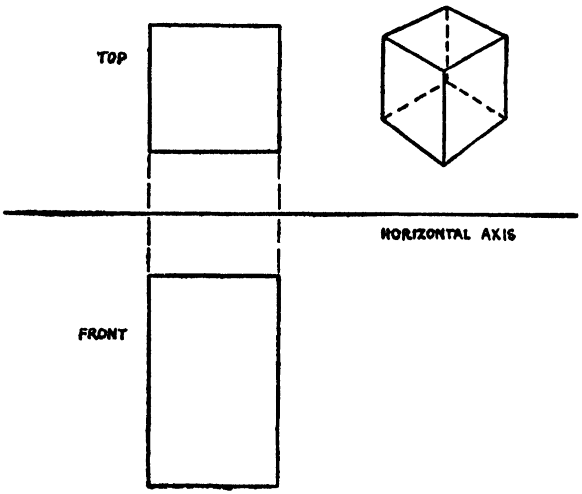


Fig. 25.—Square Prism

Your problem is to draw the top and front views of the square prism as shown in Fig. 25. Be sure that the projection lines are light. When making a drawing be careful that the views are placed on the paper in a neat and orderly manner. Never crowd your drawings.

Instructions.

1. Draw a half-inch margin on all four sides of your paper; then draw your title space one-half inch from the top margin line. Do your lettering in the title space.
2. Draw the horizontal axis line.
3. Draw the top view of the square prism 1½ inch by 1½ inch. (Note: Use the T-square for horizontal lines and the T-square plus the triangle for vertical lines.)
4. From the top view, draw light projection lines to the front view, which is 2½ inches high.
5. Complete the front view.

Example: (Fig. 26).

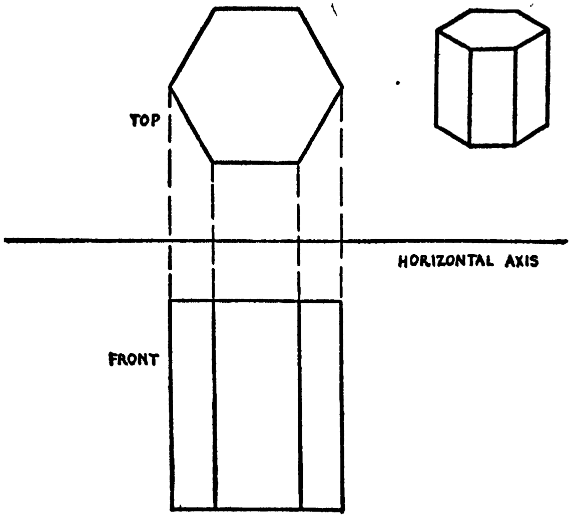


Fig. 26.—Hexagonal Prism

Instructions.

1. Draw the top view of the hexagonal prism whose sides are each 1½ inches long. These sides are drawn with the 60° triangle.

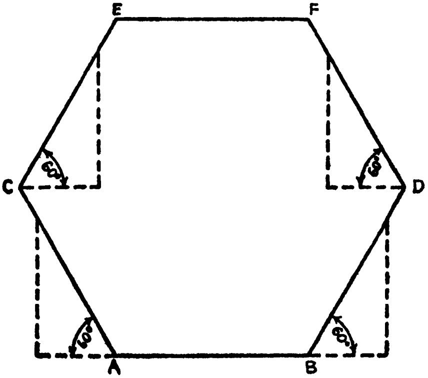


Fig. 27.

Note on Fig. 27. In order to draw the hexagon, we draw line AB equal to 1½ inches; then at points A and B, draw 60°-lines each 1½ inches long. At points C and D, draw 60°-lines each 1½ inches long; then complete by drawing line EF.

2. Draw projection lines down from the top view to the front view and complete the front view, which is 2½ inches high.

Example: (Fig. 28).

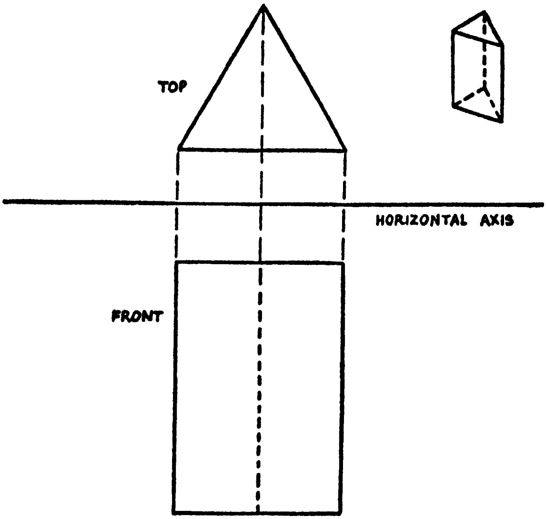


Fig. 28.—Triangular Prism

Instructions.

1. Draw the horizontal axis line.
2. Draw the base line of the triangle 2 inches long.
3. At each end of the base line draw 60° angles (see Fig. 27) and connect the resultant lines, completing the top view.
4. Project the base line to the front view, using light projection lines.
5. Draw the front view 3 inches high.
6. Draw an invisible line to show the rear edge, projected from the apex of the triangle in the top view.
7. Complete the drawing.

Example: (Fig. 29).

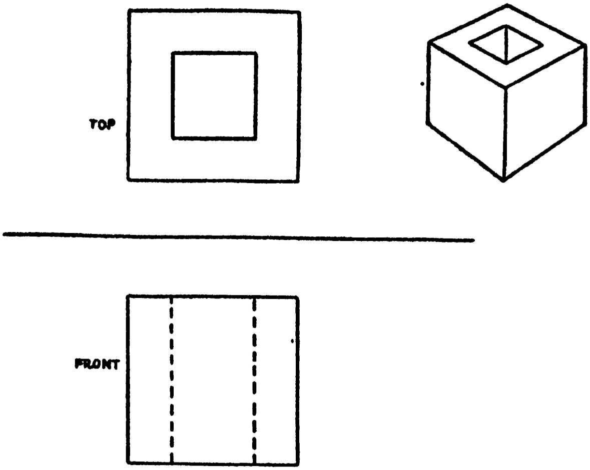
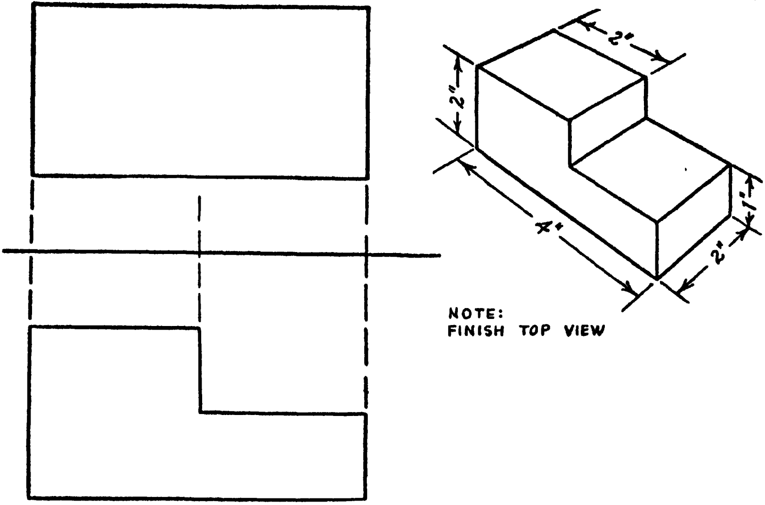


Fig. 29.—Hollow Cube

Instructions.

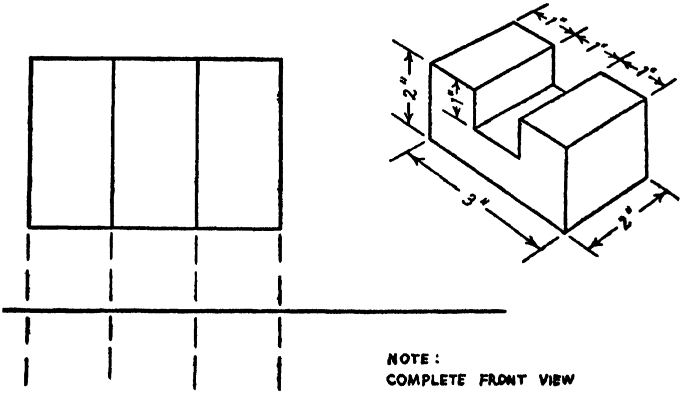
1. Draw the top view of the cube which measures 2″×2″×2″.
2. Draw the hole in the top view in the center of the top; this hole is 1″×1″.
3. Extend the length of the top view, including the length of the square hole to the front view.
4. Draw the outline of the front view; the height is 2 inches.
   Note: The top view shows the square outline of a 1″×1″ hole. When looking at the front view we cannot see the hole because of the thickness of the material around it. We say that the lines are "hidden," and show these edges as invisible on the front view.
5. Draw invisible lines to represent the length of the hole on the front view.
6. Note that the lines drawn from the top view to the beginning edge of the front view are light projection lines.
7. Complete the drawing.

In the next series of drawings, one view, or part of a view, has been omitted. The reader is to complete the entire drawing.



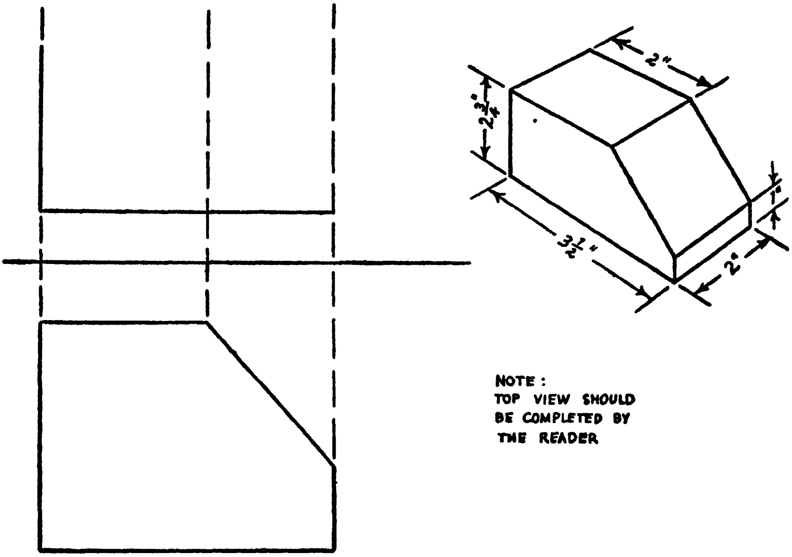
NOTE:
FINISH TOP VIEW

FIG. 30.



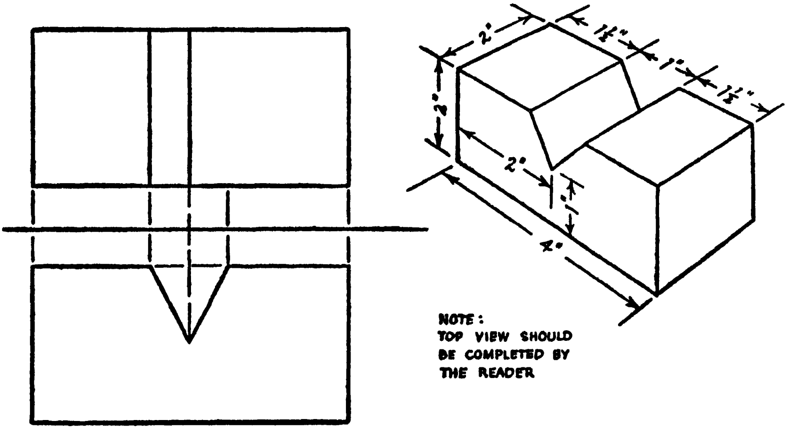
NOTE:
COMPLETE FRONT VIEW

FIG. 31.



NOTE:
TOP VIEW SHOULD
BE COMPLETED BY
THE READER

Fig. 32.



NOTE:
TOP VIEW SHOULD
BE COMPLETED BY
THE READER

Fig. 33.

Example: (Fig. 34).

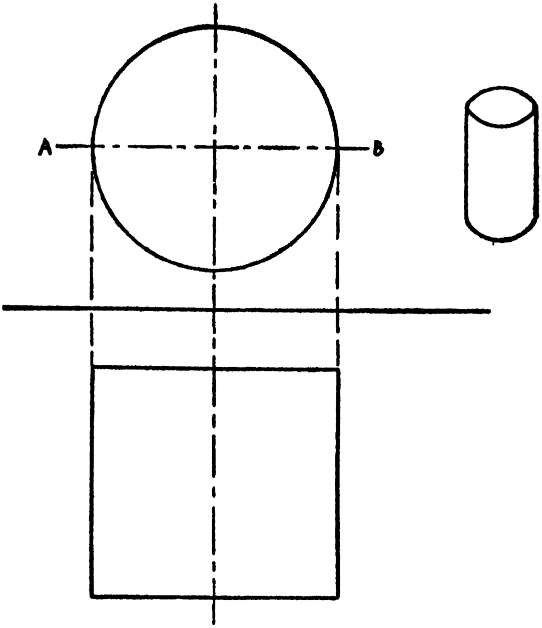


Fig. 34.—Cylinder

Instructions.

1. Draw the horizontal axis line, and then draw the vertical and horizontal center lines.
2. In the top view, place the point of the compass at the intersection of the two center lines, and with a radius of 1¼″, draw a circle.
3. Project points A and B of the circle to the front view.
4. With a height of 3 inches complete front view.

Example: (Fig. 35).

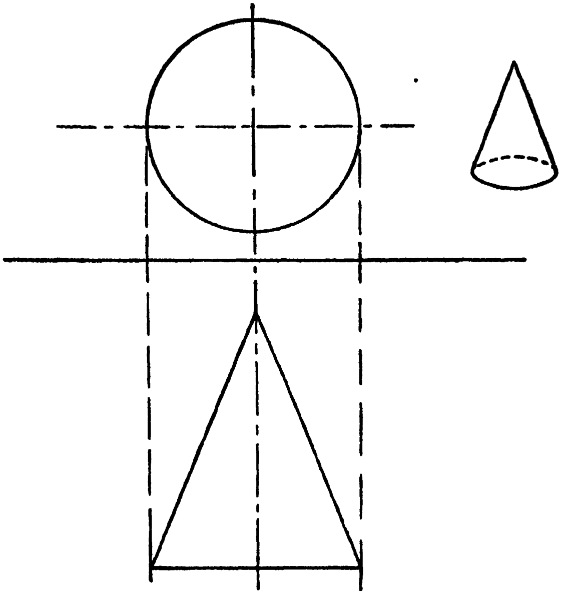


Fig. 35.—Cone

Instructions.

1. Draw the horizontal axis, and the vertical and horizontal center lines.
2. The radius equals 1¼″. Draw a circle in the same manner as in the previous problem.
3. Project the diameter to the front view.
4. With a height of 3 inches, complete the front view.

EXAMPLE: (Fig. 36).

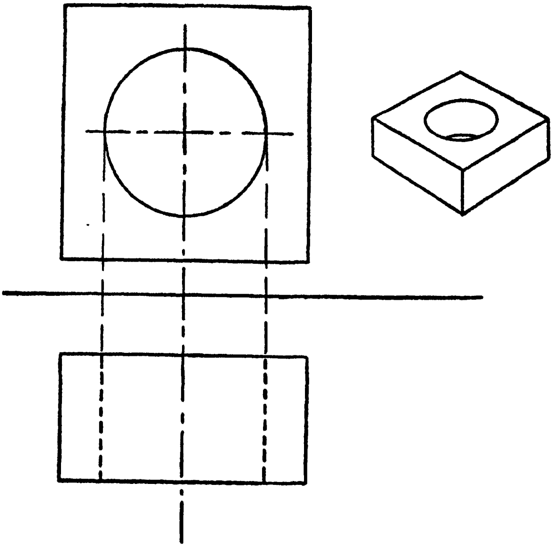


FIG. 36.—Hollow Block

Instructions.

1. Draw the horizontal axis and both vertical and horizontal center lines.
2. Draw a square 3″×3″ in the top view, so that the intersection of the center lines is in the dead center.
3. In the center of the top view, with radius of 1″, draw the top view of the circular hole.
4. Project all lines to the front view.
5. With height of 1½″, complete the front view.
   Note: Invisible lines indicate the hidden edges of the circular hole, running through the block.

Example: (Fig. 37).

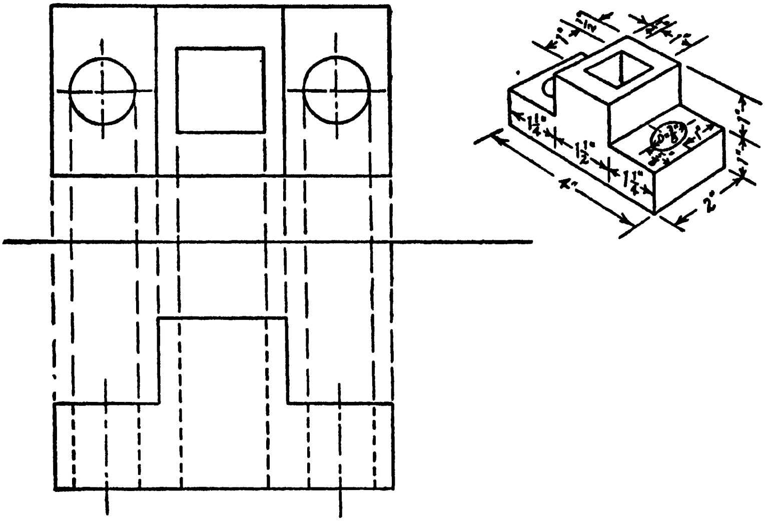


Fig. 37.—Rectangular Block

Instructions.

1. Draw the horizontal axis line.
2. Draw the top view of the bearing block, following all dimensions as shown in the drawing. Be sure to indicate all center lines.
3. Project all lines as shown to the front view.
4. Complete the front view, showing all invisible lines as indicated on the drawing.

Example: (Fig. 38).

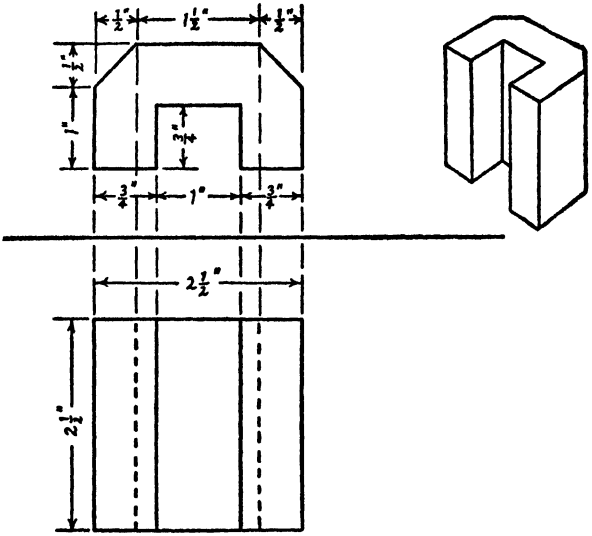


Fig. 38.—Channel Block

Instructions.

In Fig. 38, the problem is to draw the dimension lines which, up to now, have been omitted. All dimensions for laying out this drawing are indicated.

Notes:

1. All dimensions are to be drawn at least ¼" from any object line.
2. Arrows heads are to be drawn close to the line and about ⅛ inch long.
3. If there are more than two dimensions to be shown, keep them spaced at least ¼" apart.

26. THREE VIEW DRAWINGS

The work that we have done up to now has shown objects that only require two views to give us all necessary information. Sometimes it is necessary to show more than two views of an object in order to indicate its correct size and shape adequately.

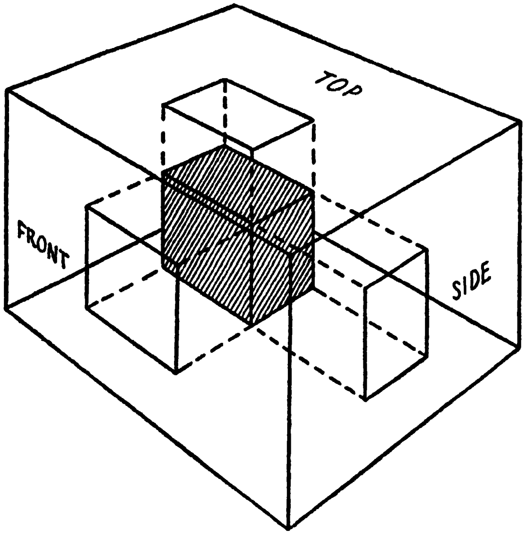


Fig. 39.

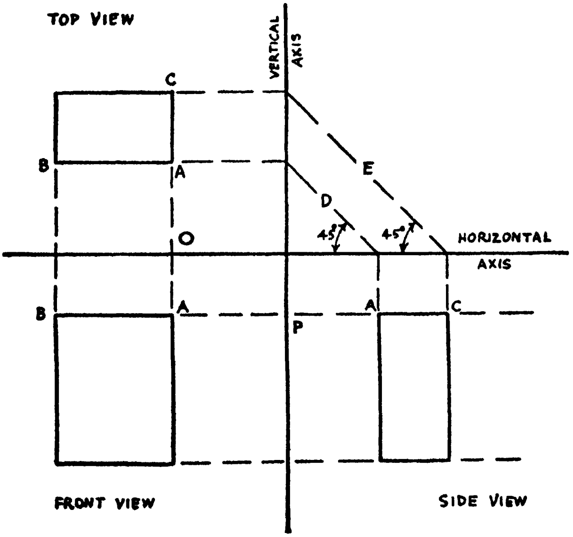


Fig. 40.

Let us go back to our glass box. We see, in Fig. 39, a rectangular cube resting in the glass frame. Notice very carefully how the three views, the front view, top view and side view, are projected to the sides of the glass frame. If we open this frame in the following manner, as indicated in Fig. 40, we will have a three-view drawing of the rectangular cube. Point A in our top view is exactly the same distance from point O on the horizontal axis, as point A of the side view is from point P on the vertical axis. Point C on the top view is exactly the same distance from Point O on the horizontal axis, as point C on the side view is from point P on the vertical axis. In fact, every point on the top view is at the same distance from the horizontal axis as its corresponding point on the side view is from the vertical axis. Lines E and D, which are projection lines of point A and C from the top view to the side view, are 45°-lines. The following instructions will aid considerably in making a drawing of three views.

Instructions. Let us assume that we have a block 2″ long, 1½″ high, and 1″ wide (Fig. 41).

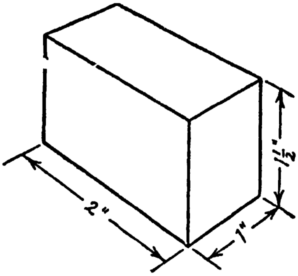


Fig. 41.

1. Draw the vertical axis line and the horizontal axis line at right angles (90°) to each other (Fig. 42).

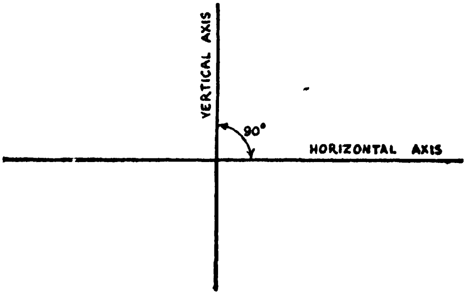


Fig. 42.

2. Draw the front view of the block (length×height) as shown in Fig. 43.

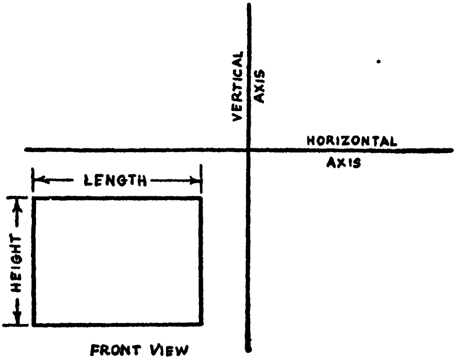


Fig. 43.

3. Extend the length by means of projection lines to the top view, and the height by means of projection lines to the side view (Fig. 44).

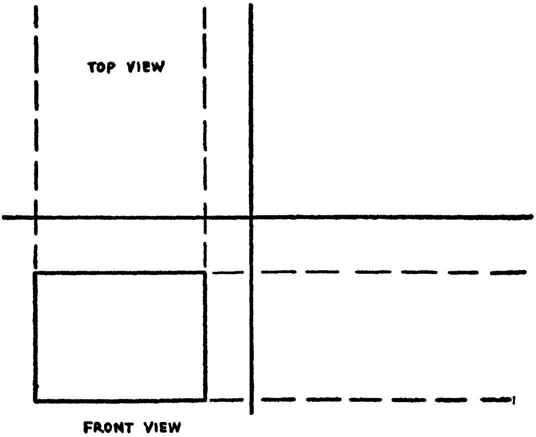


Fig. 44.

4. Draw the top view, and then project the width to the vertical axis line. Do not draw your projection lines beyond this vertical axis line (Fig. 45).

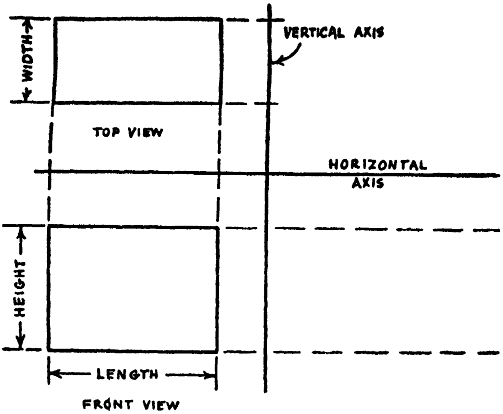


Fig. 45.

5. At points A and B, in Fig. 46, draw 45°-projection lines to the horizontal axis line.

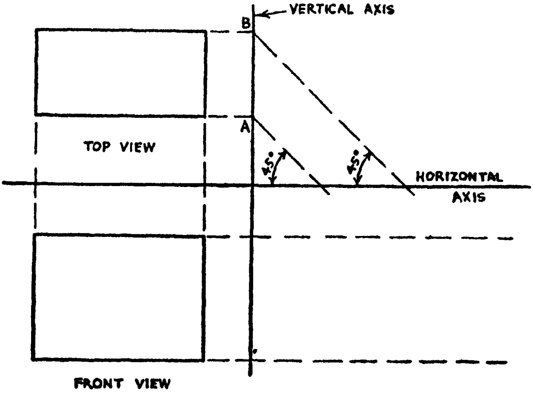


Fig. 46.

6. At points C and D in Fig. 47, draw vertical projection lines down the paper, crossing the horizontal projection lines coming from the front view. Notice that the side view is "taking shape" in between the vertical and horizontal projection lines.

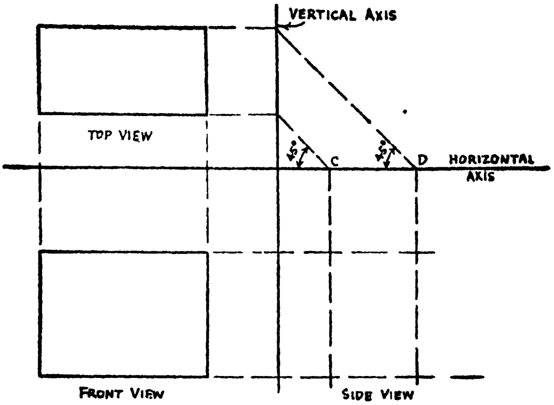


Fig. 47.

7. Complete the side view as shown in Fig. 48.

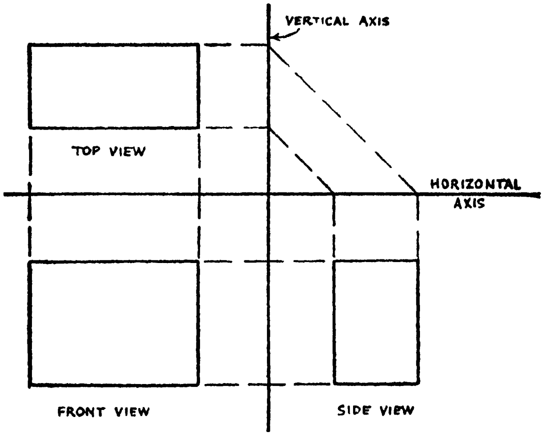


Fig. 48.

If the reader will follow carefully the simple method outlined above in the case of the rectangular block, he will find the job of making working drawings comparatively simple. The following rules should always be kept in mind:

1. Do not crowd the drawings on the paper.
2. Draw all lines lightly except the visible and invisible edges.
3. Be sure that you have drawn the vertical and horizontal axis lines before you start the actual drawing.

The rules for dimensioning three-view drawings are the same as those for the two-view drawings.

The following plates indicate how various typical problems are executed. Each plate has been completely worked out. The reader will gain considerable experience by conscientiously doing each of these plates himself, following all directions that have been set forth.

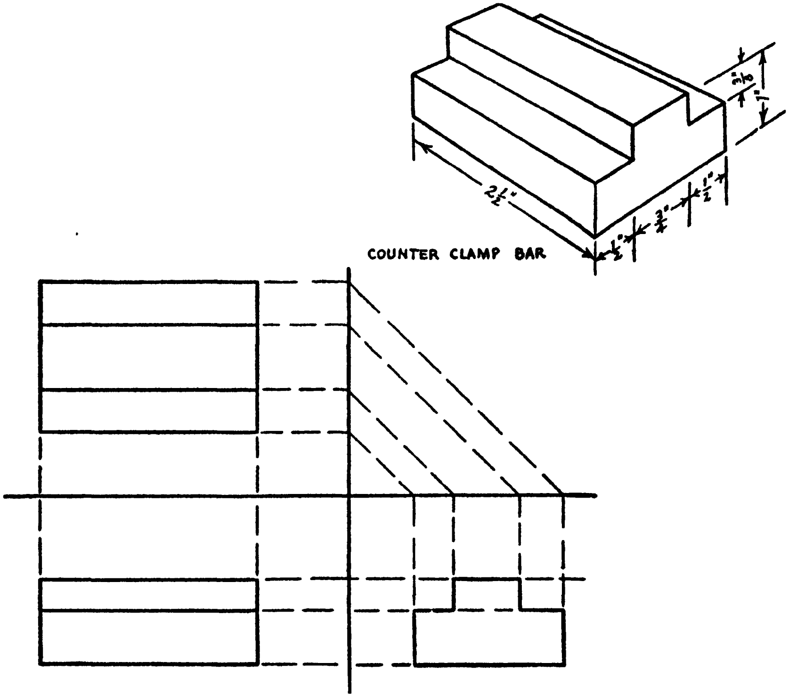
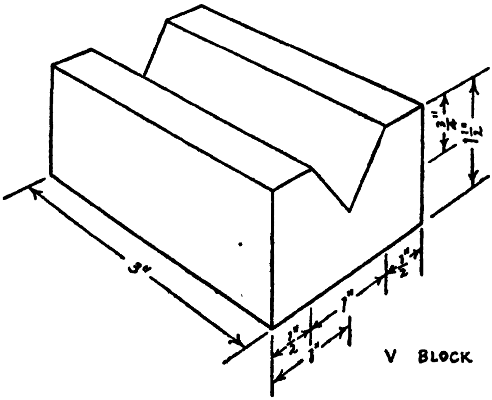


Fig. 49.—Counter Clamp Bar



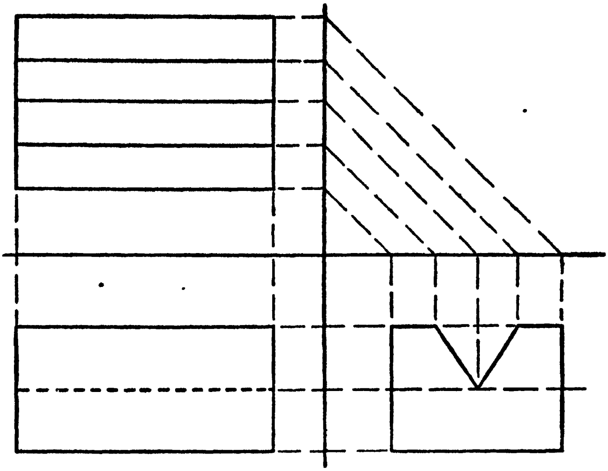


Fig. 50.—V-block

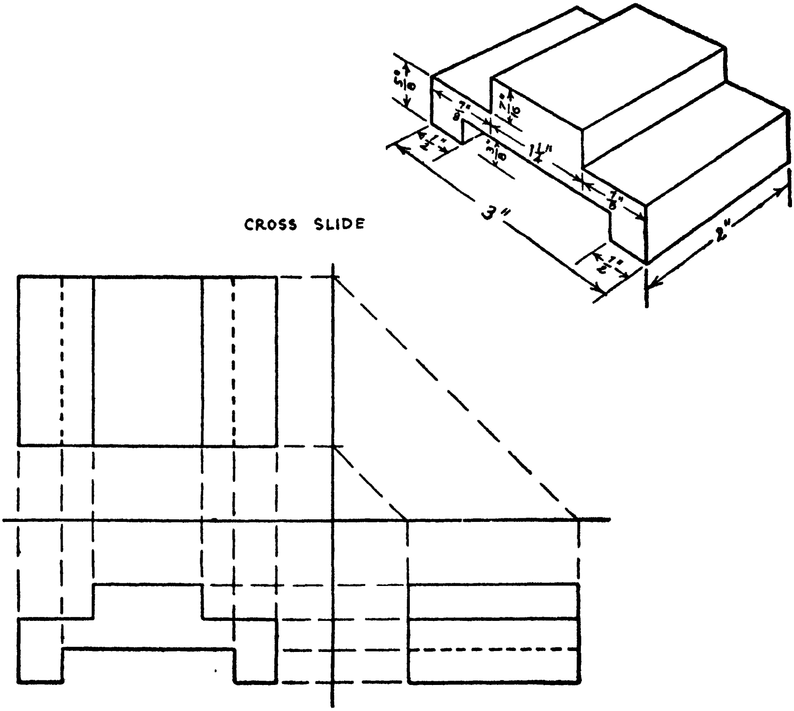


Fig. 51.—Cross Slide

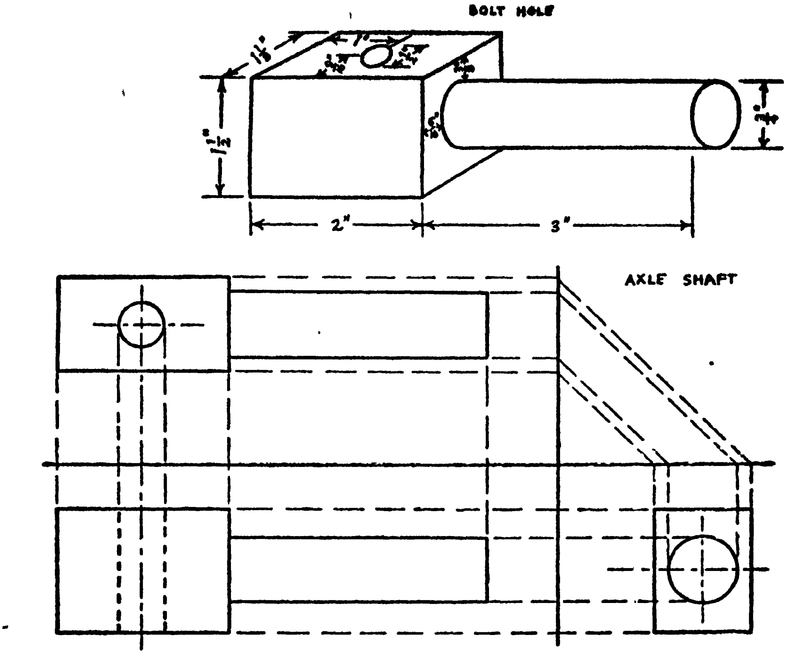


Fig. 52.—Axle Shaft

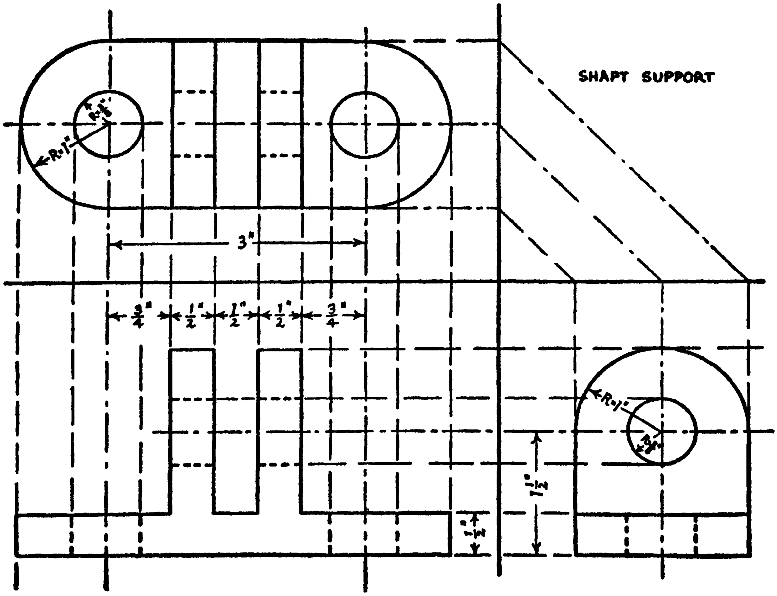


Fig. 53.—Shaft Support

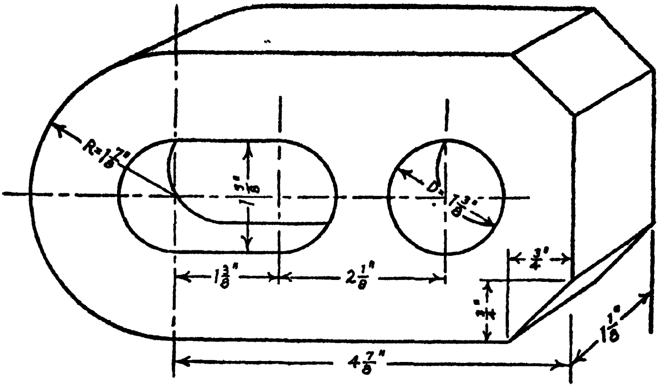


Fig. 54.—Backstop (Draw 3 views)

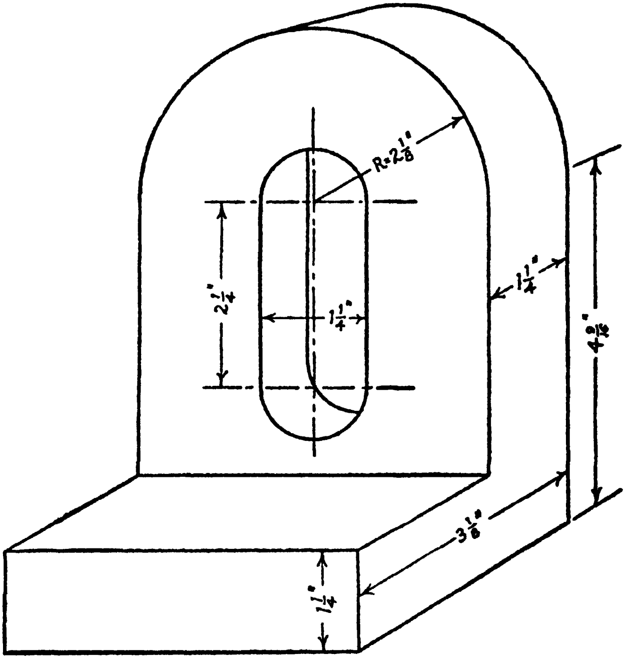


Fig. 55.—Idler Guide (Draw 3 views)

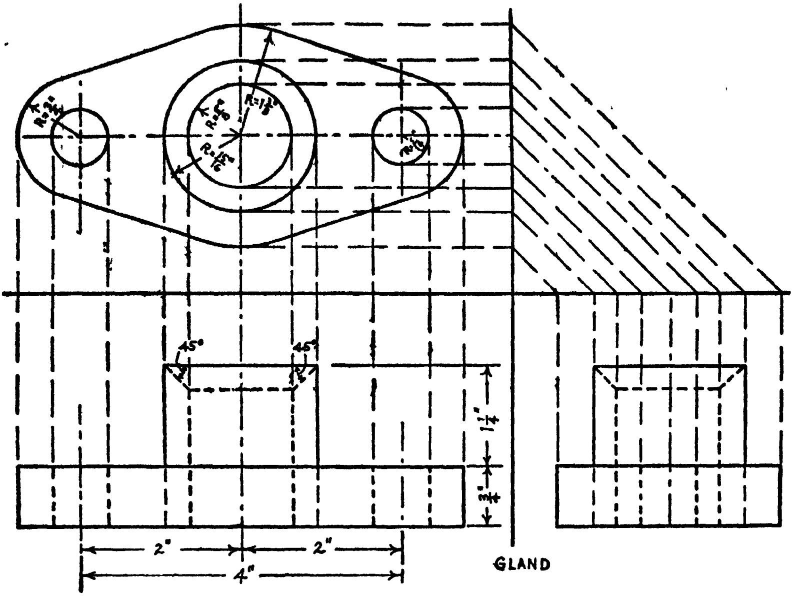


Fig. 56.—Gland

27. ISOMETRIC DRAWINGS

Purpose. In many instances, it is necessary to make an isometric drawing, i.e., one that shows three sides of an object simultaneously. All straight line dimensions in this type of a drawing are true.

This type of drawing seems to be distorted because we draw the *true lengths* of lines, even though they are receding at an angle. Isometrics are used by mechanics as shop drawings. They help a great deal in giving the mechanic a picture of what the finished object will look like. They are used also to show the relationship of one part of the object to another part of the same object. Isometric drawing is really a very simple task.

General Procedure. The following steps will aid the reader in doing this type of work:

1. All vertical edges of the object are drawn vertically on the drawing, and are parallel to each other.
2. All horizontal edges are drawn at a 30° angle, and are also parallel to each other.
3. All edges that are neither vertical nor horizontal, but at various angles, must be drawn by first locating their ends and then connecting these points.

4. Circles are drawn within isometric squares. The diameter of the circle is the size of the side of the square. This isometric square has sides drawn at an angle of 30° to the horizontal.
5. In isometric drawings, one corner of the object is always drawn in the foreground, and the two sides recede from that corner towards the background.

EXAMPLE: (Fig. 58).

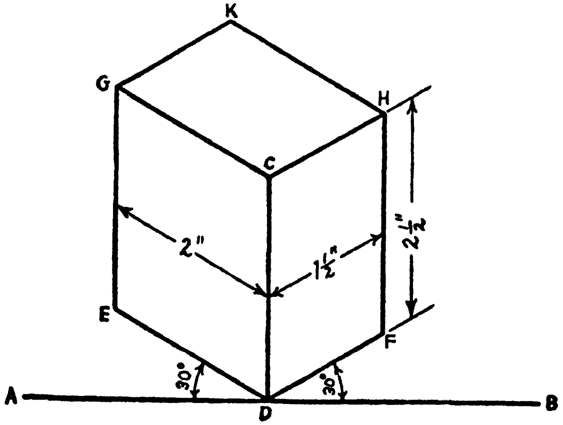


FIG. 58.

Instructions.

1. Draw the base line AB.
2. Draw the vertical line CD, which is the height of the block.
3. At point D draw the 30°-line DE (representing the length) and the 30°-line DF (representing the width).
4. Draw line CG parallel to ED, and CH parallel to DF.
5. Draw EG and HF as vertical lines parallel to line CD.
6. Complete by drawing line GK parallel to DF and line HK parallel to DE.

Example: (Fig. 59).

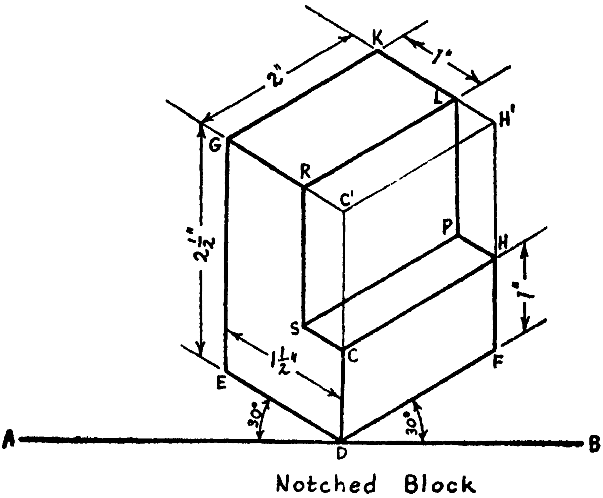


Fig. 59.—Notched Block

Instructions.

1. Draw lines DC', FH', EG, DF, C'H', GK, DE, C'G, H'K very lightly. Use the same method as in the previous example.
2. Locate point C, which is 1" above D, and draw line CH parallel to DF and line CS parallel to line ED.
3. Locate point R on line C'G, and draw line RS parallel to C'D.
4. At points R and S draw lines RL and SP parallel to G'H'.
5. At point L draw line LP parallel to C'D'.
6. Draw line PH parallel to DE and complete the drawing.

Example: (Fig. 60).

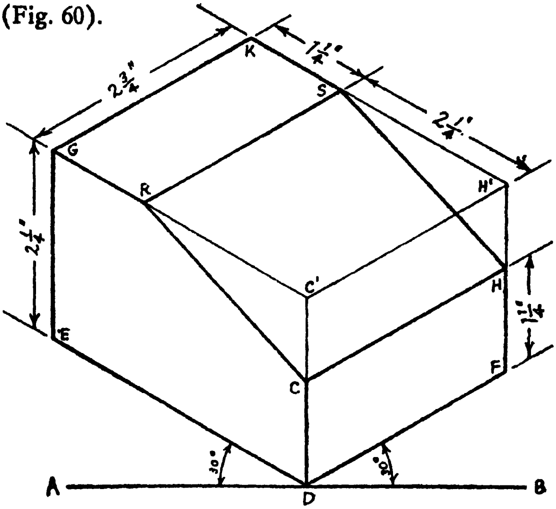
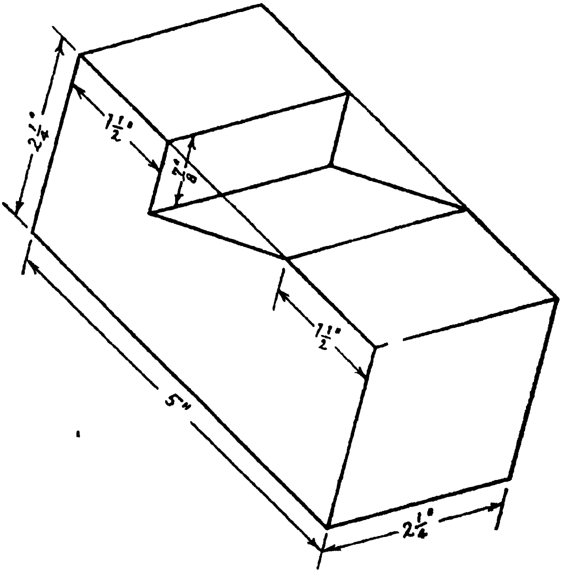


Fig. 60.

Instructions.

1. Draw the block as in previous examples.
2. Locate point R on line GC′ and point S on line KH′.
3. Draw lines RC and SH and complete the drawing.



The following group of isometric drawings is intended to help the reader perfect his drawing skill.

Fig. 61.

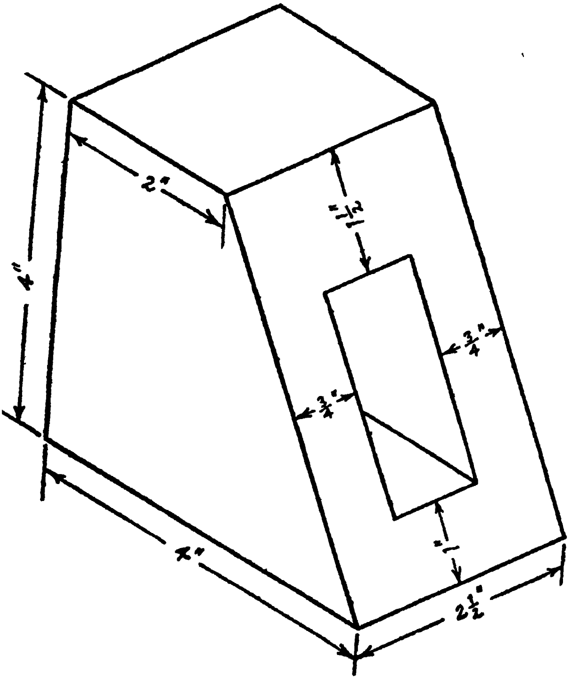


Fig. 62.

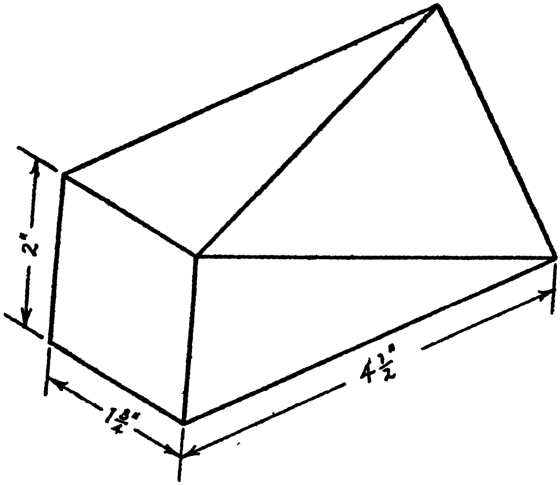


Fig. 63.

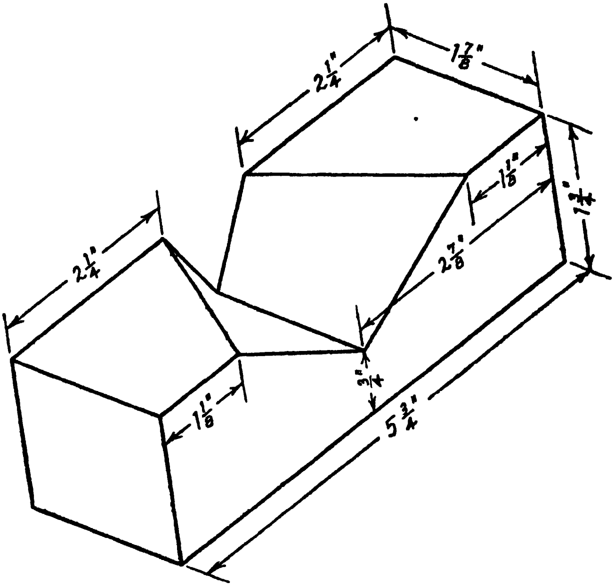


FIG. 64.

A further feature of isometric drawings is the drawing of circles; these take the form of ellipses.

EXAMPLE: (Fig. 65).

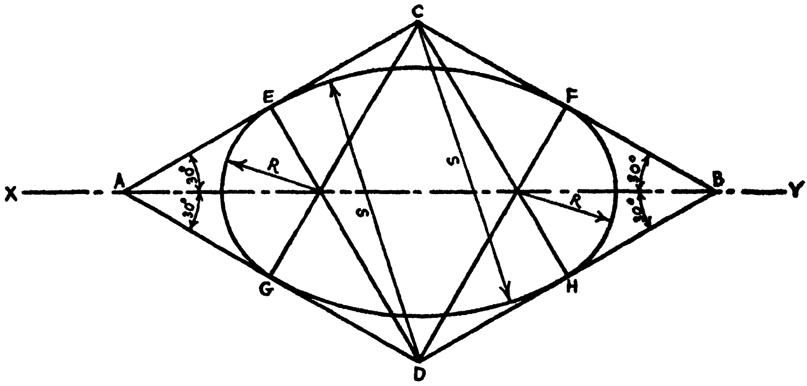


FIG. 65.

Instructions.

1. Draw the center line XY.
2. At point A, draw AC and AD at 30° to line AB.
3. On lines AC and AD lay off the desired diameter of the circle you are working with.
4. Using a compass, and with point C as center and radius equal to AC, mark off point B on line XY.
5. Draw lines BC and BD.
6. Locate the center points G, E, F and H, on their respective lines.
7. Connect lines DE, DF, CG and CH.
8. With R as radius, draw circular arcs EG and FH.
9. At points C and D, with CG as radius, draw curves EF and GH, thus completing the isometric circle.

SELECTED REFERENCES FOR FURTHER STUDY

BOGGY, H. E. Mechanical Drawing: Bk. 1, 2, & 3. McGraw-Hill Book Co.

BRAHDY, J. Blueprint Reading for Machine Shop and Related Trades. McGraw-Hill Book Co.

CROOK, T. C. Simplified Mechanical Drawing. McGraw-Hill Book Co.

FRENCH, T. E. and SVENSEN, C. L. Mechanical Drawing. McGraw-Hill Book Co.

GIESECKE, F. E., MITCHELL, A. S. and SPENCER, H. C. Technical Drawing Problems Macmillan Co.

HILLS, J. H. Pictorial Drafting. McGraw-Hill Book Co.

HOELSCHER, R. P. and MAYS, A. B. Basic Units in Mechanical Drawing (2 Vol.). J. Wiley & Sons.

McGEE, R. A. and STURTEVANT, W. W. General Mechanical Drawing. Bruce Publishing Co.

MICHELSON, H. W. and BUCK, R. O. Sketching for the Draftsman. Bruce Publishing Co.

PAULL, V. H. and SGRO, C. L. Applied Mechanical Drawing. D. Van Nostrand Co.

SPOONER, H. J. Machine Design, Construction and Drawing. Longmans, Green & Co.

THAYER, H. R. Blueprint Reading and Sketching. McGraw-Hill Book Co.

TOWNSEND, E. E. and CLEARY, S. F. Introductory Mechanical Drawing. J. Wiley & Sons.

WEIR, J. J. Blueprint Reading for Machine Trades. McGraw-Hill Book Co.

WOOLLEY, J. W. and MEREDITH, R. B. Shop Sketching. McGraw-Hill Book Co.

CHAPTER VI

APPLIED PHYSICS

Israel N. Liphshitz

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28. SOLIDS, LIQUIDS AND GASES

Introduction. Physics is the science of matter and energy and of their relations to each other. To a great extent it is that science which describes *accurately* and *quantitatively* many important phenomena of our everyday life. Its laws, its discoveries, its observations and experimental results form the bases of our mechanical processes and our industrial activities. Like most of the exact sciences, physics is not content merely with a descriptive or qualitative investigation of its field, but must have accurate measurement and quantitative exactness to enable us to study relationships and deduce physical laws. The physicist, therefore, requires definite standards of measurement, fixed units, and accurate measuring instruments. What is more, physical terms must be measurable; they must be defined in terms which can be measured or be studied.

Matter and energy have certain commonplace meanings in daily life. In physics, however, they are clearly defined quantities. Matter, or the material substance of a body, is defined in terms of its mass, a definitely measurable quantity. Energy, the ability to do work, is measured in the same physical terms in which work is measured. This method of definition we will find holds true for all physical entities.

General Properties of Matter. Every substance has a definite set of characteristics by which we are able to identify it, or distinguish it from other substances. Steel is hard, glass is brittle, wool is soft, and so on. Hammer a nail, and we are able to drive it through a wooden plank; hammer a glass rod, and it shatters. There are certain properties common to all forms of matter. Among these general properties are: extension, impenetrability, inertia, and elasticity.

All bodies extend in space, take up a certain volume. They have three dimensions—length, width and height; this we call the property of *extension.* By *impenetrability* we simply mean that two bodies cannot both occupy the same space at the same time. *Inertia* is the property of a body to resist changes in its state or condition. A body at rest, as Newton's first law of motion tells us, tends to remain at rest, and a body in uniform motion tends to continue in motion with uniform speed in the same direction, and resists any attempt to change either its state of rest or of motion. As a result, if a train or an automobile suddenly stops, the people in it invariably tend to fall forward. By *elasticity* we mean the ability of a body to return to its original shape after a force which causes it to change its shape has been removed. Release the force which stretches a steel spring, and the spring returns to its original shape and size. If a body is stressed beyond its *elastic limit,* the deformation will become permanent.

Specific Properties. Properties other than those mentioned above are characteristic only of certain substances and are not present in others. These are called *specific* or special properties. They include: hardness, ductility, malleability, and tenacity.

Hardness is the resistance which a body offers to abrasion, to being scratched by another substance. The diamond and carborundum are among the hardest substances known. Certain substances can be hammered, beaten or rolled into thin sheets. These are said to be malleable substances and possess the property of *malleability. Ductility* is the property of a substance which allows it to be elongated considerably before it will actually break. Thus copper, platinum, glass, iron are ductile substances and can be drawn into very fine wires. *Tenacity* is the resistance which a substance offers to being pulled apart. Some substances have higher tensile strengths than others, depending on their composition and molecular structure.

Three States of Matter. The three states of matter are known as the solid, liquid, and gaseous states. The *solid* state is characterized by a definite shape and volume. Any effort to change these is resisted by the material. A copper disc, a bar of steel stock, a lump of coal, a gear, are examples of the solid state. A *liquid* will take the shape of whatever vessel it is poured into. Water, oil, alcohol, gasoline in different containers are examples. A liquid, then, has a definite volume, but no definite shape. A *gas* has neither a definite shape nor a definite volume. We can pump large volumes of air into an automobile tire. We store large quantities of oxygen, hydrogen, or acetylene in tanks of varying sizes. Both the shape and volume of a gas depend upon and vary with the containing vessel. The term *fluid* is often used in the physical sciences to include both liquids and gases.

Units of Measurement. The systems of units in common use for physical measurements are the *English* and the *metric* systems. The English system is the one commonly used in this country for most industrial and applied scientific work, while the metric system is used abroad and in scientific work here. Since the fundamental physical quantities are those of length, mass, and time, the metric system is commonly called the C.G.S. or centimeter-gram-second system, while the English system is called the F.P.S. or foot-pound-second system.

The following are the units used in each of the systems for fundamental quantities:

Quantity	*English* (F.P.S.) *System*	*Metric* (C.G.S.) *System*
Length	foot (ft.)	centimeter (cm.)
Mass	pound (lb.)	gram (gm.)
Time	second (sec.)	second (sec.)

For purposes of conversion from either system to the other it is well to be familiar with the following equivalents:

1 inch = 2.54 centimeters
1 meter = 39.37 inches
1 pound = 453.59 grams
1 kilogram = 1000 grams = 2.2 pounds

Units of Area and Volume. The *unit of area* extensively used is that of the area of a square whose side is of unit length. Thus in the English system the unit is the square foot (sq. ft.), while in the metric system it is the square centimeter (sq. cm.).

1 sq. ft. = 144 square inches (sq. in.)
1 sq. in. = 6.45 sq. cm.

Likewise, the *unit of volume* or capacity is the volume of a cube whose edge is of unit length. In the English system the unit of volume is the cubic foot (cu. ft.), or the volume of a cube one foot on each side. In the metric system it is the cubic centimeter (cc.), or the volume of a cube one centimeter on each side. Since this is a very small unit, we often use the *liter* (l.), which is equal to 1000 cc. This is the volume of a cube 10 centimeters on each edge.

1 cu. ft. = 1728 cubic inches (cu. in.)
1 gallon (gal.) = 4 quarts (qt.) = 231 cu. in.
1 liter (l.) = 1000 cc.
1 liter = 1.06 quarts

Density. By *density* is meant the mass of a substance per unit volume. In the English system this would be expressed as pounds per cubic foot; in the metric system as grams per cubic centimeter. The density of a substance can always be found from the following formula:

$$Density = \frac{Mass}{Volume}, \text{ or, } D = \frac{M}{V}.$$

Thus, for example, if we have a steel casting whose volume is 2 cubic feet and which weighs 980 pounds, its density can be determined as follows:

$$M = 980 \text{ lbs.}$$
$$V = 2 \text{ cu. ft.}$$
$$D = \frac{980}{2} = 490 \text{ lbs. per cu. ft.}$$

The density is an indication of the comparative weights of different substances. Blocks of lead, copper, steel, wood, and cork of exactly the same size will differ greatly in their weights. Lead, the heaviest of these materials, is said to have the greatest density of this group; cork, the lowest.

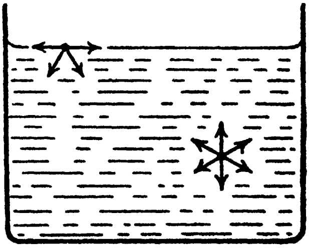
Mass and Weight. There is often some confusion in the use of the terms matter, mass and weight. By *matter* we mean anything which occupies space and has mass. It commonly refers to the material of which all bodies are composed. The *mass* of a body is the amount of matter which it contains. It is an answer to the question: how much matter is there in the body? As already mentioned, in the English system, mass is measured in pounds, and in the metric system it is expressed in grams or kilograms. By *weight,* however, we mean the pull which the earth exerts on a body of matter because of gravity. The weight of a body is thus *not* a description of what it contains, but simply a measure of *the force of gravity acting on the body.*

At sea level and at latitude 45°, which are taken as standard conditions, the mass and the standard weight of a body are numerically equal. A pound mass would weigh a pound. However, if we carry this mass to the top of a mountain where the gravitational pull on the body is less than at sea level, we will find that its weight as measured on a spring balance is less than it is at sea level. The weight as determined by the spring balance is called its "local weight." If instead of a spring balance, however, we used an equal-arm balance and a set of standard weights, we could get the standard weight of any object regardless of where we weighed it. For at the top of the mountain all bodies have the same relative decrease in gravitational pull, and in balancing the weights against the object itself we would get its standard weight. This, of course, is the method we regularly use to determine the weight of a body. When we speak of the weight of a body, we commonly mean its standard weight.

Cohesion and Adhesion. All matter is made up of very small particles called *molecules.* There are strong forces of attraction holding the molecules of a body together. It requires quite some effort to separate two clean strips of lead which have been firmly pressed together. This force of attraction which molecules of the same substance have for each other is called *cohesion.* There is also another force of attraction between molecules of different substances. Insert a spoon in water, and upon withdrawal it will be wet; a steel or wooden rod immersed in oil will emerge with a coating of oil on it; glue or cement will cause two surfaces to be held firmly together. This force of attraction between molecules of *different* substances is called *adhesion.* The one substance is said to *adhere* to the other; thus paint adheres to a wall, and cement adheres to stone.

Surface Tension. A raindrop falling through the air is very nearly spherical in shape. Molten lead allowed to fall through a sieve into a pool of water will form perfectly round little balls. In fact, this is one method of making lead shot. Soap bubbles are spherical in shape. What accounts for these phenomena? The surface of a liquid behaves very much like a stretched elastic membrane. The free surface of the liquid tends to contract and become as small as possible. This tendency is known as *surface tension.* It is surface tension, for instance, that accounts for the fact that a needle or a razor blade will float on the surface of water, though we know that the metal is much denser than water.

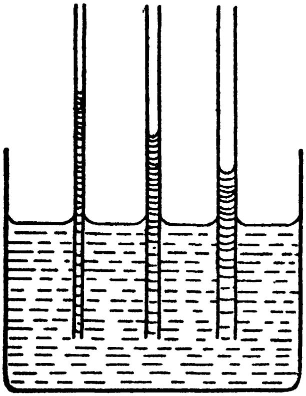
How can surface tension be explained? The molecules within a liquid are attracted equally in *all* directions by the other molecules surrounding it. The molecules on the surface, however, are attracted by the neighboring molecules only downward and sidewise, *but not upward.* Thus the surface molecules are held tightly together and are drawn toward the body of the liquid by strong cohesive forces, creating a tense surface film which acts very much like an elastic membrane.



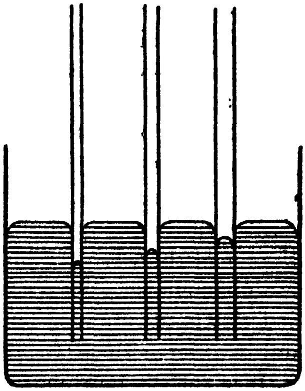
Attraction of Molecules in a Liquid

Capillary Action. When one end of a sponge is placed in water, the water rises into the entire sponge. Likewise, ink will rise into blotting paper, tea or coffee will rise into a lump of sugar held at its surface, kerosene will rise in a lampwick, and so on. These phenomena are referred to as capillary action. *Capillary action* refers to the behavior of liquids in tubes of very small diameter. In fact, the word capillary is derived from the Latin *capillus,* meaning hair, and was therefore applied to these extremely narrow tubes of almost hairlike dimensions.

When several capillary tubes of different diameters are placed into water, the water will rise into the tubes above the water level in the vessel, but it will not be at the same height in each case. The water will be highest in the tube having the smallest diameter, and lowest in the tube having the largest diameter. The top surface of the water in the tube will be concave; that is, it will turn up slightly where it is in contact with the glass.



Water Rises in Capillary Tubes



Mercury Is Depressed in Capillary Tubes

If, however, these same capillary tubes, thoroughly dry, are placed into mercury, the mercury will be depressed below the level of the mercury in the vessel, and the depression will be greatest in the smallest tube. The upper surface of the mercury will be convex, that is, turned down wherever it is in contact with the glass. The different behavior in water and mercury is due to the fact that the water wets the capillary tubes, while the mercury does not wet the capillary tubes. Experiment has shown that the following laws of capillary action hold true.

Laws of Capillary Action. 1. Liquids rise in capillary tubes which they wet and are depressed in tubes which they do not wet. In the former case, the surface of the liquid will be concave; in the latter case, convex.

2. The elevation or depression is inversely proportional to the diameter of the capillary tube.

3. As the temperature of the liquid increases, the amount of elevation or depression decreases.

PROPERTIES OF LIQUIDS

Liquid Pressure. Water standing in an open tank exerts a force at the bottom of the tank. This force is due to the weight of the water. The total force on the bottom is the weight of all the water in the tank. The pressure, however, would be the weight of water on a unit area of the tank bottom, such as a square foot, or a square inch, or a square centimeter. There is a marked difference, then, between the terms force and pressure. By *force* we mean a push or pull, and it is usually expressed in pounds, grams or kilograms. It is the total push or pull on the entire surface. By *pressure* we mean the *force per unit of area* of the surface acted upon, or the push or pull on one square inch, one square centimeter, or any other unit of area. Thus

$$Pressure = \frac{Force}{Area}.$$

Pressures are ordinarily expressed in pounds per square inch or grams per square centimeter. For example, the pressure in a steam boiler may be 200 pounds per square inch. The atmospheric pressure under normal conditions is 14.7 pounds per square inch.

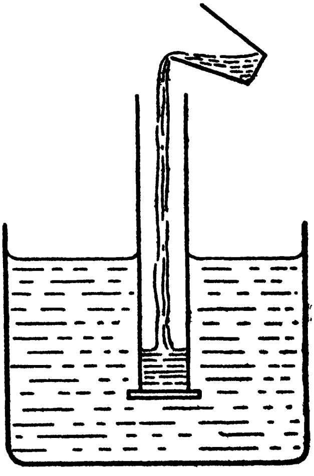
Pressure within a liquid may be caused by the following factors:

(1) The weight of the liquid itself.

(2) The application of an external force on the surface of the liquid. We will discuss case number 1 first.

Pressure Due to Weight of Liquid (Gravity Pressure). This pressure is dependent upon two factors: (1) the *depth*, and (2) the *density* of the liquid.

(1) If we place a piece of cardboard against the end of a hollow glass cylinder and then submerge it in water as shown in the accompanying figure, we find that the cardboard is held firmly against the cylinder by a strong upward force. The deeper we push the cardboard, the stronger the force trying to lift it becomes. In other words, the upward pressure of a liquid becomes greater with an increase in depth. If we pour water into the open end of the hollow cylinder, we find that the cardboard will become free when the water level on the inside of the cylinder reaches that of the level in the tank itself. The upward pressure of the water on the cardboard is then equalized by the downward pressure of the water in the hollow cylinder. The pressure exerted on an area of one square



Pressure of Liquid Dependent upon Depth

foot by a column of water one foot in depth would be equal to the weight of 1 cubic foot of water, or 62.5 pounds on a square foot. The pressure exerted by a depth of 10 feet of water would be ten times as much, or 6250 pounds per square foot.

(2) If instead of water we had used a heavier liquid, the pressure exerted on the bottom would be greater. Thus if mercury were the liquid, the pressure at any depth would be 13.6 times as great as that for water, since mercury has a density of 850 pounds per cubic foot, or 13.6 times that of water. The pressure exerted by a liquid depends then not only upon the depth of the liquid, but also upon its density.

Total Force on a Submerged Surface. The total force on a submerged horizontal surface due to a depth of liquid on it is equal to the pressure exerted by the liquid times the area covered. This may be computed by the use of the following formula:

$$F = A \times h \times d,$$

where F is the total force on the submerged surface, A is the area of the surface, h is the depth of the center of the surface below the surface of the liquid, and d is the density of the liquid. In the metric system the density (d) for water is 1 gram per cubic centimeter; in the English system it is 62.5 pounds per cubic foot.

Example: A tank with a base area of 30 square feet is filled with water to a depth of 10 feet. What is the force exerted on the bottom of the tank?

$$F = A \times h \times d$$
$$F = 30 \times 10 \times 62.5$$
$$F = 18{,}750 \text{ lb., } \textit{Ans.}$$

Summary. We may now say, by way of summary:

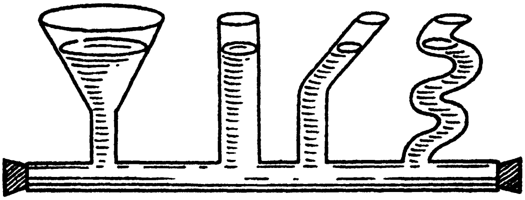
1. The pressure exerted by a liquid on a submerged surface is directly proportional to the depth of that surface below the surface of the liquid.
2. The pressure is directly proportional to the density of the liquid.
3. The total force exerted by the liquid is equal to the pressure multiplied by the area, or

$$F = A \times h \times d.$$

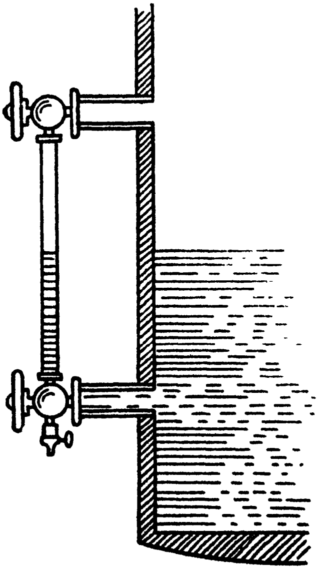
Liquid Pressure Equal in All Directions. At any point in a liquid, the pressure is the same in all directions. We have already seen that the downward and upward pressures at any point are equal, and depend upon the depth of the liquid. Experiment also shows that the sidewise pressure at that point is equal to the pressure in the other directions. The sidewise pressure of a liquid increases with an increase in depth just as the upward and downward pressures do. It is for this reason that dams are made strongest at the bottom.

Pressure Independent of Shape of Containing Vessel. The downward pressure of a liquid is not affected by the shape of the containing vessel. For the same height of liquid, the pressure exerted on a surface is independent of the shape of the container. This can be proven by a simple experiment, called the Pascal's vases experiment; we have several containers of different shapes and capable of holding different quantities of water. The base areas, however, are the same in each case. If we fill each of these vessels in turn with water to the same height, and measure the pressure exerted at the bottom with the pressure gage, we will find that the pressure in every case is exactly the same. This proves that the pressure exerted by a given liquid does not depend upon the shape of the containing vessel, but only upon the depth of the liquid.

Liquids Always Seek Their Own Level. If we pour a liquid into any one of a number of open vessels connected with one another, it will reach



Liquids Seek Their Own Level



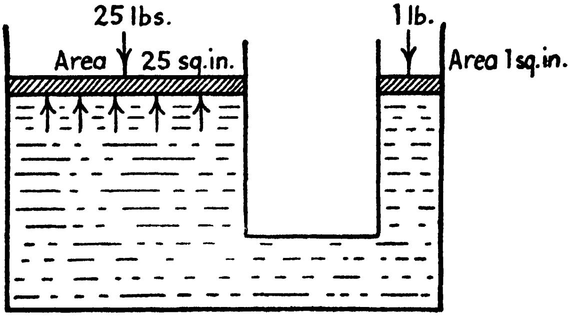
Water Gage on a Boiler

the same level in each vessel. It is in this manner that the pressure is equalized. If this were not so, liquid would flow from one vessel to another. We make use of this fact practically in the water gage on steam boilers to indicate the height of the water inside the boiler.

Practical Applications of Liquid Pressure. Water is usually distributed in city water supply systems by gravity pressure. If the city's water source and the reservoirs in which the water is stored are located at higher altitudes than the city itself, the water will flow to the point of use and will rise to levels as high as that of the source of supply. Otherwise pumping systems are used to fill large cylindrical tanks called standpipes from which the water flows through the mains and piping systems to homes, factories, hydrants, etc.

Water wheels and water turbines make use of the energy of running and falling water to generate electricity or to drive machines directly. Some of the largest sources of hydro-electric power have recently been developed by the Tennessee Valley Authority and in the Pacific Northwest at Grand Coulee and Bonneville. The water turbines in use today are highly efficient machines, reaching efficiencies of well over 90%.

Distribution of Pressure; Pascal's Law. We have discussed the pressure exerted by a liquid in an open vessel is due to its weight. How does a liquid transmit pressure in a closed vessel when it is completely confined? Pascal, a famous seventeenth century French scientist and mathematician, investigated this phenomenon and discovered the well-known principle of "Pascal's Law," namely: *Pressure applied to any part of a confined fluid is transmitted undiminished in all directions to every portion of the interior of the containing vessel.* It acts with equal force upon equal areas, and is always at right angles to the walls of the containing vessel.



Force Exerted on Two Pistons Proportional to Their Areas

If, for example, we exert a force of 1 lb. on the small piston p, which has an area of one square inch, the downward pressure is one pound per square inch. This pressure will be transmitted equally in all directions. The large piston P will, therefore, likewise have an upward pressure of one pound per square inch on every square inch of its surface. Since its area is 25 square inches, the total upward force against piston P will be 25 pounds, and we will be able to overcome a force of this magnitude, twenty-five times greater than the original force exerted on the small piston.

If the area of piston P were larger, the force exerted would be larger. In other words, *the forces exerted on two pistons are directly proportional to their areas;* thus

$$\frac{\textit{Force on large piston}}{\textit{Force on small piston}}=\frac{\textit{Area of large piston}}{\textit{Area of small piston}},$$

$$\text{or } \frac{F}{f}=\frac{A}{a}.$$

Since the areas of two circles are proportional to the squares of their diameters, the above formula may also be expressed in the following form:

$$\frac{F}{f}=\frac{D^2}{d^2},$$

where D is the diameter of the large piston and d is the diameter of the small piston. Using the above formulas, we can solve for either of the forces acting, or either of the dimensions of the pistons, depending upon what is given.

EXAMPLE: If the small piston of a hydraulic press has an area of 1.5 square inches and the large piston has an area of 150 square inches, what force must be applied to the small piston to exert a force of 5 tons (10,000 lb.) on the large piston?

SOLUTION:

$$\frac{F}{f}=\frac{A}{a}$$

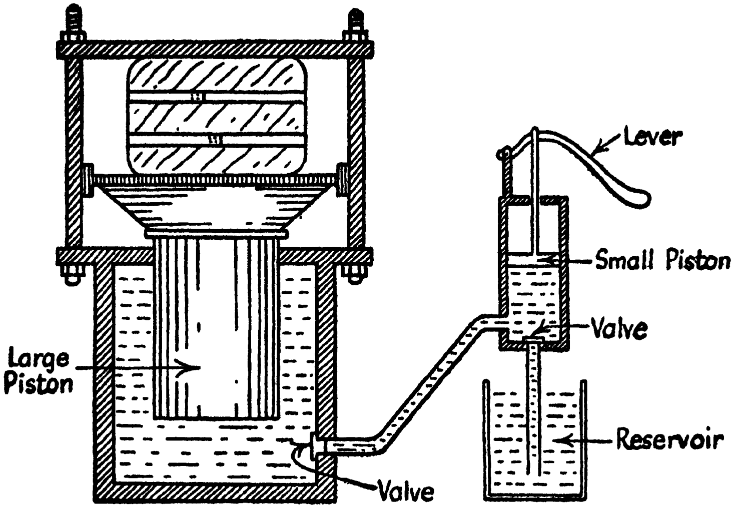
$$\frac{10{,}000}{f}=\frac{150}{1.5}$$

cross-multiplying, $150f=15{,}000$

$f=100$ lb., *Ans.*

The Hydraulic Press. Pascal's Law has many important industrial applications. Among these is the hydraulic press, a machine which makes it possible for us to exert enormous forces with comparatively small efforts. Thus, in the problem which we have just solved, we can exert a force of 10,000 pounds with an effort of only 100 pounds by using a hydraulic press.

Hydraulic presses are used for baling paper and cotton, and for extracting oil from seeds or juice from sugar cane. In the automobile indus-

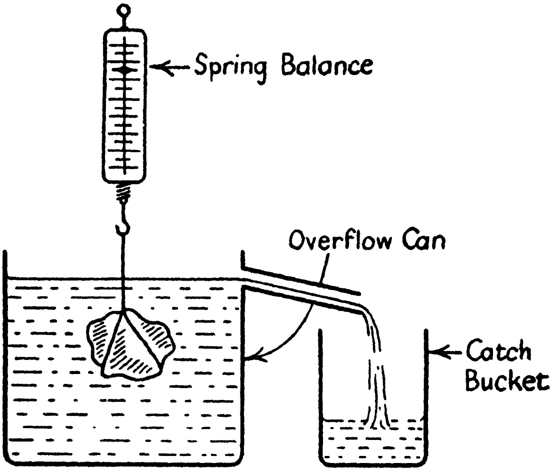


Basic Principle of the Hydraulic Press

try they are used in stamping and forming sheet steel into body tops, panels, and fenders for the cars. The hydraulic jack and the automobile lift used in so many of our service stations today are special applications of the hydraulic press. A simplified diagram of the commercial hydraulic press is given in the accompanying figure. Water or oil is pumped from the reservoir into the small cylinder. As the small piston is pushed downward, the liquid is forced through the connecting pipe into the large cylinder, causing the large piston to move upward. Since the pressure is transmitted undiminished through the liquid, the force against the large piston is extremely great as explained above.

Hydraulic Brakes. Hydraulic brakes on automobiles illustrate another important application of Pascal's principle. As the foot is brought down on the brake pedal, it forces a piston to press upon the liquid in a small cylinder. This pressure is transmitted through small tubes filled with the same fluid to a small cylinder and piston within the brake drum of each wheel. As the fluid pressure pushes the pistons outward it forces the brake shoes against the drum, producing the necessary friction to stop the car.

Buoyancy of Liquids. If we lift a stone out of the water, it appears to be a good deal heavier than it did when we held it under water. If we weigh a bar of metal immersed in water, we find it considerably lighter than when we weigh it in air. Throw a block of wood into water, and it almost immediately floats to the surface. When we dive into a swimming pool or a lake, we feel a definite force tending to lift us towards the surface. These effects are due to the upward force exerted by the water. ***Buoyancy,*** or ***buoyant force,*** is the name given to this upward force exerted by a fluid upon any object which we immerse in it. How large is this buoyant force? How much weight does a body appear to lose when immersed in a liquid? The following experiment will aid us in finding the answer.



Apparent Loss of Weight of Body Equal to Weight of Water Displaced

Weigh a lump of coal (or any other solid object) with a spring balance. Then submerge it in water in an overflow can such as shown in the figure and note the loss in weight. The overflow can has previously been filled with water up to the level of the spout, and a catch bucket of known weight placed to receive the overflow. Weigh the catch bucket with the displaced water. The weight of the displaced water is found by subtracting the original weight of the empty catch bucket from the combined weight of the bucket and overflow. We find that the loss of weight of the coal when submerged in the water is equal to the weight of the displaced water. In other words, the coal is buoyed up by a force equal to the weight of water displaced by the coal.

Archimedes' Principle. This relationship was discovered by Archimedes and is known as Archimedes' principle. In its general form the principle states: ***A body submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the body.*** Thus an object which weighs 100 grams and has a volume of 10 cc. will displace 10 cc. of water (weight 10 gm.) and will appear to weigh only 90 gm. when immersed in water.

Flotation. If a body submerged in a liquid displaces a weight of the liquid *less* than its own weight it will sink. Thus a stone or a piece of iron thrown into water will sink because the buoyancy of the water is less than the gravitational pull on the stone or the piece of iron.

If a totally submerged body displaces an amount of water *exactly equal* to its own weight it will neither sink to the bottom nor rise to the top with any portion of itself exposed; it will remain at any depth within the liquid at which it is placed and "float" there.

If, however, the body submerged in a liquid displaces a quantity of liquid *greater* than its own weight, it will be forced up so that part of its volume is above the surface of the liquid, and the body will float in this position. A piece of wood or cork, for example, submerged in water will be forced up and will float with part of itself exposed above the surface of the water.

Law of Flotation. *A body floating in a fluid displaces a weight of fluid exactly equal to its own weight.* This law can be verified by a simple experiment. An overflow can is placed on a balance and filled to the spout with water. The can and its contents are then balanced by the weights on the other pan. After providing a beaker to catch the overflow, we place a block of wood into the can and find that after the water has been displaced by the wood in reaching its floating position, the scales will again balance. This proves that the weight of the water displaced was equal to the weight of the block of wood inserted. In all cases a floating body will sink until it has displaced an amount of liquid equal to its own weight. Thus, a 10,000-ton cruiser will sink in water until it displaces 10,000 tons of water. We can determine the displacement of a ship either from the weight of the ship (including its equipment and cargo), or from the volume of the ship below the water line. A ship floats in spite of the fact that it is made of materials much heavier than water because its *average* density is less than that of water, due to the large air spaces in the compartments. When the average density is increased by filling these compartments with water the ship will sink. This is voluntarily done on submarines for purposes of submersion.

Specific Gravity. The specific gravity of a substance is the ratio of the weight of the substance to the weight of an equal volume of water; or

$$\textit{Specific Gravity} = \frac{\textit{Weight of substance}}{\textit{Weight of equal volume of water}}.$$

A piece of aluminum, for example, weighs 54 grams, and an equal volume of water weighs 20 grams; the specific gravity of aluminum is therefore $^{54}/_{20}$, or 2.7. The *specific gravity* of a substance is then a measure of the *number of times heavier that substance is than an equal volume of water.*

Although the values of the specific gravity and of the density of a given substance expressed in metric units are *numerically* equal, nevertheless specific gravity is *not* the same as density. Specific gravity is a *ratio* of the weight of a substance compared to a standard, or simply a *number* expressing relative weights. Density, on the other hand (defined as the mass per unit volume and expressed in grams per cubic centimeter or pounds per cubic foot) is a definite quantity for any particular substance and no comparison with any standard substance is necessary. The density of lead, for example, is 11.37 gm. per cc., or 710 lb. per cu. ft.; its specific gravity is 11.37. Alcohol has a specific gravity of 0.79; its density is 0.79 gm. per cc., or 49 lb. per cu. ft. Water, our standard of comparison, has a specific gravity of 1, and a density of 1 gm. per cc., or 62.5 lb. per cu. ft. In other words, in the English system, the density of a substance is numerically equal to its specific gravity multiplied by 62.5.

Specific Gravity of a Solid. From Archimedes' principle we know that a body immersed in a liquid is buoyed up by a force equal to the weight of the liquid displaced. Its apparent loss of weight in the liquid is therefore equal to the weight of the water displaced. This affords a convenient method for finding the specific gravity of a solid. For if we first weigh the solid in air, and then weigh it in water, we can easily find the specific gravity as follows:

$$\textit{Specific Gravity} = \frac{\textit{Weight of body in air}}{\textit{Loss of weight of body in water}}.$$

This is the method actually followed in finding the specific gravity of irregular solids which are not soluble in water.

If the solid is lighter than water, e.g., wood or cork, we use a "sinker" to submerge it and go through the following steps: (1) weigh the solid in air; (2) weigh the sinker in water; (3) attach the solid to the sinker and weigh them both together in water. The difference between the weight of both in water, and the sum of weights (1) and (2), will give the loss of weight of the original substance in water; from this the specific gravity can be calculated by using the formula given above.

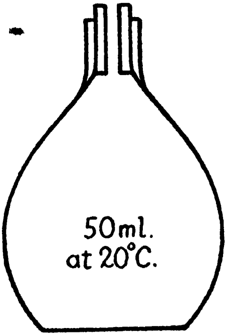
The specific gravity of a regular geometrical solid can be found by measuring its dimensions and calculating its volume. The weight of an equal volume of water is readily determined, since we know that water weighs 1 gm. per cc., or 62.5 lb. cu. ft. Having weighed the solid, its specific gravity is computed by using the general equation:

$$\textit{Specific Gravity} = \frac{\textit{Weight of substance}}{\textit{Weight of equal volume of water}}.$$

Specific Gravity of a Liquid. Several different methods for finding the specific gravity of a liquid are available. Among them are:

1. The use of the specific gravity bottle.
2. The use of the hydrometer.

Bottle Method. The specific gravity bottle is an accurately constructed glass-stoppered bottle containing a definite volume of liquid, usually 50 cc. at 20° C. In using the bottle it is first weighed when empty. Then it is weighed when filled with the liquid whose specific gravity we wish to find; finally it is weighed when filled with water. Subtracting the weight of the bottle in each case gives the weight of the liquid and the weight of an equal volume of water. The specific gravity is found by the formula:



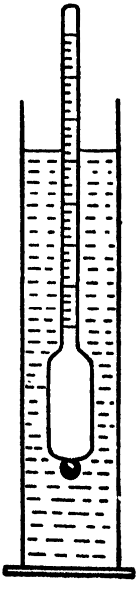
$$\textit{Specific Gravity} = \frac{\textit{Weight of liquid}}{\textit{Weight of equal volume of water}}.$$

Example: A specific gravity bottle weighs 75 grams when empty, 120.5 grams when filled with a lubricating oil, and 125 grams when filled with water, the specific gravity of the oil would be determined as follows:

Weight of oil $= 120.5 - 75 = 45.5$ gm.
Weight of water $= 125 - 75 = 50$ gm.

$$\text{Specific gravity of oil} = \frac{45.5}{50} = 0.91, \textit{ Ans.}$$

Hydrometer Method. The quickest and most convenient method of finding the specific gravity of a liquid is through the use of a hydrometer by means of which we can read its value directly from the instrument without any further calculation. The hydrometer is made of glass and consists essentially of a long stem and a bulb weighted with lead shot or mercury. The hydrometer is placed in the liquid to be tested and floats upright in it, after sinking to a depth depending upon the specific gravity of the liquid. The lighter the liquid, the deeper the hydrometer sinks in order to displace its weight of the liquid. The lower readings are therefore at the top of the scale, the higher ones at the bottom. The specific gravity is read directly off the graduated scale in the stem, at the top surface of the liquid.



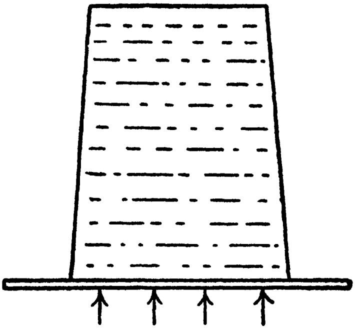
Though not quite as accurate as the bottle method, the hydrometer is used in most commercial and industrial work. It is widely used to determine the specific gravity of acids, oils, alcohols, chemical solutions and other liquids. In testing the acid solutions of storage batteries a special form of hydrometer known as the battery tester is used. Another special form called the lactometer is used to determine the percentage of whole milk.

GASES

Properties of Gases. We seldom think of the air around us as having weight. Yet we must take account of the weight of the air and the pressure it exerts in our everyday life as well as in many industrial applications. That air has weight can be shown by a very simple experiment. First weigh a stoppered flask which, of course, contains air. Then, using an air pump, exhaust as much of the air as possible, and weigh again. A definite loss in weight will be noted. When air is allowed into the flask again, an increase in weight is noted. By very accurate measurements physicists have found that a liter of air at 0° C and 760 mm. pressure weighs 1.29 grams. A cubic foot weighs about 1¼ oz., and 12 cubic feet weigh almost a pound.

Pressure of the Atmosphere. Because of its weight, the air exerts pressure. The force of the air on any horizontal surface is equal to the weight of the entire column of air directly above it, from the surface itself to the uppermost limits of the atmosphere. At any given point, the pressure of the air is the same in all directions.

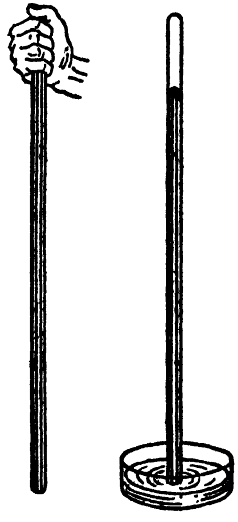
There are several simple and fairly well known experiments which show that air exerts pressure. We can, for example, boil a little water in a large tin can until the steam escapes rather freely; then cork the can tightly and stop heating it. If cold water is then poured over the outside of the can, its walls cave in and the can is badly crushed. What happened is essentially as follows: as the water turns to steam, it drives whatever air there was in the can out through the opening, and fills it with steam. When we pour water on the can after we have closed it, the steam inside the can condenses and a partial vacuum is formed. The air pressure on the outside being larger than the pressure inside the can, its walls are crushed in. Another experiment to show the pressure of the air is even simpler than the one just described. If we fill a glass with water to the brim, then place a wet piece of cardboard on top of it and invert the glass, we find that the cardboard will not fall down but will on the contrary remain in place, holding up all of the water. The air pressure on the bottom of the cardboard is sufficient to overcome the downward force of the water against the cardboard and keep the latter in place.



Pressure of the Atmosphere on the Cardboard Holds up the Liquid Pushing on the Card.

Torricelli's Experiment. During the Middle Ages it was customary for the natural philosophers to explain many phenomena, among them the rise of water in tubes through the use of pumps, by the statement that "nature abhors a vacuum." When the Grand Duke of Tuscany found that he could not pump water for the fountains on his estate from a well which was about 50 feet deep, he consulted Galileo, who found that the water would not rise more than 32 feet above the water below. Galileo is said to have stated that nature does not seem to abhor a vacuum above 32 feet.

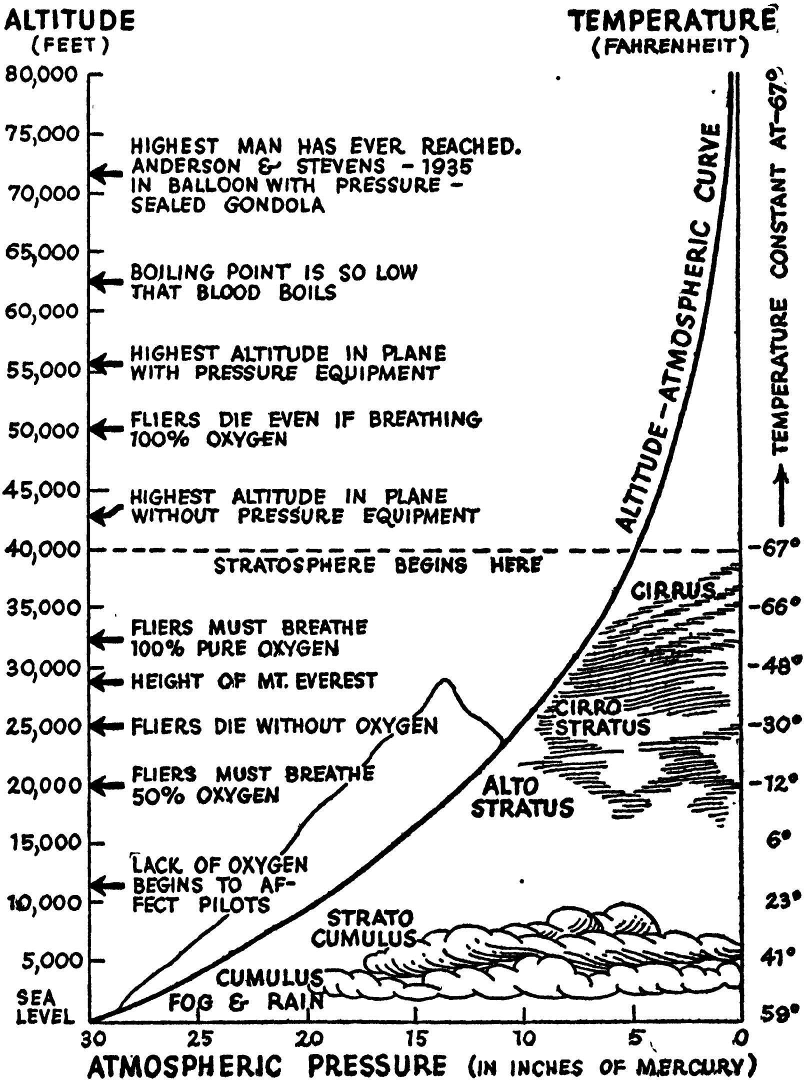
It was Torricelli, however, who first related the limits in the height of a column of liquid that can be raised to the pressure of the atmosphere. In 1643 Torricelli performed his famous experiment to find how high a column of mercury could be supported by the pressure of the atmosphere. He took a glass tube about three feet long, closed at one end, and filled it completely with mercury. Then, closing the open end with his thumb, he inverted the tube and placed its open end in a dish of mercury. Upon removing his thumb, he found that the mercury in the tube fell until its height was about 30 inches (760 mm.) above the surface of the mercury in the dish. The vacuum at the top of the tube is known as the Torricellian vacuum. This experiment, Torricelli stated, showed that the pressure of the atmosphere was sufficient to counterbalance the pressure exerted by a column of mercury 30 inches high. If the mercury column were held up by atmospheric pressure only, then at higher altitudes, where the pressure of the atmosphere is less, the height of the mercury column supported should be less. Pascal performed an experiment to test this in 1648 and found that as he ascended a high tower or a mountain, there was a decided drop in the height of the mercury column, thus bearing out his initial supposition.



(1). Filled with Mercury before it is inverted.
(2). After inverted with thumb removed.

Torricelli Experiment

Calculation of Atmospheric Pressure. How great is the pressure of the atmosphere? We can compute this easily when it is remembered that it is equal to the pressure exerted by the mercury column 30 inches high, since at sea level it supports a mercury column of this height. Since the density of mercury is 0.49 lb. per cu. in., the pressure of the column of mercury on a square inch would be



Courtesy of *Life*

$$P = A \times h \times d$$
$$P = 1 \text{ sq. in.} \times 30 \text{ in.} \times 0.49 \text{ lb. per cu. in.}$$
$$P = 14.7 \text{ lb. per sq. in.}$$

In metric units atmospheric pressure would be similarly calculated in grams per square centimeter. Since the density of mercury is 13.6 gm. per cc., and the column supported is 76 cm., the pressure on one sq. cm would therefore be

$$P = A \times h \times d$$
$$P = 1 \text{ sq. cm.} \times 76 \text{ cm.} \times 13.6 \text{ gm. per cc.}$$
$$P = 1033.6 \text{ gm. per sq. cm.}$$

Roughly then, the pressure of the atmosphere at sea level is 15 lb. per sq. in., or 1 kilogram per sq. cm. In common usage any pressure equal to this is called one *atmosphere,* even though the pressure refers, let us say, to that exerted by carbonic acid gas inclosed in a steel cylinder.

The Barometer. Atmospheric pressure is not always the same. It varies with different weather conditions. Obviously a simple way to measure variations in the pressure of the atmosphere is by using a tube of the type Torricelli used, and noting the changes in the height of the column of mercury supported. Such an instrument is actually used and is called the *barometer.* The mercury barometer consists of a glass tube containing mercury, closed at the top, and placed upright with its open end dipping into a mercury reservoir. A fixed scale is attached in back of the tube so that the height of the mercury column can be read accurately in inches or centimeters. In the Fortin cistern barometer commonly used the mercury reservoir can be adjusted to the zero point of the fixed scale by an adjusting screw which raises or lowers a leather diaphragm at the bottom of the mercury cup, so that its top surface touches the tip of an ivory pin projecting into the reservoir. We read the barometer by merely measuring the height of the mercury column above the surface of the mercury in the reservoir.

The Aneroid Barometer. At times it is inconvenient to measure atmospheric pressure with a mercury barometer because of its length and the fact that it contains a liquid. In its place we use a more compact type of barometer known as the *aneroid* or metallic barometer. In this barometer atmospheric pressure is measured by means of a hollow disc-shaped metal box with a flexible, corrugated cover from which the air has been partly exhausted. Any change in the atmospheric pressure will cause the top of the box to move in or out; this motion is magnified and transmitted through a system of levers and a chain, causing a pointer to move over a circular scale graduated in inches (or centimeters) to correspond with the readings on a standard mercury barometer. The aneroid barometer is made in various sizes, some as small as a watch, without loss in accuracy. The *altimeter* used by aviators to indicate the height of their planes is a special form of aneroid barometer which reads directly in feet or meters. Experiment has shown that for altitudes not very far above sea level, the barometer reading drops one-tenth of an inch for every 90 feet of ascent.

Uses of the Barometer. The barometer will indicate changes in the pressure of the atmosphere whether they be due to changing weather conditions or to changes in the elevation of the observer. We can, therefore, use the barometer as an altitude measuring device, as the altimeter is used, and also as an aid in foretelling the weather. In forecasting the weather, the following relationships hold good. A sudden decrease in atmospheric pressure, as shown by a *falling* barometer, indicates the approach of a storm. On the other hand, a *rising* barometer indicates the coming of fair weather. A continued high barometer indicates settled fair weather.

Buoyancy of the Air. Since the pressure of the air decreases with an increase in altitude or height, it is apparent that any object has a greater pressure exerted on its bottom surface by the air than is exerted on its top. This difference in pressure causes a lifting effect. Archimedes' principle, developed above for liquids, applies to gases also. As with liquids, the lifting or buoyant force on an object in air is equal to the weight of the air which it displaces. For most objects of ordinary size this lifting effect is very small, but for extremely large and light objects, such as balloons and dirigibles, it is very considerable, and can therefore be used practically. A balloon, filled with a very light gas such as hydrogen or helium, will rise because the weight of the air it displaces is greater than its own weight. It will continue rising until it reaches the point where its weight is equal to the weight of the air it displaces. At this point it will float. It will descend when its weight is more than the weight of the air which it displaces.

Boyle's Law; Pressure-Volume Relationship of Gases. Unlike liquids, which are practically incompressible, gases are very easily compressed. We can force a large quantity of air into an automobile tire by the use of a simple air pump. The pressure inside the tire is greater than the outside air due to the fact that we have compressed a large volume of the outer air into its comparatively smaller inner tube. If we double the pressure of a given quantity of air we find that its volume becomes one-half as great as previously; if we triple the pressure, the volume is reduced to one-third of its original volume, and so on. Robert Boyle, an Irish scientist, investigated this relationship experimentally in 1662 and found that in all ordinary cases, the volume of an enclosed mass of gas *varies inversely as the pressure exerted on it, provided the temperature remains constant.* This is the famous *Boyle's Law;* expressed mathematically, we have:

$$\frac{V_1}{V_2}=\frac{P_2}{P_1},$$

$$\text{or } P_1V_1=P_2V_2,$$

where V_1 and P_1 are the volume and pressure of the gas under the initial conditions, and V_2 and P_2 are the volume and pressure under the changed conditions.

Boyle's Law holds true for gases at ordinary temperatures and pressures. For gases under extremely high pressures, there are certain slight deviations. Using the formula given above, we can find the volume of a given mass of gas under different pressure conditions. Thus, if a gas has a volume of 10 cu. ft. at a pressure of 28 in. of mercury, what will its volume be at normal atmospheric pressure (30 in. of mercury)?

$$\frac{V_1}{V_2}=\frac{P_2}{P_1}$$

$$\frac{10 \text{ cu. ft.}}{V_2}=\frac{30 \text{ in.}}{28 \text{ in.}}$$

$$30\times V_2=10\times 28$$

$$V_2=\frac{280}{30}=9\tfrac{1}{3} \text{ cu. ft.}$$

In other words, under the increased pressure, its volume would be smaller, in agreement with Boyle's Law.

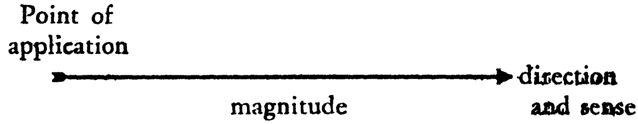
29. BODIES AT REST

Forces. A *force* is any action which tends to change the position, the condition, the velocity, or the direction of motion of a body. It is commonly defined as a push or pull on a body. Thus, a downward force is exerted by a block of steel resting on a work bench. This is overcome by the upward force exerted by the bench itself. Work held in a vise has force exerted against it by the jaws of the vise. A hydraulic press exerts a force on the metal in contact with its piston. A man pushing a hand truck exerts a force against it to keep it in motion. In lifting a weight, a force is exerted to overcome the downward pull of gravity.

The unit of force commonly used in the English system is the gravitational unit, the pound force. This is the gravitational pull exerted on a mass of one pound. In other words, in the gravitational system the units of force and weight are the same.

When the forces acting on a body are balanced, the body is said to be in *equilibrium.* A body at rest, or a body in uniform motion, is in equilibrium, and the sum of all the forces acting upon it is zero.

Graphic Representation of a Force. It is necessary that we know the magnitude, the direction, and the point of application of a force in order to determine its effect. A force may be represented by a straight line, the length of which, when drawn to scale, indicates the magnitude of the force. The point at which the force is applied is indicated by one end of the line; the direction of the force is represented by the position of the line; and the "sense" of the force is represented by an arrowhead placed at the other end of the line. Such a linear representation of a force is called a vectorial representation, and the line itself is called a *vector.*

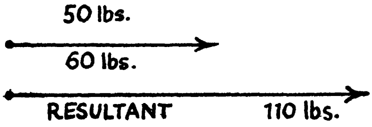


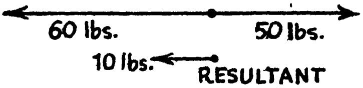
Vector Representation of a Force.

If a scale of $1'' = 10$ lb. is used, then a force of 10 lb. would be represented by a line 1 inch long; a force of 30 lb., by a line 3 inches long; and a force of 100 lb., by a line 10 inches long. The direction in which the force is acting is shown by the direction in which the arrow points.

Concurrent Forces. Forces which act together at a point are called *concurrent forces.* The process of finding the resultant of two or more forces is known as the *composition of forces.* The *resultant* is that force which, when substituted for two or more forces acting on a body, produces the same effect as the forces acting directly. The *equilibrant* is a force equal in magnitude to the resultant and acting in the opposite direction, which will counterbalance the resultant thus producing equilibrium conditions.

Composition of Forces Acting in the Same Straight Line. Two forces acting in the *same direction* along the same straight line produce *an effect equal to their sum.* Thus if a force of 50 lb. and a force of 60 lb. both act in an easterly direction against a given surface, the effect is the same as if a single force of 110 lb. were acting in that direction.



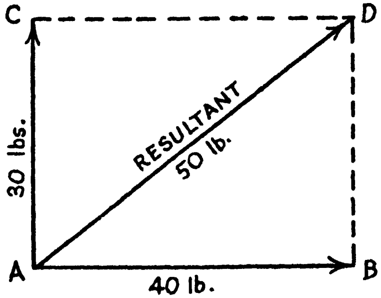


Forces acting in the same straight line, but in *opposite directions* produce *an effect equal to their difference.* If the force of 50 lb. acted in an easterly direction, and the 60-lb. force acted westward, then the effect on the surface would be the same as if one force of 10 lb. acted westward. The resulant in such a case acts in the direction of the greater force.

Composition of Forces Acting at an Angle to Each Other. When two forces act at an angle to each other, their resultant can be found by constructing a parallelogram with the two forces (drawn to a suitable scale) as sides. The diagonal of this parallelogram will indicate both the magnitude and the direction of the resultant.

EXAMPLE: A force of 30 lb. acts at right angles to a second force of 40 lb. at a certain point. Find the resultant of these forces.

GRAPHICAL SOLUTION:



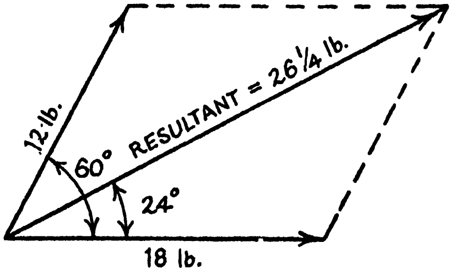
1. Select a convenient scale; let 1 inch=10 lb.
2. Draw AB 4 inches long to represent a force of 40 lb.
3. At right angles to AB, draw AC, 3 inches long, representing the force of 30 lb.
4. Complete the parallelogram by constructing CD parallel to AB, and BD parallel to AC.
5. Draw diagonal AD which is the resultant.
6. Measure AD. Its length of 5 inches, representing a force of 50 lb., indicates the magnitude of the resultant. The direction of the diagonal indicates the direction of the resultant force.

ALGEBRAIC SOLUTION: The above problem can also be solved algebraically. Since in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the arms, we have:

$$\overline{AD}^2=\overline{AB}^2+\overline{AC}^2$$
$$=40^2+30^2=1600+900=2500$$
$$AD=\sqrt{2500}=50 \text{ lb., } Ans.$$

The direction of the resultant can easily be found from the diagram.

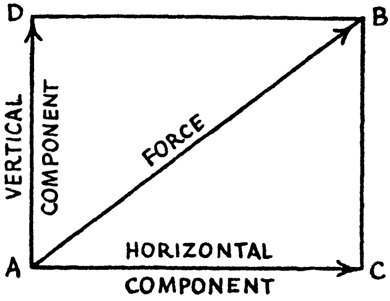
Composition of Forces Not at Right Angles. The problem just solved involved two forces at right angles to each other, but the same graphical method of solution can be used for forces acting at *any angle* to each other. If, for example, a force of 18 lb. acts horizontally and a second force of 12 lb. acts at the same point but makes an angle of 60° with the first force, what would their resultant be? The same procedure is followed. Assume a scale of 1 in.=6 lb.



The resultant is found to be 26¼ lb. in magnitude, and acts at 24° to the horizontal.

Resolution of Concurrent Forces. The composition of forces consisted in finding the resultant of two or more forces by the methods described above. It is often necessary to use the reverse procedure—in other words, to determine the component forces into which a given force may be resolved. The process of finding the components into which the given force may be resolved is called the *resolution of forces.* The parallelogram method is again used in finding the components. In the composition of forces, the sides of the parallelogram were known and the resultant was found by constructing the diagonal of the parallelogram. In the resolution of a force, it is the diagonal of the parallelogram that is known, and the sides of the parallelogram are found to represent the component forces.

Components at Right Angles. Thus, if we are given a force AB acting in the direction shown, its horizontal and vertical components may be found by drawing a horizontal line and a vertical line through point B. The component forces will be represented by AD and AC. The numerical values of these forces may be found by selecting a suitable scale in laying out the forces.

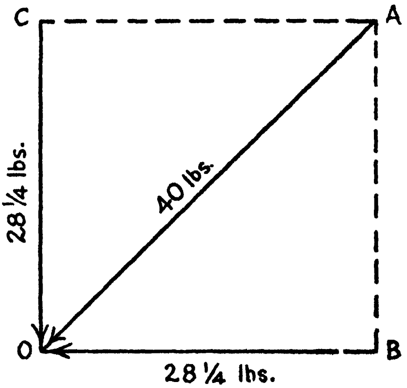


Resolution of Forces

Example: A boy pushes a lawn mower with a force of 40 lb. along the handle, which is held at an angle of 45° to the ground. What is the useful component of the force? The component of the applied force which is parallel to the ground is effective in moving the lawn mower forward.

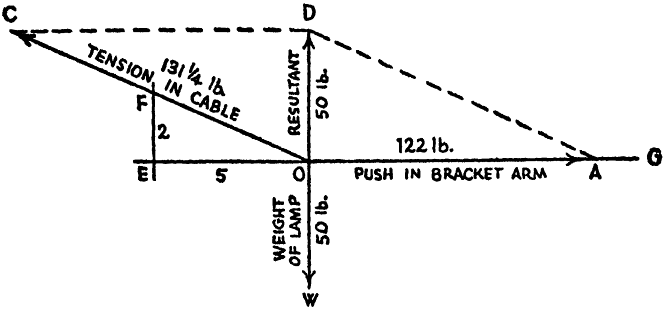
Graphical Solution:

1. Select a convenient scale. Let 1 in.=20 lb.
2. Through O, the point of application of the force, draw a line 45° to the horizontal.
3. Lay off OA=2 in. to represent the applied force of 40 lb.
4. From A, drop a perpendicular AB. Then BO will represent the horizontal component, which is effective in moving the mower forward; this is equal to 28¼ lb.

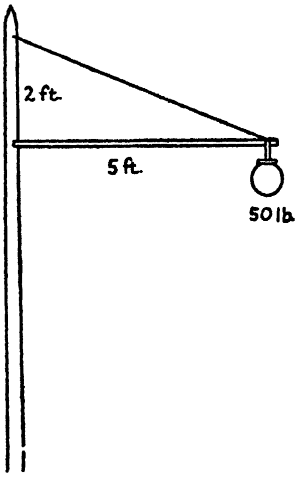


5. If it is desired to find the vertical component, construct CA parallel to OB and passing through point A. This will give us the downward component (CO) of the force, which is effective in keeping the lawn mower on the ground.

Components Not at Right Angles. The same method of resolving a single force into its components acting in different directions can be used regardless of the direction in which the components act. If, for example, the forces acting on a street lamp bracket are considered, the downward force due to the weight of the lamp is overcome by the resultant of two component forces, one exerted by the arm of the bracket, the other exerted by the cable or tie-rope. If the weight of the lamp is known, it is a simple matter to determine the forces acting. In the force diagram, the weight of the lamp is represented by OW. This is balanced by the resultant, OD, of the push of the bracket arm OA, and the tension in the cable OC. The procedure used in constructing the force diagram is as follows:



Forces Acting on a Street Lamp Bracket

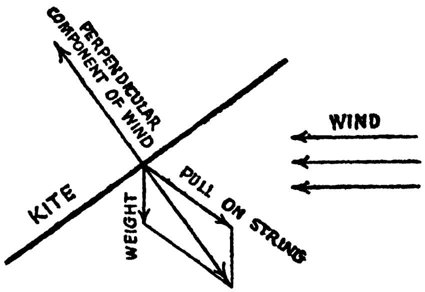


1. Lay off a vertical line OW to a suitable scale, 1 in.=50 lb.
2. Draw OD equal to OW and in the same straight line.
3. With O as a center construct a line in the direction of the cable by laying off a horizontal distance of 5 units (OE) and a vertical distance of 2 units (EF). Draw OF and extend it indefinitely. This is the direction of the cable tension.
4. Draw the horizontal OG to indicate the direction of the push of the bracket arm.

5. Complete the parallelogram by drawing CD parallel to OG, and DA parallel to OC.
6. The push on the bracket arm, found by measuring OA, is $2\frac{7}{16}$ in. × 50 lb. = 122 lb., *Ans.*
7. The tension in the cable, found by measuring OC, is $2\frac{5}{8}$ in. × 50 lb. = $131\frac{1}{4}$ lb., *Ans.*

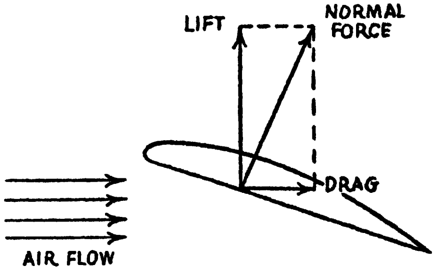
Practical Applications of the Resolution of Forces. The motion of bodies driven by the wind is an application of the resolution of forces. Such bodies do not always move in the direction in which the wind is blowing. A kite, for example, rises while the wind may be blowing horizontally; a sailboat sailing into the wind moves forward; an airplane rises from the ground when its motors are set in motion. In all these cases the actual motion in the desired direction is produced by one of the components of the force due to the wind.

There are three forces acting on a flying kite: gravity, tending to pull it downward; the wind pressure, pushing it upward; and the pull exerted along the string to balance the kite. Only that component of the wind perpendicular to the surface of the kite is effective in raising the kite. The other component is parallel to the surface of the kite and does not help in lifting it. When the kite is in equilibrium, the force of the wind pushing upward is equal and opposite to the resultant of the gravity force due to the weight of the kite and the downward pull on the string. Any variation in the wind pressure requires a change in the pull on the string to re-establish equilibrium.

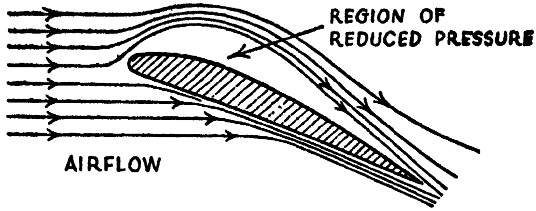


Forces Acting on a Kite

The airplane, too, is kept up by the lift exerted by the air against its wings. Since the plane is moved rapidly forward by the engine and propeller, there is a rush of air past the wings. This stream of air is deflected downward by the wing and results in a force being exerted perpendicular to the wing surface. This normal force can be resolved into two components, the *lift* and the *drag*. The lift is effective in overcoming the weight of the plane and causes it to rise from the ground. The drag resists the forward motion of the plane and is overcome by the pull

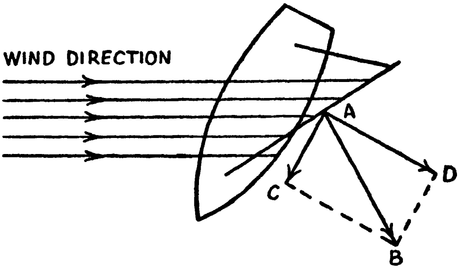


exerted by the plane's propeller. Wing surfaces are arched or *cambered* to reduce turbulance back of the leading edge of the wing. The air stream is directed upward over the wing, as shown in the diagram, before it is deflected downward. This results in a reduction of pressure on the upper surface of the wing and aids greatly in increasing the lift. The total lift will, therefore, be due both to the push on the under surface of the wing and the pull on the upper surface caused by the low pressure area.



Air Flow around Airplane Wing

In sailing a boat into the wind, the sail is held obliquely. The wind blowing against the slanting sail causes a force perpendicular to the surface, force AB in the diagram. This force may be resolved into two components, one (AC) acting to drive the boat forward, the other (AD) tending to tip the boat and move it sidewise. A deep keel or centerboard is used to overcome the sidewise motion. Only a part of the force of the wind, therefore, is effective in driving the boat forward. It is only when a boat travels directly with the wind that the entire force of the wind against the sail is effective in moving the boat forward.



Resolution of Forces in Sailing a Boat

Moment of a Force. The turning effect of a force depends not merely upon how large the force is, but also upon the perpendicular distance between its line of action and the axis of rotation. If, for example, a one-pound weight is placed on a rod supported at one end, the further the weight is moved from the point of support, the more effort is needed to prevent the rod from turning. The tendency of a force to cause rotation about a given point is called its *moment.* A given force will produce different moments when placed at different distances from the axis of rotation; the greater the distance, the greater the turning effect or moment, and the smaller the distance, the smaller the moment. The *moment of a force,* therefore, may be defined as the product of the force and the perpendicular distance from its line of action to the axis of rotation. Since the perpendicular distance between the line of action of the force and the axis of rotation is called the moment arm of the force, we may say:

Moment=Force×Moment arm of force.

The moment is measured in units corresponding to the units in which the force and moment arm are expressed. Thus, if the force is given in pounds and the moment arm in feet, the moment will be measured in *pound-feet;* if the moment arm is given in inches, the moment will be measured in *pound-inches.*

Example: What is the moment of a force of 250 pounds acting at a distance of 5 feet from the axis of rotation?

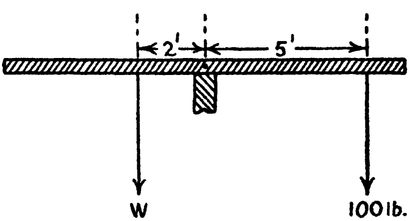
Solution: Moment=Force×Moment arm of force
=250 lb.×5 ft.=1250 lb.-ft.

Moments may be either positive or negative. A *positive* moment is one which tends to rotate the body in a *clockwise* direction. A *negative* moment is one which tends to turn the body in a *counterclockwise* direction.

Principle of Moments. A body will be in equilibrium when the sum of the clockwise moments equals the sum of the counterclockwise moments. When the sum of the clockwise moments exceeds the sum of the counterclockwise moments, there will be rotation in a clockwise direction. When the reverse conditions hold, the result is counterclockwise rotation. When the two are equal, as stated in the principle of moments, there will be no rotation at all.

Example: A beam supported at its center has a weight of 100 pounds placed 5 feet right of center. What weight must be placed at a point 2 feet from the center to balance the beam?

Solution:
Clockwise Moment=100 lb.×5 ft.
=500 lb.-ft.
Counterclockwise moment
$=W$ lb.×2 ft.
$=2W$ lb.-ft.



Since for a condition of equilibrium the sum of the counterclockwise moments must equal the sum of the clockwise moments:

Clockwise moments=counterclockwise moments
500 lb.-ft.$=2W$ lb.-ft.

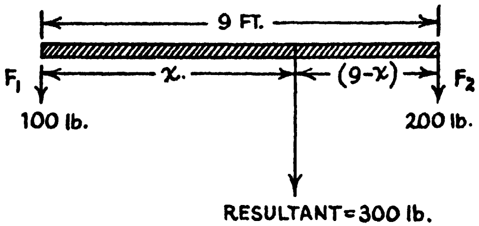
$$W=\frac{500}{2}=250 \text{ lb., } \textit{Ans.}$$

Parallel Forces. Often in supporting or moving a heavy object, two or more forces acting in the same direction are applied at different points, but in parallel directions. Such forces are known as *parallel forces.* Thus two men will lift a heavy rail or log, one man at each end; or a beam may be supported at two or more points. For parallel forces acting in the same direction, the following holds true:

1. The resultant is equal to the sum of the forces and acts in the same direction.
2. The point of application of the resultant divides the line joining the forces into two parts inversely proportional to the forces.

Example: Two parallel forces, one of 100 pounds, the other 200 pounds, act in the same direction on the ends of a beam 9 feet long. What is their resultant and where is its point of application located?
Solution: Resultant

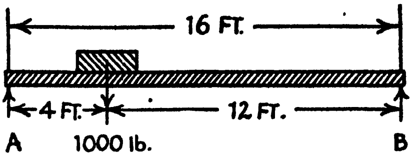
$$=F_1+F_2$$
$$=100+200=300$$



To find point of application:
Let x=distance of 100-lb. force from resultant
$9-x$=distance of 200-lb. force from resultant

$$\frac{F_1}{F_2}=\frac{D_2}{D_1}$$
$$\frac{100}{200}=\frac{9-x}{x}$$
$$100x=200\ (9-x)$$
$$100x=1800-200x$$
$$300x=1800$$
$$x=6 \text{ ft. (distance of 100-lb. force from resultant)}$$
$$9-x=3 \text{ ft. (distance of 200-lb. force from resultant)}$$

If a number of parallel forces act upon a body, that body will be in equilibrium only if the sum of the forces in one direction is equal to the sum of the forces in the opposite direction, *and* if the sum of the clockwise moments is equal to the sum of the counterclockwise moments. In solving a problem on parallel forces in equilibrium, the point of application of any one of the forces may be considered as the axis of rotation, and the clockwise and counterclockwise moments of the other forces about this point may then be considered equal to each other.



Example: A beam 16 feet long and supported at both ends has a load of 1000 pounds placed 4 feet from one end. How much of the load is carried by each of the supports?

Solution: Assume support B to be the axis of rotation. Then for equilibrium, the clockwise and counterclockwise moments about B must be equal, or,

$$(\text{Force at A})\times 16\text{ ft.} = 1000\text{ lb.}\times 12\text{ ft.}$$

$$\text{Force at A} = \frac{1000\times 12}{16} = 750\text{ lb., } \textit{Ans.}$$

If support A is now assumed to be the axis of rotation, the clockwise and counterclockwise moments about A must be equal, or,

$$1000\text{ lb.}\times 4\text{ ft.} = (\text{Force at B})\times 16\text{ ft.}$$

$$\text{Force at B} = \frac{1000\times 4}{16} = 250\text{ lb., } \textit{Ans.}$$

As a final check, the forces in one direction must be equal to the forces in the opposite direction. This is found to be true, since 250 lb. plus 750 lb. acting upward equals 1000 lb. acting downward. The beam is therefore in equilibrium.

30. BODIES IN MOTION

Motion a Result of Force. By *motion* is meant the change in position of a body with reference to some other body. Since a body may be at rest when referred to one body, and in motion when referred to another, motion itself is *relative.* Thus, a man driving an automobile is at rest with respect to the person sitting beside him, but is definitely in motion with respect to the ground.

The motion of a body is the result of the action of a force or a number of forces. When all the forces acting on a body are balanced, the body is in equilibrium, whether it be at rest or in uniform motion. If the forces are not balanced, the body changes both its position and its rate of motion. Such a body is said to have an *accelerated motion.* To disturb a condition of equilibrium of any body, the action of a force is necessary. Any unbalanced force will produce accelerated motion in a body for the period during which it acts.

A force acting upon a body may cause it to move along a fixed line, to turn about an axis, or often to have a combination of these two motions. Motion along a fixed line is known as *linear motion,* or *motion of translation.* A body having linear motion changes its position so that all points of the body move in the same direction at the same rate. Thus, the hammer of a pile driver, the piston in an automobile cylinder, a shaper tool, all have linear motion. Motion in a circular path about a fixed axis is known as *angular motion,* or *rotational motion.* The motion of a milling cutter, of a grinding wheel, of work held in a lathe, or of the flywheel of an engine are all examples of motion of rotation.

Velocity. The distance a body moves in a given direction in a unit of time is called its *velocity.* Velocity differs from speed in that it comprises both *rate* of motion and *direction* of motion of a body, while speed expresses merely its rate of motion. The units in which velocity is commonly expressed in the English system are feet per second and miles per hour. Thus a rifle bullet may have a velocity of 2500 feet per second, an automobile a velocity of 60 miles per hour, and an airplane a velocity of 200 miles per hour. Velocity is computed by dividing the distance a body moves by the time taken to cover that distance:

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}},$$

$$\text{or, } v = \frac{s}{t},$$

where v is the velocity, s the distance covered, and t the time.

EXAMPLE: An automobile travels a distance of 200 miles in 5 hours. What is its average velocity?

SOLUTION:

$$v = \frac{s}{t}$$

$$v = \frac{200}{5} = 40 \text{ miles per hour, } \textit{Ans.}$$

Uniform and Accelerated Motion. If the distances covered by a body in equal units of time are the same, the body is said to have a *uniform velocity.* If the distance covered in a unit of time changes, regardless of whether it increases or decreases, the body is said to have a *variable velocity.*

An automobile starting from rest gradually picks up speed until it reaches an operating speed of, let us say, 40 miles an hour. Its velocity during this period is variable. After reaching its operating speed, it may continue at a uniform velocity for a considerable length of time. In stopping, its velocity again becomes variable, gradually reaching zero. When a body moves with increasing or decreasing velocity, it is said to be *accelerated.* In increasing its velocity, as in starting, it is said to have

positive acceleration; when, as in stopping, its velocity is gradually decreased, it is said to have ***negative acceleration*** or ***retardation.***

Acceleration may be either uniform or variable, depending upon whether the velocity changes by equal or unequal amounts in equal units of time. Only uniform acceleration will be discussed in this section. The *acceleration* of a body is the rate of change of its velocity; thus:

$$\textit{Acceleration} = \frac{\textit{Change in velocity}}{\textit{time}},$$

$$\text{or } a = \frac{v_2 - v_1}{t}.$$

Acceleration is expressed in units of velocity per unit of time. Thus feet per second per second (abbreviated as ft./sec.^{2}) or centimeters per second per second (abbreviated as cm./sec.^{2}) are the commonly used units of acceleration.

EXAMPLE: A body starting from rest reaches a velocity of 30 ft. per sec. in 5 seconds. What is its acceleration?

SOLUTION:

$$a = \frac{v_2 - v_1}{t}$$

$$a = \frac{30 \text{ ft./sec.} - 0 \text{ ft./sec.}}{5 \text{ sec.}} = 6 \text{ ft./sec.}^2, \textit{ Ans.}$$

Formulas for Uniformly Accelerated Bodies Starting from Rest. For a body with uniformly accelerated motion, starting from rest, the velocity at any instant is the product of the acceleration and the time during which the body has been accelerated; or, $v = at$. The average velocity of such a body would be one-half the sum of the initial and final velocities. Since the initial velocity is 0, and the final velocity equals *at,* the ***average***

$$\textit{velocity} = \frac{0 + at}{2} = \tfrac{1}{2}at.$$

Since the distance traversed by a body in a given time equals the average velocity multiplied by the time, the formula for distance may be derived as follows:

$$\begin{aligned} \text{distance} &= \text{average velocity} \times \text{time} \\ s &= \tfrac{1}{2}at \times t \\ s &= \tfrac{1}{2}at^2 \end{aligned}$$

Using the above two formulas, it is possible to derive a relationship between ***velocity and distance*** without the time factor entering.

Since $v=at$, then $t=\frac{v}{a}$.

Substituting this value of t in $s=\frac{1}{2}at^2$, we obtain:

$$s=\frac{1}{2}a\left(\frac{v^2}{a^2}\right),\ \text{or}\ s=\frac{v^2}{2a}.$$

Solving, $v^2=2as$,

or $v=\sqrt{2as}$.

To find the distance (d) covered in *any* unit of time, we use the formula:

$$d=\frac{1}{2}a(2t-1).$$

Care must be taken in applying the above formulas to use consistent units.

Example 1: A train starts with a uniform acceleration of 5 ft. per sec.
(*a*) What will its velocity be at the end of 10 seconds?
(*b*) What distance will it have traveled during this time?
(*c*) How far did it go during the 10th second?

Solution: (a) $v=at$
$=5\times10$
$=50$ ft. per sec., *Ans.*

(b) $s=\frac{1}{2}at^2$
$=\frac{1}{2}\times5\times10\times10$
$=250$ ft., *Ans.*

(c) $d=\frac{1}{2}a(2t-1)$
$=\frac{1}{2}\times5\times(20-1)$
$=47.5$ ft., *Ans.*

Example 2: A projectile fired from a cannon 10 feet long leaves with a muzzle velocity of 2000 ft. per sec. (*a*) What is its acceleration inside the cannon? (*b*) How long does it take the projectile to travel the length of the cannon?

Solution:

$$v^2=2as$$
$$2000\times2000=2\times a\times10$$
$$a=\frac{4{,}000{,}000}{20}$$

$a=200{,}000$ ft. per sec. per sec., *Ans.*

$$s=\frac{1}{2}at^2$$
$$10=\frac{1}{2}\times200{,}000\times t^2$$
$$t^2=\frac{10}{100{,}000}=0.00010$$

$t=0.01$ second, or $\frac{1}{100}$th sec., *Ans.*

EXAMPLE 3: An automobile traveling at the rate of 40 miles per hour is stopped in 10 seconds by applying the brakes. (*a*) What was the negative acceleration in miles per hour per second, and in ft. per sec. per sec.? (*b*) What distance did the car move before coming to rest?

SOLUTION:

$$v = at$$

$$40 \text{ m.p.h.} = a \times 10 \text{ sec.}$$

$$a = \frac{40 \text{ m.p.h.}}{10 \text{ sec.}}$$

$$a = 4 \text{ m.p.h. per sec., } \textit{Ans.}$$

Since 1 mile=5280 feet, and 1 hour contains 3600 seconds, the velocity, $40 \text{ m.p.h.} = \frac{40 \times 5280}{3600} = 58.67$ ft. per sec. The acceleration in ft. per sec. per sec. therefore equals:

$$a = \frac{58.67 \text{ ft. per sec.}}{10 \text{ sec.}}$$

$$a = 5.867 = 5.87 \text{ ft. per sec. per sec., } \textit{Ans.}$$

$$s = \tfrac{1}{2}at^2$$

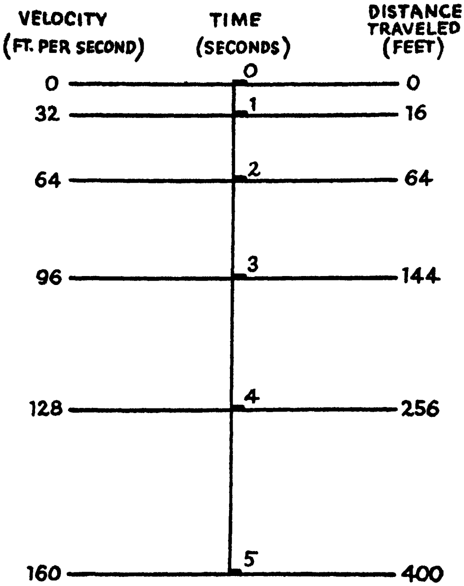
$$= \tfrac{1}{2} \times 5.87 \times 10 \times 10$$

$$= 293.5 \text{ ft., } \textit{Ans.}$$

Freely Falling Bodies. Before Galileo performed his famous experiment with freely falling bodies, it had been believed that heavy objects fell to the ground faster than light ones, and that the velocity of fall varied with the weight of a body. But Galileo showed experimentally that all freely falling bodies fall equal distances in equal lengths of time, provided that the resistance of the air was the same. The reason a piece of metal reaches the ground before a piece of paper when both are dropped from the same height is due entirely to the resistance of the air. If these objects are dropped in a vacuum, they will fall at exactly the same rate.

Freely falling bodies are a special case of uniformly accelerated motion. Since the attraction between the earth and a body near its surface is constant, a body falling freely under the action of the force of gravity will have a uniform acceleration. This acceleration of gravity has been experimentally determined to be about 32 ft. per sec. per sec., or 980 cm. per sec. per sec. While these figures vary slightly with the location, for most practical work they are sufficiently accurate.

For a freely falling body, then, the velocity at the end of the first second will be 32 ft. per sec.; at the end of the 2nd second, 64 ft. per sec.; at the end of the 3rd second, 96 ft. per sec.; etc. The distances covered are shown in the accompanying diagram. Thus a body dropped from the top of the Empire State Building, which is 1248 feet high, would reach the ground in slightly less than 9 seconds, and would have a final velocity of about 285 feet per second. Problems on freely falling bodies are solved like any other problems on uniformly accelerated motion. By substituting *g*, the acceleration due to gravity, for *a* in the formulas previously derived, the corresponding *formulas for freely falling bodies starting from rest become:*



Velocity of a Freely Falling Body

$$v=gt$$
$$s=\tfrac{1}{2}gt^2$$
$$d=\tfrac{1}{2}g(2t-1)$$
$$v^2=2gs$$

Example: A bomb is dropped from an airplane 2000 feet high. (*a*) How long will it take to reach the ground? (*b*) With what velocity will it strike? (Disregard air resistance.)

Solution:

$$s=\tfrac{1}{2}gt^2$$
$$2000=\tfrac{1}{2}\times 32\times t^2$$
$$t^2=2000/16=125$$
$$t=\sqrt{125}=11.2 \text{ seconds, } \textit{Ans.}$$

$$v=gt$$
$$=32\times 11.2$$
$$=358.4 \text{ ft. per sec., } \textit{Ans.}$$

Laws of Motion. Motion is at all times caused by the action of a force or a number of forces. The relationship of force and motion are given by Newton's three laws of motion.

Newton's First Law: Inertia. Newton's first law of motion deals with the property of matter known as *inertia.* A block of iron placed on a work bench remains there unless it is pushed or picked up. If a machine is at rest, it requires the action of some force to start it in motion; when once in motion, it tends to remain in motion until some force acts to stop it. Thus, after the power is shut off, a machine will remain in motion until frictional forces stop it. Likewise, we must apply the brakes in order to stop an automobile or train. When a train stops suddenly, the passengers in it tend to remain in motion and fall forward.

Newton expressed this property of resistance to change of motion as his first law: *A body continues in its state of rest or of uniform motion in a straight line, unless compelled by an outside force to change that state.*

Newton's Second Law: Momentum. The amount of motion which a body possesses depends both upon its mass and its velocity. Thus, it is difficult to stop the motion of a bullet fired from a gun primarily because of its great speed. On the other hand, it is difficult to stop a slowly moving railroad car because of its great mass. The quantity of motion which a body possesses is called its *momentum,* and is equal to the product of the mass of the body and its velocity:

$$\text{Momentum} = \text{mass} \times \text{velocity}.$$

Newton's second law states that: *The rate of change of momentum of a body is proportional to the force acting, and takes place in the direction in which the force acts.* Force=rate of change of momentum:

$$F = \frac{mv}{t}.$$

The greater the force acting, the greater the rate of change of momentum. If, on the other hand, the force remains the same but acts for a longer time, the momentum is accordingly increased. To "follow through" in a tennis or golf stroke results in the ball having a greater velocity because the force is applied for a longer time. From the second law it is possible to define force in terms of mass and acceleration.

Since $F = \frac{mv}{t}$, and $\frac{v}{t} = a$,

therefore, $F = m \times a$.

In other words the force acting on a body is proportional to its mass and its acceleration. This is another way of expressing Newton's second law.

In the English system, the absolute unit of force, the ***poundal,*** is that force which, when acting on a mass of 1 pound, will give it an acceleration of 1 foot per second per second. The more commonly used unit of force in the English system is the gravitational unit, the *pound force*. This is defined as the downward pull of gravity on a mass of 1 pound. The pound force is equal to 32 poundals.

Newton's Third Law: Reaction. Forces occur in pairs. When one body exerts a force upon another, the second body in turn reacts against the first body. The downward force exerted by a heavy machine on the floor is opposed by an equal upward force of the floor against the machine. When a gun is fired, the explosion in the gun barrel "kicks" the gun backward with the same momentum as it shoots the bullet forward. A swimmer pushes the water backwards in order to move forward. Likewise, the propellers of a ship push the water backward, and the reaction drives the ship forward.

Newton's third law of motion expresses this phenomenon of forces occurring in pairs as follows: *To every action there is an equal and opposite reaction.*

Centrifugal and Centripetal Forces. Circular motion, or rotation, plays an important role in most machinery. Rotating parts are acted upon by two forces, centripetal and centrifugal. Since a body in motion ordinarily tends to move in a straight line, a constant force acting toward the center of rotation is necessary to keep it moving in a circular path. Such a force is called a *centripetal force*. This force is balanced by a force pulling the body away from the center of rotation, called the *centrifugal force*.

When the centripetal and centrifugal forces are equal, and therefore balanced, the body will rotate in its circular path. When the centrifugal force exceeds and overcomes the centripetal force, the body will fly off at a tangent to the circular path in which it has been moving. It is this condition that accounts for a number of commonly observed phenomena, like the flying of water or mud from a moving wheel, or a whirling body flying off along a straight line when it breaks the string that holds it. Rotating parts, like grinding wheels or fly-wheels, sometimes break into fragments when revolving at a very rapid speed; this is simply another example of the centrifugal force exceeding and overcoming the cohesive forces in the wheels themselves.

The action of centrifugal force is usefully applied in a number of different machines used to separate liquids by rapid whirling. Such machines are known as *centrifuges*. The cream separator, used to separate cream from milk, is an example. Another is the milk and cream tester, used to determine the amount of butter fat in milk. In the refining of sugar, the sugar crystals are separated from the "mother liquor" in large centrifuges. In chemical laboratories, centrifuges are used to separate precipitates from filtrates. In laundries, clothes are dried in drying machines where the wet clothes are whirled in large vats with many perforations, through which the water particles are thrown off by centrifugal force.

31. SIMPLE MACHINES

Work and Its Measurement. If we push against a heavy load for a length of time without moving it any distance we may become tired in the effort, and commonly say we are doing work. Yet in the physical meaning of the term, we are not doing work at all. Work has a very special meaning in physics. A force does work upon a body only when it causes the body to move, i.e., when it overcomes a resistance. If we lift a heavy weight we are doing work against gravity. If we walk up a flight of stairs we are doing work in that we are lifting our own weight against gravity. A horse pulling a cart or an automobile engine furnishing the motive power for a moving automobile are both doing work against friction. No matter how great a force is exerted upon a body, *no work is done unless and until the body moves. Work* is the product of the force acting multiplied by the distance through which it acts. Expressed as a formula,

$$Work = Force \times Distance,$$

$$\text{or} \quad W = F \times D.$$

Units of Work. The units of work in common use are the foot-pound in the English system, and the gram-centimeter or kilogram-meter in the metric system. The *foot-pound* (ft.-lb.) is the amount of work done when a one pound force acts through a distance of one foot. Thus if we lift a 1-lb. weight a vertical distance of 1 ft. we do 1 foot-pound of work. If we lift the same 1-lb. weight through a distance of 10 feet, we are performing 10 foot-pounds of work. If we lift a 50-lb. weight 3 feet we are doing $50 \times 3 = 150$ ft.-lb. of work. How much work is done in lifting 25 pounds through a five-foot distance?

It is important to remember in calculating the work done, that we multiply *the force acting in the direction in which the body moves* by the actual distance covered. Thus, if it takes a 25-lb. force to push a 150-lb. machine part a distance of 20 ft., we are doing $25 \times 20 = 500$ ft.-lb. of work. The resistance we are overcoming is not the 150 lb.-machine part, but the friction between that part and the floor. The latter is overcome by the 25-lb. force exerted in the direction in which the part actually moves.

The Simple Machines. All machines, no matter how complex, can be broken down into the following six simple machines:

1. The lever
2. The wheel and axle
3. The pulley
4. The inclined plane
5. The wedge
6. The screw

More complex machines are combinations of several of the above units. The simple machines themselves have important everyday uses. Heavy machinery and machine parts, pianos, safes are easily lifted with the aid of a block and tackle, although it would be impossible to budge them without the use of such a system of pulleys. A pair of pliers, a claw hammer, a crowbar, and a wheelbarrow are all examples of the lever. The jack-screw used in lifting heavy loads is an application of the screw. The windlass and capstan are examples of the wheel and axle. Barrels too heavy to be lifted from the ground can be loaded on a truck or a platform by rolling them up a plank or skid; this is an application of the inclined plane.

In all these cases the machine enables us to overcome a force or resistance which we should probably be unable to overcome otherwise. The machine enables us to multiply the force or effort which we can exert, and to change its direction if necessary. In fact, by definition a machine is a device used to increase the effectiveness of a force either by:

(1) Multiplying the magnitude of the force applied,
(2) Changing the direction of the force applied, or
(3) Increasing the speed of an operation.

Mechanical Advantage. The force applied to a machine is called the *effort;* the force which the machine exerts or is capable of overcoming is called the *resistance.* The mechanical advantage is a measure of the effectiveness of the machine, or the increase in the resistance over the effort applied. By definition, the mechanical advantage of a machine is the ratio of the resistance to the effort:

$$\textit{Mechanical Advantage} = \frac{\textit{Resistance}}{\textit{Effort}},$$

$$\text{or } M.A. = \frac{R}{E}.$$

Thus, if the effort applied to any simple machine is 100 pounds and the resistance overcome is 500 pounds, the mechanical advantage would be

$$M.A. = \frac{R}{E} = 500/100 = 5.$$

Machines, we know, are also used to increase the speed with which a certain task can be accomplished. This is done at the expense of lessening the force which we apply. In a bicycle, for example, the rear wheel rotates faster than the sprocket wheel. In the case where increase in speed is the desired factor, the mechanical advantage is determined by the formula:

$$\textit{Mechanical Advantage} = \frac{\textit{Speed of resistance}}{\textit{Speed of effort}}.$$

The Principle of Work. In all machines a resisting force in one part is overcome by the effort applied in another part of the machine. The machine may allow us to apply a relatively small effort to overcome a large resistance. Yet in every case, if we neglect friction losses, the work put into the machine is equal to the useful work output of the machine. Under no circumstances can *more work* be done by a machine than is done on it. In nature we never get something for nothing. If a small effort overcomes a large resistance, the effort must move through a greater distance than that through which the resistance moves, so that the work input will in all cases equal the work output, plus whatever friction losses there may be. This general statement is called *the principle of work;* or

Work Input = Work Output

Since the input in foot-pounds is the effort multiplied by the distance through which it moves, and the output in foot-pounds is the resistance multiplied by the distance through which it moves, we may also write:

$$E \times D = R \times d,$$

wher E is the effort, D the distance through which the effort moves, R the resistance, d the distance through which the resistance moves. This formula is known as *the general law of machines.* We shall now see the application of this general law to each of the simple machines in the following pages.

The Lever. One of the simplest of the simple machines is the lever. Essentially the *lever* is a rigid rod or bar, free to move about a fixed point called the *fulcrum.* The fulcrum or point of support of the lever may be placed in different positions with relation to the resistance and effort, giving rise to three classes of levers.

In *first-class levers* the fulcrum is located between the resistance and the effort, as in the crowbar, the beam balance, and a pair of pliers or scissors. The ordinary beam balance has its two pans at equal distances from the fulcrum. The weights needed on each pan are equal, since equal weights will balance only when at equal distances from the fulcrum.

When, however, two unequal weights are suspended from a first-class lever, the weights will balance only if they are placed so that the smaller weight is at an appropriately greater distance from the fulcrum than the larger one is. Experiment shows that the two unequal weights will balance when the weight of the first multiplied by its distance from the fulcrum equals the weight of the second multiplied by *its* distance from the fulcrum. This, of course, is merely a proof of the principle of moments. We can then state the general law of the lever:

$$\text{Effort} \times \text{Effort arm} = \text{Resistance} \times \text{Resistance arm},$$

$$\text{or,} \quad E \times D = R \times d.$$

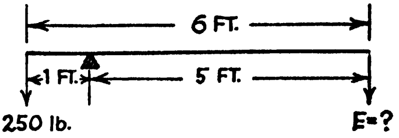
Thus, if a 2-lb. weight (resistance) is placed 1 foot from the fulcrum, it will be balanced by a 1-lb. weight (effort) placed 2 feet from the fulcrum.

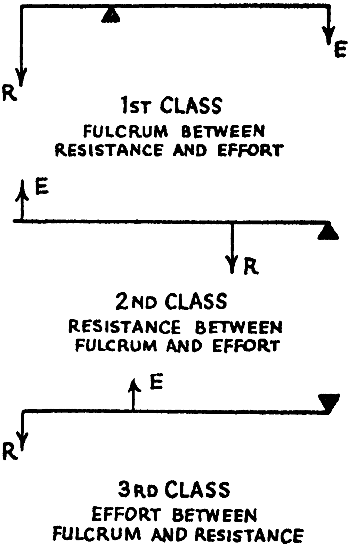
$$1 \text{ lb.} \times 2 \text{ ft.} = 2 \text{ lb.} \times 1 \text{ ft.}$$
$$2 \text{ ft.-lb.} = 2 \text{ ft.-lb.}$$

In other words, the moments of the two forces are equal.

Example: A crowbar 6 feet long is used to lift a weight of 250 lb. If the fulcrum is placed 1 ft. from the weight, how much effort must a man exert if he applies his effort at the end of the crowbar?

$$E \times D = R \times d$$
$$E \times 5 \text{ ft.} = 250 \text{ lb.} \times 1 \text{ ft.}$$
$$5E = 250$$
$$E = \frac{250}{5} = 50 \text{ lb., } \textit{Ans.}$$




E
R
1st CLASS
FULCRUM BETWEEN RESISTANCE AND EFFORT
E
R
2nd CLASS
RESISTANCE BETWEEN FULCRUM AND EFFORT
E
R
3rd CLASS
EFFORT BETWEEN FULCRUM AND RESISTANCE

The Three Classes of Levers

In *second-class levers* the resistance is located between the fulcrum and the effort. The wheelbarrow and the nutcracker are common examples of this type of lever. In *third-class levers* the effort is placed between the resistance and the fulcrum, as in the case of sugar tongs, grass shears, safety valve rods, and tweezers. The general law of the lever developed above applies to all classes of levers.

Mechanical Advantage of the Lever. The mechanical advantage of a lever, like that of any other machine, is the ratio of the resistance to the effort. However, from the general law of the lever it will be seen that the resistance and effort are to each other inversely as their distances from the fulcrum; or,

$$\frac{\textit{Resistance}}{\textit{Effort}} = \frac{\textit{Effort arm}}{\textit{Resistance arm}}.$$

Therefore, we can also compute the mechanical advantage of a lever by using the following relationship:

$$\textit{Mechanical Advantage} = \frac{\textit{Effort arm}}{\textit{Resistance arm}}.$$

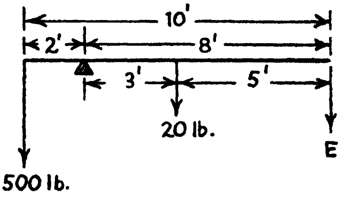
Thus, in the crowbar problem solved above, the distances of the effort and the resistance from the fulcrum are 5 ft. and 1 ft., respectively. The mechanical advantage would therefore be $\frac{5}{1}$, or 5. In other words, for each pound of effort exerted it is possible to overcome a resistance of 5 pounds. This agrees with our results, since we found a 50-lb. effort needed to overcome a 250-lb. resistance.

We have been assuming a *weightless lever* in our discussion thus far. However, the lever itself has weight, in many cases large enough to be considered in solving a problem. Where the weight of the lever is to be taken into account, we consider it concentrated and acting at the center of gravity of the lever. The center of gravity is that point on the lever where it may be balanced by a single upward force. The problem may then be solved by the principle of moments, namely: for equilibrium, *the sum of the clockwise moments must be equal to the sum of the counterclockwise moments.*

EXAMPLE: A uniform bar 10 ft. long and weighing 20 lb. is used to lift a weight of 500 lb. The fulcrum is placed 2 ft. from the weight. What effort is needed at the other end of the bar? Since the bar is uniform, the total weight may be considered as concentrated at its center, in other words, 5 feet from either end. The forces acting are then shown in the accompanying figure:

Clockwise Moments=
Counterclockwise Moments

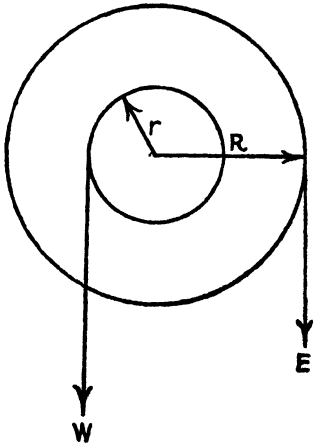
$$(E\times 8)+(20\times 3)=(500\times 2)$$
$$8E+60=1000$$
$$8E=1000-60=940$$
$$E=117.5 \text{ lb., } \textit{Ans.}$$



The Wheel and Axle. This consists essentially of a large wheel or crank rigidly fastened to an axle or drum so that both turn about a common axis. The effort is applied at the circumference of the wheel to overcome the resistance which is applied at the circumference of the axle. The wheel and axle is really a modified form of the lever. The radius of the wheel (R) is the effort arm, and the radius of the axle (r) is the resistance arm. By the principle of moments, the effort multiplied by the effort arm must be equal to the resistance multiplied by the resistance arm; thus

Effort$\times$*Radius of the wheel*=
Resistance$\times$*Radius of the axle,*

or $E\times R=W\times r$.



The Wheel and Axle

The *mechanical advantage of a wheel and axle* can be derived from the above equation as follows:

$$\frac{W}{E}=\frac{R}{r}.$$

In other words, the ratio of the resistance to the effort of a wheel and axle, which by definition is its mechanical advantage, is equal to the ratio of the radius of the wheel to the radius of the axle, or:

$$\text{Mechanical Advantage}=\frac{\text{Radius of wheel}}{\text{Radius of axle}}.$$

Since both the diameter and the circumference are directly proportional to the radius, we may also determine the mechanical advantage as follows:

$$M.A.=\frac{\text{Diameter of wheel}}{\text{Diameter of axle}}=\frac{\text{Circumference of wheel}}{\text{Circumference of axle}}.$$

EXAMPLE: The radius of the wheel in a wheel and axle unit is 1 foot; the radius of the axle is 3 inches. What effort must be applied to raise a resistance of 100 pounds?

SOLUTION: $E\times R=W\times r$

$E\times 12$ inches$=100$ lb.$\times 3$ inches

$12\,E=300$

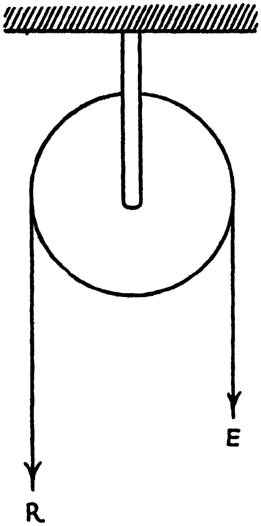
$E=25$ lb., *Ans.*

This can easily be checked by noting that the mechanical advantage of the wheel and axle (which is the radius of the wheel divided by the radius of the axle) is 12 inches divided by 3 inches, which is equal to 4. Thus for each pound effort we can lift 4 pounds; with a 25-lb. effort we can therefore lift 100 pounds.

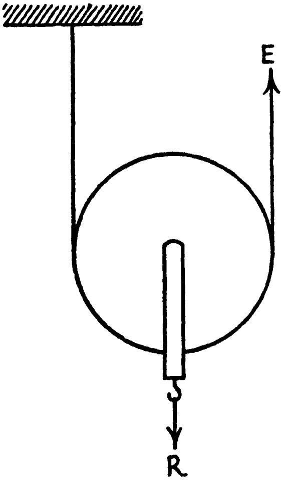
The wheel and axle has many practical applications. Thus the capstan used on ships, the windlass used in drawing water from a well, the steering wheel on a ship or on an automobile, the door knob, the meat grinder, the grinding wheel, etc., are all applications of the wheel and axle. Gears and shafts, cone pulleys, and clockwork mechanisms are also common examples.

Pulleys. A *pulley* is a grooved wheel or sheave free to rotate about an axle mounted in a frame or block. If the axis of the pulley does not change its position, we call it a *fixed* pulley; if the pulley moves with the weight lifted, we call it a *movable* pulley.

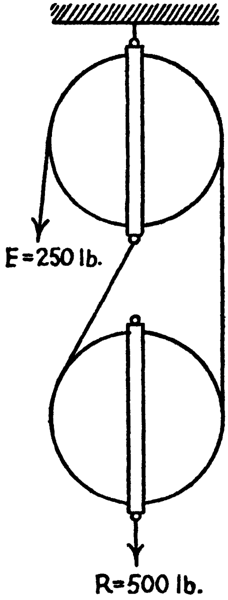
The mechanical advantage of a *single fixed pulley* is 1; that is, the effort exerted to lift a weight must be equal to the weight itself. This type of pulley is used mainly to change the direction of the force applied. It is more convenient to pull downward to lift a body than it is to pull upward; in other words, to lift from the ground it is more convenient to pull from some point overhead instead. The fixed pulley is a special type of first-class lever with the resistance arm equal to the effort arm.



Single Fixed Pulley
M.A.=1



Single Movable Pulley
M.A.=2

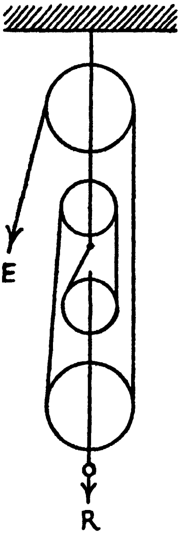


Single Fixed and
Single Movable
Pulley
M.A.=2

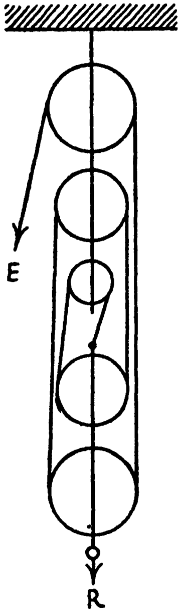
In the *single movable pulley,* the pulley itself is lifted with the weight. The load is supported by two strands of rope. In other words, half the load is supported by each rope, so that the *effort required is only one-half the weight.* The mechanical advantage of a pulley of this type is 2. Thus to lift a weight of 100 lb., we need only exert an effort of 50 lb. However, we move the effort *through twice the distance* that the resistance moves, so that the work input and the work output are equal, as we should expect from the principle of work. The single movable pulley is a special type of second-class lever with the effort arm twice the resistance arm.

Pulley Systems. Pulleys are usually used in a set or combination of fixed and movable pulleys, often called a block and tackle. Such pulley combinations are used in lifting safes, pianos, etc. If we combine a single fixed and single movable pulley we can get the mechanical advantage of 2 and have the convenience of a downward pull as well. Thus a 500-lb. weight can be supported by an effort of 250 lb. With a system of two fixed and two movable pulleys the weight is supported by four strands of rope. If we disregard the weight of the movable pulleys, the pull on each rope would be one-fourth of the load, so that the effort required would be one-fourth the resistance, giving us a mechanical advantage of 4.

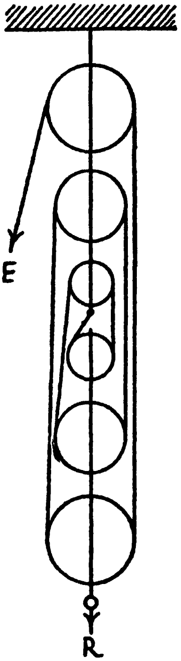
The mechanical advantage of a combination of pulleys is in all cases equal to the number of strands of rope supporting the resistance. What would the mechanical advantage be in the following pulley systems?



TWO FIXED
AND TWO MOVABLE
MA=?

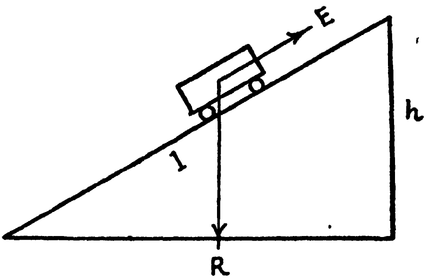


THREE FIXED
AND TWO MOVABLE
MA=?



THREE FIXED
AND THREE MOVABLE
MA=?

The Inclined Plane. The *inclined plane* is any plane surface making an angle with the horizontal. Instead of lifting a body vertically, it can be rolled, dragged, or pushed up the inclined plane with far less effort. Thus, heavy barrels are rolled up sloping planks or skids. Ramps are used to get a loaded truck into a building above or below the street level. Streets



The Inclined Plane

$$\frac{R}{E}=\frac{l}{h}$$

and roadways are graded. All these are examples of the inclined plane in use.

The law of the inclined plane may be derived from the principle of work. According to this principle, the work input of any machine is equal to the work output. In rolling a weight up an inclined plane, the work done equals the effort exerted multiplied by the length of the plane; or, $E\times l$. The weight meanwhile has been lifted the height of the plane, or h feet, a work output of $W\times h$. This is the equivalent of lifting the weight W through a vertical distance of h feet. Now since *Work Input = Work Output,* we have:

Effort × length of plane = Resistance × height of plane,

$$\text{or } E\times l=R\times h.$$

This is the law of the inclined plane. From it we can determine the mechanical advantage of the inclined plane as follows:

$$\frac{\textit{Resistance}}{\textit{Effort}}=\frac{\textit{length of plane}}{\textit{height of plane}},$$

$$\text{or } \frac{R}{E}=\frac{l}{h}.$$

The mechanical advantage of an inclined plane is, then, equal to the ratio of its length to its height.

Example: An inclined plane 20 ft. long and 5 ft. high is used to raise a 200-lb. weight. What effort is required, and what is the mechanical advantage of the plane?

Solution:

$$E\times l=R\times h$$

$$E\times 20=200\times 5$$

$$E=\frac{1000}{20}=50 \text{ lb., } \textit{Ans.}$$

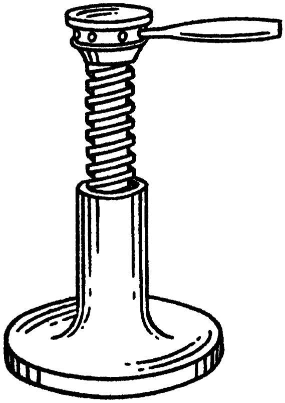
$$\text{Mechanical Advantage}=\frac{l}{h}=\frac{20}{5}=4, \textit{ Ans.}$$

While the effort required to lift the weight is only ¼ the weight itself, it must be applied over a distance four times as great as the weight is lifted, so that the work put into the machine is equal to the work output in accordance with the principle of work. A work output of 1000 ft.-lb. requires a work input of 1000 ft.-lb., neglecting friction.

The Wedge. The *wedge* is a special form of inclined plane. Instead of pulling the load up an incline as is done with the inclined plane, we drive the wedge under the load. The wedge can overcome extremely large resistances. Most cutting and piercing instruments, such as axes, chisels, knives, nails, etc., are applications of the wedge. Wedges are used to split logs of wood. Cams, used in machines to lift valve rods or other machine parts, are rotating wedges. Because of the considerable variations in friction when using the wedge, its mechanical advantage varies and there is no definite relationship between the effort required to overcome a given resistance.

The Screw. In lifting very large weights or exerting extremely large forces for other purposes, the screw is widely used because of the large mechanical advantage obtained with it. Thus the jackscrew can lift a building, raising it off its foundations. Screws are used in the large compression and tension testing machines. Presses of various kinds, the bookbinder's press for example, make use of the screw. The machinist's vise is a combination of the screw and the lever. The screw, of course, has other uses as well. On the lathe the lead screw is used primarily to transmit motion. In the micrometer caliper, it is used as a precision measuring device. Its widest use is as a fastening and holding device.

The screw itself is a cylinder whose outer surface has a helical projection called the thread. It can be considered as an inclined plane wrapped around a cylinder. The *pitch* of a screw is the distance between two successive threads measured parallel to the axis. For each complete revolution, the screw advances a distance equal to its pitch. The screw is ordinarily turned by a rod attached to or fitting into the cylinder. A small effort exerted at the end of the rod and acting through its circumference results in a large lifting force acting through a very small distance. The work input is the effort multiplied by the distance the effort travels. For each complete turn of the screw the resistance is raised a distance equal to the pitch, so that the work output equals the resistance multiplied by the pitch of the screw.



Jackscrew

$$\text{Since}\quad \text{work input} = \text{work output},$$

$$\mathit{Effort} \times \mathit{Circumference\ turned\ through} = \mathit{Resistance} \times \mathit{pitch},$$

$$\text{or}\quad E \times 2\pi r = R \times p.$$

The mechanical advantage of the screw is the ratio of the circumference through which the effort travels in one revolution, to the pitch of the screw; or

$$\frac{R}{E} = \frac{2\pi r}{p}.$$

Example: A jackscrew having a pitch of 1/4″ and a handle 2 ft. long has an effort of 20 lb. applied to it. How great a load can it lift?

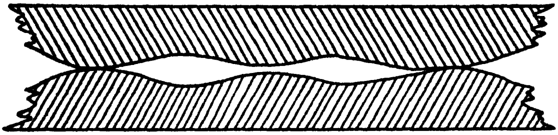
Solution:

$$E \times 2\pi r = R \times p$$

$$20 \times 2 \times 3.14 \times 24'' = R \times 1/4$$

$$R = 12{,}048 \text{ lb., } \mathit{Ans.}$$

Friction. If we try to slide or roll one object upon another, we find that some effort, often a considerable amount, is necessary. We say this effort is needed to overcome the friction between the two bodies. *Friction is the force resisting the motion of one body over another.* The two main types of friction are known as *sliding friction* and *rolling friction.* In the former, the surfaces of the two bodies in contact slide over each other, while in the latter, one body rolls over the other.



Sliding Friction—Flat Surfaces Highly Magnified

In some cases friction is a very useful phenomenon. Brakes and clutches on different machines depend upon friction for their operation. It is friction too that prevents belt slippage on pulleys, and that holds threaded parts together. Bolts and nuts, screws, screw-jacks are held in place because of friction. Friction between the wheels of a train and the rails, or an automobile and the ground, is essential for traction. Without it we have slip of the wheels, the car skids and is to a certain extent out of control. As a matter of fact, even walking would be difficult without friction. Walking on ice or on a highly polished floor where friction is at a minimum is ample proof of this.

On the other hand, friction may also be a liability. Thus friction in the bearings of any machine results in heat, with an accompanying loss in work output; furthermore, if the bearings become overheated an actual breakdown of the machine may result. In such cases friction is definitely undesirable, and efforts are made to reduce it as much as possible. Some of these methods will now be discussed.

Factors Affecting Friction. When apparently smooth surfaces are examined under a magnifying glass, it will be found that they are in reality fairly rough, and that they consist of many minute ridges or projections. It is the rubbing of these surface irregularities and their deformation which cause friction.

Friction between two bodies depends upon several factors, among them the speed, the nature of the surfaces in contact, and the load applied.

1. Friction is greater when starting a body in motion than when the body is actually sliding over another. Though it is generally stated that friction does not depend to any great extent upon velocity, there is a definite decrease in friction as the speed increases.

2. Friction increases with an increase in the load being pulled or the pressure being exerted. This increase in friction is in almost direct proportion to the increase in load; the greater the weight being pulled, the greater the force needed to overcome friction.

3. Friction is independent of the surface areas in contact. As long as the weight is the same, the area of the surface in contact has virtually no effect on the friction force.

4. Friction depends on the nature of the rubbing surfaces if these are non-lubricated. Two hard, polished surfaces will have less friction resistance than two rough unpolished surfaces.

These principles are not too exact or rigid. Because of the many factors which influence friction, the different attendant conditions, the varieties of types of surfaces, there are no rigid laws of friction. These are, however, good approximations, and for practical purposes can be said to hold, in general, for dry surfaces.

Reduction of Friction. Several methods are in common use to decrease friction and frictional losses. Ball and roller bearings are used to substitute rolling friction for sliding friction, and to reduce the surface areas of the rotating parts in contact. With ball bearings, the contact area is nominally a point, while with roller bearings the contact area is reduced to a straight line.

Where large areas are in contact, the practice is to make the contact surfaces of unlike metals. Steel shafts, for example, run in brass or Babbitt metal bearings. Unlike metals are used so that the surface projections of the contact areas do not fit into each other.

Another method for reducing friction is to make the rubbing surfaces as smooth as possible. Honed, ground, or lapped surfaces are often used.

The most common method of reducing friction is by lubricating the contact surfaces with oil, grease, or graphite. These materials spread out in a thin film and serve to prevent actual contact between the rubbing surfaces. It is important that the proper lubricant be selected in each case. Different rubbing surfaces and different operating conditions require different types of lubricants. In an automobile, for example, we use several types of lubricants: a cylinder oil in the crankcase for cylinder wall lubrication; grease in the wheel bearings; a heavy semi-solid lubricant for the differential; graphite paste for the leaf springs; etc.

The factors affecting friction which we discussed above applied to dry surfaces. With lubricated surfaces, other considerations enter. Friction no longer depends entirely on the nature of the rubbing surfaces, but on the type and effectiveness of the lubrication. For lubricated surfaces, friction does not vary with the pressure. It does, however, increase with increased surface areas, and with increased velocities.

Coefficient of Friction. As stated earlier, for non-lubricated surfaces friction increases with an increase in load or pressure; the greater the load, the greater the frictional resistance. The ratio of the friction to the force causing it is known as the *coefficient of friction;* or

$$\textit{Coefficient of friction} = \frac{\textit{Friction}}{\textit{Force}}.$$

The coefficient of friction varies with the type of surfaces in contact. Thus the coefficient of friction for metal surfaces in contact will vary from 0.15 to 0.20; for stone surfaces, from 0.50 to 0.65; for wooden surfaces, from 0.25 to 0.50. For any two given materials under similar operating conditions, however, the coefficient of friction is essentially constant.

EXAMPLE: If a force of 50 lb. is needed to pull a 150-lb. oak chest across a hardwood floor, what is the coefficient of friction between the two surfaces?

$$\text{Coefficient of friction} = \frac{\text{Friction}}{\text{Force}} = \frac{50}{150} = .33, \textit{Ans.}$$

Efficiency of a Machine. In actual practice no machine ever converts all the energy put into it into useful work. There are at all times some losses due to friction no matter how well the machine is designed or how carefully it is operated. The output, therefore, is always somewhat less than the input. The efficiency of a machine is a measure of the effectiveness of the machine in performing useful work. By efficiency is meant the ratio of the work done *by* a machine (the output) to the work done *on* the machine (the input); or

$$\textit{Efficiency} = \frac{\textit{Output}}{\textit{Input}}.$$

Efficiency is expressed as a per cent, and obviously no machine under practical conditions can have an efficiency of 100%. The closer we approach this limit, however, the more effective the machine is in converting the energy input.

Example: Using one of the simple machines, a man exerting a force of 50 lb. acting through a distance of 120 ft. lifted a weight of 500 lb. through a distance of 10 ft. What was the efficiency of the machine?

Solution:

$$\text{Work output} = R \times d = 500 \times 10 = 5000 \text{ ft.-lb.}$$

$$\text{Work input} = E \times D = 50 \times 120 = 6000 \text{ ft.-lb.}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}}$$

$$\text{Efficiency} = \frac{5000 \text{ ft.-lb.}}{6000 \text{ ft.-lb.}} = .83\tfrac{1}{3} = 83\tfrac{1}{3}\%, \textit{ Ans.}$$

In other words, the useful work output of the machine is 83⅓% of the work done on the machine.

All the simple machines studied in this section have losses due to friction and other causes. The greater the losses, the larger must be the work input to overcome a given resistance, and the smaller the efficiency of the machine. While there are few friction losses in levers, overcoming the weight of the lever itself causes a loss in useful work output for a given input, with a consequent loss in efficiency. In pulley systems energy is lost in friction at the bearings and where the ropes rub against the sheaves, as well as in lifting the movable pulleys. The wheel and axle likewise has its efficiency decreased because of friction of the bearings and ropes. There are friction losses in moving a body up an inclined plane, requiring greater work input. In the screw and in the wedge frictional forces must also be overcome.

In more complex machines the losses are even greater. The effectiveness of our larger machines and engines in converting the energy supplied to them into useful work can be compared by an analysis of their efficiencies. The simple non-condensing steam engines used primarily in steam locomotives have efficiencies of from 6% to 8%. Steam turbines and condensing steam engines used in power plants have efficiencies ranging from 15% to 27%. The efficiencies of the gasoline engines used on our airplanes, automobiles, etc., run from about 22% to 28%. The Diesel engine used increasingly in locomotives, ships, trucks, and smaller power plants is about the most efficient of our heat engines. Its efficiencies average from 32% to 38%. Electrical machines have the highest efficiencies, running from about 80% in large motors and generators to as high as 99% in transformers.

32. ENERGY, WORK, AND POWER

Work and Time. In the section on work we found that work was done on a body only when a force acting upon it caused the body to move a certain distance. The work done was equal to the product of the force itself and the distance through which it acted, and was measured in foot-pounds or gram-centimeters. Time did not enter into the amount of work done. Thus if by using a block and tackle a 1000-pound weight was lifted a distance of 50 feet in 5 minutes, we were doing the same amount of work as if we lifted this weight through the same distance in 1 minute, or 10 minutes, or 1 hour. The work done in each case was 1000 lb.×50 ft., or 50,000 ft.-lb. Yet the *rate* at which the work was done was in each instance quite different. The power required to do the job in 1 minute was 5 times that required to do it in 5 minutes. It is with *power* that the time element enters.

Power. *Power is the rate of doing work.* It is equal to the work done divided by the time taken in doing it;

$$\text{Power} = \frac{\text{Work}}{\text{Time}}.$$

The unit of power is the quantity of work done in a unit of time. Thus in the English system, it is the foot-pound per second or the foot-pound per minute. The power required to lift the 1000-lb. weight mentioned above a distance of 50 feet in 5 minutes (300 seconds) would be:

$$\frac{1000\times 50}{300} = 166\tfrac{2}{3} \text{ ft.-lb. per second.}$$

Horsepower. To know how quickly a machine can perform a certain task or piece of work is extremely important. Ordinarily we wish to know not only whether the machine can do the work, but also how quickly it can do it. Machines then are rated in terms of power, and in selecting a machine to meet our needs we must know the power requirements. Since most industrial machines do work in much greater quantities than that represented by the ft.-lb. per sec., a larger unit of power and the ft.-lb. per sec. is desirable. The horsepower is therefore commonly used. The *horsepower* (Hp.) is the performance of work at the rate of 33,000 ft.-lb. per min., or 550 ft.-lb. per sec. Therefore to find the horsepower of a machine, we merely divide the work done per minute by 33,000, or the work done per second by 550. Thus

$$\text{Horsepower} = \frac{\text{Work}}{33{,}000\times\text{time (minutes)}},$$

or

$$\text{Horsepower} = \frac{\text{Force}\times\text{Distance}}{33{,}000\times\text{time (minutes)}}.$$

If the time is expressed in seconds, the following formula is used:

$$\textit{Horsepower}=\frac{\textit{Force}\times\textit{Distance}}{550\times\textit{time (seconds)}}$$

Example: An electric hoist raises a machine weighing 500 pounds 55 feet in 10 seconds. What horsepower motor is required?

Solution:

$$\text{Hp.}=\frac{\text{Force}\times\text{Distance}}{550\times\text{time (sec.)}}$$

$$=\frac{500\text{ lb.}\times 55\text{ ft.}}{550\times 10\text{ sec.}}=5\text{ hp., } \textit{Ans.}$$

Electrical power is measured in *watts* and *kilowatts*. A *kilowatt* is equal to 1000 watts. One horsepower is equal to 746 watts, or roughly ¾ of a kilowatt. These units will be discussed at greater length in the chapter on electricity.

Transmission of Power. Power must be transmitted from the point of origin to the point of use. Where it is generated at a central station power plant, whether of the hydro-electric or steam-electric type, electrical transmission is often used to supply power to points many miles away.

Within any plant, however, there may be driving devices like the steam engine or turbine, the gasoline engine, Diesel engine, or the electric motor, all of which are used to provide power for the other machines in the plant—presses, machine tools, and manufacturing machinery of all types. There must then be some method of transmitting the power from the *driving* to the *driven* machines. If the driving and driven machines are not too far apart, shafting, pulleys, belts, chains, or gears may be used to transmit the power. When a belt or chain is used, it passes over two pulleys; the pulley on the drive shaft is known as the driving pulley, and it actuates the driven pulley by means of the belt or chain. With gears, there is direct contact between the driver and the driven through the meshing of the respective gear teeth. The speeds in both cases are inversely proportional to the diameters. If the small pulley were ¼ the diameter of the large pulley, for example, it would turn 4 times as fast.

Energy. The energy a body possesses is its capacity for doing work. Thus the water behind a dam, an electric storage battery, the elevated hammer of a pile driver or a steam hammer, or a wound-up spring are all able to do work and are said to possess energy. Work and energy are very closely related and are sometimes confused. We measure energy in the same units as we do work. The two entities differ in that work represents an actual accomplishment, the action of a force through a distance; on the other hand, energy represents the ability to do a task rather than the actual performance.

A body possesses energy only because work has previously been done upon it. This stored work becomes available for future use. Among the different forms of energy are heat, light, sound, and electricity. When we examine the various manifestations of energy, we find that they can be grouped in two main classes:

1. Energy due to the position or condition of a body, known as *potential energy.*
2. Energy due to the motion of a body, known as *kinetic energy.*

Potential Energy. When a body is lifted to any height we are doing work against gravity. The body in its elevated position is able to do work in return if it is allowed to fall. It is precisely on this principle that the pile driver and steam hammer operate. Likewise a wound-up spring can turn a clock or a set of gears in unwinding and returning to its original position. In all these cases, energy has been stored in the bodies because of the work that has previously been done upon them.

In many instances potential energy is measurable. Since the potential energy of a body is equal to the work done on the body in bringing it to its elevated position or stressed condition, we can use the following formula in calculating the potential energy of an elevated body.

$$\textit{Potential Energy} = \textit{Weight of body} \times \textit{Vertical Distance through which it has been lifted,}$$

$$\text{or} \quad P.E. = W \times h.$$

For example, in lifting the hammer of a pile driver which weighs 2000 lb. a distance of 15 feet, 2000×15 or 30,000 ft.-lbs. of work are required. The hammer in its elevated position, therefore, has 30,000 ft.-lbs. of potential energy stored in it. The hammer is able to do that much work in return when it is released.

Kinetic Energy. The energy which a body possesses because of its *motion* is known as kinetic energy. Thus a bullet fired from a gun is able to do work because of its rapid motion. A falling weight, a moving automobile, a waterfall all possess kinetic energy. Each of these moving bodies is capable of performing a definite amount of work against any resisting force which stops it. The energy it possesses depends upon two factors, its *mass* and its *velocity.* A loaded 10-ton truck, for example, crashing into a parked car can do much more damage than a small passenger car traveling at the same speed because of its larger mass.

On the other hand, the energy possessed by a moving bullet, a cannonball or a baseball is almost entirely due to its velocity. Drop a bullet from one hand to the other, and very little energy is available. Fire the same bullet from a rifle, and because of its great speed, the bullet becomes the effective instrument it is.

The kinetic energy of a body may be computed by the following formula:

$$K.E. = \tfrac{1}{2}mv^2,$$

where m is the mass measured in pounds, v is the velocity measured in feet per second, and $K.E.$ is the kinetic energy expressed in *foot-poundals*. To convert this into foot-pounds we merely divide by 32. To calculate the kinetic energy in foot-pounds directly, the following formula is used:

$$K.E. = \frac{Wv^2}{2g},$$

where v is the velocity expressed in feet per second, W is the weight in pounds, and g is the acceleration of gravity which equals 32 feet per second per second. The two formulas are equivalent, since $m = \frac{W}{g}$.

Example: A projectile weighing 24 lbs. is fired from a gun with a muzzle velocity of 1600 feet per second. What is its kinetic energy in foot-pounds?

Solution:

$$K.E. = \frac{Wv^2}{2g}$$

$$K.E. = \frac{24 \times 1600 \times 1600}{2 \times 32}$$

$$= 960{,}000 \text{ ft.-lb., } \textit{Ans.}$$

Transformation of Energy. Energy is constantly being changed from one form to another. When the hammer of a pile driver is at its highest position it has a maximum potential energy and, since it is at rest, no kinetic energy. As soon as we release the hammer it starts losing height and gaining speed. In other words, it loses potential energy and gains kinetic energy. Just before striking the pile it has its maximum speed and highest kinetic energy, but no potential energy. The potential energy of the hammer in its elevated position has thus been converted into the kinetic energy of the moving hammer which is directly effective in driving the pile into the ground.

In the central station power plant which plays so vital a role in modern industry we encounter a far more complex series of energy transformations. It is here that electricity is generated by large generators or dynamos, and from here that it is transmitted over systems of transmission lines to the point of use. What takes place in the power plant from an energy standpoint? Coal or oil is burned in furnaces. The potential chemical energy of the fuel is converted into heat energy, which in turn is used to heat water and turn it into steam at high pressure. This is a change from heat energy to potential energy. The steam is then piped to the steam turbines or engines where the kinetic energy of its moving particles is effectively used in driving machines; this is a change to mechanical energy. The turbines or engines drive electric generators transforming the mechanical energy into electrical energy. This electrical energy is transmitted over transmission lines to the point of use where it may run motors (mechanical energy), heat electric heaters, irons, furnaces (heat energy), light buildings (light energy), run radios (sound energy), etc. These are typical illustrations of transformation of energy from one form to another.

Conservation of Energy. Great nineteenth-century scientists such as Mayer, Helmholtz, and Joule found that while energy can readily be changed from one form to another, it cannot however be created or destroyed. Where one form of energy seems to disappear, it actually reappears in some other form in an equal amount. The total energy available in the universe is a fixed quantity. It changes only in the forms in which it appears, and in its distribution. This has been stated in one of the most significant scientific generalizations ever made, *the law of the conservation of energy: Energy can neither be created nor destroyed, but can be changed from one form to another.*

Although modern physicists have found that under certain special conditions it is possible to convert matter into radiant energy and vice versa, the law of the conservation of energy still holds in the field of practical experience, and is still probably the most important principle in science and technology.

Dissipation of Energy. While it is true that energy cannot be destroyed, the form in which it exists may be such as to render it unavailable for further use. For example, in any engine some of the energy input is used to overcome friction in the bearings and the moving parts. This energy is converted into heat which is radiated to the surrounding atmosphere. Some of the mechanical energy has thus been converted into heat energy, and has become unavailable since we can no longer change this heat energy back into its previous form. In such a case we say the energy has been dissipated. Similarly when we burn a ton of coal in the furnace of a power plant, not all the heat value in the fuel is used to heat the water in the boilers and convert it to steam, and certainly not all of it is effective in driving the engines. Some of the heat is lost in radiation from the furnace; a goodly portion escapes with the flue gases; there are further losses from radiation in the steam pipes; additional friction losses occur in the engine itself; hence it is clear that only a relatively small portion of the energy originally available is finally delivered as useful work, all the other heat energy having become unavailable for further use.

While we can account for almost every thermal unit of energy we started out with in the coal, we end with the bulk of it in such a form as to be of very little use. Lord Kelvin, a brilliant nineteenth-century British scientist, was among the first to prove that in every energy transformation some of the energy is dissipated and becomes quite useless or otherwise unavailable. Our energy resources, then, may well become exhausted despite the fact that energy cannot be destroyed. It is for this reason that we must be so careful with our natural resources, our supplies of coal, oil and other fuels, as well as our water resources and other sources of energy.

33. HEAT AND LIGHT

Nature of Heat. Heat is a form of energy arising from the motion or vibration of the molecules within a substance. All bodies are made up of very small particles called *molecules* which are in a state of rapid motion. According to the kinetic theory of heat, when the speed of these molecules is increased, the body becomes hotter; when the molecular speed is decreased, the body becomes cooler. In other words, heat may be considered as the kinetic energy of the molecules of which a substance is composed.

Experimental proof of this view is given by the fact that a body expands when it is heated. The increased energy of the molecules results in their colliding more vigorously. This forces the molecules further apart if they are not confined in a container. This behavior is exhibited by all substances, whether solids, liquids, or gases. If, however, a confined volume of any gas is heated, its pressure is increased. The molecules of the gas, moving with greater velocity, strike the walls of the container with greater force and cause the increase in pressure. As the gas is cooled, the pressure decreases. Any means of increasing the molecular motion of a body is, therefore, a source of heat. Other forms of energy are readily convertible into heat.

Sources of Heat. Heat may be obtained from a variety of sources. The most important source is the sun itself. It is when the sun's heat received in a given locality decreases that buildings must be heated, warmer clothing must be worn, etc. Motors to utilize directly the heat of the sun are being experimented with.

The interior of the earth is a source of heat that must in many cases be reckoned with. The deeper we descend in mine shafts, the warmer they become. Volcanoes and hot springs are other indications of this heat source.

Heat is also produced by *friction.* The bearings of machinery, if not properly lubricated, become hot; often the heat is so great that they are actually damaged. Rubbing two bodies together results in their becoming warmer. Filing metal, sawing wood or metal, drilling a hole all result in the development of heat. In fact, in every case where work is done against friction, heat is produced.

Impact, or the rapid colliding of two objects, is still another source of heat. Hammer a nail or an iron rod vigorously and it becomes hot. Percussion caps on cartridges are exploded by the heat developed upon impact.

In a Diesel engine no spark plug and ignition system is used to ignite the oil as it is injected into the cylinder. The air in the cylinder becomes so hot upon being compressed, that the heat developed is sufficient to ignite the fuel as soon as it is injected. *Compression* then is another source of heat. The rubber tube of an automobile tire pump becomes hot when the pump is in use because of the air being warmed in the process of compression.

By far the most important source of heat industrially and in everyday life is the *combustion* or burning of coal, oil, gas, wood and other fuels. This is the method used to evolve the heat needed in power plants, industrial and home heating, automobile and airplane engines, other transport equipment, etc. Production of heat by this method is caused by the chemical combination of the fuel with the oxygen in the air, or, as it is known chemically, by oxidation.

Chemical action is also a source of heat in reactions other than oxidation. Thus, if cold water is added to lime, heat is evolved. The same results occur in many other chemical combinations.

The use of *electricity* as a source of heat is widespread. Pass an electric current through a wire and it becomes hot. Electric heaters, ovens, irons, toasters, lamps, and soldering irons are common applications of this source of heat.

Temperature. Temperature is the degree of hotness of a body. It is the condition of the body upon which depends its ability to *give off heat* or to *take in heat* from surrounding bodies. A hot body will give heat to its surroundings, while a cold body will absorb heat until thermal equilibrium is reached, that is, until the body and its surroundings are at the same temperature.

The temperature of a body must not be confused with the *quantity* of heat which it contains. A kettle of boiling water, for example, is at the same temperature as a glass of water poured from it, yet it has far more heat because of the greater quantity of water which it contains. Temperature is measured in degrees, while heat is measured in calories or British thermal units.

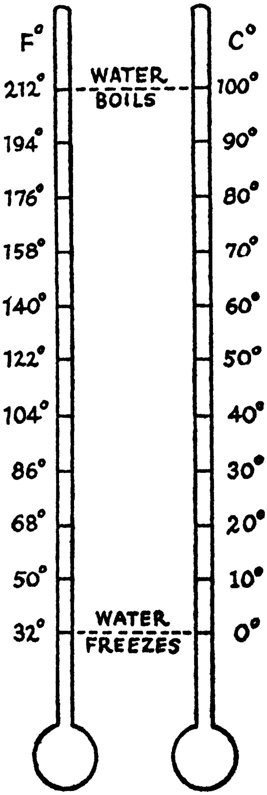
Measurement of Temperature. We cannot depend upon our senses to determine the temperature of a body. A bowl of warm water will seem to be hot if we place our hand in it after having held it in cold water, while the same bowl of water will seem cold if our hand has previously been in hot water. Likewise, if we enter a room from the outside on a winter day, the room may feel warm; enter the same room from another section of the house, and it may feel rather cold.

To measure temperature accurately an instrument called a *thermometer* is used. The commonly used mercury or alcohol thermometers depend for their operation on the fact that these liquids expand uniformly when they are heated and contract when they are cooled. The thermometer consists essentially of a glass bulb filled with the liquid and a thick-walled stem graduated in degrees. The freezing point and boiling point of pure water under normal atmospheric pressure are known as the *fixed points* of the thermometer. The freezing point is found by placing the bulb of the thermometer in finely crushed melting ice and noting the point at which the mercury becomes stationary. The boiling point is found by placing the thermometer in steam from pure water at normal atmospheric pressure, and noting the point at which the mercury column becomes stationary.

Thermometer Scales. The two temperature scales in common use are the *Centigrade* scale and the *Fahrenheit* scale. The Fahrenheit scale is used in the English-speaking countries for both industrial and everyday work, while the Centigrade scale is generally used abroad, and for most scientific work the world over.

On *the Centigrade scale,* the freezing point of water is marked zero degrees (0°) and the boiling point one hundred degrees (100°). The distance between these points is divided into 100 equal parts called Centigrade degrees. Temperatures below the freezing point are referred to as degrees below zero. On *the Fahrenheit scale,* the freezing point is 32° and the boiling point 212°. The distance between these points is divided into 180 equal parts called Fahrenheit degrees.

Since 100 Centigrade degrees are equivalent to 180 Fahrenheit degrees, the Centigrade degree is evidently larger than the Fahrenheit degree. One Centigrade degree is equal to $\frac{180}{100}$ or $\frac{9}{5}$ Fahrenheit degrees.



Conversion of Thermometer Readings. To convert a Fahrenheit reading to the Centigrade scale, we first subtract 32° and then multiply by $\frac{5}{9}$; or,

$$C=\frac{5}{9}(F-32)$$

Example 1: The temperature of a room is 68° F. What is its Centigrade equivalent?

$$C=\frac{5}{9}(68-32)=20^\circ$$

To convert a Centigrade reading to the Fahrenheit scale, we multiply by $\frac{9}{5}$ and then add 32°.

$$F=\frac{9}{5}C+32$$

Example 2: A given liquid boils at 90° C. What is its boiling point in Fahrenheit degrees?

$$F=\frac{9}{5}\times 90+32=194^\circ$$

Types of Thermometers. The standard *liquid thermometers* described above contain either mercury, or alcohol colored red for easier reading. Since mercury freezes at —39° C, and alcohol at —130.5° C, thermometers used to measure very low temperatures should be alcohol thermometers. On the other hand, mercury thermometers can be used to measure much higher temperatures than alcohol thermometers, since mercury boils at 357° C, while alcohol boils at 78.4° C.

A *metallic thermometer* based on the expansion of a "compound bar" is used to measure the temperature of ovens and other heating devices. It is easily read though not quite as accurate as a good liquid thermometer.

The *maximum and minimum thermometer* is used to record the highest and lowest temperatures at a certain place during some particular time interval. Such thermometers use two liquids, mercury in the lower bend, and either alcohol or glycerin in the remaining tube space. The mercury is moved when the alcohol or glycerin contracts or expands, depending on the outside temperature, pushing the index to a higher position in either arm. The maximum and minimum temperatures reached during the time interval are read directly from the position of the indexes. To reset the thermometer, a magnet is used to bring the indexes directly above the mercury column.

Expansion Due to Heat: Solids. When a body is heated its molecules are set in more rapid motion. This causes the molecules to push each other further apart, resulting in an expansion of the body as a whole. Most bodies, therefore, expand when they are heated and contract when they are cooled. Substances whose molecules are held most strongly together expand less than those whose molecules are not so strongly held. Solids, for example, expand much less than either liquids or gases.

The expansion of solids can be shown by a simple experiment performed with a metal ball and ring, so fitted that the ball will easily pass through the ring when both are cold. If the ball is then heated it expands and is unable to pass through the ring; when it cools it is again able to pass through. Different solids expand unequally for the same rise in temperature, depending upon their molecular structure. For comparative purposes we use the expansion of the solid lengthwise, known as its linear expansion.

Coefficient of Linear Expansion. The rate of expansion of a substance is called its ***coefficient of linear expansion.*** This is the expansion of a unit length of the substance for a one degree rise in temperature. The coefficient of linear expansion can be determined experimentally for different materials by measuring the expansion of a definite length of the material for a known temperature rise, and then computing as follows:

$$\textit{Coefficient of linear expansion} = \frac{\textit{Total Expansion}}{\textit{Original length} \times \textit{Rise in temperature}}.$$

Thus if a copper rod, originally 20 inches long, measures 20.034″ when its temperature is raised 100° C, the coefficient of expansion of copper is found as follows:

$$\text{Total expansion} = 20.034'' - 20'' = 0.034'';$$

$$\text{Coefficient of expansion} = \frac{0.034''}{20'' \times 100^\circ\, C} = 0.000017.$$

The coefficients of expansion of several common metals are given in the following table. These coefficients refer to Centigrade degrees. For use with Fahrenheit temperatures, they should be multiplied by 5/9.

COEFFICIENTS OF LINEAR EXPANSION

(Per Degree Centigrade)

Lead	0.000029
Aluminum	0.000023
Tin	0.000022
Silver	0.000019
Brass	0.000018
Copper	0.000017
Steel	0.000011
Nickel Steel (Invar)	0.0000009

These coefficients have been accurately determined by experiment in order that scientists and engineers should be able to compute the proper allowance for expansion whenever needed in their design work. The total expansion of any body may be found by multiplying its length by its coefficient of expansion and by its temperature rise. Thus,

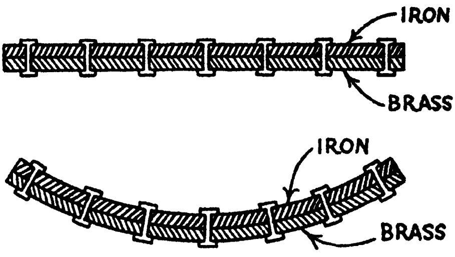
Total Expansion = Coefficient of expansion × length × temperature rise.

EXAMPLE: What will be the expansion of a steel rail 25 feet long, for a rise in temperature of 30°?

$$\text{Total expansion} = 0.000011 \times 25 \times 30 = 0.00825 \text{ ft.}$$

Applications of Expansion of Solids. Allowance must be made for the expansion of metals in many industrial and engineering projects. Railroad tracks are laid with a small space between their ends to allow for expansion in the summer. In the design of bridges and other steel structures the expansion of the metal must be taken into account and provided for. Expansion joints are used in steam piping to take up the expansion due to the heating of the pipes. In the construction of riveted joints on boilers, tanks, buildings, bridges, or ships, the rivets are set when they are red hot. As they cool, they contract, thus drawing the metal plates together, resulting in a firm tight joint.

Use is made of the *unequal* expansion of different metals in the thermostat, the metallic thermometer, and other devices. These are applications of the *compound bar,* a device formed when two different metal strips are welded or riveted together. Such a bar will bend when heated because of the unequal expansion of the metals. Thus, if brass and iron are the metals used, the bar when heated will curve with the brass on the outside, since brass exhibits a greater amount of expansion than iron.



Unequal Expansion in a Compound Bar When Heated

Expansion of Liquids. Liquids also expand when they are heated and contract when cooled. Thus, if a glass flask is filled with a liquid, and a rubber stopper and glass tube are inserted, the liquid when heated will expand up into the tube. On cooling it will contract to its original volume. Liquids expand far more than solids do, but not as much as gases. Different liquids expand unequally for the same rise in temperature. Alcohol, ether, and glycerin, among others, expand far more than water does.

For liquids and gases, cubical expansion, or the increase in volume, is measured rather than the linear expansion. The *coefficient of cubical* *expansion* for a liquid or gas is the expansion per unit volume for a one-degree temperature rise. Liquids, however, do not expand *uniformly*. At some temperatures they expand more than at others. Their coefficients of cubical expansion, therefore, vary at different temperatures.

Unusual Expansion of Water. Unlike other liquids, water contracts upon addition of heat from 0° C to 4° C, and then expands continuously. At 4° C, therefore, water is at its *maximum density*. That is, at this temperature, its weight per unit volume is greatest. Ice, being lighter than water, will float at its surface. If water is cooled, it will contract in volume until it reaches 4° C, and then will expand until it freezes. At freezing it will expand still further. It is for this reason that anti-freeze is used in automobile cooling systems to prevent freezing of the water and consequent cracking of the cylinder block. It is for this reason also that water pipes burst in winter, that milk pushes the cap off the bottle upon freezing, etc.

Expansion of Gases. Of all the states of matter, gases have the greatest rate of expansion upon the addition of heat. *All* gases expand *equally* for equal increases in temperature, and contract equally for equal decreases in temperature. It has been experimentally determined that the *coefficient of expansion* of any gas at constant pressure is $\frac{1}{273}$ of its volume at 0° C. If, for example, air at 0° C is heated until its temperature is 1° C, its volume becomes $\frac{1}{273}$ greater than its original volume. If it is heated 20° C, its volume becomes $\frac{20}{273}$ greater than its original volume.

Absolute Temperature. If air at 0° C is cooled 1°, its volume decreases by $\frac{1}{273}$ of its volume at 0° C. It is apparent that if a gas were cooled to 273° below zero (—273° C), its volume would become zero. This, however, is quite impossible, since all known gases liquefy before they reach this temperature. The temperature, —273° C, is called the *absolute zero* and is of great importance theoretically. At this temperature all molecular motion would cease, and since heat depends upon molecular motion, a body at this point would have no heat whatever. If temperatures are measured from the absolute zero instead of from 0° C (as is done on the Centigrade scale) a new and extremely useful temperature scale, called the *absolute scale,* is obtained. It is also known as the Kelvin scale. Any reading on the Centigrade scale can be converted to the absolute scale by adding 273° to it algebraically:

$$T=t+273,$$

where T is the absolute temperature, and t is the Centigrade temperature. Thus, 0° C equals 273° Abs.; 100° C equals 373° Abs.; —10° C equals 263° Abs.; etc.

Charles' Law. Charles' Law states that the volume of a given weight of gas is directly proportional to its *absolute temperature*, provided that the pressure remains constant. Expressed mathematically,

$$\frac{V_1}{V_2}=\frac{T_1}{T_2}$$

where V_1 is the volume of the gas at absolute temperature T_1, and V_2 is the volume of the gas at absolute temperature T_2.

EXAMPLE: A given quantity of air at 27° C occupies 150 cc. What volume will it occupy at 0° C if its pressure is kept constant?

SOLUTION:

$V_1=150$ cc. $T_1=27°+273°=300°$ Abs.

$V_2=?$ $T_2=0°+273°=273°$ Abs.

$$\frac{150}{V_2}=\frac{300}{273}$$

$$V_2=\frac{150\times 273}{300}$$

$V_2=136.5$ cc., *Ans.*

Since *both* pressure and temperature affect the volume of a gas, Boyle's Law and Charles' Law are combined to give us a general law covering the relationship between pressure, volume, and absolute temperature:

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}$$

where P_1, V_1, T_1, are the initial conditions of the gas and P_2, V_2, T_2, are the final conditions.

EXAMPLE: At standard conditions, 760 mm. of mercury pressure and 0° C temperature, a given gas occupies 200 cc. What will its volume be at 30° C and 740 mm. of mercury?

SOLUTION:

$P_1=760$ mm. $V_1=200$ cc. $T_1=0°+273°=273°$ Abs.

$P_2=740$ mm. $V_2=?$ $T_2=30°+273°=303°$ Abs.

$$\frac{760\times 200}{273}=\frac{740\times V_2}{303}$$

$$V_2=\frac{760\times 200\times 303}{273\times 740}$$

$V_2=228$ cc., *Ans.*

Transmission of Heat. Heat is transmitted from a warm place to a cooler place by one of the following methods:

1. Conduction
2. Convection
3. Radiation

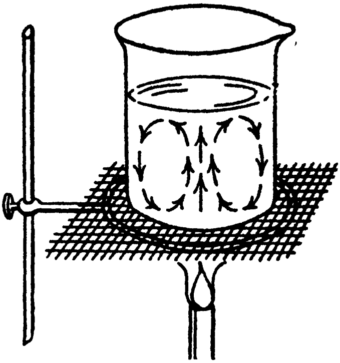
Conduction. Conduction is the transfer of heat by the collision of molecules. If one end of an iron rod is heated, after a while the other end also becomes hot. The molecules of iron in contact with the source of heat are set in rapid vibration and hit the cooler molecules next to them, setting these cooler molecules into more rapid motion. This process of molecular collision goes on until all the molecules in the rod are set into rapid vibration, causing the entire rod to become hot. Not all substances conduct heat equally well. If, for example, one end of a wooden rod were heated, in place of an iron rod, the other end would not become very warm.

A substance which transmits heat readily by conduction is called a *good conductor* and is said to have a high *conductivity* of heat. Metals, in general, are good conductors, though they vary in their conductivity. Among the best conductors, in the order of decreasing conductivity, are silver, copper, aluminum, brass, and iron. It is because it conducts heat well that iron is used in furnaces, stoves, boilers, and radiators, and that aluminum is used for cooking utensils, etc.

Insulating Materials. Most non-metals are poor conductors of heat. Thus asbestos, cork, wool, rock wool, and wood resist the passage of heat through themselves by conduction, and are called good *heat insulators.* Liquids and gases are likewise poor conductors, gases being among the poorest. In fact, fibrous materials like asbestos, cork, and wool are good insulators largely because of the many air spaces they contain, preventing the transfer of heat through them.

Good insulators serve to prevent the loss or gain of heat in many practical applications. The walls of homes and industrial buildings are either built with an air space between them or are insulated with rock wool or a similar material. Likewise, refrigerators, ice houses, fireless cookers are built with double walls filled with insulating material to reduce heat transfer to a minimum. Steam pipes are covered with layers of asbestos or magnesia to retain their heat. Furnaces are jacketed with an insulating material for the same reason. Carpets, blankets, woolen clothes, and furs are used because they are good heat insulators. Here, too, the numerous air spaces between the fibers of the materials serve to reduce heat flow and thus keep us warm.

Convection. When one wishes to warm his hands, he holds them over a radiator or an open fire. A warm stream of air rises and warms them. The transfer of heat by moving currents of a fluid is called *convection.* Liquids and gases are in the main heated by this method. Convection depends on the fact that liquids and gases expand when they are heated. Thus, if a vessel of water or any other liquid is heated, only the bottom layer (in direct contact with the flame) receives the heat. As this layer is heated, it expands and becomes less dense. Since this hot layer is lighter than the liquid above it, it rises to the top, and the heavier, cold layer above it comes down to take its place. This layer is in turn heated, expands, decreases in density, and rises to the top. Thus a current of hot liquid rising and cold liquid coming down to take its place continues until all the liquid in the vessel is heated to the same temperature. The same process takes place when a gas is heated.



Convection Currents in a Liquid

Applications of Convection Currents in Heating and Ventilating. Most of our heating systems transmit their heat by the convection method. Thus in the ***hot-water system of heating,*** water is heated to a temperature somewhat below its boiling point by a furnace in the basement, and being less dense than the cold water, it is pushed up through pipes to radiators in the different rooms in the building. Here the water rapidly gives off heat to the surrounding air and is therefore cooled. The cooled water is carried back by return pipes to the furnace for reheating. Each individual room is heated by convection currents set up by the hot radiator. The air above the radiator is warmed, rises, and cooler air flows down to take its place and be heated. This convection process continues and the room is fairly uniformly heated.

In the ***hot-air system of heating,*** the air is heated in a jacket surrounding the furnace in the basement and rises to the different rooms through flues as cold air comes in to be heated. As the warm air loses its heat to warm the rooms, it cools, becomes more dense, and returns to the furnace to be reheated through a return flue. Additional cold air, drawn in from the outside through a special flue, provides a sufficient amount of fresh air for ventilation.

The ***draft in a chimney*** of a furnace is another example of convection currents. The cold fresh air supplied to keep the fuel burning forces the hot burned gases (which are lighter than the cold air) up the chimney, providing proper draft for the fires. Though a large amount of heat is carried off by the flue gases, proper draft is necessary to keep the fires going.

Radiation. If a person stands in front of a hot stove, a fireplace, or a radiator, he becomes warm. The heat could not have reached him by convection, since convection currents would rise upward. Nor could it have been transmitted by conduction, since the air is a poor conductor of heat. The heat in this instance is transmitted by the third method of heat transfer known as *radiation.* Any hot body is constantly losing heat into space in the form of radiant energy. When this energy is absorbed by another body it is converted into heat. Thus, the earth is warmed by radiant energy received from the sun. A person near a heat source is warmed by heat radiation emitted by that source.

Radiation is the transfer of radiant energy through space, and its conversion to heat when absorbed by matter. Radiation can take place in a vacuum as well as in air. When the medium of transmission is a homogeneous substance, radiation travels in straight lines. The further a body is from the heat source, the less heat radiation it receives. This phenomenon is known as the law of inverse squares; thus the intensity of heat radiation is inversely proportional to the square of the distance from the source. A highly polished surface reflects radiant heat, while a rough, black object absorbs it. Good absorbers of heat are also good radiators, and poor absorbers are poor radiators.

The *thermos bottle,* used primarily to prevent the transmission of heat, consists of a double-walled flask from which the air has been exhausted. The flask has both its walls silvered to prevent radiation either to or from the bottle. The vacuum between the double walls prevents the flow of heat since it is a very poor conductor. Convection is also prevented by the vacuum space since there is no medium for convection currents. Hence liquids placed in such a bottle will be kept at almost their original temperature for a considerable length of time because of the almost complete heat insulation.

Quantity of Heat. The units in which quantity of heat is measured are the calorie and the British Thermal Unit. The *calorie* is the quantity of heat required to raise the temperature of 1 gram of water 1° C. Where a larger unit is desired, we use the *kilogram calorie,* which is defined as the heat needed to raise the temperature of 1 kilogram of water 1° C. The calorie and kilogram calorie are the units used in Europe, in scientific laboratories, and wherever the metric system is used. In the English-speaking countries heat is generally measured in British Thermal Units. The *British Thermal Unit* (*B.T.U.*) is the quantity of heat required to raise the temperature of 1 pound of water 1° F. One kilogram calorie is roughly equal to 4 B.T.U. (3.97 B.T.U., more exactly).

Specific Heat. Different substances vary in the quantity of heat needed to raise their temperature by a definite amount. Thus, while 1 calorie will raise the temperature of 1 gram of water 1° C, it will raise the temperatures of 1 gram of each of the following substances by entirely different amounts, as indicated: air, about 4° C; aluminum, about 4½° C; copper, about 10° C; mercury, about 30° C. The capacity of a body to absorb heat is indicated by its specific heat. Hence the *specific heat* of a substance is defined as the number of calories required to raise the temperature of 1 gram of the substance 1° C. As this is equivalent to the number of B.T.U. needed to raise the temperature of 1 lb. of water 1° F, we can also use this definition for specific heat.

Of all common substances, water has the greatest specific heat. This can be seen from the table below which gives the specific heats of several substances as determined by accurate experiment.

SPECIFIC HEATS

Water	1.000	Copper	0.093
Ice	0.502	Tin	0.055
Air	0.242	Mercury	0.033
Aluminum	0.212	Lead	0.031
Iron	0.113		

Change of State. Most substances can exist in the solid, the liquid, or the gaseous state. In changing from solid to liquid, or liquid to gas, or vice versa, a body is said to undergo a *change of state.*

Melting and Freezing. The *fusion or melting* of a substance is its change from the solid to the liquid state. Crystalline substances change into liquids at a fixed temperature, that is, they have a *definite melting point.* Thus, ice will melt at 0° C, tin at 232° C, copper at 1083° C. Amorphous or non-crystalline substances, however, do not have a fixed melting point, but will soften gradually when they are heated before they actually liquefy. Iron, glass, wax, tar, fats, and similar substances are examples of such materials.

During fusion, heat will be absorbed by a substance without any change in temperature. The entire melting process for a crystalline substance takes place at the same temperature. In melting 1 gram of ice, 80 calories of heat are added. Yet a thermometer placed in the melting ice will continue to read 0° C until *all* the ice has melted. The heat required to melt 1 gram of a solid without change in temperature is called the *heat of fusion* of the substance. This is sometimes referred to as the latent heat of fusion, since it is not indicated by a rise in temperature. When a substance *solidifies* or *freezes,* the heat of fusion is released.

In changing from the liquid to the solid state (solidification), some substances contract while others expand. Lead, gold, and silver, for example, contract when they become solid. It is difficult to cast them into definite shape because they will shrink away from the mold upon solidifying. Several alloys like type metal and cast iron, as well as some metals like bismuth and antimony, expand when solidifying. These substances can therefore be cast into definite forms and will take the exact shape of the mold. Type metal is an alloy of lead, antimony, and copper.

Water, too, expands upon freezing. It is this expansion (about 9%) that accounts for the burst water pipes in winter and for the cracked cylinder blocks and radiators in automobiles in which anti-freeze solutions have been forgotten. It is because of this expansion, also, that ice, having become less dense than water, will float at its surface.

In general the freezing point of a liquid is lowered by dissolving another substance in it.

Boiling, Evaporation, and Condensation. The change from the liquid to the gaseous state is called *vaporization.* As in the melting or fusion process, heat is required to change a substance from a liquid to a vapor. Thus, if a vessel of water is heated, its temperature rises until it reaches its boiling point, 100° C. Any heat added thereafter does not change the temperature, but serves to change the water into steam at 100° C. This heat energy, called its heat of vaporization, is sometimes referred to as the latent heat of the steam. The quantity of heat required to change 1 gram of a liquid to the vapor or gaseous state without change of temperature is called its *heat of vaporization.* For water this has been experimentally found to be 540 calories per gram, or 970 B.T.U. per pound. The heat of vaporization is released when the vapor is condensed, i.e., changed back to the liquid form.

Boiling. The change from the liquid to the gaseous state may take place by either boiling or by evaporation. If a liquid is heated sufficiently, vapor bubbles form throughout the liquid, rise to the surface, and escape rapidly from the liquid. At this stage the liquid is said to *boil.* The *boiling point* of a liquid is defined as the temperature at which its vapor pressure is equal to the pressure of the atmosphere. Every liquid has a definite boiling point. Thus, under normal atmospheric pressure, water will boil at 100° C, ethyl alcohol at 78° C, ether at 35° C, mercury at 357° C, and ammonia at —34° C.

Any change in pressure results in a change in the boiling point of a liquid; an increase in pressure raises the boiling point, while a decrease in pressure lowers the boiling point. Thus water boils at 100° C under normal atmospheric pressure. Its boiling point is almost doubled at a pressure of 200 pounds per square inch, at which it boils at 194.5° C. The following table gives the boiling points of water at several different pressures:

Pressure (lbs. per sq. in.)	*Boiling Point of Water* F°	C°
5	162.3	72.2
10	193.2	89.5
14.7	212.0	100.0
30	250.3	124
100	327.8	164.5
200	381.2	194.5

Steam boilers in large modern power plants generate steam at extremely high pressures. In the Ford River Rouge plant, steam is generated at 1400 lb. per sq. in. pressure; other plants use steam at comparable pressures.

Since a decrease in pressure results in a decrease in the boiling point, water will boil at less than 100° C at high altitudes where the pressure is less than atmospheric. In order to cook food quickly and thoroughly at such points, *pressure cookers* are used to raise the pressure and therefore increase the boiling point. Pressure cookers are also used at low altitudes to speed up the cooking process by boiling foods at temperatures higher than 100° C, and are commercially used in the sterilization of canned foods. In the extraction of gelatine from bones, high pressure cookers known as digesters are employed.

Evaporation. Any liquid left out in the open slowly vaporizes or *evaporates.* Evaporation, which takes place at the surface of the liquid only can occur at any temperature. An increase in temperature increases the vapor pressure and hastens the evaporation process.

What takes place during evaporation may be explained by the kinetic theory. Some of the molecules at the surface of the liquid have sufficient velocity to break through the surface and enter the space above it. The liquid, losing its most active molecules, has its temperature slightly lowered and begins to absorb heat from its surroundings. As the speed of its molecules increases, several again leave the surface, and this process continues until the air above it has become saturated. When the latter condition is reached, as many molecules enter the surface of the liquid as leave it per unit of time.

Since evaporation can continue only if heat is received from its surroundings, the evaporation of a liquid has a cooling effect not only on the liquid itself but on its surroundings as well. Thus perspiration evaporating from our bodies seems to cool them. Alcohol, which evaporates more quickly than water, absorbs body heat more quickly, and an alcohol rub has a more decided cooling effect. Spraying the ground on a hot day cools the surrounding air when the water upon evaporating absorbs heat from its surroundings.

Laws of Evaporation. 1. The rate at which a liquid evaporates increases as *the temperature of the liquid increases.* The higher the temperature of the liquid, the greater the speed of its molecules, and the more rapidly they leave the surface of the liquid.

2. The rate of evaporation increases with *an increase in the area of the exposed surface of the liquid.* Thus the water in a large shallow pan will evaporate more quickly than the same quantity of water in a small deep container, because of the larger surface from which the molecules can escape. In salt-making, the salt water is evaporated in basins of extremely large area.

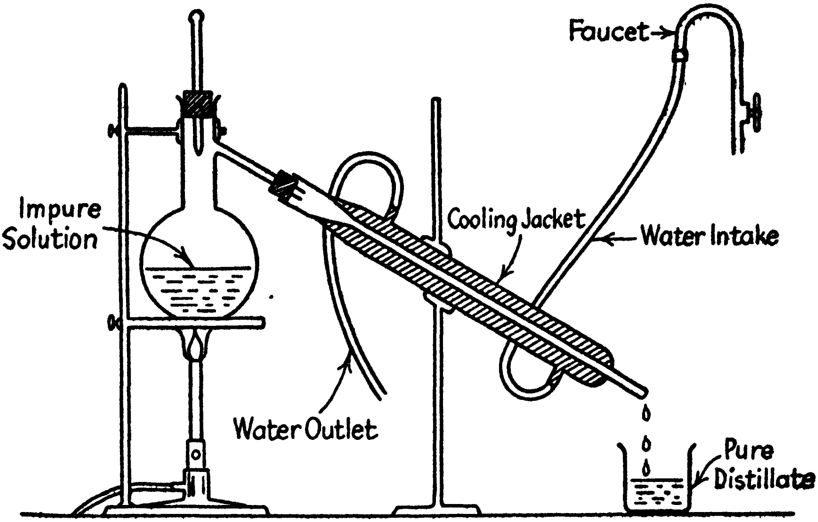
3. The rate of evaporation is hastened *by decreasing the pressure on the evaporating surface.* This serves to reduce the interference of air molecules in the space above the liquid, and allows more rapid escape of the molecules from the surface. In the refining of sugar, for example, the sugar solution is heated in large vacuum pans. The lower pressure hastens the evaporation process.

4. The rate of evaporation increases as *the air above the liquid is changed.* As the molecules of the liquid escape into the air above it, the air tends to become saturated. At this point it has absorbed just as much vapor as it can hold, thus cutting down any further evaporation unless this saturated air is removed. Wet clothes dry quickly on a windy day because of the constant flow of air carrying off the evaporated water.

Condensation. If the heat of vaporization is removed from a vapor, it will liquefy. This process of liquefaction is called condensation. Vapors and gases can be liquefied by cooling, or by compression, or by a combination of both processes. In steam heating, steam is generated in a boiler and piped under pressure to radiators throughout the building to be heated. Here the heat of vaporization of the steam is given off to warm the rooms and the steam is condensed. The hot water formed by the condensing steam is returned to the boiler by gravity.

Distillation. By distillation is meant the boiling of a liquid and the subsequent condensing of its vapor. The liquid is boiled in one vessel and its vapor is led through a condenser, which is essentially a tube surrounded by a jacket through which cold water is circulated. The cold water absorbs the heat of the vapor, thus liquefying it. Distillation is used in separating liquids from impurities, or from solids dissolved in them. Water containing impurities can be made fit to drink by distillation. The pure water is distilled off and condensed, while the impurities remain in the original flask in which it is boiled.

Mixtures of liquids having different boiling points can be separated by distillation. The mixture will boil at a point somewhere between the boiling points of each of its constituent liquids, so that the vapor which passes off will consist predominantly of the substance having the lower boiling point. By condensing this vapor and redistilling it, it is possible to separate almost completely one of the liquids from the other. This process, known as *fractional distillation,* is used in separating gasoline, benzine, kerosene, and naphtha from crude petroleum.



Distillation Apparatus

Artificial Refrigeration. The various methods of artificial refrigeration depend largely upon the heat absorption that takes place during vaporization. In most commercial and industrial refrigerating systems, like ice-making plants, cold storage plants, refrigerating plants, etc., ammonia is the refrigerant commonly used.

Liquid ammonia is allowed to escape through an expansion valve to a lower pressure. In expanding, the liquid vaporizes rapidly, its temperature falling. The low temperature vapor enters the expansion coils or evaporator, where it expands still further, absorbing heat in the process. In the evaporator coils the refrigerant absorbs heat from the region or substance which it is desired to cool. These coils may be placed directly in the region which is to be cooled (direct expansion system), or they may be placed in and absorb heat from a brine solution with a low freezing point, which is in turn circulated to the region that is to be cooled. In this latter type of process, known as indirect expansion refrigeration, the extremely cold brine solution then acts to absorb heat from the region to be cooled. The absorption of heat energy by the refrigerant will cause it to evaporate still further. After expansion in the evaporator coils, the ammonia vapor passes to a compressor where it is compressed and sent on into a condenser; here it is cooled and condensed by cold circulating water. The liquid ammonia is now ready to be sent once more through the expansion valve, and the entire refrigeration process is repeated.

In the manufacture of ice, smaller tanks of pure water are immersed in the brine tanks. The water in the small tanks freezes completely in about forty hours or so, after which the tanks are pulled up and the cakes of ice removed. For cold storage rooms, the cold brine solution is circulated through pipes directly to the regions which are to be refrigerated.

A variety of fluids are available as refrigerants in the *vapor compression* systems of refrigeration. For large commercial installations, ammonia is almost universally used. In small self-contained household refrigerators, sulfur dioxide vapor is often employed. Carbon dioxide is used in a great many marine installations. Ethyl chloride, propane and other hydrocarbons are also used as the refrigerants in a number of systems.

The gas refrigerators used in some household installations are an example of another type of refrigeration known as the *vapor absorption* system.

LIGHT

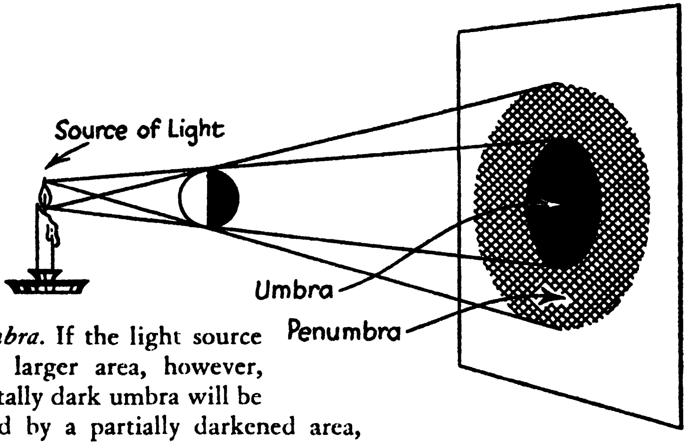
Nature of Light. Light, another of the many forms of energy, is a series of waves resulting from the motions of electrons. It is transmitted through the ether with a velocity of 186,000 miles per second, or about 300,000 kilometers per second. Light waves are transverse and vary in length from 0.00004 cm. to about 0.00007 cm. They are the only electromagnetic waves capable of affecting the optic nerve, thus causing the sensation of sight. Other electromagnetic waves, similar to light in character, but which do not affect the optic nerve, are heat waves and radio waves, which are *longer* than light waves, as well as ultra-violet waves, cosmic rays and X-rays, which are *shorter* than light waves.

Luminous and Illuminated Bodies. When light from an object enters the eye and forms an image on the retina, the object is said to be *visible*. This light may be emitted from the object itself, or it may be reflected to the eye by the visible body. A *luminous* body is one which emits light produced by itself. The sun, the stars, a flame, and an incandescent wire are all luminous bodies; that is, they are visible by their own light. An *illuminated* body is one which is visible by light reflected from some other source. The moon, the planets, and most everyday objects are seen by reflected light and are, therefore, illuminated bodies. Illuminated bodies may transmit, reflect, or absorb light.

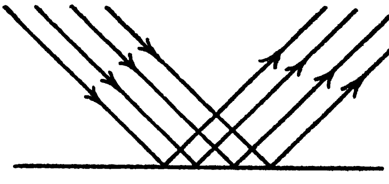
A *transparent* body is one which allows light to pass through it. Glass, air, water are transparent bodies. Objects can be seen clearly through such bodies. *A translucent* body allows some light to pass through it, but diffuses the light so that objects cannot be seen clearly through it. Ground glass, tissue paper, window shades and curtains are examples of translucent bodies. An *opaque* body is one through which no light will pass. Steel, wood, stone, rubber are opaque bodies. ·

Transmission of Light. Light travels in straight lines in all directions, so long as it is transmitted through the same medium.

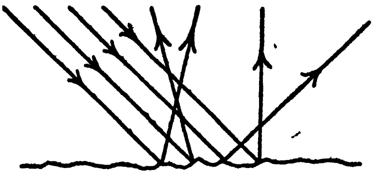
Shadows. If an opaque body is placed between a source of light and a screen, it will exclude the light from the space on the side away from the light source, causing a darkened space known as a *shadow.* If the light source is very small, e.g., an electric arc lamp, the shadow will be of uniform intensity and will have a sharp outline. Such a shadow is called an *umbra.* If the light source has a larger area, however, the totally dark umbra will be fringed by a partially darkened area, called the *penumbra.* The umbra is caused by the almost total exclusion of light from the space behind the opaque body, while the penumbra is caused by the partial exclusion of light. Eclipses of either the sun or the moon are shadow phenomena. An eclipse of the moon occurs when the moon enters the earth's shadow. An eclipse of the sun occurs when the moon is between the sun and the earth, cutting off the sun's light so that the umbra of the moon's shadow touches the earth.



Reflection of Light. When light falls on a polished surface, the greater part of it is bent back or reflected. Reflection may be either regular or diffused, depending upon the nature of the surface. *Regular reflection* occurs when light falls upon a smooth, highly polished surface. In such cases the light is not scattered, but is reflected in a definite direction.



Regular Reflection

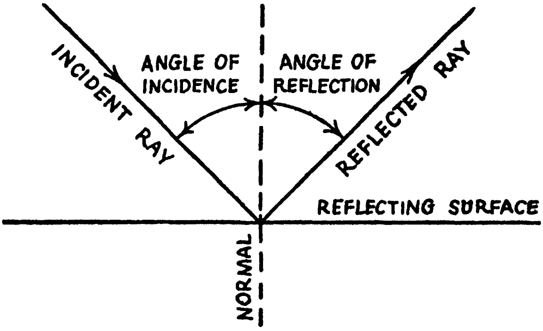


Diffused Reflection

If, however, the light strikes a rough surface, it is not regularly deflected, but is scattered instead in all directions because of the irregularities on the reflecting surface. This type of reflection is called *diffused reflection.* Most everyday objects are visible because they are diffuse reflectors. It is by diffusely reflected light, also, that areas with no direct illumination are lighted.

Laws of Reflection. A *ray* is a single line of light emitted from a light source. Strictly speaking, light rays do not actually exist, since light spreads in traveling from any source. Nevertheless, the concept of the ray or line of light is useful in studying the subject.

When light passes through a small opening, a narrow band called a *beam* results. A beam may be considered as composed of a number of parallel rays of light. A light ray striking any surface is called an *incident ray,* and the angle which this ray makes with the normal, or perpendicular, is known as the *angle of incidence.* A light ray bent back from a polished surface is called a *reflected ray,* and the angle which this ray makes with the normal is the *angle of reflection.* Careful experiment has established the following *laws of reflection:*

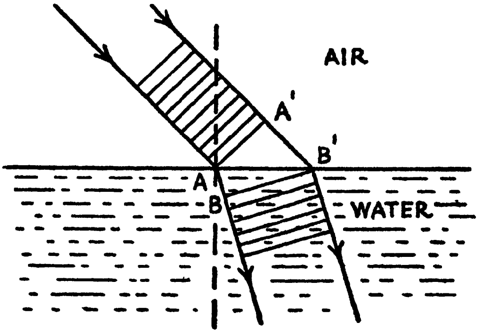


1. The angle of reflection is at all times equal to the angle of incidence.
2. The incident and reflected rays lie in the same plane perpendicular to the reflecting surface.

Refraction of Light. An oar or a fishing rod held in the water appears bent. Any object in the water appears to be nearer the surface than it actually is. These effects are caused by the bending of light in the water. It is for the same reason that objects often appear distorted when viewed through plate glass or window glass. Light is bent whenever it passes from one substance to another having a *different density.* It travels in straight lines only when it is passing through the same substance. The bending of a ray of light when it passes obliquely into a medium of different optical density is called *refraction.* Experiments have established the following laws of refraction:

1. When light enters a denser medium obliquely, it is bent ***toward*** the normal.
2. When light enters a rarer medium obliquely, it is bent *away* from the normal.

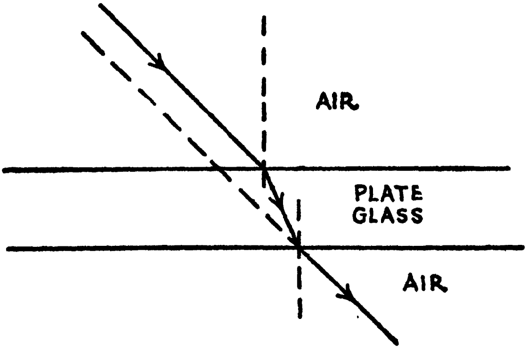
Refraction may be explained in the following manner. When a beam of light passes from one medium to another its speed is changed, depending upon whether the second medium is optically denser or rarer than the first. If, for instance, a beam of light passes from air into water, its speed is reduced. The under edge of the beam, which strikes the surface of the water first, has its speed reduced while the upper edge is still in the air traveling at its previous speed. The upper edge, therefore, gains on the lower edge and bends around it. Thus refraction occurs because one part of the beam is retarded before the other is. The reverse effect takes place when the light emerges from the water; in other words, one end will be accelerated before the other and will cause bending away from the normal. When a beam of light enters a new medium *perpendicular* to the surface, there is no refraction, since all parts of the beam are retarded equally.



Refraction of Light

(Under edge of beam travels from A to B while upper edge travels from A′ to B′)

When a ray of light falls obliquely on a glass plate whose faces are parallel, it will bend toward the normal on entering the glass, and bend away from the normal on leaving the glass. The direction of the ray on leaving the glass will, however, be parallel to its original direction. It is for this reason that objects viewed through plate glass appear slightly displaced.



Refraction of Light through Plate Glass

Index of Refraction. The index of refraction of a substance is the ratio of the velocity of light in air to its velocity in that substance. It can also be defined as the ratio of the sine of the angle of incidence to the sine of the angle of refraction. The angle of incidence is the angle the incident ray makes with the normal, while the angle of refraction is the angle the refracted ray makes with the normal.

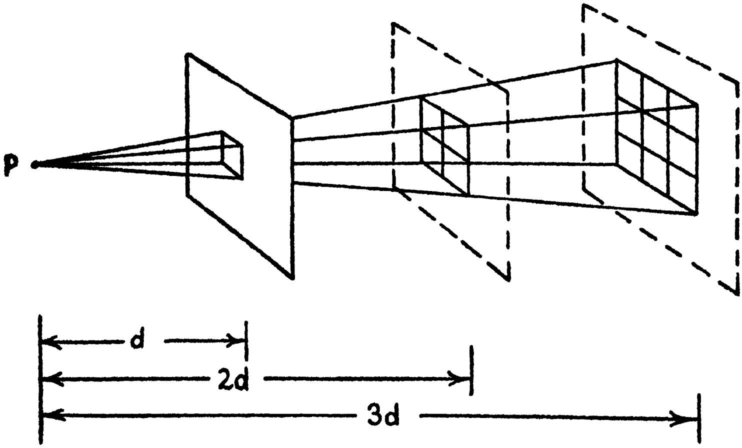
The index of refraction for several common substances, as determined by careful experiment, is given below:

INDEX OF REFRACTION

Crown glass	1.53	Water	1.33
Flint glass	1.67	Alcohol	1.36
Diamond	2.47	Carbon disulphide	2.47

The index of refraction is constant for any two given substances no matter what the angle of incidence may be. This statement is often referred to as the third law of refraction.

Illumination. The closer an object is to a lamp or any other light source, the more brightly it is illuminated. The amount of light that falls on a unit area of that object is known as the *intensity of illumination;* it depends upon the distance from the light source as well as upon the candle power of that source. A *candle power* is the rate at which light is emitted by a standard sperm candle ⅞ inches in diameter and burning 120 grains per hour. The illuminating power of a light source, measured in candle power, is determined by comparing it with a lamp of known candle power through the use of an instrument known as a photometer.



Law of Inverse Squares

Law of Inverse Squares. Experiment shows that the intensity of illumination of any surface varies inversely as the square of the distance from the light source. If, for example, a substance is 2 ft. away from a light source, its intensity of illumination is ¼ of what it would be 1 ft. from the source; if 3 ft. away, its intensity is ⅑ of what it would be at 1 ft.; etc. This is known as the *law of inverse squares.* The intensity of illumination decreases because the further the light travels from its source, the greater becomes the area over which it spreads.

The intensity of illumination on a surface is measured in foot-candles. A *foot-candle* is defined as the intensity of illumination on a surface 1 foot from a standard candle. The amount of illumination desirable for different types of work varies from about 2 to 50 foot-candles. Thus, for lighting a hallway, 2 or 3 foot-candles are sufficient; for an office, 10 foot-candles; for a store, about 20 foot-candles; and for ordinary reading, from 8 to 10 foot-candles. A drafting room should have about 20 foot-candles of illumination. For general factory work about 10 foot-candles are required; for finer work, about 15 foot-candles. For high precision work, from 20 to 50 foot-candles may be used.

34. MAGNETISM AND ELECTRICITY

Magnets and Magnetic Substances. Magnetite (Fe_8O_4), one of the important ores of iron, has the property of attracting small pieces of iron and steel. Pieces of this ore, called lodestones, have been used as *natural magnets* for thousands of years. This ability to attract iron, steel, and several other substances is known as *magnetism.* Not all substances, however, are attracted by a magnet. Rubber, glass, wood, copper, zinc, etc., when placed near a magnet are not attracted at all. Such substances are called *non-magnetic* substances. Substances which are attracted by a magnet are called *magnetic* substances. The most important of these are, of course, iron and steel. Cobalt and nickel possess this property of magnetic attraction to a smaller degree.

If a piece of iron or steel is placed in contact with a natural magnet, it in turn acquires magnetic properties, and has the ability to attract other bodies to it. By its contact with the lodestone, it has become a magnet itself. Bars of iron and steel which have been magnetized in this way, and which possess the same properties of magnetic attraction as the natural magnets, are called *artificial magnets.* In general, artificial magnets are either straight steel bars, called bar magnets, or bars which have been bent into a horseshoe shape, called horseshoe magnets.

Magnetic Poles. Magnets possess the property of *polarity.* If a bar magnet is placed in iron filings, most of the filings will be attracted to its ends, and very little to its sides. The ends of the magnet, where the magnetic attraction is greatest, are known as the *poles* of the magnet. If the magnet is suspended so that it can move about freely, one end (known as the north pole of the magnet) will point northward, the other end (known as the south pole of the magnet) will point southward.

Magnetic Attraction and Repulsion. If a magnet is mounted so that it can move freely, and the north pole of a second magnet brought near its north pole, the poles will repel each other. If the two south poles are brought together, they will likewise repel each other. If, however, the north pole of one magnet is brought near the south pole of the second magnet, or vice versa, the two will attract each other. From this it can be seen that: *Like magnetic poles repel each other; unlike magnetic poles attract each other.* It has been found experimentally that the force of attraction or repulsion between two poles decreases with an increase in the distance between them. In fact, these forces vary inversely as the square of the distance between the poles. The force of attraction or repulsion also increases with an increase in the strength of the magnetic poles.

Magnetic Induction. If an unmagnetized bar of soft iron is placed near some iron filings, it will not attract them. However, if a strong magnet is placed near it, the soft iron bar will become magnetized and be able to attract the iron filings. As soon as the magnet is removed, the soft iron bar loses its magnetic properties and drops the filings. The process of magnetizing a substance by bringing it near a magnet is known as *magnetic induction.* In this process, two poles are induced in the substance. The magnet induces an unlike pole in the portion of the iron bar nearest it, and a like pole in the portion of the magnet farthest away from it.

Soft iron is quickly magnetized by induction, but it also loses its magnetism readily when the magnetic field is removed. On the other hand, hardened steel, which is not easily magnetized, retains its magnetism for a considerable length of time. Hardened steel is therefore used where we require magnets which retain their magnetic properties (permanent magnets), as, for example, in telephones and magnetos. Soft iron is used where temporary magnets are required as in the various electromagnets. Soft iron is said to have a small *retentivity,* while hardened steel, giving a more permanent magnet, is said to have great retentivity.

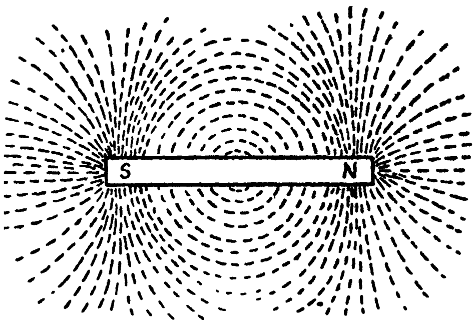
Methods of Magnetizing a Substance. Magnetic substances may be magnetized by several different methods:

1. *Magnetization by induction.* This is the method described above, where a piece of iron or steel when placed in the field of a magnet becomes a magnet itself.

2. *Magnetization by contact.* If a piece of steel is rubbed from one end to the other consistently in the same direction with one pole of a magnet, it will in turn become magnetized.

3. *Magnetization by electricity.* If a coil of wire is placed around a piece of iron or steel, and an electric current passed through the coil, the iron or steel core will become magnetized.

The Magnetic Field. A magnet exerts influence in the space around it. The *magnetic field* is the name given to the space surrounding the magnet in which its magnetic action is felt. If iron filings are sprinkled on a sheet of paper over a magnet, and the paper gently tapped, the filings will arrange themselves in a definite pattern of lines. From this phenomenon Faraday deduced the concept of lines of force around a magnet.

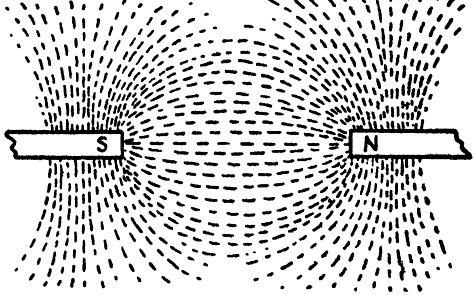


Pattern of Lines of Force about a Bar Magnet

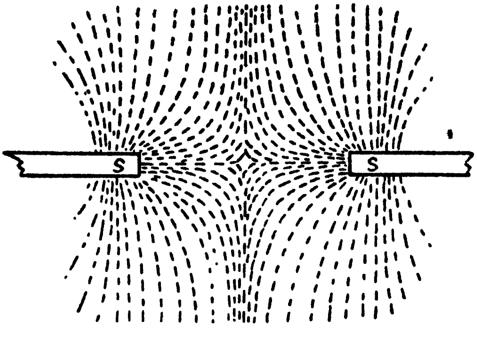
A *magnetic line of force* is the path along which an independent freely moving north pole would move under the influence of the attractions and repulsions of the magnetic poles acting upon it. Lines of force are curved lines, thought of as emerging from the north pole and entering the south pole of the magnet, completing their circuit through the magnet itself. Faraday pictured these lines of force as acting very much like stretched elastic fibers which consistently tend to contract or shorten lengthwise, and at the same time separate or swell sidewise.

Magnetic lines of force, therefore, can be said to have the following properties:

1. They form closed curves, emerging from the north pole and entering at the south pole.
2. They never intersect each other.
3. They tend to shorten longitudinally.
4. They repel each other laterally.



Pattern of Lines of Force between Two Unlike Poles



Pattern of Lines of Force between Two Similar Poles

It is interesting to examine the lines of force set up between like poles and unlike poles. If two like poles are placed together, the lines of force set up about them will repel each other, as shown in the diagram. The space in the center would be relatively free from lines of force. If, however, two unlike poles are placed close together, they will attract each other and set up lines of force acting to draw the poles together.

Magnetic Flux and Permeability. *Magnetic flux* is the name given to the total number of lines of force in a particular magnetic field. The strength of a magnetic field is measured by the number of lines of force acting in a unit area, i.e., by the concentration of the lines of force.

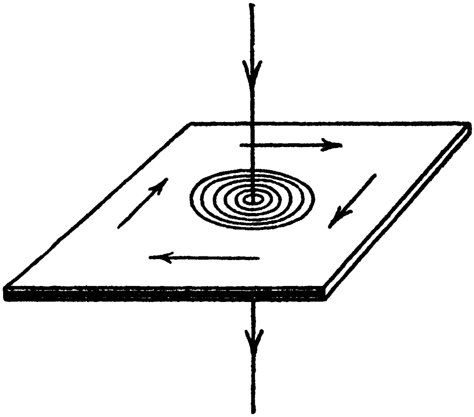
Some substances have a greater ability to concentrate lines of force than others. A piece of soft iron, for example, placed in a magnetic field will concentrate the lines of force in the vicinity through it. Soft iron is thus said to have a high permeability. The *permeability* of a substance is the ease with which lines of force may be established in it compared with the ease with which they are set up in a vacuum.

While there is no insulator for magnetic lines of force, substances with high permeability, such as soft iron, are often used as magnetic screens, since they serve to divert the lines of force from other media through themselves. The opposite of permeability, the resistance which a substance offers to the passage of lines of force, is known as *reluctance.* Iron has a permeability several hundred times greater than that of air. Permalloy, an alloy of iron and nickel, has a permeability about thirty times greater than that of soft iron.

The Theory of Magnetism. The phenomenon of magnetism has been explained by a fairly simple theory based on the following assumptions. The molecules of a magnetic substance are themselves magnets. When the substance is in an unmagnetized state, these molecules making up an iron or steel bar do not point in any definite direction, but are arranged haphazardly, neutralizing each other. When a substance is magnetized, however, its molecules are so arranged that all their north poles face in one direction, resulting in a north pole at one end of the bar and a south pole at the opposite end. This theory is borne out experimentally. If, for example, a magnet is broken into any number of parts, each part is itself a magnet with a north and south pole. If a magnet is heated, jarred, or hammered, it is weakened considerably. This is explained as due to the fact that the molecules have been disarranged from their magnetic line-up. Or again, a glass tube full of iron filings can be easily magnetized; when shaken, however, its magnetism is lost for the same reason, namely, the magnetic line-up of its particles has been disarranged.

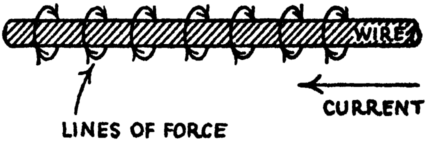
ELECTROMAGNETISM

Magnetic Field Surrounding a Conductor. A conductor carrying an electric current is surrounded by a magnetic field. As early as 1819 Oersted discovered that if a wire through which a current flowed was held over a compass, the compass needle was deflected. When placed under the compass, the needle was deflected in the reverse direction. To study the magnetic field around a conductor, we may send a strong current through a vertical wire which passes through a horizontal piece of cardboard. If iron filings are sprinkled on the cardboard, they will arrange themselves in concentric circles about the wire, indicating the positions of the lines of force. The directions of the lines are found by placing a small compass at various positions on the cardboard.



Lines of Force around a Conductor

The rule for determining the direction of the lines of force about a wire carrying a current is known as the *thumb rule.* If a wire is grasped with the right hand so that the thumb points in the direction in which the current is flowing, the fingers will point in the direction in which the lines of force encircle the wire. Conversely, if the direction of the lines of force is known, the same method can be used to find the direction in which the current flows. For, if the wire is grasped with the right hand so that the fingers point in the direction of the magnetic field, the thumb will point in the direction in which the current is flowing.



Thumb Rule for Wire Carrying a Current

Magnetic Field around a Coil of Wire. If a current is passed through a *solenoid,* or coil of wire, there will be several circular lines of force about each turn of wire, but in general the lines of force will thread the whole coil and be set up outside the coil very much like those of a bar magnet. In fact, the solenoid carrying current behaves like a bar magnet. When a core of soft iron is placed inside the solenoid, we have an *electromagnet.* While the current is flowing, the soft iron core will become strongly magnetized and be able to attract magnetic substances. As soon as the current is stopped, the soft iron core loses its magnetism. This ability to control the magnetic action by controlling the current is widely used in a number of important industrial and commercial applications.

Strength of an Electromagnet. This depends upon several factors. It increases with an increase in each of the following:

1. The number of turns of the coil.
2. The current flowing through the coil.
3. The permeability of the iron core.

The product of the number of amperes flowing through the coil multiplied by the number of turns in the coil is called *ampere-turns.* The strength of an electromagnet is therefore directly proportional to the number of ampere-turns and to the permeability of the core.

To determine the *polarity* of an electromagnet, another right-hand rule is used, namely, the *thumb rule for a coil.* If the coil is grasped with the right hand so that the fingers point in the direction in which the current is flowing, the thumb will point to the north pole of the coil.

Practical Applications of the Electromagnet. Electromagnets are widely used in various electrical devices, machinery and equipment. They are found in the telegraph, the telephone, the electric bell, etc. They are used to generate the magnetic field of motors and generators. Large electromagnets called magnetic hoists are used for handling large masses of iron and steel.

Although electromagnets are made in different shapes, the form most commonly used is the horseshoe electromagnet. This consists essentially of two coils of wire wound upon cylindrical iron cores which are connected by a soft iron yoke. The coils are wound so that the current will pass through them in opposite directions resulting in opposite poles at the ends of the cores. One of the most common applications of the electromagnet is the ordinary electric bell, described in detail in Chapter XIV.

STATIC ELECTRICITY

Static Electricity. It is a matter of common experience that a hard rubber comb will attract small bits of paper after having been used to comb one's hair. Over 2500 years ago, the Greeks found that amber when rubbed would attract other light objects. Objects which have acquired the property of attracting other light bodies are said to be *electrified,* or to have acquired an *electric charge.* Electrified bodies are able to attract bodies which are non-electrified. Thus, a glass rod rubbed with silk, or a rubber rod rubbed with cat's fur, is able to attract bits of paper, cork, etc. Electrification is not the same as magnetization. In none of the cases mentioned were the electrified bodies magnetic substances.

Two Kinds of Electrification. Experiment shows that there are two distinct kinds of electrical charges. If, for example, an electrified glass rod is suspended so that it can turn freely, and another electrified glass rod is brought near it, the rods will repel each other. However, if a rubber rod electrified with flannel is brought near the glass rod, the two will attract each other. Evidently there appear to be two kinds of electricity, one which will repel a charged glass rod, the other, which will attract it. By agreement among scientists the following definitions have been accepted. The charge which appears on glass when rubbed with silk is called a *positive charge,* and substances which behave like glass when electrified are called positively electrified substances. The charge which appears on rubber or sealing wax when rubbed with flannel is called a *negative charge,* and substances behaving like them are called negatively electrified substances.

From the experiment previously described, it can be seen that bodies charged with the same kind of electric charge repel each other, while bodies charged with opposite kinds of electric charges attract each other. It should also be noted that when two substances are rubbed together, both kinds of electricity are developed. One of the substances becomes positively charged, while the other becomes negatively charged. Thus, if hard rubber is rubbed with flannel, the negative charge on the rubber will be quantitatively equal to the positive charge on the flannel.

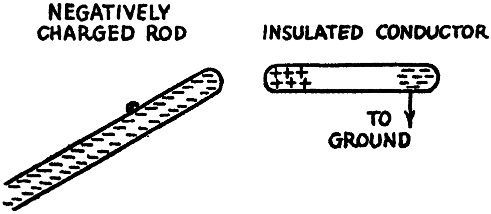
Conductors and Insulators. Not all substances will conduct electricity. Those substances which transmit electricity readily are called *conductors.* Metals, graphite, and water solutions of acids or salts are among the good conductors. Those substances which do not transmit electricity so well are called *insulators.* Thus rubber, glass, paraffin, paper, silk, mica, pure water, and dry air are all extremely poor conductors of electricity, i.e., are examples of insulators. It is important to note that insulators may have electric charges. They are insulators because they *cannot transmit* or *conduct* the electric charges easily. Thus glass or hard rubber are very easily electrified by friction. The charges, however, will remain and will not flow away as they do in a good conductor.

The Electron Theory and Electrification. The electron theory is a distinct aid in understanding what occurs during the process of electrification of a substance. According to this theory, the atoms of all elements are composed of positive and negative electricity. The negative electrical particles, called electrons, are grouped about a positive nucleus. If the atom is unelectrified, as it is in its normal state, the sum of the negative charges of the electrons equals and balances the positive charge of the nucleus. If a body is *negatively charged,* it contains more electrons than its usual number. When *positively charged,* a body contains fewer electrons than its usual number. In other words, it has lost electrons from some of its atoms. The charging of a substance electrically consists essentially in adding or removing electrons from that substance. This theory is explained more fully in Chapter VIII.

Electrification by Contact and by Induction. A body may be electrified or charged either by contact or by induction.

Contact. If an unelectrified body is brought in contact with a charged body, it will acquire the *same* charge as that body. If the charged body is a rubber rod, the latter will release its excess electrons causing the body contacted to become negatively charged. A glass rod, positively charged, will absorb electrons, leaving the body contacted with a positive charge.

Induction. A body may also be charged by bringing it near an electrified body, but *not in actual contact* with it; such a body is said to be *charged by induction.* What occurs is the following: If a negatively charged rod is brought near an insulated conductor, the negatively charged electrons in the conductor are repelled toward the far side of the conductor, leaving a positive charge on the side near the rod. If the conductor is grounded by touching its far side with a finger, the electrons will flow into the ground, leaving the conductor positively charged. It can thus be seen that the charge produced on a body by induction is always opposite to that of the charging body.



Charging by Induction.

The Electroscope. In order to determine whether a body is electrically charged, and what the nature of its charge is, a special device called an *electroscope* is used. An electroscope consists essentially of a brass rod inserted through an insulating stopper into a glass container. The brass rod has a knob, also of brass, at its outer end, and two strips or leaves of gold or aluminum foil suspended from its lower end. When an electrically charged body is brought near the knob of the uncharged electroscope, the leaves will diverge since they both become similarly charged and therefore repel each other. The greater the charge, the further apart will the leaves be repelled. Upon removing the charged body, the leaves will collapse.

An electroscope can be charged by induction in the manner previously described. If, for example, a positively charged rod is brought near the knob of the electroscope, the latter will become positively charged, and its leaves will diverge. If the knob is then touched with the hand, electrons will flow from the ground through the body to the electroscope, neutralizing its positive charge. The leaves will then collapse. The electrons in the knob are held bound by the rod. When the hand and then the rod are withdrawn from the knob, the excess of electrons in the knob will be distributed over the electroscope causing it to be negatively charged, and resulting in its leaves diverging.

The charged electroscope can readily be used to determine the type of charge on a body brought near it. If the electroscope is negatively charged, a negatively charged body brought near it will cause its leaves to diverge still further. A positively charged body brought near it will cause the leaves to collapse, since it will draw away most of the electrons from the leaves to the knob. In general, the leaves of a charged electroscope will diverge if a similarly charged body is brought near, and will collapse if an oppositely charged body is brought near.

Construction and Uses of a Condenser. A condenser is a device used to increase the capacity of a conductor for holding electricity. It consists of two conductors separated by an insulator, or a dielectric, as it is usually called. The Leyden jar developed in about 1745 is a simple form of condenser used primarily in experimental work. It consists of a glass jar coated both inside and outside with tin foil for about two-thirds of its height. A brass rod and knob, inserted through its insulated stopper, are connected to the inner coating by means of a chain. The glass jar separating the two layers of tin foil acts as the dielectric.

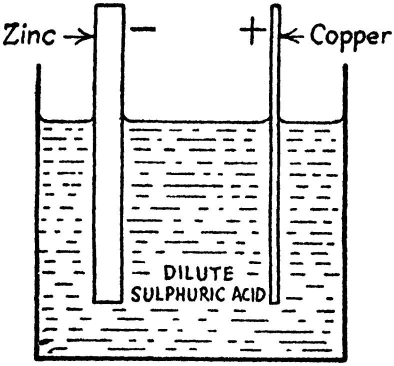
The Leyden jar is charged by grounding the outer tin foil coating, either by holding it with the hand or by means of a wire, and bringing the knob in contact with the source of electric charges. If the electrically charged body is negative, electrons will flow down the rod and chain to the inner coating, producing a negative charge on it. The outer grounded coating becomes positively charged by induction, since the inner electrons repel the electrons from the outer coating and cause them to pass off through the ground connection. The inner and outer coatings, being oppositely charged, bind each other so that neither can be discharged separately. The jar can be discharged by bringing one end of a conductor in contact with its outer coating, and the other end on the conductor near the knob causing neutralization of the charges.

Commercial condensers are much more compact units than the Leyden jar, but the principle is essentially the same. The *capacity* of a condenser, or its ability to store electrical charges, is directly proportional to the size of the condenser plates, and inversely proportional to the distance between them. It depends also upon the substance used as the insulator or dielectric. If mica, glass, or waxed paper is used as the dielectric, the capacity is greater than if air is the insulating material. Capacity is usually expressed in *microfarads*. Condensers are widely used in the electrical communications industries in radios, telephones, radio telegraphy, etc.

CURRENT ELECTRICITY

Potential Difference. To have an electric current flow from one point to another in a circuit, it is necessary that a *difference of potential* exist between these two points. Electricity will continue to flow from the point at the higher potential to that at the lower potential until they are both at the same potential, just as heat will flow from a body at a high temperature to one at a lower temperature until thermal equilibrium is established, or a liquid will flow from a tank at a high level to one at a lower level until it is at the same level in both tanks. An *electromotive force,* or electrical pressure, as it were, exists between the two points at different potentials. Electromotive force (abbreviated e.m.f.), difference of potential, and voltage are the names given to the electrical pressure existing between two points in a circuit.

The Simple Voltaic Cell. In order to keep an electric current flowing continuously, there must be some method of setting up and maintaining a potential difference. One of the simplest ways of accomplishing this is by chemical action as in the Voltaic cell. This type of cell consists of a strip of zinc and a strip of copper immersed in dilute sulphuric acid. The zinc is called the *negative electrode,* the copper the *positive electrode,* and the acid is called the *electrolyte.* When the two electrodes are connected by a wire, an electric current flows through the wire from the copper to the zinc plates.



The Voltaic Cell

What happens within the cell is as follows: zinc particles enter into solution in the form of positively charged ions, leaving an equal number of electrons, or negative charges, on the zinc plate. The positive zinc ions repel the positive hydrogen ions of the acid, which travel toward the copper plate where they are liberated in the form of gaseous hydrogen, after absorbing electrons from the copper plate. The copper plate becomes positively charged because of its loss of electrons to the hydrogen ions. The chemical action in the cell results in an excess of electrons on the zinc plate, and a deficiency of electrons on the copper plate. It is because of the crowding of electrons off the zinc plate and the lack of electrons at the positive plate, that the electromotive force of the cell is maintained.

When the circuit is closed by connecting the two electrodes with a conductor, the electromotive force will cause a current to flow in the conductor from the positive copper terminal to the negative zinc terminal. After the current has been flowing for some time, a decrease in voltage occurs. This is due to the *polarization* of the cell, or the accumulation of hydrogen bubbles on the copper plate. The hydrogen bubbles increase the resistance of the cell and also set up an electromotive force in the direction opposite to that of the cell, therefore decreasing its effective voltage. Polarization may be remedied by surrounding the copper electrode with an oxidizing agent such as manganese dioxide, chromic acid, or copper sulphate. The hydrogen will readily be disposed of by combining chemically with the oxygen in any of these materials. Different types of cells can be made using other materials, providing two different metals are used as electrodes, and an appropriate electrolyte is selected.

The Dry Cell. The dry cell is one of the most convenient types of cells, and is used extensively for open-circuit or intermittent work, as in operating bells, signal devices, flash lights, etc. In this type of cell, the negative electrode is a zinc can which also serves as the containing vessel for all the other contents. The zinc can is lined with an absorbent layer of blotting paper saturated with a solution of ammonium chloride. The positive electrode is a carbon rod, either cylindrical or fluted, placed in the center of the cell. The space between the zinc can and the carbon rod is filled with a paste made of manganese dioxide, granulated carbon, ammonium chloride, zinc chloride, and water. The cell is sealed at the top with a pitch composition to prevent evaporation of the water. The voltage of such a cell is about 1.5 volts. In using the cell, ammonia gas and hydrogen gas are liberated at the positive carbon rod. The ammonia to a great extent dissolves in the water, but the hydrogen polarizes the cell. The hydrogen gradually combines with the oxygen from the manganese dioxide, slowly depolarizing the cell. It is because of the polarization that takes place, that a dry cell should be used for intermittent service only, and not be left on a closed circuit for any length of time.

Electrical Units. All substances oppose the flow of an electric current through them. The opposition which a conductor of electricity offers to the passage of current is known as its *resistance*. Different materials have different electrical resistances. The unit in which resistance is measured is the *ohm*. By definition, the ohm is the resistance of a column of mercury one square millimeter in cross section and 106.3 centimeters long at a temperature of 0° C.

The rate of flow of an electric current is the quantity of electricity passing a definite point in one second. Since the unit of quantity of electricity is the coulomb, an *ampere* may be defined as the strength of a current carrying one coulomb per second. The ampere is the unit of intensity of current, or rate of flow of current. By international agreement, one ampere is also defined in terms of its chemical effect, as that current which will deposit 0.001118 gram of silver per second when passing through a solution of silver nitrate.

Electromotive Force. It has been shown that in order to have an electric current flow through a resistance, it is necessary that a difference in potential exist between the two ends of that resistance. The electrical pressure resulting is known as the electromotive force. The unit of electromotive force, the *volt,* is the difference in potential needed to send a current of one ampere through a resistance of one ohm.

Ohm's Law. The relationship between current, electromotive force, and resistance was discovered by Ohm in 1827. Ohm found as a result of a number of experiments that *the current flowing in a circuit is directly proportional to the electromotive force and inversely proportional to the resistance.* This is the famous *Ohm's Law:*

$$Current=\frac{Electromotive\ force}{Resistance},$$

$$\text{or,}\quad amperes=\frac{volts}{ohms}.$$

Expressed in symbols,

$$I=\frac{E}{R},$$

where I is the intensity or rate of current flow measured in amperes, E the electromotive force measured in volts, and R the resistance measured in ohms.

Example 1: What current flows through a resistance of 10 ohms if the voltage across it is 110 volts?

$$I=\frac{E}{R}=\frac{110}{10}=11 \text{ amperes, } \textit{Ans.}$$

If both the current and the resistance of a circuit are known, the voltage may be found by using Ohm's Law in the form $E=I\times R$, or, *volts=amperes×ohms.*

Example 2: What voltage is needed to send ¼ ampere through an incandescent lamp which has a resistance of 450 ohms?

$$E=I\times R=\tfrac{1}{4}\times 450=112\tfrac{1}{2} \text{ volts, } \textit{Ans.}$$

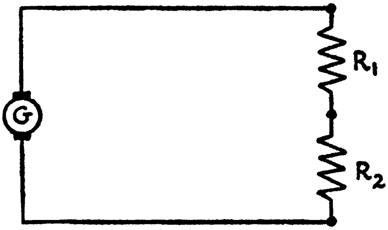
If it is desired to find the resistance needed to maintain certain conditions of current and voltage, Ohm's Law may be used in its third form:

$$R=\frac{E}{I};\quad \text{or, } ohms=\frac{volts}{amperes}$$

Example 3: What is the resistance of an electric iron which takes 5 amperes on a 110-volt line?

$$R=\frac{E}{I}=\frac{110}{5}=22 \text{ ohms, } Ans.$$

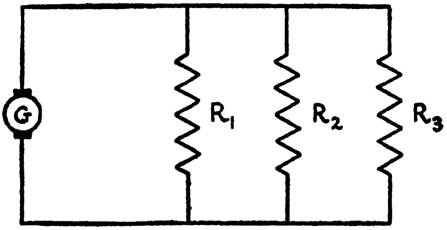
Series Circuits. A series circuit is one in which the pieces of electrical equipment are connected one after the other so that the same current will flow through each of them in turn. In a series circuit the following holds true:



Series Circuit

1. The *current* through every part of the circuit is the same.
2. The *resistance* of the circuit is equal to the *sum* of the separate resistances.
3. The *voltage* across the circuit is equal to the *sum* of the voltages across the separate resistances.

Parallel Circuits. A parallel circuit is one in which the pieces of electrical equipment are connected side by side so that the current is divided among them. For parallel circuits we have:



Parallel Circuit

1. The *voltage* across the circuit is the same as the voltage across each branch.
2. The total *current* in the circuit is the sum of the individual branch currents.
3. The *resistance* of the entire circuit is *less* than that of the smallest branch resistance.

The combined resistance of a number of parallel resistances can be found by the following relationship: The reciprocal of the combined resistance is equal to the sum of the reciprocals of the individual resistances; or,

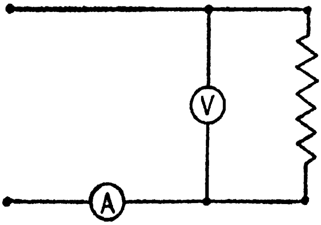
$$\frac{1}{R}=\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}+\frac{1}{r_4}+\cdots$$

Ohm's Law and Circuit Problems. Ohm's Law can be used in solving for the current, voltage, and resistance of a complete circuit, or any part of a circuit. Care must be taken to select the proper and consistent values of current, voltage and resistance. For a *complete* circuit, the current flowing in the line is equal to the voltage across the entire circuit, divided by the resistance of the entire circuit. For any *part* of a circuit, the current flowing in that part is equal to the voltage across that part divided by the resistance of that part. Corresponding statements can be made for both voltage and resistance.

Measuring Instruments. A *galvanometer* is an instrument used to detect the existence of an electric current and to indicate its strength. Voltage or electromotive force is measured by means of a *voltmeter,* a high resistance galvanometer graduated to read in volts. A voltmeter is always connected in *parallel* with that portion of the circuit across which we wish to measure the potential drop. Current is measured by means of an *ammeter,* which is essentially a low-resistance galvanometer graduated to read in amperes. An ammeter is always connected in *series* with the circuit. Resistance can easily be determined by the voltmeter-ammeter method. An ammeter is connected in series with the device whose resistance we wish to determine. The voltmeter is placed across it. The resistance can then be found by applying Ohm's Law, $R=E/I$.

EXAMPLE: A voltmeter placed across an electric device reads 110 volts; an ammeter placed in the line reads 4 amperes. What is the resistance of the device?

$$R=\frac{E}{I}=\frac{110}{4}=27\tfrac{1}{2}\text{ ohms, } \textit{Ans.}$$



Factors Affecting the Resistance of a Conductor. The resistance of a conductor depends upon the following four factors: length, cross-sectional area, material, and temperature.

It has been experimentally found that *resistance is directly proportional to the length of the conductor.* The longer the conductor, the greater will be its resistance. Thus, a #10 copper wire 50 feet long has a resistance twice that of 25 feet of the same wire, and 5 times that of 10 feet of the wire.

The resistance of a conductor varies inversely as its cross-sectional area. Since the area is equal to 3.14 times the square of the diameter, resistance also varies inversely as the square of the diameter of the conductor. The greater the diameter of a wire, therefore, the lower its resistance. A conductor 1 inch in diameter would have one quarter the resistance of one

½ inch in diameter. Since wires carrying current are almost invariably circular in cross-section, a special unit called the *circular mil* is used to express area instead of square inches. A mil is equal to one one-thousandth of an inch (0.001″). Thus, a wire 0.001″ in diameter is also 1 mil in diameter, and by definition has an area of 1 circular mil (1 c.m.). The area of a wire in circular mils is equal to the square of its diameter in mils. A wire 10 mils in diameter has a cross-sectional area of 100 circular mils.

Resistance depends also upon the material of the conductor. Different materials of the same length and cross-sectional area have different resistances. For purposes of comparison, the resistance of a piece of wire 1 foot long and 1 circular mil in cross-section, known as the *mil-foot,* is used. The resistance of a mil-foot is called the *specific resistance,* or *resistivity,* of the material. Copper has the lowest specific resistance of any metal except silver, and is therefore almost always used in electrical work.

SPECIFIC RESISTANCE OF COMMON METALS

(*in ohms per mil-foot at 20° C.*)

Silver	9.9–11.0	Zinc	37.4
Copper	10.4	Platinum	60.2
Gold	14.7	Iron	66.5–81.0
Aluminum	17.0	Lead	132.5

The resistance of a wire is readily found by multiplying its specific resistance by its length in feet and dividing by its area in circular mils. Expressed mathematically,

$$R = \frac{kL}{c.m.},$$

where k is the specific resistance, L the length in feet, $c.m.$ the area of cross-section expressed in circular mils.

The resistance of a conductor also varies with its temperature. For most metals, the resistance increases as the temperature rises. Several substances, among them carbon, porcelain, glass, and electrolytes, decrease in resistance with a temperature increase.

Heating Effect of an Electric Current. Whenever electricity flows through a conductor heat is produced. This heating effect of electricity is utilized in a number of important electrical appliances, e.g., electric heaters, irons, toasters, ovens, furnaces, etc. The incandescent lamp is another application; electricity passing through its fine tungsten filament of extremely high resistance causes the filament to become white hot, thereby emitting light. In electrical welding, the two pieces of metal to be joined are placed together and an electric current passed through them. The resistance is greatest at their point of contact, and enough heat is developed here to raise the metals to their welding temperature. They are then pressed firmly together.

Joule found that the heat produced in a conductor by an electric current is proportional to the square of the current, to the resistance of the conductor, and to the time during which the current flows. A current of one ampere flowing through a one-ohm resistance for 1 second produces 0.24 calorie of heat energy. One watt-second, therefore, is equivalent to 0.24 calorie. Joule's findings can be expressed mathematically as follows:

$$H=0.24I^2Rt,$$

where H is the heat in calories, I the current in amperes, R the resistance in ohms, and t the time in seconds.

EXAMPLE: How much heat is developed per hour in an electric iron which uses 5 amperes and has a resistance of 20 ohms?

$$H=0.24I^2Rt,$$
$$H=0.24\times5\times5\times20\times3600=432{,}000 \text{ calories.}$$

Electric Power and Energy. The *power* of an electric circuit depends upon two factors: the current flowing, and the voltage. It can be found by multiplying the current in amperes by the electrical pressure or potential in volts.

$$Power=Current\times Voltage$$

The unit of electric power, the *watt,* is defined as the power required to keep a current of one ampere flowing under a potential difference of one volt. The above equation can therefore be rewritten in terms of units:

$$Watts=Amperes\times Volts, \text{ or } W=I\times E.$$

Thus, if an electric iron uses 4 amperes on a 110-volt line, the power consumed is 4×110, or 440 watts. Since the watt is a rather small unit for most industrial and commercial power requirements, a larger unit, the *kilowatt* (kw.), is more commonly used. A kilowatt is equal to 1000 watts. Thus 5000 watts are equal to 5 kilowatts, and 500 watts to ½ kilowatt. Kilowatts may be found directly by the following formula:

$$Kilowatts=\frac{Amperes\times Volts}{1000},$$

$$\text{or } kw.=\frac{I\times E}{1000}$$

In order to convert electrical power units into the corresponding mechanical units, it is important to know the relationship between the horsepower, the watt and the kilowatt. We have:

1 horsepower=746 watts=0.746 kilowatts
1 kilowatt=1.34 horsepower

For most practical purposes, and for estimating generally, the following are sufficiently accurate:

1 *hp.*=¾ *kw.*
1 *kw.*=1⅓ *hp.*

Example 1: A motor operating at 220 volts requires 25 amperes. What power does the motor use in kilowatts? In horsepower?

Solution:

$$\text{Kw.}=\frac{\text{I}\times\text{E}}{1000}$$

$$=\frac{25\times 220}{1000}=\frac{5500}{1000}=5.5\text{ kw., } \textit{Ans.}$$

$$5.5\text{ kw.}=5.5\times 1.34=7.37\text{ hp., } \textit{Ans.}$$

The kilowatt, like any other power unit, is a measure of the *rate* of doing work. To find the *electrical energy* consumed in running a machine or any other electrical device, it is necessary to know not only the power supplied, but also the *length of time* during which that power was supplied. Electrical energy is equal to the product of electrical power and the time, and is expressed either in watt-hours or kilowatt-hours. A motor consuming 10 kw. for 5 hours uses 50 kw.-hrs. of electrical energy; in 10 hours it will use 10×10, or 100 kw.-hrs. of electrical energy. The energy used in any circuit can be calculated directly by the use of the equation:

$$kw.\text{-}hrs.=\frac{amperes\times volts\times time}{1000}$$

or

$$kw\text{-}hrs.=\frac{I\times E\times t}{1000}.$$

Example 2: An electric motor when operating on a 110-volt line requires 20 amperes. (*a*) How many kilowatt-hours of energy are used in 10 hours? (*b*) At 2¢ per kw.-hr., what is the cost of the energy used?

Solution:

$$\text{kw.-hrs.}=\frac{20\times 110\times 10}{1000}=\frac{22{,}000}{1000}$$

$$=22\text{ kw.-hrs., } \textit{Ans.}$$

$$\text{Cost}=22\text{ kw.-hrs.}\times \$.02=\$0.44, \textit{ Ans.}$$

Electromagnetic Induction. In 1831 Faraday in Great Britain and Joseph Henry in America independently discovered the principle of electromagnetic induction, namely, that when a conductor is moved in a magnetic field so as to cut across lines of force, an electromotive force is produced in the conductor. It is primarily upon this discovery that the modern electrical industry is built. Electricity is generated in large dynamos or generators, and utilized in operating electric motors to run various types of machinery, as well as for lighting, for heating, etc. Both the generator and the motor are based upon these early discoveries in electromagnetic induction.

Faraday found that if two coils of wire are wound on a cylinder, and an electric current passes through one of them, the other coil will have a current momentarily induced in it. Likewise, when the current in the first coil is stopped, a current will be induced in the second coil in the opposite direction.

Whenever lines of force are cut by a conductor, an electromotive force is set up in the conductor. The current flowing in the circuit as a result of this induced electromotive force is called an *induced current.* If a strip of wire is connected to a galvanometer and the wire moved downward across the poles of a strong electromagnet, the galvanometer will be deflected, indicating a flow of current. If the wire is moved upward, the galvanometer will be deflected in the opposite direction. The quicker we move the wire across the magnetic field, the greater will be the galvanometer deflection. However, if the wire is held at rest between the two poles, no current will flow through it. If the magnetic field is increased by sending a greater current through the electromagnet, and the wire is again moved upward and downward, the deflection will be greater than previously. If instead of a single wire, a *coil* of the same wire is moved through the poles, the induced electromotive force is even more greatly increased.

From these experiments, the *laws of induced currents* are deduced:

1. Any change in the number of lines of force passing through or cut by a conductor results in an electromotive force being induced in the conductor, and therefore causing a current to flow through it.

2. The magnitude of the induced electromotive force is increased with an increase in the speed with which the lines of force are cut, with increased strength of the magnetic field, and with an increase in the number of turns of the coil, if a coil is used.

Fleming's Rule. The direction of an induced electromotive force depends upon the direction of the magnetic field and the direction of motion of the conductor. The relationship of these directions is given by *Fleming's rule:* Hold the thumb, forefinger, and center finger of the right hand so that they are at right angles to each other. If the thumb points in the direction in which the conductor is moving, and the forefinger in the direction of the magnetic flux, then the center finger will automatically point in the direction of the induced current.

Lenz's Law. Whenever a current flows in a conductor, a magnetic field is set up about the conductor. This happens also whenever induced currents are set up in a wire or coil. It has been found experimentally that the direction of an induced current is always such that its magnetic field opposes the motion which causes the induced current to flow. It was Lenz who first stated this principle, which is now called Lenz's Law.

Electric Generators and Motors. The fact that an electromotive force is induced when a conductor moves across a magnetic field is utilized industrially in the production of electric energy in our large dynamos and electric generators. Here, instead of one conductor, there are hundreds of conductors wound on an armature, which continuously cut through a magnetic field produced either by electromagnets in large generators, or by a magneto in some small generators. The result is the development of an electromotive force and the steady flow of an electric current. The electric generator transforms the mechanical energy supplied to it into electrical energy. The electric motor, in construction quite similar to the generator, utilizes electrical energy to drive other machines. The motor transforms electrical energy into mechanical energy.

SELECTED REFERENCES FOR FURTHER STUDY

ELEMENTARY

BANKS, C. W. Applied Science. J. Wiley & Sons.

BLACK, N. H., and DAVIS, H. N. Elementary Practical Physics. Macmillan Co.

CLARK, J. A., GORTON, F. R., and SEARS, F. W. Physics of Today. Houghton-Mifflin Co.

DOOLEY, W. H. Science Training for Metal and Wood Trades. Ronald Press Co.

FULLER, R. W., BROWNLEE, R. B., and BAKER, D. L. First Principles of Physics. Allyn & Bacon.

SMITH, L. R. Industrial Physics: Mechanics. McGraw-Hill Book Co.

SMITH, L. R. Industrial Physics: Heat. McGraw-Hill Book Co.

SMITH, L. R. Industrial Physics: Elementary Industrial Electricity. McGraw-Hill Book Co.

ADVANCED

ANDERSON, W. B. Physics for Technical Students. McGraw-Hill Book Co.

BLACK, N. H. An Introductory Course in College Physics. Macmillan Co.

HAUSMANN, E., & SLACK, E. P. Physics. D. Van Nostrand Co.

KIMBALL, A. L. College Textbook of Physics. H. Holt & Co.

LEMON, H. B. From Galileo to Cosmic Rays. University of Chicago Press.

MENDENHALL, C. E., EVE, A. S., and KEYS, D. A. College Physics. D. C. Heath & Co.

SPINNEY, L. B. A Textbook of Physics. Macmillan Co.

SWANN, W. F. G., and FREEMAN, I. M. Physics. J. Wiley & Sons.

CHAPTER VII

STRENGTH OF MATERIALS

William L. Schaaf

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35. PHYSICAL PROPERTIES OF MATERIALS

Use of Materials in Trade and Industry. Obviously the activities and products of the shop and factory involve the use of a wide variety of materials. These include basic structural materials, such as iron, steel, wood, brick, stone, cement, plaster, concrete, etc.; it includes metals and alloys used both in the tools and the machines which are needed to fabricate the products, as well as the products themselves, as for example, brasses and bronzes, solders, bearing metals, tool steels, beryllium copper, etc.; and miscellaneous other materials, such as glass, porcelain, and various ceramics; textile products such as hose material, felt padding, screening material, filter cloths; and scores of others such as asbestos, rubber, cork, plastics. It can readily be seen, then, that a knowledge of the physical properties of these materials is indispensable if we are to work with them effectively.

These physical properties are concerned (1) with the physical aspects of the material as such, and (2) with its physical behavior under varying external circumstances and conditions. Examples of (1) would be melting point, heat conductivity, plasticity, hardness, specific gravity, density, molecular structure (i.e., crystalline, amorphous, etc.); examples of (2) would be the behavior of a substance when subjected to temperature changes, blows, pressure, rubbing, vibration and, in general, forces of all kinds. It is to be understood that these two aspects are not always sharply distinguishable nor are they entirely unrelated. However, for convenience, we shall first discuss some of the general physical properties, and then briefly examine their reactions to external forces. It is because of these two aspects of the problem that the phrases "strength of materials" and "mechanics of materials" are used more or less synonomously.

Hardness. One of the most commonly observed features of a substance is its hardness; thus stone and steel are hard, putty and rubber are soft. But what about other materials? How hard is glass? tin? is lead harder than gold? how much softer than oak is pine wood? Just what is hardness? Hardness is generally regarded as the capacity of a material to resist abrasion (scratching), plastic deformation (molding), or relatively small indentation (penetration). Actually, hardness is not a *single property,* but rather a combination of several properties. This explains why a number of different methods are available for measuring degrees of hardness, although, strictly speaking, there is no single measure of hardness. For the present, let us think of the hardness of a substance as the degree to which it resists penetration by another substance. Hardness should not be confused with brittleness; a *brittle* substance is one which shatters readily when subjected to impact or blows. Furthermore, neither brittleness nor hardness are directly related to elasticity and resiliency; the latter have to do with recovery from deformation, whereas brittleness and hardness have to do with permanence of deformation, i.e., with scratching or penetration, fracture or shattering. Several methods for measuring degrees of hardness are in common use.

Moh's Scale of Hardness. This method is based upon the relative ease of scratching by various minerals. The "modified" Moh Scale consists of a series of minerals, each of which can be scratched by the mineral next above it in the scale, as follows:

MOH SCALE (*Modified*)

#1—Talc
#2—Gypsum
#3—Calc spar
#4—Fluor spar
#5—Apatite
#6—Orthoclase
#7—Vitreous silica
#8—Quartz
9—Topaz
#10—Garnet
#11—Fused Zirconia
#12—Fused alumina
#13—Silicon carbide
#14—Boron carbide
#15—Diamond

Needless to say, the differences in the degree of hardness between any two numbers on the scale are not numerically equivalent. The relative hardness of some common materials is given below:

Bell metal	4.0	Mica	2.8
Brass	3.5	Phosphor-bronz ...	4.0
Copper	2.8	Platinum	4.3
Glass	5.5	Silver	2.8
Gold	2.8	Tin	1.5
Iron	4.5	Wood's metal	3.0

Brinell Method. This is a penetration test, based on an invention by a Swedish engineer, J. A. Brinell (1900). A hard steel ball 10 mm. (.3937 in.) in diameter is forced under pressure into a flat surface on the sample whose hardness is to be tested. For iron and steel samples, the pressure used is 3000 kg. (approx. 6,600 lb.), and the ball is allowed to rest on the sample for 10 seconds; for "soft" metals, the pressure used is 500 kg. (1100 lb.), and the pressure is maintained for 30 seconds. The diameter of the slight spherical indentation thus made is then measured under a microscope. The hardness is expressed as the ratio of the pressure used to the area of the indentation made; this ratio is known as the *Brinell hardness number;* or

$$B=\frac{P}{\frac{\pi D}{2}\left(D-\sqrt{D^2-d^2}\right)},$$

where P=pressure used in kg., D=diameter of ball (10 mm.), d= diameter of the indentation made in mm., and B=Brinell number.

The Scleroscope. This is an instrument which consists essentially of a vertical glass tube, inside of which a small cylinder of very hard steel is free to slide. This cylinder is pointed at one end, and is called the hammer. The hammer is allowed to fall about 10″ on the sample to be tested. The distance it rebounds is then taken as a measure of the hardness of the sample; a scale on the tube is divided into 140 equal parts, and the hardness is expressed by the number on the scale to which the hammer rebounds. Comparative values of some common substances are given below.

Glass	130
Porcelain	120
Steel (very hard)	110
Cast iron	40
Wrought iron	20
Brass (soft)	12
Babbitt metal	8
Lead	2

Rockwell Hardness Tester. The Rockwell hardness tester indicates the hardness of metals by measuring the residual depth of penetration made by a steel ball or a diamond cone. In use the penetrator, or the brale, is first seated on the specimen by applying a minor load of ten kilograms. The major load is then applied and the penetrator or brale is forced into the specimen. When the major load is removed, the elasticity of the material will raise the penetrator to some extent, but the minor load still keeps the penetrator in contact with the impression. The depth of impression remaining after the elasticity has exerted its "lift" is the residual depth.

The principal parts of the tester are: the table or anvil, upon which the specimen is placed; the penetrator, which may be a steel ball or a conical diamond which is pressed into the specimen; the gage, which measures the depth of impression; the elevating screw, which raises the anvil; the dashpot, which regulates the speed of application of the major load; the crank-handle, which releases the major load; and the levers and weights which determine the force applied to the penetrator.

The gage dial has two scales marked B and C. The C scale is used for hard materials with a 150-kilogram load applied through the brale; the B scale is used for softer material with a 100-kilogram load and the steel ball penetrator. All Rockwell readings must be prefixed with the letter of the scale used; the reading is not merely a number, but a letter plus a number.

Elasticity. Whenever a body is subjected to external forces the effect of which is to modify its size or shape, or both, the body is said to be subject to *deformation*. It may resist this tendency to deformation in varying ways and to different degrees. When a given material allows itself to be deformed and then returns to its original shape upon removal of the force or load it is said to be *elastic*. The measure of elasticity is the ability to recover its original state. All materials possess a greater or lesser degree of elasticity; some substances, of course, are far more elastic than others. Rubber, for example, in the form of the familiar elastic band, comes to mind at once; but how elastic is wood? or steel? or the hard rubber of which your fountain pen is made? Most substances are elastic only within very narrow or restricted limits. Thus steel is elastic up to a certain point, coming back to its original state, as when used in springs; but beyond this point steel loses much of its elasticity and the material only partially returns to its initial form upon removal of the force. Or again, glass is fairly elastic up to a point; a long glass tube or a good-sized window pane can stand an appreciable amount of bending before the stage is reached where it shatters and the deformation is permanent. Billiard balls, made of ivory, illustrate an exceptionally high degree of elasticity; due to hardness, the deformation due to impact is not readily observable (although with recently developed methods of high speed photography the deformation of billiard balls, golf balls, tennis balls, racquet strings, etc., are vividly portrayed). *Resiliency* is a property closely related to elasticity, though by no means synonomous with it; it refers to the "give" or "springiness" of a material. More technically, resilience is the stress energy which may be recovered when the load causing the stress is removed. Within certain limits, the resilience equals the external work that was originally required to deform the body. Thus the gut strings of a tennis racquet are highly resilient; similarly the resilience of wood in certain musical instruments, both woodwinds and string instruments, explains in large part some of their pleasing qualities. A wooden floor is more resilient than a stone floor or a cement pavement, as anyone accustomed to much standing or walking very well knows.

Ductility and Malleability. A substance is generally referred to as *ductile* if it lends itself rather well to being *drawn out* into a thin rod or wire; thus silver, copper, steel, quartz, and glass manifest relatively high degrees of ductility, since they can all be *extruded,* i.e., drawn out or spun into wires or threads. On the other hand, materials are said to be *malleable* if they readily admit of being rolled, hammered or pressed into comparatively thin sheets or plates. Thus lead, tin, gold, copper are common instances of malleable substances; gold *leaf,* tin *foil, sheet* lead, copper *plate* are familiar articles whose very names suggest their high degree of malleability. Actually, ductility and malleability are rather similar properties; they are both an indication of the ability of a material to undergo comparatively large *permanent* deformation without rupture (i.e., without breaking). Many ductile substances are also malleable, and vice versa. The chief distinction between ductility and malleability is that in the case of the former the deformation results from what is known as *tension,* or pulling apart, while in the latter it results from *compression,* or pressing together.

ORDER OF DUCTILITY OF METALS
Gold
Platinum
Silver
Iron
Copper
Aluminum
Nickel
Zinc
Tin
Lead

The degree of ductility of materials may be measured in several ways: (1) by the tension test; (2) the bending test; and (3) the cupping test. In the first case, the ductility is given in terms of the percentage of elongation and the percentage of contraction in cross-sectional area. The bending test depends upon a determination of the amount of bending that a material can withstand, under specific conditions, before it shows outside cracking. The third method is particularly useful for metals which are too brittle or which rupture too easily to permit their true ductility to be measured by pulling or bending; it depends upon the readiness with which a material responds to "shaping" under the force of a plunger when a sheet of the material to be tested is interposed between the plunger and an appropriate die (i.e., a hard, molded "pattern").

Plasticity and Brittleness. When a substance is of such a nature that even a very slight force or load produces a permanent deformation, the material is said to be highly plastic. A perfectly plastic material, theoretically, is totally lacking in elasticity; it has no "breaking-point." Examples of such comparatively non-elastic materials (at ordinary temperatures) are wax, paraffin, asphalt, etc. Lead, although not ordinarily thought of as such, is also quite inelastic. So, also, are the various forms of "plastics," those recently developed synthetic organic compounds that are coming into wider use daily; their great plasticity and lack of well-defined internal molecular structure permit these materials to be molded freely in a variety of shapes in the first place, and allow them to be worked easily afterward, that is, cut, drilled, shaped, etc. *Brittleness* is the opposite of plasticity. A brittle material is one which can be deformed to only a very slight degree before rupture or breaking occurs; thus (at ordinary temperatures) glass and hard rubber are extremely brittle; among structural materials, cast iron, stone, and brick are also examples of comparatively brittle materials. In short, highly brittle substances have no plasticity, and extremely plastic materials are not brittle. The degree of plasticity is measured by the amount of reduction in area at the place where a material breaks when subjected to stress; a large reduction in area shows high degree of plasticity or inelasticity, while a small reduction in area shows a high degree of brittleness.

Stiffness. This refers to the ability of a *body,* such as an iron *beam,* a concrete *column,* a wooden *plank* or a steel *plate,* to resist deformation under stress rather than to any particular physical property of the material itself. *Toughness* is the ability of a material to resist *intense* deformation under *intense* load or force, where "intense" refers to the amount of deformation or load per unit of area of material. Thus while lead is ductile, hickory is tough.

Creep of Metals. The phenomenon of *creep* refers to the tendency of the particles of a solid substance to *flow* over one another. It is perfectly typical of plastic materials at ordinary temperatures, as already mentioned in connection with wax or asphalt, which can be deformed with a minimum of force. However, even at ordinary temperatures, certain metals such as tin, zinc and lead reveal a tendency to "creep" under moderate stress; and at higher temperatures, practically all metals creep under stresses. The amount of creep depends both upon the amount and nature of the stress, as well as upon the temperature; in general, the higher the temperature, the less the amount of stress required to cause creep. This can be visualized by considering the relative ease of working asphalt or tar when hot as compared to when it is cold; or by observing the edges of a burning wax candle. The exact nature of creeping in metals is not yet too well understood, but its significance is apparent when we consider that many machine parts and structural members are subjected to rather high temperatures while in actual use; e.g., furnace walls, boiler tubes, superheaters, jackets, baffle plates, parts of turbines, piston rods, etc.

Defects and Flaws. Many materials, due either to their intrinsic nature or origin, or the way in which they were fabricated, possess certain structural weaknesses or lack of strength. Thus the knots in lumber constitute weakness in wooden timbers, joists and boards; similarly the internal condition of improperly annealed glass, insufficiently mixed concrete, and a heavily corroded metal bar, are all further examples of diminished strength of materials. Sometimes such weaknesses are not easily detected, particularly in the case of the internal condition of metals. Thus it is extremely important to be aware of the presence of such defects as included grains of sand, undue porosity, minute or pinhole cavities, blow-holes, gas cavities, included pieces of slag, segregation and imperfect blending of ingredients, actual or incipient cracks, etc. In recent years, however, through research and experimentation, there have been developed methods for detecting such flaws by means of X-rays; since X-rays can pass through the defects more easily than through thoroughly sound metal, the presence and location of the defects is revealed by the X-ray photograph, which clearly outlines the defective area. In this way it is possible not only to control the quality of the finished metal product—whether it be a machine casting, a steel rail, or a piece of armor plate—but also to modify and improve the techniques used in their fabrication.

Effect of Heat on Materials. Thus far nothing has been said concerning the effect of heat on materials of construction and production. While not strictly an aspect of the strength of materials, this is nevertheless an appropriate place to discuss the matter.

As you have already learned, heat causes solids, liquids and gases to change in volume; as the temperature increases, the volume increases, and as the temperature drops, the volume decreases. Gases change in volume more than liquids, and liquids more than solids. The change in solids, although relatively slight, is nevertheless one which must be taken into account in dealing with materials of engineering and industry. As a matter of fact, when solids change as a result of an increase in temperature, both the increase in length and in volume are important. The *coefficient of linear expansion* we saw was equal to

$$k=\frac{L_t-L_0}{L_0 t},\ \text{or}\ L_t=L_0\,(1+kt),$$

where L_0=length at 0° C, L_t=length at t°C, and t=any given temperature (C°). For ordinary work the so-called *mean coefficient of expansion* (k_m) between any two temperatures t_1 and t_2 is often used:

$$k_m=\frac{L_2-L_1}{L_1\,(t_2-t_1)};$$

for moderate ranges of temperature, say from 0° C to 100° C, the difference in between k and k_m for the same material is negligible. Moreover, when we consider the expansion in volume, it turns out that the *cubical coefficient of expansion* (k') may (except in extremely delicate and refined work) be taken equal to be three times the value of the linear coefficient; i.e., $k'=3k$. This means that to find the volume V at any temperature t, we make use of the formula

$$V_t=V_0(1+3kt).$$

For reference and comparative purposes we give below a list of linear coefficients of some commonly used materials.

LINEAR EXPANSION OF TYPICAL MATERIALS
(*Unit length increase per C° rise in temperature*)

Porcelain	.000003	Copper	.000017
Glass	.000008	Brass	.000019
Cast iron	.000010	Silver	.000019
Steel	.000011	Tin	.000021
Wrought iron	.000012	Aluminum	.000025
Gold	.000014	Zinc	.000026
Lead	.000028		

Another important physical property of materials related to heat effect is the capacity of various substances to conduct heat, or their *thermal conductivity*. Flow of heat through a substance by conduction is measured in terms of the amount of heat passing per unit of time through a unit of area with unit uniform temperature—gradient. While the physicist uses the centimeter-gram-second system, the technologist generally uses the English system of units. You will recall that a British thermal unit (B.T.U.) is the amount of heat required to raise 1 lb. of water 1° F; it is equal to 252 calories. For purposes of comparison and study, the relative thermal conductivities of common substances is given below; the figures represent (approximately) the number of B.T.U. per hour per sq. ft. of material 1″ thick per degree (F°) difference in temperature between the two faces.

RELATIVE THERMAL CONDUCTIVITIES

Silver	2900	Brickwork	4.0
Copper	2600	Magnesia	.5
Steel	300	Cork	.3
Building stone	13	Mineral wool	.3
Concrete	6	Asbestos	.29
Plate glass	5	Hair felt	.26

Physical Properties of Materials: Summary. The following tabular outline will perhaps summarize and clarify the salient features of physical properties that have to do with the strength and use of materials in technical work.

PROPERTY	CHARACTERISTIC FEATURE	TYPICAL MATERIALS	TYPICAL USES
Ductility	Can be drawn out or stretched without breaking.	Wrought iron Soft steel Copper Cartridge brass	Drawn articles Deep pressings Wire and cables
Flexibility	Can withstand considerable bending without rupture.	Hickory Spring steel	Springs Gears Tool handles
Toughness	Capable of absorbing heavy shocks.	Wrought iron Manganese steel Mild steel	Structural purposes Moving machine parts
Malleability	Capable of being hammered into thin sheets without rupture.	Copper Lead	Tank lining Tin foil Gold leaf Extruded products
Hardness	Offers considerable resistance to permanent deformation.	Bronze White cast iron Manganese steel	Cutting tools Wearing parts of machines
Brittleness	Can stand only a very slight amount of deformation before breaking.	Cast iron Hard steel Brick Concrete	Water pipes (mains) Engine cylinder blocks Dies for stamping Supports for dead loads

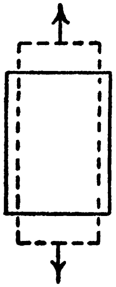
36. MECHANICS OF MATERIALS

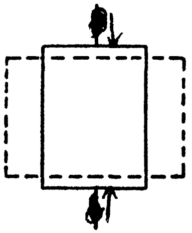
Simple Internal Stresses. Even though a body may be at rest, as when the external forces acting upon it are in equilibrium and no motion of the body as a whole results, nevertheless the material of which the body is made is subject to the influence of such external forces. The *internal* forces set up within the body to resist the effect of external forces are known as internal *stresses.* These are the stresses that were referred to in connection with deformation in the previous discussion. The *total internal stress* may then be described as the resultant internal force which resists or counteracts any tendency of the body to change in shape or size when acted upon by external forces. An alteration in shape or size begins as soon as the external force or load is applied; it is said to stop when the counteracting internal forces or stresses are in equilibrium with the external forces. If the maximum internal forces that can be developed within the material are unable to balance the external forces, changes in shape and size will occur relatively rapidly, and the material is said to "fail," i.e., it breaks or ruptures.

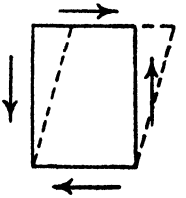
Kinds of Stresses. A number of kinds of stresses are recognizable. The following are *direct stresses:*

1. Tension. 2. Compression. 3. Shear.

Tension, also called tensile stress, is the internal force which resists the tendency of a body to elongate as a result of external forces acting in opposite directions *away* from its ends; it is caused by the attempt to pull an object apart, and as a result, there is a tendency within for the body to resist being torn into two pieces. *Compression,* or compressive stress, on the other hand, is the internal force resulting from the resistance to external pressure rather than pulling action, or external forces which tend to decrease the length of the body and push it together. Thus compression occurs in an object whenever the external forces act in opposite directions *toward* its ends. *Shear,* or shearing stress, is somewhat more


(a) Tension


(b) Compression


(c) Shear

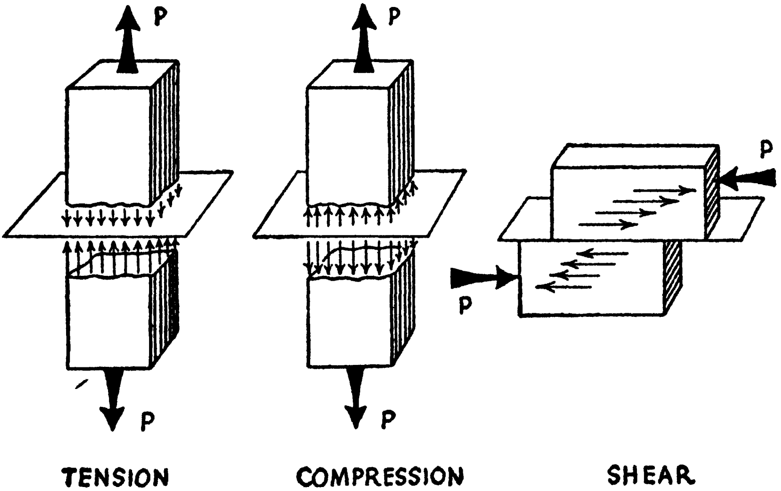
complicated; it is the internal resistance tending to overcome the effect of *internal sliding* of the particles of the material over one another as a result of external parallel forces acting in different, but parallel, planes, and in opposite directions.

These three types of stresses within a body may be considered as acting either separately or simultaneously; if considered singly, they are referred to as *simple stresses.* In addition, other stresses also to be considered include:

4. Transverse stresses (bending).
5. Tortional stresses (twisting).

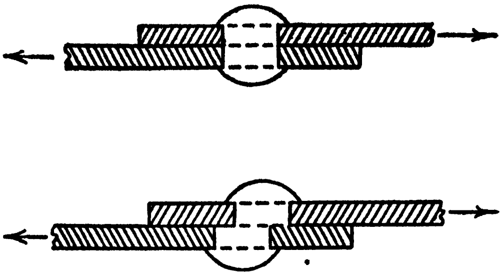
Unit Stress. The amount of internal stress on a unit of cross-sectional area is called the *unit stress;* it is the intensity of stress at any point within the material, and often varies as we go from one point to another. Unit stresses are expressed in terms of lb. per sq. in., or tons per sq. ft. Whenever the total stress, P, is distributed evenly or uniformly over the entire area, A, then the value of the unit stress at any point in the area is $s=P/A$.

Deformation under Stress. The amount of alteration in shape and size of a body resulting from the application of external forces is known as its *deformation.* As already suggested, when a body is subjected to tension, the deformation is the extent of its *elongation;* when in compression, it is the amount of *shortening* in length; when in shear, it is the amount of slipping within the body, or *detrusion.* These deformations are measured in terms of the same units as the original linear measurements, viz., inches or centimeters. Thus the so-called *unit deformation* is found by



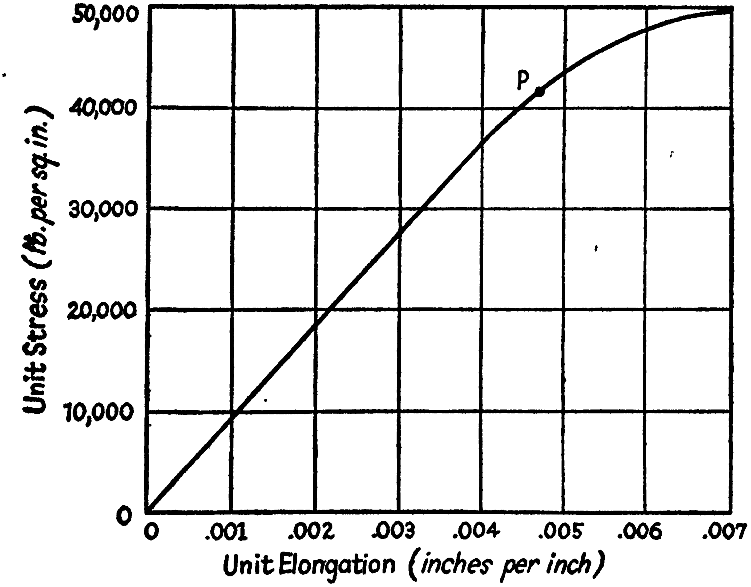
Internal Stresses

dividing the total amount of deformation by the original length of the body before the force or load was applied; for example, if a metal rod 6.50 in. long is subjected to tension and stretched until it has become 6.62 in. long, the total elongation=6.62—6.50=0.12 in., and the unit deformation=0.12÷6.50=.018 inches per inch of original length. In the past, the word *strain* was commonly used when referring to deformation, but at present the latter form is to be preferred.



Practical Example of Shear

Hooke's Law; Proportional Limit. It has long been known that when a material is subjected to external forces or an external stress that the *amount of the resulting deformation is directly proportional to the amount of the force applied,* within, however, certain limitations. This is known as *Hooke's Law.*

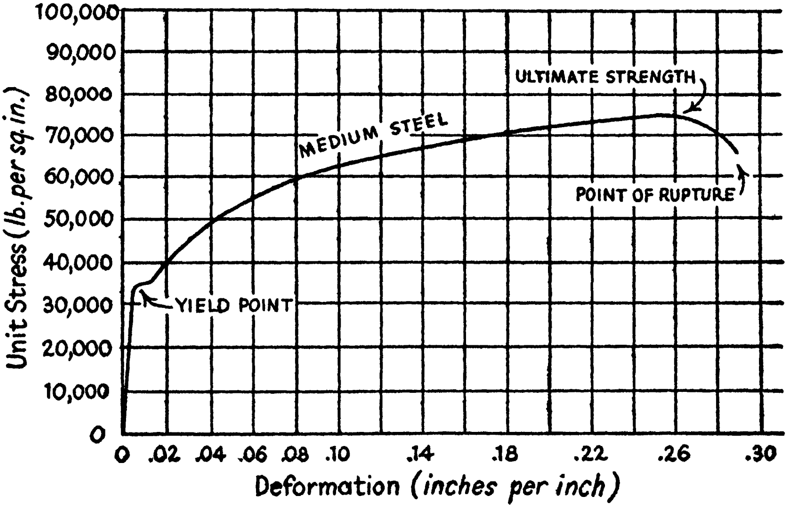


Typical Elongation of Metal Rod or Wire

The restriction applying to Hooke's Law is that the unit stress must not exceed a certain value, known as the *proportional limit,* which varies for different materials. The proportional limit is the maximum unit stress for which the deformation is proportional to the stress. Or, putting it another way, the proportional limit is that unit stress at which the unit deformation begins to increase more rapidly than the unit stress. On the accompanying graph this is represented by point P. Up to this point, Hooke's Law applies, for the graph is a straight line, indicating that the stress and deformation are proportional; beyond this point, however, the steepness of the graph diminishes, showing that the elongation per unit stress is larger than it was up to this point.

Elastic Limit. By the *elastic limit* is meant the maximum unit stress which may be applied to a material and still allow it to return to its original state when the stress is removed. If a body is stressed beyond its elastic limit, it will only partly recover its original size and shape; this is known as a permanent deformation or *set.* It will be seen that to determine the elastic limit carefully requires a considerable number of repeated tests, each with an increased load, until a permanent set has appeared. Because this is a rather laborious procedure, and also because for many materials (especially metals) there is no significant difference between the elastic limit and the proportional limit when measured experimentally, these two limits are sometimes regarded as equivalent for practical purposes.

In the case of some materials, the curve showing the rate of increase in the deformation or elongation as the load is increased does not change abruptly from a straight line to a curved line; instead, the change is rather gradual, so that the proportional-elastic limit is not clearly indicated. In such cases the value sometimes used is *Johnson's Apparent Elastic Limit,*



Stress-Deformation Curve (Tension)

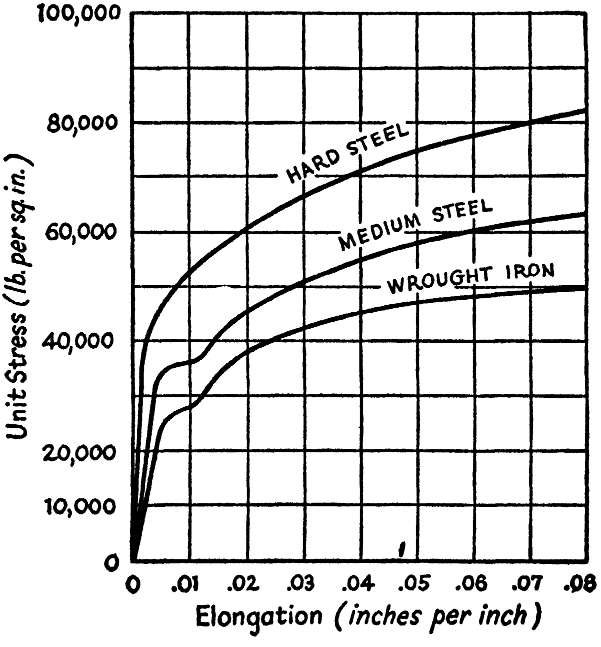
which is taken as the point on the stress-deformation curve at which the rate of deformation is 50% greater than it is at the origin.

Stress-Deformation Diagram. The various relations between stresses and deformations are conveniently shown on a graph known as the stress-deformation diagram ("stress-strain" diagram).

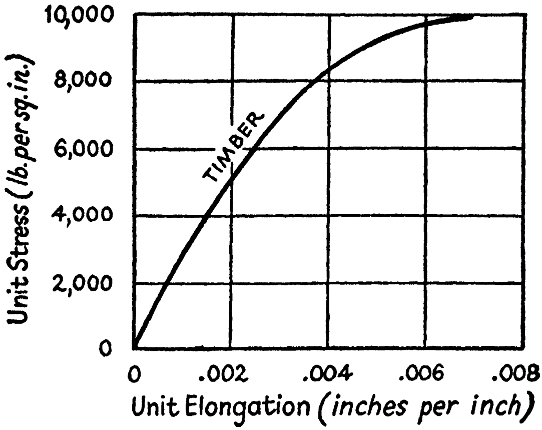
The shape of a stress-deformation curve will of course vary with the particular material tested; many metals and alloys show the more or less typical characteristics shown by the curve for steel, with fairly well defined elastic limits and yield points; others (e.g., cast iron in tension) do not show as clearly marked elastic limits or yield points; and still others (e.g., wood and concrete) frequently show no such points at all, having, in fact, no part of the curve at all that is straight.

Ultimate Strength of a Material. The *tensile strength* of a material is the maximum tensile stress which it can withstand without rupture; it is expressed in lb. per sq. in., and is found by dividing the maximum load carried during a tension test (conducted until the material fails) by the original cross-sectional area. The *compressive strength,* similarly, is the maximum comprehensive stress to which a material can be subjected without failure. These are also commonly known as the *ultimate strength.*

Yield Point. The unit stress at which a considerable increase in deformation occurs without an increase in stress is called the *yield point.* If the



curve for a particular material shows a sharp "knee" or plateau where the curve is briefly and approximately horizontal, the yield point is well-defined and can readily be determined experimentally. However, for some materials the stress-deformation curve is smooth and gradual in the "region of yield." In such cases the *yield strength* is used instead of the yield point; the yield strength is defined as the unit stress at which the material exhibits a *specified* limiting permanent set, or a *specified* elongation, when under a load. The yield strength is thus a convenient measure of the useful limit of such materials. Where a material yields without increase in stress the yield strength and yield point are essentially equivalent. The *breaking strength* is the unit strength at which the material breaks or ruptures. In some instances, e.g., steel, the breaking strength is slightly less than the ultimate strength.



Stress-Deformation Curve—Timber

Coefficient of Elasticity; Young's Modulus. The ratio of the unit stress to unit deformation for values of the stress below the proportional limit is known as the *modulus of elasticity.* It is also called the coefficient of elasticity and Young's modulus. It is the measure of the "stiffness" of a body or the degree to which it can resist deformation within the proportional limits of that particular material.

By using Young's modulus the amount of deformation can be computed. Suppose a bar of length l and cross-sectional area A is subjected to a load, and if

P=axial load applied (lb.),
q=total deformation (inches),
s=unit stress (lb. per sq. in.),
e=unit deformation,

then the modulus of elasticity is

$$E=\frac{\textit{unit stress}}{\textit{unit deformation}}=\frac{s}{e}=\frac{P/A}{q/l}=\frac{Pl}{Aq},$$

$$\text{or } E=\frac{Pl}{Aq}.$$

$$\text{From this, } q=\frac{Pl}{AE}, \text{ or } q=\frac{sl}{E}.$$

In other words, the total deformation produced by a force of P lb. equals the product of the unit stress $(P \div A)$ and the length (l), divided by Young's modulus. The greater the value of Young's modulus, the less will be the deformation resulting from any unit stress, provided, of course, that it is within the elastic limit.

Relative Strength of Materials. More for the purpose of comparison and study than for actual practical use, the comparative strengths of several commonly used materials are given below. These values are only approximate, since there are many grades of iron and steel, for example, and many varieties of wood; thus the mechanical strength of steel depends among other factors upon the carbon content; the characteristics of oak and hickory are considerably different from those of yellow pine.

APPROXIMATE RELATIVE STRENGTH OF COMMON MATERIALS

Material	Ultimate Strength		Modulus of Elasticity (lb. per sq. in.)
	Tensile Strength (lb. per sq. in.)	Compressive Strength (lb. per sq. in.)	
Steel	50,000–100,000	——	30,000,000
Cast iron	20,000– 45,000	80,000–150,000	12,000.000
Wrought iron	45,000– 50,000	——	27,000,000
Brass	40,000– 60,000	60,000	15,000,000
Concrete	——	20,000– 25,000	3,000,000
Wood	——	2,500– 4,000	1,400,000

MODULUS OF ELASTICITY (*Tension*)
FOR CERTAIN MATERIALS
(*approximate values for E in lb. per sq. in.*)

Lead (cast)	700,000
Tin (rolled)	5,600,000
Glass	8,000,000
Brass	9,000,000
Duralumin	10,000,000
Copper (cast)	11,000,000
Bronze	13,000,000
Copper (drawn)	18,000,000
Tungsten	60,000,000

Endurance; Fatigue of Metals. Whenever parts of a machine or members of a structure are subjected to *repeated stresses,* a new factor enters into consideration known as the *endurance of the material.* Repeated stress means the application of a load or force of at least a million times or more; such repetitions of stress commonly occur in the case of piston rods in engines, crankshafts, shafts in motors and generators, valve parts, turbine blades, and the like. Under stresses of this sort it is believed that minute particles of the metal are stressed beyond the elastic limit. As the stress is repeated, an imperceptible *fatigue crack* is formed; this crack, microscopic at first, grows progressively under still further repetitions of the load, until suddenly, and without warning, failure occurs. The metal is said then to have passed its *endurance limit.* The phrase "fatigue of metals" is still used in this connection, although, of course, the material in no sense gets tired; neither is failure due to increased crystallization or brittleness, as was once thought.

The determination of the endurance limit of a metal is rather difficult. Laboratory tests, such as repeated-impact tests on a machine, are tedious and unsatisfactory. Some results have been obtained with methods of testing endurance involving measurement of a rise in temperature and also of electrical resistance. It has been fairly well established that:

(1) there is no necessary relationship between the endurance limit on the one hand and the yield point or the proportional limit on the other hand;
(2) in the case of wrought iron and wrought steel, there exists an endurance limit stress below which these metals are capable of withstanding an unlimited number of repetitions of stress;
(3) for cast iron and cast steel the endurance limit is somewhat less than half their tensile strength.

Strength of Timber. The strength of wood as used for structural purposes cannot be determined as accurately as in the case of other materials. In general, its tensile strength is greater than its strength in compression or shear; tensile strength perpendicular to the grain (fibers) is more significant than tensile strength parallel to the grain. The strength of timber is affected chiefly by the following factors:

1. Species of tree.
2. Rate of growth.
3. Moisture content.
4. Defects (knots, radial cracks, and cracks between annular rings).
5. Deterioration (dry rot, fungi, insects, bacteria).

The effect of drying timber is to decrease its weight, increase its strength, and increase its resistance to decay. The strength of wood is roughly directly proportional to its specific gravity; the larger the specific gravity, the larger modulus of rupture.

Strength of Concrete. Concrete, a mixture of cement, sand and stone, has no well-defined elastic limit. Its tensile strength is much less than its compressive strength; the ultimate tensile strength of concrete is approximately 1/10 of its ultimate compressive strength.

The strength of concrete naturally varies with different proportions of the ingredients used in the mix. For a "rich" mixture (high per cent of cement) the value of Young's modulus is about five times as great as for a "lean" mixture. The strength of concrete increases

(1) with increase in the quantity of cement in a unit volume;
(2) with increased density of the concrete;
(3) with increased size of the coarsest stones used.

Excessive amounts of sand decrease the strength of concrete. The maximum strength is attained when it is about one year old.

APPROXIMATE STRENGTH OF CONCRETE

Proportions of Mixture	*Age*	*Compressive Strength*
1:1:2	1 month	3500 lb. sq. in.
	1 year	4500 " " "
1:2:4	1 month	2000 " " "
	1 year	2500 " " "
1:3:6	1 month	1000 " " "
	1 year	1500 " " "

Working Stresses; Factor of Safety. Working stresses are the unit stresses to which the material of a machine or a structure is subjected. The *allowable unit stress* is the *maximum* working stress to which a material should be subjected. Building codes and specification standards vary somewhat, but for general structural work the usual maximum allowable working stress for a steady load is about 16,000 lb. per sq. in.

The *factor of safety* of a material is the ratio of its ultimate strength to its working stress, or

$$\textit{Factor of Safety} = \frac{\textit{Ultimate Strength}}{\textit{Unit working stress}}.$$

This is also known as the *apparent* factor of safety; it is more realistic to design materials with respect to their elastic limit rather than their ultimate strength. Hence

$$\textit{“Real Factor of Safety”} = \frac{\textit{Elastic limit}}{\textit{Unit working stress}}.$$

For values of elastic limit and ultimate strength see the Table below.

EXAMPLE 1: A steel rod 2½″ in diameter is subjected to a pull of 50,000 lb. Find the factor of safety.

SOLUTION: The cross-sectional area $= \frac{\pi d^2}{4} = 4.91$ sq. in.

Hence, unit working stress $= \frac{50,000}{4.91} = 10,180$ lb. per sq. in.

Apparent Factor of Safety $= \frac{65,000}{10,180} = 6.4$, *Ans.*

Real Factor of Safety $= \frac{35,000}{10,180} = 3.4$, *Ans.*

EXAMPLE 2: A square beam 6″×6″ supports a load of 12,000 lb.; what is the factor of safety?

SOLUTION: Cross-section $= 6^2 = 36$ sq. in.

Unit working stress $= \frac{12,000}{36} = 333$ lb./sq. in.

Apparent Factor of Safety $= \frac{3500}{333} = 10.5$, *Ans.*

Real Factor of Safety $= \frac{3200}{333} = 9.6$, *Ans.*

AVERAGE STRENGTH OF MATERIALS

MATERIAL	TENSILE STRENGTH		MODULUS OF ELASTICITY Tension and compression (lb./sq. in.)
	Elastic limit (lb./sq. in.)	Ultimate Strength (lb./sq. in.)	
Wood	3,200	3,500	1,400,000
Cast Iron	15,000	22,000	12,000,000
Wrought Iron	28,000	50,000	27,000,000
Medium Steel	35,000	65,000	30,000,000
Rail Steel	50,000	100,000	30,000,000

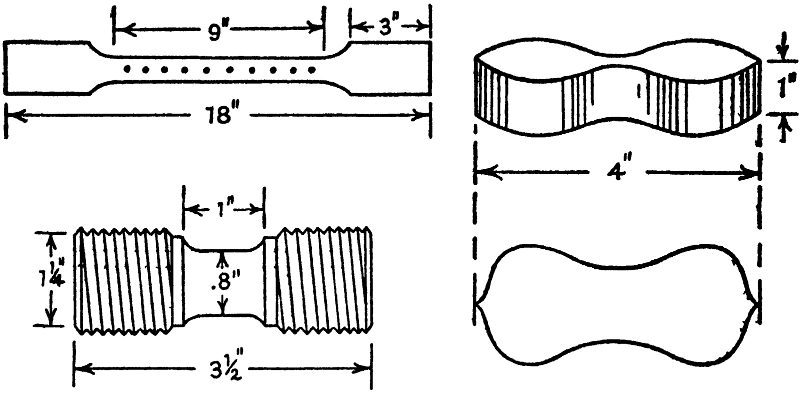
In actual practice, when designing structures or machine parts, the factor of safety is used the other way about; that is, instead of computing the factor of safety from the known load and the known strength, we select an appropriate factor of safety, and, with the known load and known strength, determine the *most appropriate and economical size* of member which will still give the selected factor of safety. The value used for the factor of safety depends not only upon the selection of material and the amount of the load, but also upon the *nature* of the load, i.e., whether it is a *dead load,* a *live load,* or a *shock load.* Typical average values of the apparent safety factors generally used are given below:

FACTORS OF SAFETY (Average values)

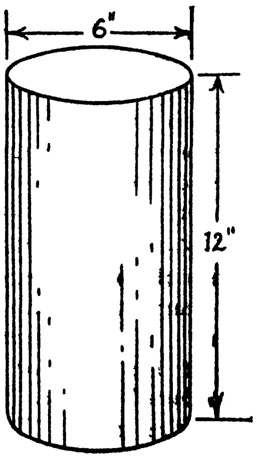
MATERIAL	DEAD LOAD (Buildings)	LIVE LOAD (Bridges)	SHOCK LOADS (Machine parts)
Structural steel Wrought iron	4	6	10–12
Cast iron	5	8	15–20
Timber	8	12	15–20
Masonry	15	25	——

Testing the Strength of Materials. The importance of testing the materials of engineering and industry before they are used is of course obvious. Some of the tests for physical properties have already been discussed or suggested (e.g., testing hardness). For testing the strength of materials, tension and compression tests are among the most commonly used.

To test the tensile strength, standardized test specimens of the material are subjected to enormous pulls, in specially designed machines, until the breaking point is reached. The ends of the specimen are gripped in the jaws of the machine or inserted by threaded ends which permit it to be screwed into nuts in the jaws. The power is supplied by an arrangement of screws and levers, or by hydraulic power. Typical shapes for testing specimens in tension are shown below.



Standard Tension Test Pieces
(*Dimensions approximate*)



Metals and wood are commonly tested for both tensile and compressive strength. To determine its compressive strength, the specimen is subjected to great pressure until failure occurs. Compression tests are commonly made on wood, concrete, stone and brick. For concrete compression tests the specimen usually consists of a cylindrical form 6″ in diameter and 12″ high.

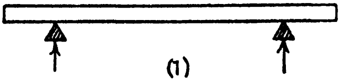
There are a number of testing machines used in practice. A small machine, such as used for testing wire or cement, will furnish a force of some 2000 lb. per sq. in.; medium machines, used for metals and concrete, for example, will furnish from 100,000 to 150,000 lb. per sq. in.; and large machines will furnish upwards of 750,000 lb. per sq. in. These gigantic machines are capable of testing the strength of a steel bar or a fine strand of wire with equal sensitivity.

37. THEORY OF BEAMS

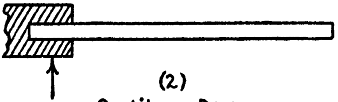
Mechanical Structures. The obvious application of our knowledge of strength of materials and of the theory of mechanics is to the design of structures and machines. This includes the shape, dimensions and choice of material for structural members in buildings and frameworks, such as beams, columns, channel bars, plates and the like, as well as machine parts such as rods, pins, shafts, blades, bearings, gears, bolts, etc. The general theory of such construction design would take us beyond the scope of this book, but a few principles concerning the theory of beams will perhaps give the reader some insight into this fascinating but technical field.

Beams. Technically a *beam* is any bar, rod or structural member subjected to transverse loads that tend to bend it. Beams are thus generally intended to carry a vertical load. A *simple* beam is one which rests on two supports at, or near, the ends.

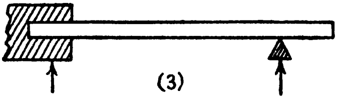
A *cantilever* beam is a beam in which one end projects beyond the point of support, and the other is fixed (or constrained) in a wall, etc., so that it remains horizontal. A *fixed* beam is one in which one end is fixed and the other merely resting on a support, or both ends are fixed; here again "fixed" means *constrained* so that the fixed end must remain in a horizontal position. A beam fixed at one end and supported at the other is sometimes called a "propped cantilever." A *continuous* beam is one which has more than two supports; a continuous beam with only one additional support beside the end supports *may be* partly a cantilever, depending upon the particular conditions.



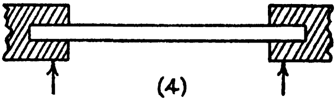
(1)
Simple Beam



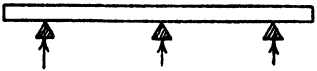
(2)
Cantilever Beam



(3)
Beam fixed at one end



(4)
Beam fixed at both ends



(5)
Continuous Beam

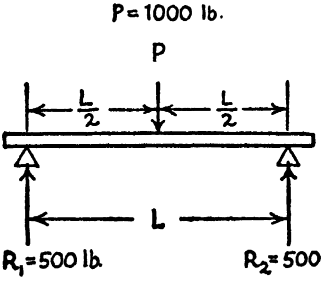
Types of Beams

Types of Load. Regardless of the style of beam, the load carried may be either (1) a *concentrated load,* acting at one point, or (2) a *uniformly distributed* load, acting equally along its entire length. The weight of the beam itself is thus a uniformly distributed load, and while in actual practice the weight of the beam cannot be neglected, for the sake of simplicity we shall disregard it here. The length of a beam between points of support, expressed in inches, is designated by *L;* the amount of a concentrated load, in pounds, is denoted by *P;* and the amount of the distributed loads is given by *w* lb. per inch, so that the entire load $W=wL$.

Beam Reactions. The upward forces (at the supports) which resist the load are called beam *reactions.* For a beam to be in equilibrium, two conditions must be fulfilled:

(1) The algebraic sum of the vertical forces must equal zero; or $\Sigma(fv)=0$.

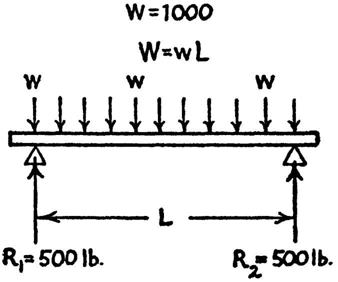
(2) The algebraic sum of the external turning moments and the internal bending moment must equal zero; or $\Sigma(m)=0$.



Simple Beam
concentrated load in center

$R_1 = \frac{1}{2}P$

$R_2 = \frac{1}{2}P$



Simple Beam
uniformly distributed load

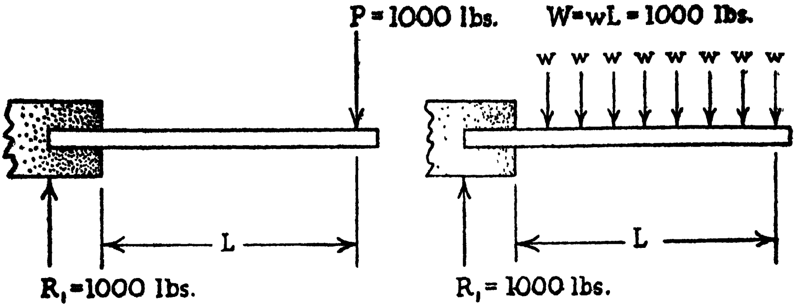
$R_1 = \frac{1}{2}W = \frac{1}{2}wL$

$R_2 = \frac{1}{2}W = \frac{1}{2}wL$

Thus in the simple beams illustrated above,

$$P-R_1-R_2=0, \text{ or } P=R_1+R_2;$$
$$W-R_1-R_2=0, \text{ or } W=R_1+R_2.$$

We shall not go into the turning moment equations; but the external turning moments are resisted by the internal bending moments (causing shear). When the load is concentrated at the center, the maximum bending moment is also at the center, and it equals $\frac{PL}{4}$; for the distributed load, the maximum bending moment is also at the center and it equals $\frac{WL}{8}$. Similarly in the cantilever beams shown below:



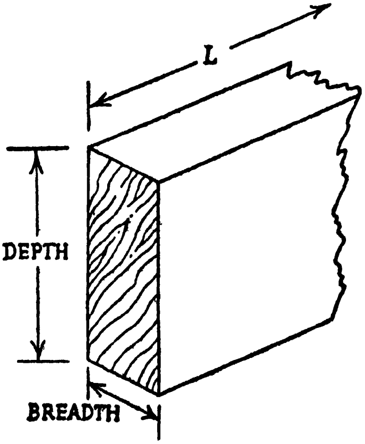
$$P - R_1 = 0, \text{ or } P = R_1$$
$$W - R_1 = 0, \text{ or } W = R_1$$

Here the maximum bending moments are at the fixed end, and are equal, respectively, to PL and $\frac{WL}{2}$.

Strength of a Beam. The supporting strength of a beam is indicated by its maximum bending moment. The maximum bending moments are given in Table I, column (1), for several types of beams and loadings. By studying these formulas carefully, it will be seen that, for all beams, regardless of their cross-section, the following is true:

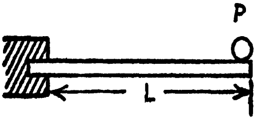
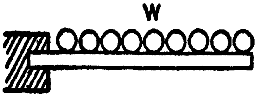
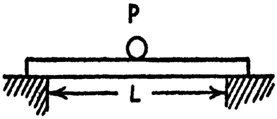
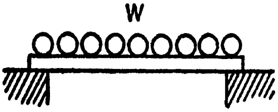
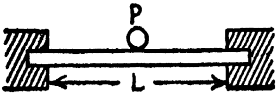
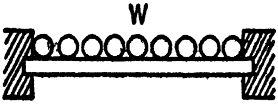
(1) the strength of a beam varies inversely as the length; i.e., the longer the beam, the less the load it will carry.

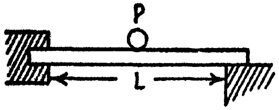
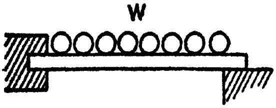
(2) the strength of a beam is twice as great under a uniformly distributed load than under an equivalent concentrated load.



In the special case of a beam with a rectangular cross-section, the strength varies directly as the breadth and directly as the *square* of the depth. That is why such beams are generally placed in the position shown; the strength increases more rapidly as the depth is increased than when the breadth is increased.

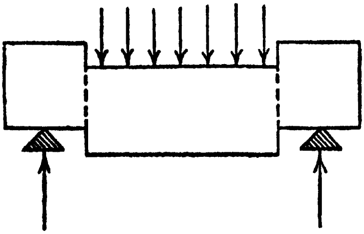
I. BENDING MOMENTS AND DEFLECTIONS FOR BEAMS OF UNIFORM SECTION

Kind of Beam and Load		Maximum Bending Moment (M)	Maximum Deflection (d)
I. 	*Cantilever Beam:* concentrated load at end	$P \times L$	$\frac{PL^3}{3EI}$
II. 	*Cantilever Beam:* uniform load	$\frac{W \times L}{2}$	$\frac{WL^3}{8EI}$
III. 	*Simple Beam:* concentrated load at center	$\frac{P \times L}{4}$	$\frac{PL^3}{48EI}$
IV. 	*Simple Beam:* uniform load	$\frac{W \times L}{8}$	$\frac{5WL^3}{384EI}$
V. 	*Beam fixed at both ends:* concentrated load at center	$\frac{P \times L}{8}$	$\frac{PL^3}{192EI}$
VI. 	*Beam fixed at both ends:* uniform load	$\frac{W \times L}{12}$	$\frac{WL^3}{384EI}$

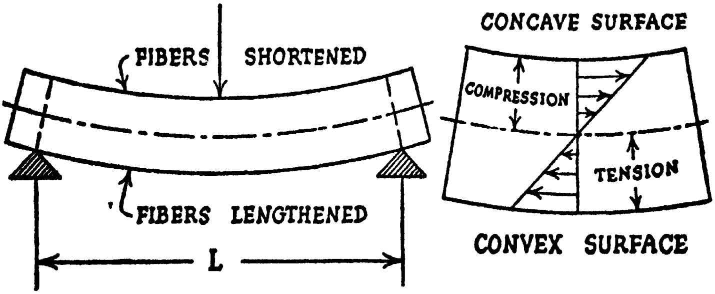
VII.  	*Beam fixed at one end:* concentrated load at center	$\frac{3P\times L}{16}$	$\frac{7PL^3}{768EI}$
VIII.  	*Beam fixed at one end:* uniform load	$\frac{W\times L}{8}$	$\frac{WL^3}{185EI}$

P, W=total load in pounds
L=length of beam in inches
E=coefficient of elasticity (Young's modulus)
I=moment of inertia
d=deflection in inches

Flexure of Beams. When a beam is subjected to a load, it tends to bend. This bending action is called *flexure,* and is present in all beams. It jointly causes a combination of tension, compression and shear. Ordinarily the tendency to shear is less than the tension and compression that results, and can be neglected except for very short beams. For longer beams, flexure results in an actual *deflection* of the beam, as shown below.



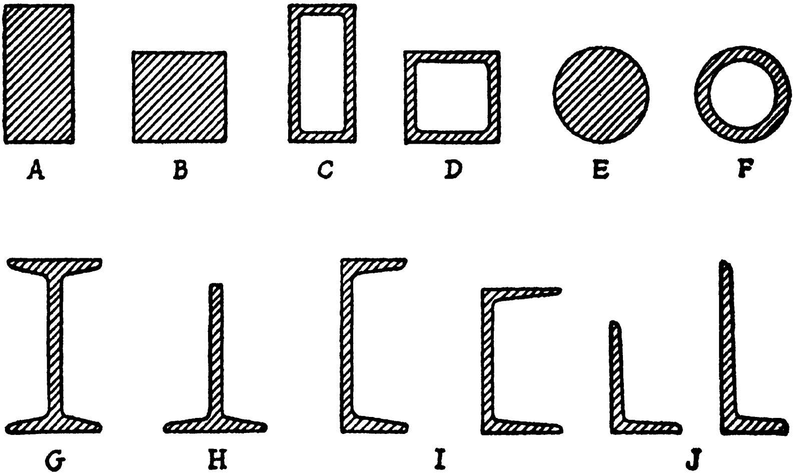
Shear Tendency in a Beam under Transverse Load



Beam Deflection (exaggerated)

The fibers on the concave side are shortened, causing compression; those on the convex side are elongated, causing tension. These changes in length cause the beam to deflect; all points on the beam except those directly over the supports fall below their original position. The so-called *neutral axis* (really a plane through the center of gravity) is the only portion that suffers no change in length. The capacity of a beam to resist deflection is called its *stiffness;* the capacity to support a load is its *strength.*

Design of a Beam; Varying Cross-Sections. The stiffness of a beam depends not only upon its length, but also upon its size and shape, i.e., its cross-sectional area and geometric design. Typical cross-sections of beams are shown herewith:

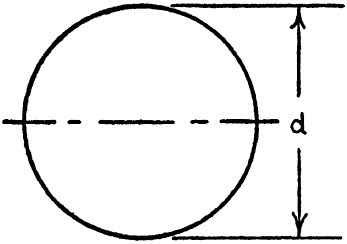
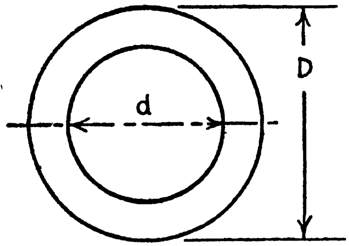
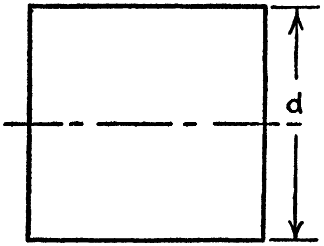
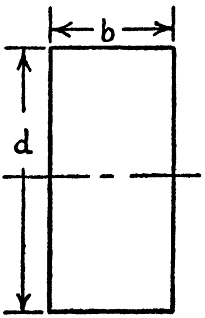


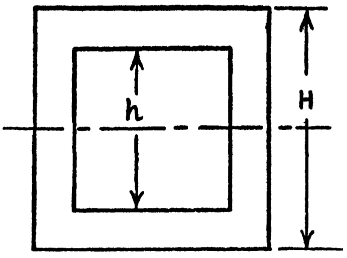
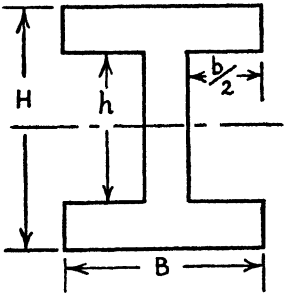
Types of Cross-Sections

A Solid Rectangle. *B* Solid Square. *C* Hollow Rectangle. *D* Hollow Square. *E* Solid Circle. *F* Hollow Circle. *G* I-Beam. *H* T-Beam. *I* Channel Beams. *J* Angle Beams.

It will be seen that by varying the cross-section, the distribution of material is shifted with respect to the principal axes, of which there are generally two, since a beam can usually be placed in one of two ways. It is thus possible, for mechanical reasons which cannot be explained here, that one beam can have greater stiffness than another beam, even though the first has less material, i.e., smaller cross-sectional area. The degree to which the distribution of the area contributes to the stiffness of the beam is expressed mathematically by a quantity (I) known as the *moment of inertia;* the mathematics involved is too complicated to consider here, but the numerical value of I changes with varying shapes of sections. The numerical values of the moments of inertia of a few typical cross-sections, in terms of their dimensions, are shown in Table II; the values given refer to the horizontal axis through the center of gravity.

II. SECTION ELEMENTS

Shape of Section	Name	Moment of Inertia (I) (*horizontal axis*)
	Solid Circle	$\frac{\pi d^4}{64}$
	Hollow Circle	$\frac{\pi(D^4-d^4)}{64}$
	Square	$\frac{d^4}{12}$
	Rectangle	$\frac{bd^3}{12}$

	Hollow Square	$\frac{H^4-h^4}{12}$
	I-beam	$\frac{BH^3-bh^3}{12}$

As will be seen from Table I, column (2), the amount of the deflection of a beam depends upon the load (*P* or *W*), upon the length of the beam (*L*), upon the moment of inertia (*I*) of the section (which in turn depends upon the size and shape of the section), and upon the material of which the beam is made (E=coefficient of elasticity). In actual practice, when designing beams, the maximum allowable deflection is taken as $\frac{1}{360}$ of the length of the beam expressed in inches.

Example 1: A simple beam of timber with a square cross-section 6″×6″ sustains at the center a concentrated load of 900 lb. If the beam is 16 ft. in length, find the maximum deflection.

Solution:

$P=900$ lb.

$L=16$ ft. $=192$ in.

$E=1{,}400{,}000$ lb. per sq. in.

$$I=\frac{d^4}{12}=\frac{(6)^4}{12}=108$$

$$d=\frac{PL^3}{48EI}$$

$$=\frac{(900)(192)^3}{(48)(1{,}400{,}000)(108)}=.88 \text{ in., } \textit{Ans.}$$

A deflection of .88″ is greater than a deflection of $\frac{1}{360} \times 192''$ or .53″, the maximum allowable deflection for safety; this beam is therefore not safe with a load of 900 lb.

Example 2: A round steel beam with a diameter of 3½″ and 20 ft. long is subjected to a uniformly distributed load of 70 lb. per linear foot. If the beam is fixed at both ends, what is its maximum deflection?

Solution:

$w = 70$ lb. per ft.

$W = (70)(20) = 1400$ lb.

$L = (20)(12) = 240''$

$E = 30{,}000{,}000$ lb. per sq. in.

$$I = \frac{\pi d^4}{64} = \frac{\pi(3\tfrac{1}{2})^4}{64} = 7.35$$

$$d = \frac{WL^3}{384EI}$$

$$= \frac{(1400)(240)^3}{(384)(30{,}000{,}000)(7.35)} = .23 \text{ in., } \textit{Ans.}$$

The maximum allowable deflection would be $\frac{1}{360} \times 240 = .67''$; hence a deflection of .23″ is well within the safe limit.

SELECTED REFERENCES FOR FURTHER STUDY

Breneman, J. W. Strength of Materials. McGraw-Hill Book Co.

Fairman, S. & Cutshall, C. S. Engineering Mechanics. J. Wiley & Sons.

Kottcamp, J. P. & Harper, A. C. Strength of Materials. J. Wiley & Sons.

Laurson, P. G. & Cox, W. J. Properties and Mechanics of Materials. J. Wiley & Sons.

Leigh, C. W. & Mangold, J. F. Practical Mechanics and Strength of Materials. McGraw-Hill Book Co.

Merriman, M. & Merriman, T. Strength of Materials. J. Wiley & Sons.

Moore, H. F. Textbook of the Materials of Engineering. McGraw-Hill Book Co.

Poorman, A. P. Strength of Materials. McGraw-Hill Book Co.

Pulver, H. E. Materials of Construction. McGraw-Hill Book Co.

Roys, F. W. Materials of Engineering Construction. Ronald Press.

Seely, F. B. Resistance of Materials. J. Wiley & Sons.

Swain, G. F. Structural Engineering: Fundamental Properties of Materials. McGraw-Hill Book Co.

Wines, W. E. Strength of Materials. McGraw-Hill Book Co.

CHAPTER VIII

PRACTICAL CHEMISTRY

Monroe M. Offner

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38. BASIC CONCEPTS OF CHEMISTRY

Introduction. Chemistry is the science that deals with changes in matter; changes that result in the formation of new substances with new properties. From limestone, coal, salt and water chemists are producing a synthetic rubber that is superior in many ways to the natural product. Using only air and water we can prepare nitric acid, particularly important in war time. Fats can be converted into soap and glycerine by boiling them with lye. Sweet, syrupy glycerine, used in many cough mixtures, is changed by the action of nitric acid into dangerous, explosive nitroglycerine or dynamite. However, before we can understand such chemical changes, we must first be able to recognize purely physical changes.

Physical Changes. Let us consider the changes that iron goes through as it is manufactured into, say, sash weights from pig iron. First the pigs are melted; then they are poured into the molds where they solidify, cool further and shrink somewhat. The iron thus changes from the solid state to the liquid and back to the solid again, it cools and contracts; yet it is iron all the time, even though its temperature, size, shape and state have been altered. Or again, if water is poured on a hot object, like a red hot casting, the latter may crack (another physical change) and the water will change to steam. In this case a liquid has changed to the gaseous state; but water, whether it be in the form of a solid called ice, a liquid called water, or a gas called water vapor or steam, is always the same substance, identified by the chemist as H_2O. In all of the examples given, only *physical changes* have been described: in *no* case was a *new substance* with new properties formed, and besides, the changes *can readily be reversed.*

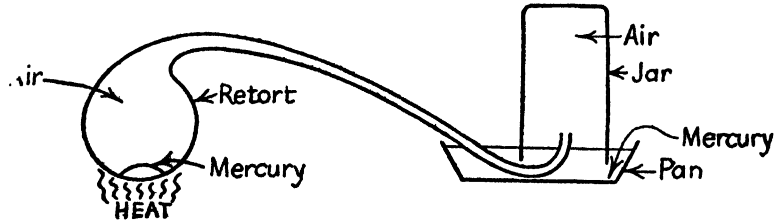
Chemical Changes. Everyone has observed that objects made of iron will rust when exposed to air unless they are properly protected. Now instead of a gray solid with a metallic luster, a new substance with new properties has been formed. The rust itself is a powdery solid, varying in color from yellow to red, and in fact so different from the original iron that a metallurgical process, usually carried out in a blast furnace, would be required to restore it to its original form.

A beginner who uses a forge for the first time to heat up a piece of wrought iron, to soften it for shaping on the anvil, must always be cautioned not to overheat his material. For iron burns too, and in so doing it changes into a scaly, dark gray substance. This brittle solid is identical with the ore called magnetite or lodestone from which iron may be manufactured. As these examples illustrate, *chemical changes are changes that result in new substances with new properties.*

Burning. Chemical changes like rusting and burning were not understood until Lavoisier, a Frenchman, just around the time of our American Revolution, carried out some remarkable experiments. His experiments are noteworthy because they were carried out quantitatively, that is, with painstaking accuracy in weighing and measuring. They are important because they led him first, to the discovery of the true nature of burning, and secondly, to the formulation of the law of the conservation of matter.

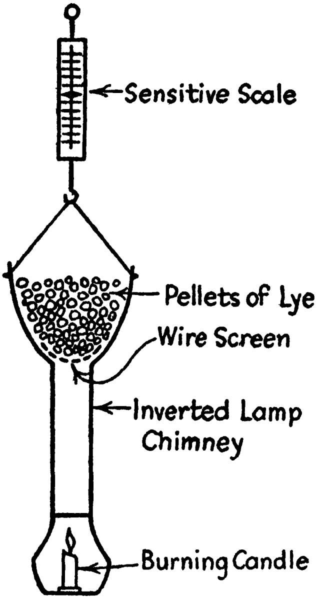
Discovery of Oxygen. Priestley, an Englishman, had previously discovered oxygen, a gas that makes up approximately one fifth of the air we breathe. He had heated mercury, that silvery liquid metal used in thermometers, until a red ash was formed. Then on heating the red powder to a slightly higher temperature, he found that it changed back again to quicksilver; in addition, a gas was liberated which made a glowing splint burst into flame. That gas was *oxygen.*

True Nature of Burning; Conservation of Matter. Lavoisier repeated Priestley's work. However, he first carefully weighed the mercury. He then heated it in air in such a way as to measure just how much of the air was used up in this change. He found that one fifth of the air was consumed, and that the red *powder weighed more than the mercury he had burned.* The air left could not support combustion. The experiment was continued by reassembling the apparatus; but this time only the red powder was placed in the retort, and the heating was more intense. As a result, the volume of *air* was *restored to* its *original amount,* and again supported combustion. Mercury was again found in the retort. Lavoisier



Lavoisier's Famous Experiment on the Nature of Burning then weighed the *restored mercury,* and just as he had anticipated, it had the *same weight as the original!* Many similar experiments were carried out, using other metals as well as mercury. Each time a similar result was obtained. He concluded that *burning is a chemical change during which oxygen combines with the metal being burned.* This accounts for the fact that the *ash always weighs more than the original metal.* His work (later abundantly corroborated by other scientists) also proved that *matter can be neither created nor destroyed.*

One interesting experiment that is easy to perform answers some other puzzling questions about burning. Coal burns, leaving relatively little ash. A burning candle seems to disappear completely. How can we explain this? What is often overlooked is the fact that fuels burn to form gases and vapors. These, too, have weight. In the case of the candle, we can catch the water vapor and carbon dioxide, its products of combustion. The accompanying diagram shows how we can do this, and how the increase in weight can be measured. With Lavoisier's discovery of the true nature of burning and of the law of the conservation of matter, chemistry became a science.



Experiment Showing That the Products of Combustion Always Weigh More Than the Fuel.

Organizing Facts. The scientist collects his carefully made observations (his facts) and always finds it necessary to classify them. He speaks of energy, and then says it can appear as *potential,* or stored up energy, and as *kinetic energy,* or energy in motion. He classifies further, and speaks of energy in the form of *light, heat, electrical, magnetic, sound, radioactive, chemical* and *mechanical* energy. He then looks at matter, and finds a bewildering variety. But careful work discloses that he can classify all matter into three groups.

Elements. Into the first group he places all *substances* that *cannot be changed by any known chemical means into anything simpler.* He calls these *elements.* Carbon, sulfur, nitrogen, iron, gold, mercury and neon are examples of the total of ninety-two elements known to comprise all matter.

Mixtures. Reinforced concrete used for engine foundations is usually made up in the proportions of 1 bag of cement to 2 of sand and 4 of gravel. For ordinary floors, the mix is 1:3.5:5. Yet we always call it concrete, even though its composition varies widely. Air, too, is a mixture; each breath we take alters it as we use up some of its oxygen, each gust of wind mixes more dust with it, a sudden temperature drop may cause it to release most of its moisture in the form of rain. Not only do mixtures vary in composition, but their parts can be readily recognized and easily separated by mechanical methods.

Compounds; Law of Definite Proportions. The third class of materials, called compounds, is made up of substances whose composition never varies by the slightest degree. *A compound is a substance made up of two or more elements, chemically combined in definite proportions by weight.* Its constituent parts cannot be recognized nor mechanically separated.

Look at a piece of dry ice, that intensely cold, white solid used so extensively as a refrigerant especially where weight must be kept at a minimum, as in airplanes. Who would imagine that that white solid is really carbon dioxide, CO_2? Yet it actually is composed of black carbon chemically united with colorless oxygen gas.

There are approximately half a million different compounds known today, each of which has been carefully analyzed. In each case the composition is found never to change. Water (H_2O) is a compound made up of two elements, hydrogen and oxygen. *No matter what the source of the water,* whether it be rain water, or the water obtained by distilling the salt water of the seas, or condensed in the exhaust of a gasoline engine, *so long as it is pure, every 9-pound sample* of it is *invariably* made up of *1 pound of hydrogen* and *8 pounds of oxygen.* Or again, ammonia (NH_3) is a useful gas manufactured by combining nitrogen, which is taken mechanically from the air, with hydrogen, which is electrolyzed out of water. Nature makes it through the decay of organic matter. But whichever way it is produced, it is *always* composed of 14 parts by weight of nitrogen to 3 parts by weight of hydrogen.

The chief differences between mixtures and compounds therefore are:

CHARACTERISTICS OF MIXTURES	CHARACTERISTICS OF COMPOUNDS
1) Their composition varies.	1) Their composition is always the same.
2) Their separate parts can be recognized.	2) Their constituent parts cannot be recognized.
3) Their parts can be separated mechanically.	3) Their parts can be separated only by chemical means.

ATOMIC THEORY

Law of Multiple Proportions. Science cannot thrive on facts alone, however; not even organized facts are sufficient. Science requires theories to explain the facts, and to help in predicting as yet undiscovered facts. Dalton, an English schoolteacher, early in the nineteenth century was wondering why compounds never varied in composition. He was puzzled, too, by another law he had discovered, the *law of multiple proportions.* He wanted to know why, whenever he analyzed a series of compounds made up of the same elements, the *weight of each element in one compound* bore *a simple ratio to its weight in the other compounds.* This simple ratio could always be expressed in small whole numbers; it was never necessary to use fractions. To make this clear, study the table below:

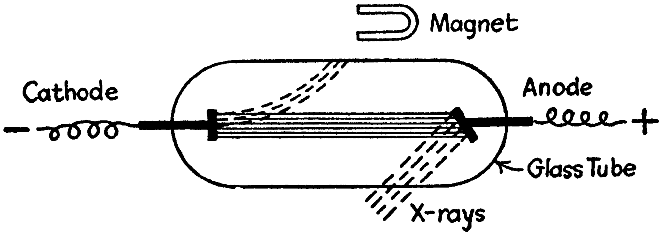
NAME	FORMULA	PARTS BY WEIGHT	RATIO
	SERIES I		
Water	H_2O	2:16	1
Hydrogen peroxide	H_2O_2	2:32	2
	SERIES II		
Laughing gas	N_2O	28:16	1
Nitric oxide	NO	28:32	2
Nitrogen peroxide	NO_2	28:64	4
Nitrogen pentoxide	N_2O_5	28:80	5

Atoms. As a result of his pondering, Dalton gave us his atomic theory to explain the laws of definite and multiple proportions. His theory states that all *matter* can be regarded as being *made up of small indivisible particles* which we call *atoms.* The atoms making up one element are all alike in weight, size and all other properties, but they differ from the atoms that make up the rest of the elements. Atoms of one kind tend to combine with the atoms of many other elements. Once this theory was understood, chemists accepted it. Even though it has been modified in recent years, we still use it to give us a mental picture of matter and of chemical changes. It also led to formula-writing, the chemist's shorthand, some examples of which you have already seen.

Molecules. Observations have convinced the scientists that atoms must be held together in tiny units, each of which has *all the properties of the large sample* of the substance. These units we call *molecules.* When we speak of a molecule of common salt, sodium chloride, we mean the smallest particle of salt that can exist. If it were divided any further it would no longer be salt but instead one atom of sodium and one atom of chlorine. Actually, sodium chloride, NaCl, can be decomposed by the electric current into a very soft metal, sodium, and a poisonous gas, chlorine.

Atoms Are Divisible. A little over forty years ago the belief in Dalton's atom received several severe jolts. Startling experiments were performed, which led in turn to others, continuing to the present day. Taken together they give us a new picture of the atom and of chemical changes. We still think of the atom as retaining its individuality through all chemical changes, even though the atom itself is now considered as being made up of three kinds of very minute particles.

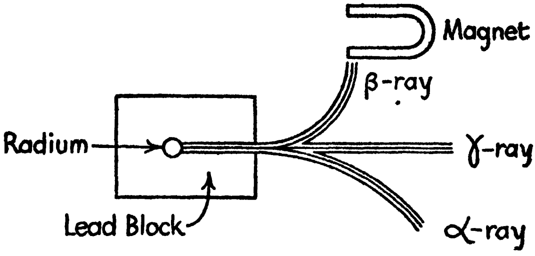
Electrons. The complete story of the development of the electron theory of matter would fill a very large volume; here we can only discuss a few of the high lights. If a high voltage is applied to a glass tube from which practically all the air has been exhausted, two phenomena occur. By suitable means we can detect a stream of particles traveling from the *cathode,* or the negative electrode, towards the *anode,* or positive electrode. The diagram depicts this stream moving horizontally from left to right. When a magnet is brought near the tube, the stream, called the cathode ray, is deflected in a direction which shows it to be made up of *particles of negative electricity.* These are the *electrons.*



Cathode Ray Tube

X-Rays. From the part of the anode struck by the electrons come rays of a kind of invisible light that has remarkable penetrating power. They are the well-known X-rays. They are not influenced by the magnet; and, like ordinary light, they affect the photographic plate. X-rays are now being widely used in industry to detect hidden flaws in metal parts, just as doctors and dentists use them on human beings.

Radio-activity. When Madame Curie discovered radium she gave the element that name because it sent out radiations. If radium or any radium compound is enclosed in a thick lead block with a slot cut into it, the ray emitted can be studied. Under the influence of a magnet the ray



Effect of Magnet on Radium-Ray

divides into three rays. One, the *beta-ray,* is identical with the cathode ray; that is, it consists of a stream of electrons. The second, called the *gamma-ray,* is uninfluenced by the magnet, and is similar to the X-ray. The third, called the *alpha-ray,* is made to curve by the magnet in the direction that shows it to be a stream of positively charged particles. Incidentally, other elements besides radium are radio-active.

Transmutation. The alchemists used to dream of discovering the magic "philosopher's stone" which would enable them to change cheap metals into gold. Today, the physicist with his modern electrical devices is daily *changing one element into another.* He is doing this to learn still more about the structure of matter. It appears extremely unlikely, however, that it will ever become commercially profitable to transmute lead, for example, into gold.

Protons. It has been known since Franklin's day that there were two kinds of electrical charges. He called the charge produced on hard rubber by stroking it on fur a *negative* charge. The charge acquired by glass rubbed with silk he called *positive.* Unless charged artificially, all matter is electrically neutral. Since matter is capable of giving off negative electricity, but has no charge ordinarily, the balancing positive charge must exist in each atom. Experiments show this to be so. The positive charges, called *protons,* are found to be located in the center of the atom. When Alpha-rays are passed through matter they are rarely hindered. This indicates that atoms are mostly empty space with a positively charged *central nucleus* around which, at a distance, the electrons revolve. Each atom is thus like a miniature solar system, with electrons in place of the planets, and a nucleus instead of the sun at the center.

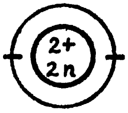
Atomic Numbers. Moseley, a young English physicist who was killed during the first World War, showed us how to find the *number of protons in the nucleus* of each kind of atom. Refer again to the diagram of the cathode ray tube. The kind of X-rays given off in such a tube depends on the nature of the substance used to make the anode. To find out how many protons the silver atom contains, simply use a silver anode.

As you know, sunlight can be analyzed into its component rainbow colors by directing it through a triangular glass prism towards a white screen. In a similar way, X-rays can be analyzed by sending them through a salt crystal towards a photographic plate. In this way a characteristic pattern is obtained for each element tested. Calculations based on the X-ray photographic pattern tell the number of protons in the nucleus. In this way it is now possible to give each element a number indicating how many protons it contains. Since the unit positive proton charge just neutralizes the unit negative electron charge, the atomic number consequently also tells the number of electrons outside each nucleus. Using ingenious methods too complicated to explain here, scientists have found that the electron has negligible mass (weight), one proton being as heavy as 1850 electrons.

Neutrons. Only ten years ago, a third fundamental particle, the *neutron,* was discovered in the nucleus. It has the *same mass as the proton,* but, as the name implies, it has *no electric charge.* Our picture of the atom is now almost complete.

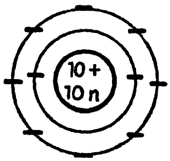
Inert Elements. It remained for two American scientists, Lewis and Langmuir, to complete the picture. Their theory helps us to understand why elements behave as they do. One of the bases of the electron theory is the fact that there happen to be six gaseous elements *not one of which can be made to combine with any other element.* Helium is the first of these inert elements. It is the most desirable gas for inflating dirigibles because it is very light and cannot burn or explode. The second inert element is *neon,* the gas whose orange-red glow you see in so many electric display signs. *Argon* is another inert element.

Arrangement of Electrons. The theory states that these gases are inert because their electrons are arranged so as to give their atoms perfect stability. The reason other elements are chemically active is that they each tend to gain electrons, or get rid of electrons, in order to acquire perfect stability. Here is how we represent the structure of the inert elements we are discussing. A plus sign stands for a proton, a minus sign for



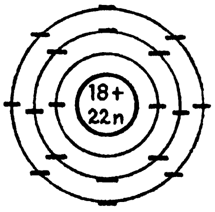
Helium

Atomic number=2
Atomic weight=4



Neon

Atomic number=10
Atomic weight=20



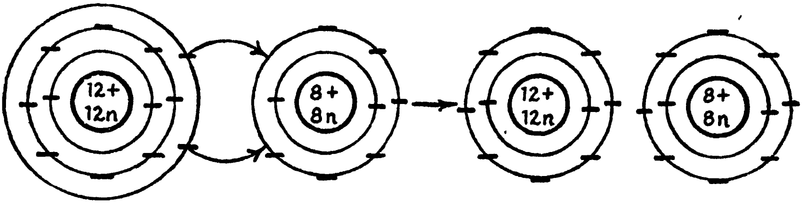
Argon

Atomic number=18
Atomic weight=40

an electron, and an *n* for a neutron. Note that, according to the electron theory, ***all matter is composed of just three fundamental particles*** (neutrons, protons and electrons), and that the only difference between the elements lies in the number and arrangement of these particles.

Chemical Change. Let us see how this theory explains the burning of magnesium, the element used in tracer bullets and for incendiary bombs. Magnesium (atomic number=12, atomic weight=24) has a nucleus containing 12 protons plus 12 neutrons to bring its weight to 24. To balance its 12 positive charges, it must have 12 electrons whirling around outside its nucleus (see diagram below). They must be arranged, always starting with the innermost shell and working out filling each as you go, *2 in the first shell, 8 in the second,* and *8 electrons in the third shell.* Unless the element is inert, the outermost or valence shell is never complete.

The atom tends to have a *complete outermost shell,* and therefore either *lends electrons or borrows them* from another atom, *whichever is easier.* Once it has done this, it is no longer electrically neutral; it has a charge opposite to that of the other atom. They attract each other and combine to form a compound.



Mg^0	O^0	Mg^{++} $O^=$
Magnesium atom Two electrons to lend Valence+2	Oxygen atom Needs two electrons Valence—2	Magnesium ion, Oxygen ion Valence shells complete Magnesium oxide molecule

Diagrammatic Representation of the Burning of Magnesium, According to the Electron Theory.

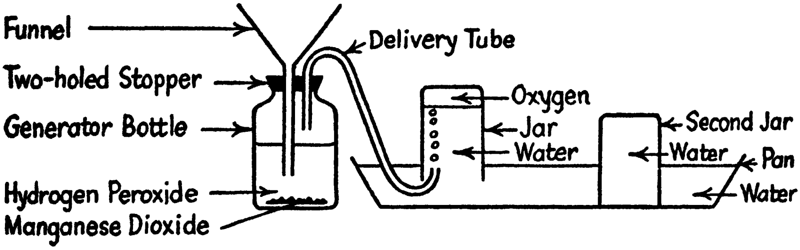
Practice drawing the atoms of elements 1 to 18, using the nearest whole numbers for the weights. Refer to the Table of Elements in the Appendix for the necessary information.

39. CHEMICAL SUBSTANCES AND CHANGES

Oxygen and Oxidation. The most plentiful element on the earth is *oxygen.* It makes up fifty per cent of the weight of the earth's solid crust, almost ninety per cent of the weight of water, and over twenty per cent of the weight of the air.

Commercial Preparation. Oxygen is extracted from the air by a simple mechanical process. Air which has been freed of moisture, dust and carbon dioxide is compressed in several stages and cooled until it becomes liquid. This liquid is a mixture of almost pure oxygen and nitrogen. Since nitrogen has a lower boiling point than oxygen, when the liquid air is allowed to evaporate, the nitrogen comes off first, leaving the oxygen behind. A less important industrial method for preparing oxygen is the electrolysis of water which we shall study later.

Laboratory Preparation. Oxygen can readily be prepared by heating mercuric oxide, HgO, the red powder that Priestley and Lavoisier used for this purpose over one hundred and fifty years ago. A simpler and cheaper method, that can easily be carried out in the home, consists in causing hydrogen peroxide, H_2O_2, to decompose rapidly into water and oxygen. This reaction is speeded up, due to the presence of a little manganese dioxide, MnO_2, which acts as a catalyst here. (Catalysts are substances which change the speed of chemical reactions without, themselves, being used up. They are of great importance in industrial processes.)



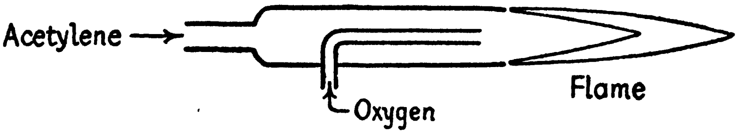
Preparation and Collection of Oxygen

Set up the apparatus as shown in the diagram above. Have the manganese dioxide in the generator. Use the ordinary hydrogen peroxide sold by the druggist as a germicide, or the double strength peroxide he sells as a hair bleach. Pour enough through the funnel to insure that the bottom is below the liquid level. As the hydrogen peroxide is used up, merely add more through the funnel if you want to generate more oxygen. The gas displaces the water in the collecting bottles.

Properties. Examination of the gas you have collected shows oxygen to be a colorless, tasteless, odorless gas, only slightly soluble in water, and more dense than air. (When we speak of the density of a gas we are comparing the weight of a bottle of it with the weight of air filling a bottle of the same size, both gases being at the same temperature and under the same pressure.) The outstanding chemical property of oxygen is its ability to *support combustion.* Anything that burns in air burns much more rapidly in oxygen. A glowing splint bursts into flame when thrust into it. Steel wool that burns with difficulty in air, once ignited, will behave like a Fourth-of-July sparkler when placed in oxygen.

Uses. All living things, both plants and animals, must have oxygen for respiration. Without it there would be no life, nor could fires burn. The medical profession uses it for patients suffering from pneumonia, gas poisoning, and even heart disease. At high altitudes airplane pilots cannot operate efficiently without an auxiliary oxygen supply.

Shipped in steel cylinders about 4 feet in height and 8 inches in diameter, and under a pressure of 2000 pounds per square inch, oxygen is consumed by industry to the extent of several billion cubic feet a year. Most of this is used for the familiar oxy-acetylene flame. Provided that the acetylene gas and oxygen are mixed in the right proportions, the flame can reach a temperature of over 5400°F. The special burner used has valves for regulating the flame size and the gas proportions. Oxy-acetylene welding is used for joining metal parts: hard abrasion-resistant alloys are welded to oil-well drilling tools to make them last longer; high pressure oil and gas pipe lines are made leak proof by welding; chromium-molybdenum steel tubing, used for the framework of airplanes, is joined in this way too. There are few repair shops in the country that do not find frequent use for the oxy-acetylene blowpipe to repair damaged parts. The oxy-acetylene cutting blowpipe uses a second jet of cutting oxygen to burn through the metal where the flame has already heated it up. With this device a 6-inch steel shaft can be cut through in one minute.



Oxy-Acetylene Burner

Rapid Oxidation. Any chemical reaction that gives off noticeable light and heat is called a *combustion*. Ordinary burning is the most common example. In the illustrations already given, you have observed that oxides are always formed: thus iron combines with oxygen to form iron oxide; magnesium forms magnesium oxide; and candle wax, a mixture of carbon and hydrogen compounds, ends up as carbon dioxide and hydrogen oxide, water.

Kindling Temperature. To start a coal fire, you first ignite paper which heats the wood above it, and this in turn starts the coal off. You do not attempt to ignite the coal by holding a match flame under it. Clearly then, materials differ in the degree of heat necessary to start them burning. The *lowest temperature at which a substance takes fire and burns* is called its *kindling point.*

Burning. Briefly summarized, the *conditions necessary for combustion* are:

1. A combustible material.
2. A supporter of combustion.
3. Enough heat to reach the kindling point.

Explosions. A sudden, great expansion in volume is called an *explosion.* Uncontrolled explosions spell disaster, whereas controlled explosions do much of the work of man. Controlled explosions are utilized in the internal combustion engines of automobiles and airplanes. Gasoline is the commonly used combustible; air supplies the supporter of combustion; and the spark plug furnishes the igniting heat. We have not specified all the conditions necessary for an explosion, however. Every gasoline engine must have a carburetor; its function is to make sure that the *fuel and air* have been *thoroughly mixed* in the *right proportions.* If gasoline is not mixed with air, it burns quietly instead of exploding. If too much air is mixed with it, it will not explode; the mixture is too lean. If the mixture is too rich, that is, if the ratio of gasoline to air is too high, then instead of occurring in the cylinder, the explosion takes place in the exhaust pipe when the unexploded vapors reach the needed amount of air. This explains the sharp, pistol-like report of the "back fire."

Explosives. Gunpowder, dynamite and other explosives contain combustible materials well mixed in the right proportions with a chemical to supply oxygen. A detonator is used to start the reaction. Mining and quarrying, clearing land of tree stumps, blasting through stone mountains to make tunnels are but a few additional examples of where controlled explosions are used to help man.

Unfortunately, all the conditions necessary for an explosion are frequently present where men have to work. A poorly ventilated mine may have combustible gases seeping in; an oil refinery may be filled with gasoline vapor; a flour mill, even a metal buffing room, may have air laden with combustible dust. A flame or spark sets off the explosion. Knowing the conditions necessary for an explosion, it is possible to prevent many catastrophes. Adequate ventilation will get rid of combustible dusts and gases. Open flames, and electric switches or motors that spark, should be avoided in such places; tools made of beryllium-copper rather than steel tools are preferable, because the former do not strike sparks.

Slow Oxidation. The starch in the bread and the sugar in the candy that we eat can be made to burn if we bring them to their kindling points. If they are burned completely (not merely charred) they are changed into carbon dioxide and water vapor. In our bodies, too, they can be oxidized into the same compounds, but then the *reaction takes a much longer time,* and the *heat therefore comes off much more slowly.*

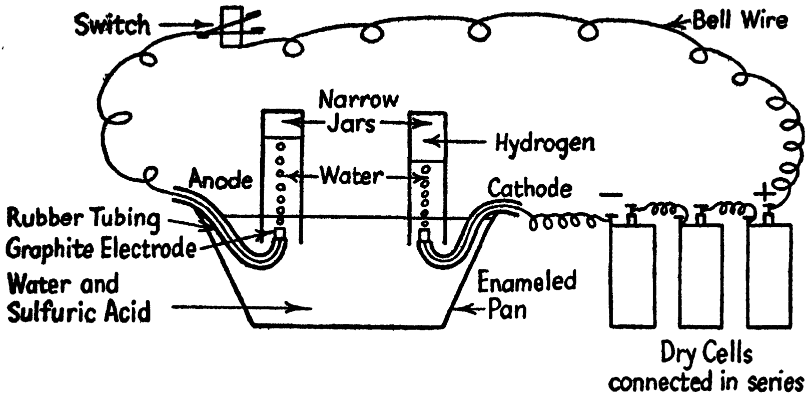
Corrosion. The rusting of iron is another example of *slow oxidation;* it costs this country millions of dollars every year. Other metals corrode or tarnish as well, but usually the corrosion layer on them acts as an excellent protective coating to keep out the *moisture, oxygen and other corrosive agents in the air.* Not so with iron rust; it actually catalyzes further rusting.

To protect iron from rusting, it is necessary to oil or grease it, paint it, or cover it with some other metal. Iron is often galvanized by dipping it into molten zinc; sometimes it is tinned. Frequently an enamel surface is baked on. Playing steam on red hot iron gives it a fairly good protective layer of the black oxide. Some alloy steels, that is, mixtures of iron with other metals, resist corrosion very well. The famous "18–8 Stainless Steel" is made by adding 18 pounds of chromium and 8 pounds of nickel to each 74-pound batch of the molten iron.

Spontaneous Combustion. As everyone knows, the drying of water-color paint or of white wash consists simply of the evaporation of the water present. When oil paints "dry," however, the linseed oil vehicle slowly oxidizes into a hard film. We usually do not notice the heat of oxidation because it is slowly dissipated into the air. Many "mysterious" fires have been started by careless workmen who threw paint rags or oily waste into closets or bench drawers, instead of using the metal cans provided. In a wooden drawer, the *heat from the slow oxidation of the oil accumulates until the rags reach their kindling point and begin to burn.* Precautions must always be taken against spontaneous combustion whenever large amounts of combustile materials are stored, for slow oxidation is going on continuously. This applies especially to coal and hay which are notorious in this respect.

Hydrogen and Water. Never found free in the air except in infinitesimal amounts, hydrogen is nevertheless available in large amounts in many common materials. Water is 11% hydrogen by weight. Oils, fats, coal and protoplasm, the material of living cells, all contain hydrogen.

Electrolysis of Water. Most industrial hydrogen is made from *water gas,* an important fuel gas that we shall study later. Both commercially and in the laboratory, hydrogen is frequently prepared by the electrolysis of water. Chemically pure water conducts electricity so poorly that it is considered a non-conductor for all practical purposes. For this reason, sulfuric acid, H_2SO_4 (a catalyst here), is added to the water to aid in the passage of the current. The diagram clearly shows the arrangement of the ap-



Arrangement of Apparatus for the Electrolysis of Water

paratus, but a word about the electrodes may not be amiss. They must be made of a conductor that will not be affected by the acid or by oxygen. Short lengths of graphite from an old dry cell will do nicely; solder them to the copper wire. Protect the connection with rubber tubing that covers the parts of your wire in the acid solution. As soon as the switch is closed you will notice bubbles rising from each electrode. In a little while it will be evident that twice as much gas is accumulating above the cathode as is being formed in the anode jar. When enough gas has collected, open the switch, cork the jars and test the gases. You already know the properties of oxygen; the characteristics of hydrogen will be discussed after a quicker method of preparation has been described below.

Chemists summarize reactions by writing equations. In a later chapter, the writing of equations will be explained in greater detail. In the meantime, it is worth your while to acquire a little familiarity with them. The equation for the electrolysis of water is:

$$2H_2O \rightarrow 2H_2 + O_2;$$

this is read "water yields hydrogen plus oxygen."

Reaction Between a Metal and an Acid. Using exactly the same apparatus and arrangement that we used for preparing and collecting oxygen (see page 476), we can readily obtain plenty of hydrogen to study. In the generator place some pieces of zinc, iron or aluminum. Through the funnel pour dilute hydrochloric acid (HCl). Both the metal and the acid are used up in this reaction. For convenience an excess of metal is usually used, so that if the reaction ceases before you have collected all the hydrogen you need, you simply pour a little more acid through the funnel. When the reaction has stopped, filter the liquid remaining in the generator to separate it from any unconsumed metal. Evaporate the solution to dryness by boiling off the water, and you will find a solid left. It is a salt; that is, a compound formed after a metal has replaced the hydrogen of an acid. If zinc has been used, then the salt remaining is zinc chloride, and the equation for the reaction is:

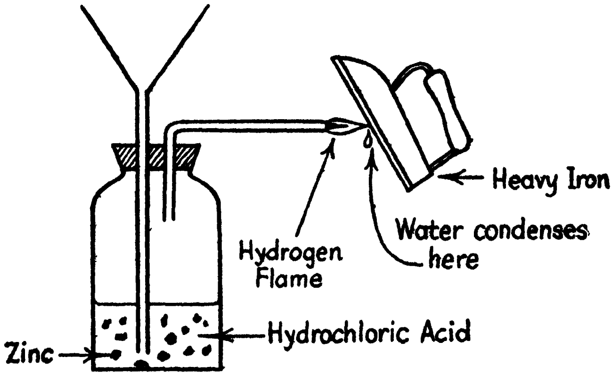
$$Zn + 2HCl \rightarrow ZnCl_2 + H_2;$$

or "zinc and hydrochloric acid yield zinc chloride and hydrogen."

Physical Properties. Now that we have our *hydrogen* let us examine it. The sample obtained by electrolysis is pure. It is *colorless, odorless, tasteless, the lightest gas* known to man, and only *very slightly soluble in water.* The hydrogen prepared by the reaction between a metal and an acid usually has an odor, due to impurities in the metal. (It is interesting to note that if the metal were pure, no reaction would take place; the impurities act as catalysts.)

Chemical Properties. Take two unstoppered jars of hydrogen. Let one stand, mouth up, on a table for a minute, while you hold the other up in the air, mouth down. Now thrust a burning splint down into the first one. Then light the splint again and thrust it up into the second jar and *hold it there* in spite of the slight explosion you hear. Soon you will notice that the splint is burning again at its middle at the mouth of the jar. This surprising experiment shows that hydrogen is less dense than air; it escapes quickly from the first jar. Hydrogen *does not support combustion.* That is why the light on the splint was extinguished in the second jar. *But hydrogen burns,* and this accounts for the slight explosion heard and also for the strange re-igniting of the splint at the mouth of the jar. In the daylight it is difficult to see the pale blue flame of hydrogen as it burns at the mouth of the jar.

Synthesis of Water. Set up your hydrogen generator again, this time using a copper delivery tube bent as shown in the diagram. Add acid in



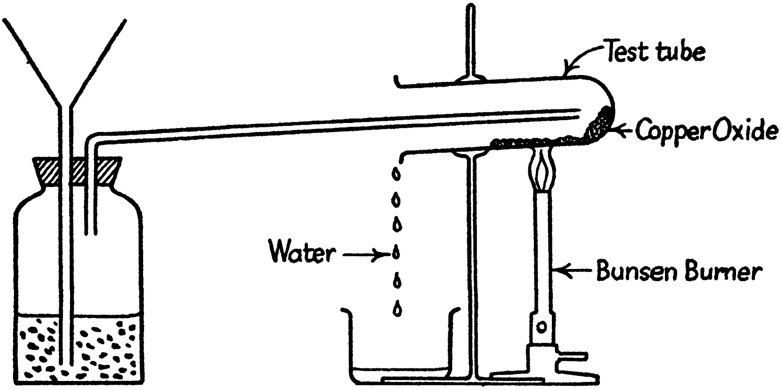
Arrangement of Apparatus for the Synthesis of Water sufficient amount to give a vigorous reaction. When you feel sure all the original air has been swept out of the bottle by the hydrogen, add more acid, and *wrap a towel around the generator.* Now light the jet of issuing gas and hold a heavy iron in the flame. You will observe drops of water forming. Then, completely fill the generator with water to stop the reaction.

$$2H_2+O_2 \rightarrow 2H_2O$$

Hydrogen plus oxygen yields hydrogen oxide (water).

Analysis Versus Synthesis. You have just synthesized water; that is, water has been made from its elements by direct combination. Look back now to the discussion of the electrolysis of water. There the reverse was done; water was analyzed, or decomposed, into its elements. Measurements show that the electrical energy consumed in electrolyzing one pound of water is exactly equal to the heat energy released when one pound of water is synthesized.

Oxidation and Reduction. Hydrogen is also a *reducing agent,* that is, it can deoxidize, or take away the oxygen from metallic oxides. To demonstrate this reducing property of hydrogen, set up a test tube containing black copper oxide as shown in the diagram. After a few minutes water drips out of the test tube and the copper oxide has been reduced to red metallic copper. This reducing reaction illustrates also what is called a *simple replacement* or *substitution* reaction. Look at the equation:



Reduction of Copper Oxide by Hydrogen

$$H_2+CuO \rightarrow Cu+H_2O$$

Hydrogen+copper oxide→copper+water (hydrogen oxide). You see that the hydrogen has *replaced* the copper. Notice, too, that while the copper oxide has been deoxidized or reduced, the hydrogen has been oxidized. This is always true: *when one substance is reduced, the reducing agent itself becomes oxidized of necessity.*

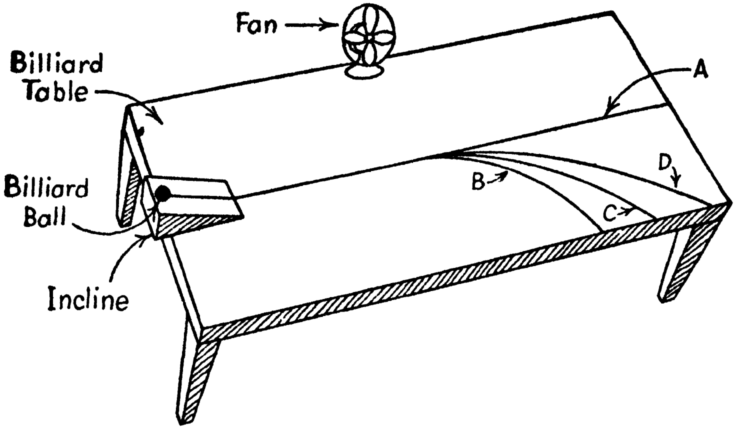
Uses. Because hydrogen is so light, and in spite of its inflammability, it is still used for filling military balloons when helium is not available. Its lightness makes it the best heat conductor of all gases, and for this reason hydrogen is circulated through large electrical generators to keep them cool. Within furnaces where metals are being annealed or deoxidized, hydrogen is used to furnish a reducing atmosphere. The oxy-hydrogen blow torch, though not giving as hot a flame as the oxy-acetylene burner, is used for welding alloys that are harmed when they absorb carbon from the acetylene. The manufactured fuel gas piped to city homes is 50% hydrogen by volume. On a large industrial scale now, hydrogen is being used in the catalytic synthesis of wood alcohol, formaldehyde and ammonia.

Hydrogenation. Lard substitutes, like Crisco and Spry, are made from vegetable oils by the process of *hydrogenation*. Both vegetable and animal oils can be converted into solid fats by causing hydrogen to combine with them in the presence of a nickel catalyst. Coal and petroleum tars are being hydrogenated into gasoline substitutes by hydrogen under very high pressures.

40. ELEMENTARY CHEMICAL THEORY

Ionized Atoms. You will recall from our discussion of the electron theory that all atoms are electrically neutral. When, however, an electron is knocked off an atom, the latter acquires a positive charge, and becomes an ion, or charged atom. In the popular neon sign, this is just what is done; high potential electrical energy forces an electron out of neon's outermost shell. When the electron later jumps back into place, energy in the same amount must be emitted; but it is emitted in the form of the familiar red-orange light you see.

Physicist's Method of Determining Atomic Weights. Any atom thus ionized in a vacuum tube will move towards the cathode with a speed proportional to the voltage applied. As you already know, a powerful magnetic field will cause a stream of charged particles to curve. Knowing the charge on the atom, its speed, the strength of the magnetic field and the shape of the curve, it is possible to calculate the weight of the atom. This is the latest, most accurate method of determining atomic weights. An analogy may help to make this method clear. If a billiard ball is allowed to run down an incline, as shown in the illustration below, it will travel in a straight line along the billiard table. If an electric fan blows at right angles across the table, the path of the ball curves. The lighter the ball (atom) the greater the effect of the fan (magnetic field) on the ball's (atom's) path.



Billiard Analogy to the Mass Spectrograph Method of Determining Atomic Weights

Straight line, A, is the path of any ball (charged atom) when the fan (magnetic field) is not turned on; line B is the path of a light ball when the fan is turned on; C, the path of a heavier ball, and D, the path of a still heavier ball.

Atomic Weights by the Chemist's Method. One of the older methods for determining atomic weights consists simply in first weighing a sample of the gaseous element. The number of molecules in the sample can be accurately counted in any one of several ways. (It is of interest to note that experiments show that equal volumes of gases, all at the same temperature and pressure, contain the same number of molecules. In other words, if five quart bottles are filled, one with oxygen, one with carbon dioxide, one with air, one with helium and one with acetylene, each bottle contains exactly the same number of molecules.) We can also discover the number of atoms in each molecule. Knowing the weight of the sample, and the number of atoms it contains, simple division gives the weight of the atom.

Actual and Comparative Atomic Weights. As you have already probably concluded, the actual weight of any atom is an inconveniently small number. For example, the weight of even the heaviest atom, *uranium,* is only 0.000,000,000,000,000,000,000,393 grams (a U.S. nickel weighs 5 grams). For this reason, in all practical chemical work *comparative* atomic weights are used. Oxygen has been selected as the standard because it combines with most of the other elements. Its atomic weight is given as 16 in order that the (comparative) atomic weight of hydrogen, the lightest element, should not be less than unity. The atomic weights given in the

Table of Elements are, therefore, only comparative atomic weights, not actual weights. (See Appendix.)

Molecular Weights. Oxygen gas has two atoms in its molecule. We indicate this by writing its symbol with the subscript 2, like this: O_2. Its molecular weight therefore is 2×16, or 32. In scientific work, the gram is used instead of the pound as the unit of weight. The *gram-molecular weight* of oxygen is 32 grams. It so happens that 32 grams of oxygen gas, measured at 0°C (32°F) and under a pressure of 76 cm. (30 inches) of mercury, that is, under normal pressure and temperature, *occupies a volume of 22.4 liters* (about 1 cu. ft.). Since, as we have learned, equal volumes of gases all contain the same number of molecules, it follows that 22.4 liters of any gas will contain one gram-molecular weight of it. The weight, therefore, of 22.4 liters of hydrogen gas (H_2) is 2 grams; of carbon dioxide (CO_2), it is $12+(2\times16)$, or 44 grams; and of helium (He) it is 4 grams.

The chemical engineer uses ounce-molecular weights and ounce-molecular volumes, in which case it happens that 32 ounces of oxygen gas occupy 22.4 cu. ft. (That the gram-molecular volume and the ounce-molecular volume are numerically the same is sheer accident, due to the fact that there are approximately 28 grams in 1 ounce, and 28 liters in one cubic foot.) An understanding of molecular weights and volumes enables the chemical engineer to plan the sizes of his tanks, pipes and chimneys; to design his equipment to withstand expected gas pressures with safety, and to calculate just how much raw material he needs to turn out the desired amounts of his product.

Chemical Symbols. In order to save time and effort we often write U.S.A. to symbolize United States of America. Similarly, chemists use symbols to stand for the names of elements. For convenience, the symbol also represents one atom. Usually the first letter only of the element's name is used, *capitalized*. For example: O represents oxygen; H, hydrogen; N, nitrogen; C, carbon; A, argon. Frequently the names of several elements begin with the same letter. To distinguish these elements from one another, some symbols require a second letter, not capitalized. The symbol for aluminum is Al, not A, since that stands for argon; Cl is the symbol for chlorine, not C, which is used for carbon. When this system was adopted some of the elements were still called by their old Latin names. That is why the symbol for iron is Fe; for sodium, Na; for mercury, Hg. You will find it worth while to memorize the symbols of the elements encountered as we go along. Consult the Table of Elements (see Appendix) for the symbols of all 92 elements.

Valence. As mentioned earlier, the chemist writes formulas in place of names to designate compounds. To do this he must know both the symbol and the valence of each element. You recall that by valence is meant the number of electrons an atom either gives or takes in order to end up with

FORMULA CHART

Ions	**Chloride** Cl^-	**Hydroxide** $(OH)^-$	**Nitrate** $(NO_3)^-$	**Carbonate** $(CO_3)^=$	**Oxide** $O^=$	**Sulfide** $S^=$	**Sulfate** $(SO_4)^=$
Hydrogen H^+							
Sodium Na^+	NaCl						
Silver Ag^+							
Ammonium $(NH_4)^+$					X		$(NH_4)_2SO_4$
Mercury Hg^{++}							
Copper Cu^{++}							
Zinc Zn^{++}	$ZnCl_2$						
Calcium Ca^{++}		$Ca(OH)_2$					
Magnesium Mg^{++}					MgO		
Lead Pb^{++}				$PbCO_3$			
Iron(ous) Fe^{++}							
Iron(ic) Fe^{+++}							
Aluminum Al^{+++}					Al_2O_3		

a complete outermost shell. The formula chart shown below gives the valence of some of the more common ions. Make a copy of this chart and fill it in as you study the directions below.

Variable Valence. Some of the heavier atoms exhibit more than one valence; that is, they may, under certain conditions, be able to get rid of more valence electrons than under other conditions. Iron, for example, has this property. It sometimes lends two electrons, becoming a ferr*ous* ion, Fe^{++}; when it lends three electrons it becomes the ferr*ic* ion, Fe^{+++}.

Radicals. Frequently, a *group of atoms* acts as a *single ion* through many chemical reactions. A compound ion or group of atoms with an electric charge is called a *radical.* The ammonium ion $(NH_4)^+$ and the sulfate ion $(SO_4)^=$ are examples of radicals.

Simple Formulas. If you review now the electron theory explanation of the burning of magnesium, you will notice that the compound magnesium oxide, MgO, is itself electrically neutral. This is because magnesium has the valence $+2$, while oxygen has the valence -2. It is easy to write the formulas of compounds like this one, made up of oppositely charged ions, having numerically equal valences. You merely write the ions together, the positive one always first.

EXAMPLES: sodium chloride, NaCl; lead carbonate, $PbCO_3$; aluminum phosphate, $AlPO_4$. Now using the formula chart, write in the formulas of all the compounds whose .ions have numerically equal valences. You should have written 38 formulas, excluding illustrative formulas already given. Check to see that you have.

Trial Method. To write the formulas of the rest of the compounds needed to complete the formula chart, always remember *that the compound must be electrically neutral.* In other words, every electron lent by a positive ion must find a place in the negative ion of the compound. Let us try to write the formula of aluminum oxide. The chart shows aluminum to be Al^{+++} and oxide to be $O^=$. AlO is *not correct* because one electron from aluminum has not been taken care of. AlO_2 is also *incorrect* because the two oxygen ions need four electrons, and aluminum can furnish only three. Al_2O_2 is *wrong* since now we have two aluminums supplying together six electrons, whereas the two oxygens can accept only four. Al_2O_3 *is correct* because the six electrons from the aluminums can be received by the three oxygens.

Short Method. Fortunately, we do not have to go through a trial and error procedure every time we try to write a formula. There is a very easy method which you should now learn. Step 1: Write the ions, showing their valence charges, viz. Al^{+++} $O^=$. Step 2: Bring down the numerical value of the valence of each ion, writing it as the subscript of the *other ion.* (This is called the *criss-cross method.*) In this example, the valence, 3, of Al is written as a subscript to O, and the valence, 2, of O is written as the subscript to Al, giving us the formula, Al_2O_3. (This is read, "A, l, two, O, three.")

What is the formula of ammonium sulfate? Step 1: $(NH_4)^+(SO_4)^=$. Step 2: $(NH_4)_2SO_4$. (This is read, "N, H, four taken twice, S, O, four.") It is not necessary to write or say, "taken one time" when the formula, as in this case, calls for just one sulfate ion.

What is the formula of slaked lime, calcium hydroxide? Step 1: $Ca^{++}(OH)^-$. Step 2: $Ca(OH)_2$. Note that the subscript 2 affects the *whole* hydroxide ion, not the hydrogen alone. The $Ca(OH)_2$ molecule therefore contains five atoms: one Ca, two O's and two H's. $(NH_4)_2SO_4$ contains fifteen atoms: two N's, eight H's, one S, four O's.

Now complete your formula chart by writing in the 83 formulas still missing; (there does not exist any compound of NH_4 and O). Use the criss-cross method, being particularly careful not to forget the parentheses in those formulas which include radicals taken more than once.

At this point it must be realized that formulas are not merely shorthand ways of writing chemical names. They are much more than that. They tell exactly what elements make up the compound, and in exactly what proportions the elements are combined. Each formula also *suggests* how the compound may react, and even how it can be made or synthesized.

Writing and Balancing Chemical Equations. You have already encountered a few examples of chemical equations. They are the chemist's way of stating briefly and accurately the substances that react and those that result from the chemical reaction under discussion. They also tell the relative weights of all the substances involved. In the case of gases, equations also indicate the relative gas volumes.

Note 1: All equations must represent actual reactions based on experimental work! Merely writing, "argon plus oxygen yields argon oxide" does not make it so. Argon is an inert element. It cannot form an oxide or any other compound for that matter.

Note 2: With the exception of the inert gases, all the common elementary gases have two atoms in their molecules. They are therefore written with the subscript 2 whenever they are free. For example, oxygen gas is written O_2; hydrogen, H_2; nitrogen, N_2.

Let us illustrate how equations are written by using reactions with which we are familiar. The rusting of iron serves well as the first example.

Step 1: *While you are learning* to write equations, the *first step* should be a *word equation,* like:

iron+oxygen→rust (which analysis shows to be ferric oxide)

Step 2. Under each substance write its correct formula:

$$Fe+O_2 \rightarrow Fe_2O_3$$

Step 3. Since matter can be neither created nor destroyed every equation must *balance.* There must be the *same number of atoms* of each element

on the left as on ***the ride side.*** Start with the first element. Here, there is one atom of iron on the left, but there are two on the right. Put a 2 in front of the Fe. (Numbers placed in front of formulas are called ***coefficients.*** The coefficient multiplies everything that follows it, up to the next sign.) We now have:

$$2\ \overset{\surd}{\mathrm{Fe}}+\mathrm{O_2}\rightarrow\overset{\surd}{\mathrm{Fe_2}}\mathrm{O_3}$$

The "irons" check, but there are only two oxygen atoms on the left while there are three on the right. You might be tempted to change the subscript 2 on the left into a 3 in order to make the equation balance, but that would be wrong. Remember that you must ***never change subscripts.*** For changing a subscript really means changing the substance into some other substance which might actually not even exist. (It happens that O_3 stands for ozone, something quite different from the O_2 (of the air). Coefficients, however, may be changed when necessary. They merely signify different amounts of substances. If you use the coefficient 1½ in front of O_2, the equation balances, for now there are 3 O's on each side:

$$2\overset{\surd}{\mathrm{Fe}}+1\tfrac{1}{2}\overset{\surd}{\mathrm{O_2}}\rightarrow\overset{\surd}{\mathrm{Fe_2}}\overset{\surd}{\mathrm{O_3}}.$$

This *equation,* however, is *still not satisfactory.* A fractional coefficient would indicate that you can weigh out a fraction of a molecule. To overcome this objection, multiply the entire equation by 2. You now have a correctly balanced equation:

$$4\mathrm{Fe}+3\mathrm{O_2}\rightarrow2\mathrm{Fe_2O_3}.$$

Practice writing the equations for the burning of hydrogen, copper, magnesium and aluminum. Be sure your formulas are correct. Check them against your formula chart. Follow the procedure given above to balance each equation. Now copy *just* the word equations below and work out the balanced chemical equations on your own sheet, ***without consulting the book.*** Then refer to the steps given here to see that you have learned the method.

Step 1. mercuric oxide→mercury+oxygen

Step 2. $\overset{\surd}{\mathrm{Hg}}\overset{\times}{\mathrm{O}}\rightarrow\overset{\surd}{\mathrm{Hg}}+\overset{\times}{\mathrm{O_2}}$

Step 3. $\overset{\times}{2\mathrm{Hg}}\overset{\surd}{\mathrm{O}}\rightarrow\overset{\times}{\mathrm{Hg}}+\overset{\surd}{\mathrm{O_2}}$

$\overset{\surd}{2\mathrm{Hg}}\overset{\surd}{\mathrm{O}}\rightarrow\overset{\surd}{2\mathrm{Hg}}+\overset{\surd}{\mathrm{O_2}}$ (Balanced)

Step 1. zinc+hydrochloric acid→zinc chloride+hydrogen
(hydrogen chloride
solution)

Step 2. $\overset{\checkmark}{Zn}+\overset{\times}{HCl}\rightarrow\overset{\checkmark}{ZnCl_2}+\overset{\times}{H_2}$

Step 3. $\overset{\checkmark}{Zn}+\overset{\checkmark\checkmark}{2HCl}\rightarrow\overset{\checkmark\ \checkmark}{ZnCl_2}+\overset{\checkmark}{H_2}$ (Balanced)

Step 1. copper oxide+hydrogen→copper+water

Step 2. $\overset{\checkmark}{Cu}\overset{\checkmark}{O}+\overset{\checkmark}{H_2}\rightarrow\overset{\checkmark}{Cu}+\overset{\checkmark}{H_2}\overset{\checkmark}{O}$ (Balanced)

Solutions. Though mixtures have already been discussed, there are certain ones called *solutions,* that are so important that some space must be devoted to them. As you know, water can form solutions of a solid like washing soda, of a liquid like alcohol, or of a gas like ammonia. Besides such *liquid* solutions, there are *solid* solutions of which glass and solder are examples, and *gaseous* solutions typified by air. They are unique because they are homogeneous (uniform throughout), usually clear and transparent; and their parts do not separate on standing. *Solutions are mixtures,* however, not compounds, because their compositions vary. When a teaspoonful of sugar is stirred sufficiently in a glassful of water, a sugar solution results. Here the water is the *solvent,* and the sugar the *solute.* In *tincture* of iodine, *alcohol* is the solvent and *iodine crystals* the solute.

Dilute and Concentrated Solutions. A pinch of salt dissolved in a cupful of water makes a dilute solution. Any solution containing a relatively *small amount of solute* is called *dilute.* A solution made by dissolving an eighth of a pound of salt in a cup of water is a concentrated solution. Whenever a solution contains a relatively *large amount of solute* it is called *concentrated.* Concentrated ammonia water may contain as much as 700 quarts of ammonia gas dissolved in one quart of water.

Saturated Solutions. By shaking blue vitriol crystals with water for some time, *until no more dissolves,* even though some *undissolved solute* crystals still remain *in contact with the solution,* you can prepare a *saturated solution* of copper sulfate. Using the same procedure, you can prepare saturated solutions of any soluble substance. A solution containing less than the amount of solute needed to saturate it is called an *unsaturated solution.*

Supersaturated Solutions. Prepare a saturated solution of photographer's hypo. Then bring the solution to a boil. Add to the boiling solution, the same amount of solute as you originally used. This, too, dissolves quickly. Now cover the container to keep dust out, and allow the solution to cool to room temperature without disturbing it. Examination of this cool, supersaturated solution reveals nothing strange; it looks like any other clear water solution. If now one small crystal of hypo is dropped into the solution, the crystal grows before your eyes. This goes on until all the excess hypo has been precipitated, and a saturated hypo solution remains above the crystals. Other salts besides hypo form *supersaturated solutions.* When their hot solutions are cooled, *more solute remains in solution than you could get to dissolve* directly by stirring at the *lower temperature.* Supersaturation occasionally presents a problem in industries producing crystalline products. And in the laboratory the analytical chemist may not detect the precipitate that should, but does not come out of solution.

Gases can also form supersaturated solutions. Precautions must be taken to avoid this in petroleum refining plants. Seltzer and pop bottles have been known to explode violently when shaken, for the same reason.

Conditions Influencing Solubility. From what has already been said it is obvious that temperature has an important effect on solubility. While it is true that the solubility of most solids *increases with a rise in temperature,* some solids are hardly more soluble and a few are even less soluble in hot than in cold water. All gases, on the other hand, are more soluble in cold than in hot solvents. To get a large amount of gas to dissolve, force it in under high pressure, for the *solubility of a gas in a liquid goes up as its pressure increases.* The solubility of solids and liquids is practically uninfluenced by pressure. You have probably observed that a lump of sugar or a piece of rock candy takes longer to dissolve than granulated sugar. You know, too, that unless coffee is stirred, the sugar added may remain undissolved at the bottom of the cup. To make solutions quickly then, break up or *grind the solute,* and *keep mixing it with the solvent.* Heating, of course, helps too in most cases.

Sand, as you know, does not dissolve in water; it is *insoluble.* Silver does not dissolve in water either, but it does dissolve in mercury. A grease spot can be removed with benzine but not with water since grease is not soluble in water. To be accurate, then, when you speak of solubility and insolubility you must mention the solvent you have in mind.

Suspensions and Emulsions. When clay is shaken up with water, in which it is insoluble, a turbid mixture is obtained, the ingredients of which separate on standing. This is an example of a *suspension.* In industry, where time is an important consideration, it is not always possible to wait until the suspended particles settle out, and so a filter press is used instead. Kerosene and water if shaken vigorously together form a milky mixture which also separates into its parts after a short time. Here we have an example of an *emulsion.*

Collodial Suspensions. Because of the problems they present, *very finely divided particles* have been studied separately and been given the name *colloids.* Colloidal particles are so small that they are not visible under the ordinary microscope. (Yet they are much larger than the *molecules* that are the *solute* in *true solutions.*) They do not settle out if suspended in water, and go right through the pores of filter paper or filter cloth when an attempt is made to purify the water in this way. Sometimes the addition of a salt to the colloidal suspension causes the colloidal particles to clump together and settle out. In sugar refining, the brown colloidal coloring matter in the impure sugar solution is removed by powdered bone black, a kind of charcoal made by charring animal bones. Valuable industrial dusts and smoke particles that formerly fouled the air in the vicinity of cement mills and metallurgical plants are now being coagulated in the smokestacks by electrical devices. Most of these recovered dusts are being sold at a profit.

Properties of True Solutions. There is only one way by which both solvent and solute can be separated and recovered pure from a solution. Distillation, a process to be discussed later, must be used. Settling or filtration will not work because the true solute, say the sugar in a sugar solution, is dispersed uniformly as separate sugar molecules.

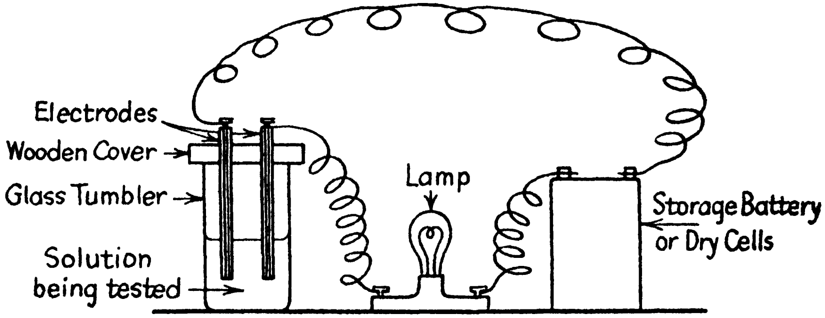
Solutions do not boil or freeze at the same temperatures as the solvent alone. Pure water boils at 212° F at sea level. A water solution, however, must be heated hotter before it begins to boil. For example, a solution made by dissolving a gram molecular weight of glycerine (92 grams of $C_3H_5(OH)_3$) in about a quart of water boils at 213°F. A gram molecular weight of *sugar* (342 grams of $C_{12}H_{22}O_{11}$) *dissolved* in the same amount of *water* also *raises its boiling point* by 1 degree. Three times 93 grams, or 276 grams of glycerine raises the boiling point 3 degrees. One half a gram molecular weight raises the boiling point ½ a degree.

The temperature at which pure water freezes is 32° F. A water solution, however, must be cooled more to get it to freeze. A solution of *alcohol,* for example, containing one gram molecular weight (46 grams of C_2H_5OH) in about ⅛ quart of water freezes at 31° F. Four gram molecular weights *lower the freezing point 4 degrees.* This explains why alcohol, Prestone and other anti-freezes are added to the radiator water of cars in the winter. If you live in a region of the country which does not experience very cold weather, you do not have to buy as much anti-freeze as the man who runs a car in the northern part of the country. Solder, a fifty-fifty mixture of lead and tin, has a lower melting point than either lead or tin alone. You would expect this to be the case, knowing solder to be a solid solution, and knowing also that solutes lower the freezing point of the solvent.

Abnormal Freezing Points and Boiling Points. When you try the effect of bases, acids and salts on the freezing and boiling points of water you get surprising results. For example, one gram molecular weight of lye (40 grams of NaOH) has twice the effect that sugar or glycerine has. A gram molecular weight of sulfuric acid (98 grams of H_2SO_4) reduces the freezing point of water 3 degrees, whereas you recall that a g.m.w. of alcohol reduces the F.P. of the same amount of water by only 1 degree.

One g.m.w. of common salt (NaCl) has twice the effect; a g.m.w. of aluminum nitrate, $Al(NO_3)_3$ four times the effect; and one g.m.w. of ferric sulfate $Fe_2(SO_4)_3$ five times the effect on freezing and boiling points that one g.m.w. of sugar has. This peculiar behavior of acids, bases and salts in solution disturbed chemists for many years.

Electrolytes and Non-Electrolytes. Another way in which acids, bases and salts show themselves to be in a different class from compounds like sugar and alcohol is made clear by the following experiment. Make up separate solutions of the compounds to be tested, by dissolving one tenth of a gram molecular weight of each in half a glass of water. Set up the apparatus as depicted in the diagram. Notice that the current cannot flow, and therefore the lamp in series cannot glow, unless some conductor of electricity closes the circuit between the two electrodes. Now when the compounds listed in the table below are tested, the results shown in the second column are obtained. Notice that the solutions of those substances which have the *normal effect* on boiling and freezing points *do not conduct* the current. However, those compounds whose solutions *do conduct* the current are the very ones which affect the boiling and freezing points *abnormally.*



Apparatus to Test the Conductivity of Solutions

TABLE OF SOME STRONG AND WEAK ELECTROLYTES AND NON-ELECTROLYTES

Compound Tested	*Brightness of Lamp*	*Formula*	*Free Ions*
distilled water alone	no glow	H_2O or HOH	none
sodium chloride solution	bright	$NaCl \rightleftarrows$	$Na^+ + Cl^-$
aluminum nitrate "	"	$Al(NO_3)_3 \rightleftarrows$	$Al^{+++} + 3(NO_3)^-$
ferric sulfate "	"	$Fe_2(SO_4)_3 \rightleftarrows$	$2Fe^{+++} + 3(SO_4)^=$
sodium hydroxide "	"	$NaOH \rightleftarrows$	$Na^+ + (OH)^-$

ammonium hydroxide solution	dull	NH_4OH	$\rightleftarrows (NH_4)^+ + (OH)^-$
hydrochloric acid “	bright	HCl	$\rightleftarrows H^+ + Cl^-$
sulfuric acid “	“	H_2SO_4	$\rightleftarrows 2H^+ + (SO_4)^=$
pure sulfuric acid alone	no glow	H_2SO_4	none
acetic acid solution	dull	$H(C_2H_3O_2)$	$\rightleftarrows H^+ + (C_2H_3O_2)^-$
alcohol “	no glow	C_2H_5OH	none
sugar “	“	$C_{12}H_{22}O_{11}$	none
glycerine “	“	$C_3H_5(OH)_3$	none

Arrhenius' Theory of Ionization. A great Swedish chemist, Arrhenius, has given us a theory to explain the behavior of acids, bases and salts. The ionic equations in columns three and four summarize his theory. According to it, there are two classes of compounds: first, a group called *electrolytes* which can furnish ions; and second, a group of *non-electrolytes* which cannot ionize. NaCl depresses the freezing point twice as much as sugar does because the former in solution separates (ionizes) into *two ions* whereas the sugar remains as a molecule. The *ions conduct the electric current* in solutions.

The more ions present the greater the amount of current that can flow. $Fe_2(SO_4)_3$ raises the boiling point five times as much as glycerine does because it dissociates into five ions, while the glycerine *does not conduct* the current because it remains *unionized*. Sulfuric acid, H_2SO_4, if not dissolved in water, does not conduct the current. It has no free ions. Water itself is a non-electrolyte, but it enables electrolytes to ionize.

Acids and Bases. Any compound which ionizes to furnish an H^+ion is an *acid*. It is the H^+ion that gives to all acids their *sour taste*, and causes them to change the color of a purplish dye called *litmus* to *red*. The *hydrogen ion of an acid is replaceable by active metals*. You made use of this fact when you added hydrochloric acid to zinc, producing hydrogen gas and the salt, zinc chloride. *Acids neutralize bases*, also, because of their H^+ions. Compounds which ionize to furnish $(OH)^-$ ions are called *bases*. It is the *hydroxide ion* that makes them *feel slippery* and taste *bitter*. The $(OH)^-$ ion *turns litmus blue*. Bases, because of their $(OH)^-$ ions, *neutralize acids*.

Strong and Weak Electrolytes. We shall study a few acids in detail a little later. At this point, however, it is worth noting that *strong acids* are those that ionize almost completely in solution. *Weak acids* like vinegar, or acetic acid, furnish relatively few H^+ions. Refer to the previous Table again. Notice that acetic acid solution is so poor a conductor of electricity that the lamp can merely glow dully. Precise measurements show that only one or two molecules out of a hundred of acetic acid are ionized at one time. The arrows in the ionic equations tell you at a glance the strong and the weak electrolytes.

Neutralization. To a solution of caustic soda, $NaOH$, add hydrochloric acid, a drop at a time. Use a glass stirring rod to mix the solutions after each addition, and to touch a drop of the mixture to a piece of blue and a piece of red litmus paper. At the start, the liquid does not change blue litmus, but it changes the red litmus to blue. Finally, when exactly the right amount of acid has been added, a drop of the liquid affects *neither* blue nor red litmus. It is neutral. The solution is neither sour nor bitter; it is salty. Boil off the water, and crystals of salt remain. In this particular case the equation is:

$$NaOH+HCl \rightarrow NaCl+HOH$$

$$\text{(Ionic Equation)}Na^{+}+(OH)^{-}+H^{+}+Cl^{-} \rightarrow Na^{+}+Cl^{-}+H_2O$$

Note that the $(OH)^-$ of the base and the H^+ of the acid *combined* to form *unionized water*. In general:

$$\text{BASE}+\text{ACID} \rightarrow \text{SALT}+\text{WATER}$$

Salts. The product besides water, formed when a base neutralizes an acid, is a *salt*. The following equations illustrate that the salt produced depends on both the acid and the base used.

$$2NaOH+H_2SO_4 \rightarrow Na_2SO_4+2HOH$$

You complete and balance the following equations:

$$Ca(OH)_2+HNO_3 \rightarrow$$
$$NH_4OH+H_3PO_4 \rightarrow$$

41. SOME IMPORTANT ELEMENTS AND COMPOUNDS

SODIUM AND ITS COMPOUNDS

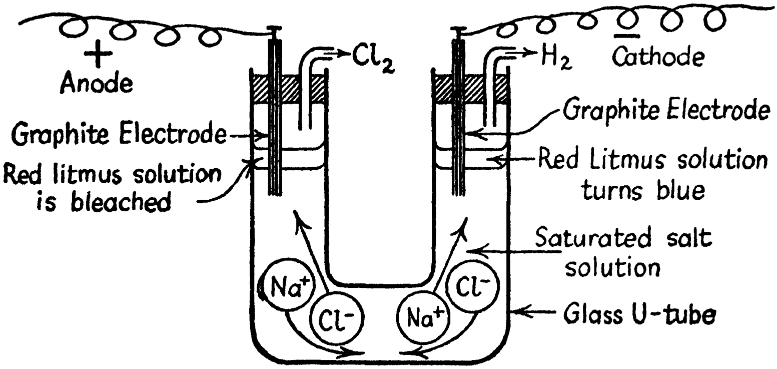
Occurrence and Properties of Sodium. Because of its great chemical activity, metallic sodium is never found free in nature. In the form of its compounds, chiefly as common salt, it is however widely distributed all over the earth. Sodium is prepared by the electrolysis of molten sodium hydroxide and also by the electrolysis of fused sodium chloride. Only in recent years has the industrial demand warranted really large scale production. Now metallic sodium is being shipped in tank car lots to the synthetic rubber factories and to the manufacturers of sodium vapor highway lamps.

When you cut through this soft metal, its newly exposed silvery surface quickly tarnishes as it reacts with the oxygen, moisture and carbon dioxide of the air. For this reason it is stored under kerosene. Dropped on water, sodium melts, spins around on the surface, hissing and spattering with such violence that it often bursts into its characteristic bright yellow flame. As it does this, it is replacing one of water's hydrogens, and forming a solution that turns red litmus blue. The equation explains the reaction taking place.

$$2Na + 2HOH \rightarrow 2NaOH + H_2\uparrow^*$$

Sodium Chloride. The amount of common salt contained in the oceans alone, even though they average less than 3% sodium chloride, is sufficient to make a solid block about *five million cubic miles* in size. Besides this there exist numerous salt lakes and solid salt deposits throughout the world. Practically all the salt of commerce is obtained from these latter sources. The impurities always found in natural salt make purification necessary. This process consists simply in recrystallizing the salt from water. When a saturated solution of natural salt is made, we find that it is saturated only with respect to sodium chloride. The magnesium chloride and other impurities are present in small concentrations only. Therefore, as the water is evaporated off, pure sodium chloride crystals come out of solution, while the impurities remain in the mother liquor. The purified sodium chloride crystals are then separated from the solution, just before it becomes saturated with the impurities.

It would be difficult to exaggerate the importance of salt. The blood streams of human beings and animals are dilute salt solutions, which is one of the reasons it is so important as food. Large quantities of salt are used by the tanning, textile, fertilizer, ceramic, food preserving, refrigeration and soap industries. The electro-chemical industry makes metallic sodium, caustic soda, chlorine, hydrochloric acid and other products from it. In fact it is the source from which all manufactured sodium compounds are made.



Electrolysis of Brine

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*To indicate that a gas is given off as the result of a reaction, we place an upward pointing arrow ↑ next to the formula for the gas. If an insoluble substance, a precipitate, separates from solution because of a reaction, an arrow pointing down ↓ is used to bring out that fact.

Sodium Hydroxide. *Caustic soda,* or lye, the common names by which sodium hydroxide is known, is prepared commercially by the electrolysis of brine. Many different types of apparatus are used to permit of continuous operation. A simple experiment that we can easily perform enables us to understand the principles involved.

The diagram shows how a direct current can be passed through the brine. A little red litmus solution floats on the more dense salt water. When the switch is closed bubbles of gas come off from both electrodes. Soon the litmus on the anode side is bleached, as the liberated chlorine gas acts on it. At the cathode, hydrogen can be detected, and the litmus turns blue. What happens can be explained as follows. The salt solution contains Na^+ions and Cl^-ions. The negative ion is attracted to the positive electrode (the anode), where it gives up its extra electron. It thus becomes a chlorine atom. Two join and escape as Cl_2, chlorine gas, a bleaching agent. The positive ion is attracted to the cathode, where it receives the electron it needs to become a sodium atom. But metallic sodium cannot exist as such in water. It immediately reacts to liberate H_2, and to form NaOH, which stays in solution and turns the litmus blue. The equation for the *electrolysis of brine* summarizes these reactions as follows:

$$2NaCl + 2HOH \rightarrow 2NaOH + H_2\uparrow + Cl_2\uparrow$$

Over half a million tons of caustic soda are used annually in the United States. Almost one hundred thousand tons of this are consumed in neutralizing the sulfuric acid that is used in petroleum refining. About the same amount goes into the soap factories.

Sodium Bicarbonate and Sodium Carbonate. In spite of the fact that extensive deposits of both these salts are found in nature, practically all we use is made by the Solvay process. In the laboratory we can duplicate the factory process by bubbling carbon dioxide and ammonia through a saturated salt solution. The precipitate that settles out is bicarbonate of soda, $NaHCO_3$. The equation for the reaction is:

$$CO_2 + NH_3 + H_2O + NaCl \rightarrow NaHCO_3\downarrow + NH_4Cl$$

The sodium bicarbonate is washed free of ammonium chloride and then heated to redness, whereupon it changes to sodium carbonate:

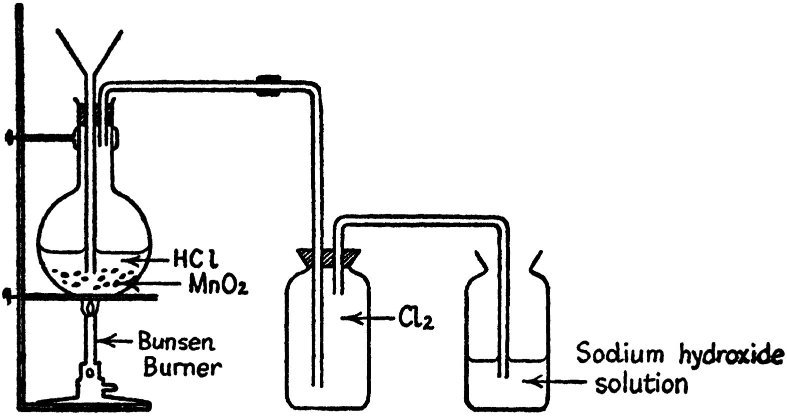
$$2NaHCO_3 \rightarrow Na_2CO_3 + H_2O\uparrow + CO_2\uparrow.$$

The manufacture of glass and other industries consume about three million tons of sodium carbonate yearly. Crystallized sodium carbonate is the familiar washing soda, or sal-soda. Sodium bicarbonate is the well known baking soda.

Sodium Nitrate. In Chile, located on mountains thousands of feet high, there are found extensive deposits of saltpeter. Until man learned how to "fix" atmospheric nitrogen, the world was almost wholly dependent on these 100-mile wide and 500-mile long beds of impure nitrates for its nitrogen fertilizers and explosives.

CHLORINE AND ITS COMPOUNDS

Preparation and Properties of Chlorine. We have already learned how chlorine is made commercially by the electrolysis of brine. In the laboratory, the apparatus is set up as shown in the diagram. When concentrated hydrochloric acid is poured on manganese dioxide (which here acts as an *oxidizing agent*) and heated gently, bubbling is observed in the generator. A greenish-yellow gas soon fills the flask and the collecting bottle. The reaction can be understood by examining the equation, which shows that first a double replacement occurs; then the unstable manganic chloride decomposes into manganous chloride and chlorine:



Laboratory Preparation of Chlorine

$$
\begin{array}{l}
4HCl + MnO_2 \rightarrow 2H_2O + MnCl_4 \\
\qquad\qquad\qquad\qquad\qquad\quad \downarrow \\
\qquad\qquad\qquad\qquad\quad MnCl_2 + Cl_2\uparrow
\end{array}
$$

Chlorine is collected as shown because it is *soluble in water and denser than air.* Chlorine has a very *irritating odor,* and is, in fact, quite *poisonous.* The second bottle is provided to prevent the escape of any excess hydrogen chloride or chlorine into the air.

Chemical Properties. Chlorine is one of the most active non-metals. By this is meant that it combines very readily with metals by taking an electron from them. It is therefore a powerful *oxidizing agent,* as well as a supporter of combustion. A piece of warm metallic sodium held in a bottle of chlorine burns brilliantly, producing by this reaction between two poisonous elements, a perfectly harmless salt, sodium chloride.

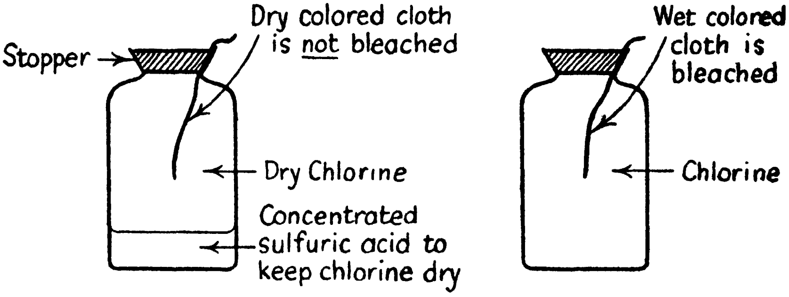
$$2Na+Cl_2 \rightarrow 2NaCl$$

In fact, most of the commonly known metals form chlorides in the same way. A jet of burning hydrogen continues to burn in a bottle of chlorine just as it does in the air. However, instead of hydrogen oxide (H_2O), here the product is hydrogen chloride (HCl):

$$H_2+Cl_2 \rightarrow 2HCl$$

A burning candle also continues to burn in chlorine. In this case, however, the carbon of the wax does not combine with the chlorine; it forms soot, while the hydrogen part forms HCl. Chlorine reacts with water, too, in a way that will be discussed below.

Uses of Chlorine. Easily compressed to a liquid, chlorine is shipped in steel cylinders. It was from such cylinders, when the wind was blowing toward the enemy, that the Germans, during the first World War, directed their original chlorine poison gas attack. Oddly enough, in the same war, thousands of lives were saved by a newly discovered chlorine-containing germicide now being sold as *Zonite*. Chlorine is also shipped as chloride of lime which we study in the chapter on calcium compounds. Dry chlorine is quite inactive. You can demonstrate this by trying the experiment illustrated in the diagram. Tests show that chlorine reacts with water to form two acids:

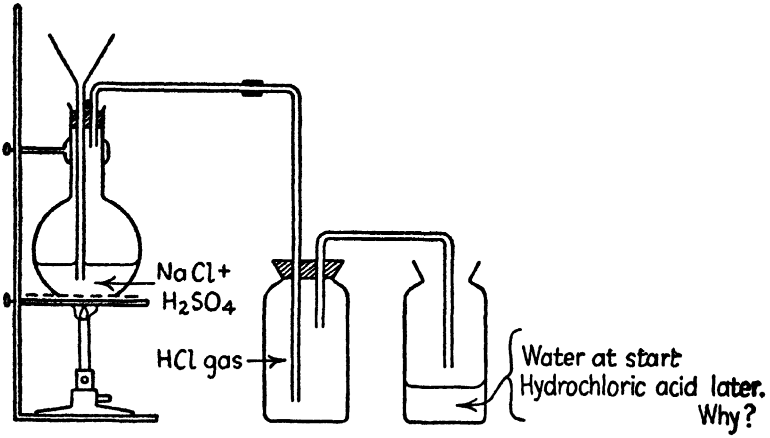

Stopper
Dry colored cloth is not bleached
Dry Chlorine
Concentrated sulfuric acid to keep chlorine dry
Wet colored cloth is bleached
Chlorine

Chlorine Bleaches Only When Wet

$$Cl_2+H_2O \rightarrow HCl+HClO$$

It is the HClO, hypochlorous acid, which does the bleaching and kills the germs by releasing oxygen in a very active form. Over two thirds of the chlorine produced are used for bleaching wood pulp for paper and rayon manufacture, and cotton and linen for textiles (not silk or wool, which are ruined by it). Large amounts are also used in water purification. The chemical manufacturer uses chlorine in making bleaching powder, household bleaches like Javelle water (NaOCl), chloroform ($CHCl_3$), and many other important compounds, including toxic war gases like phosgene, "tear," "sneeze" and "mustard" gases.

Hydrogen Chloride. Most of the hydrogen chloride of commerce is made by the direct combination of hydrogen with chlorine obtained by the electrolysis of brine. Its water solution is *hydrochloric acid,* or *muriatic acid,* as it is sometimes called. Using the same apparatus that we used for the laboratory preparation of chlorine, we can prepare hydrogen chloride gas. Common salt, NaCl, is placed in the flask and concentrated sulfuric acid poured on it through the funnel. Heat strongly. A *colorless* gas, *denser than air,* having a *sharp,* penetrating odor, and *extremely soluble* in water is obtained. The equation for the reaction is important, because it illustrates a *general method for making acids.* Use a *salt of the acid desired, plus sulfuric acid.* Here we used the cheapest salt of hydrochloric acid. To make, say, phosphoric acid, we use its cheapest salt, $Ca_3(PO_4)_2$, plus H_2SO_4:



Laboratory Preparation of Hydrogen Chloride and Hydrochloric Acid

$$NaCl + H_2So_4 \rightarrow NaHSO_4 + HCl\uparrow$$

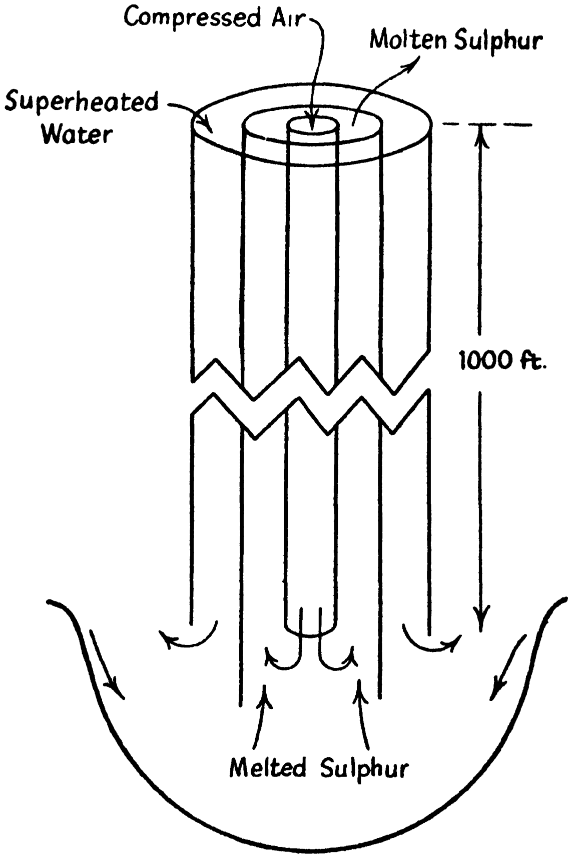
In industry where the apparatus can be brought to a red heat, all the sulfuric acid, not just half as above, is utilized:

$$2NaCl + H_2SO_4 \rightarrow Na_2SO_4 + 2HCl\uparrow$$

Hydrochloric Acid. A water solution of hydrogen chloride is a typical acid. It tastes sour, turns litmus red, neutralizes bases and reacts with active metals to liberate hydrogen and to form a chloride salt of the metal. It is present in the gastric juice, enabling us to digest meats and other proteins. Industry uses it for many purposes, including the conversion of animal skins and bones into gelatin and glue, and in "pickling" iron and steel to clean the surfaces before galvanizing or tinning.

SULFUR AND ITS COMPOUNDS

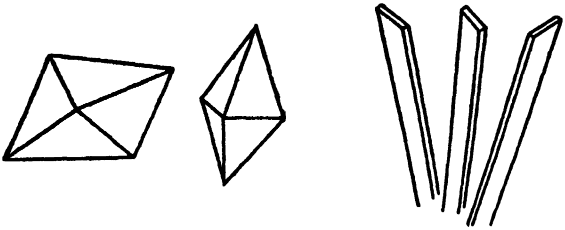
Occurrence of Sulfur (Brimstone). Since sulfur is one of the constituents of protoplasm, it is part of all living things. It also occurs combined in



Frasch Process for Extracting Sulfur from Deep Underground Deposits many important ores and minerals. The sulfur of commerce, however, is the free or native (uncombined) sulfur found both on the surface in volcanic regions, and from 500 to 1500 feet below the surface in the world's largest deposits located in Texas and Louisiana. Most of the more than two billion tons produced yearly come from these U.S. deposits.

Frasch Process. The sulfur from these underground sulfur beds is brought to the surface by a unique mining operation. Using oil-well-drilling methods, three concentric pipes are sunk. Through the outermost pipe, superheated water under pressure is forced down to melt the sulfur. Compressed air is pumped down through the smallest (innermost) pipe to make a light frothy mixture with the sulfur, and to force it up to the surface. The melted sulfur, mixed with air bubbles, gushes up through the middle pipe (see diagram p. 501).

Physical Properties of Sulfur. Sulfur is a yellow, brittle solid, denser than water, and insoluble in it. It is, however, soluble in carbon disulfide. When the latter evaporates from such a solution, the sulfur is left in the form of *rhombic crystals* which look like two pyramids base to base. Heat sulfur in a test tube, slowly, to avoid overheating. You notice that sulfur melts to form a light amber, watery liquid. Pour the liquid into a paper cone, watch it begin to solidify and then quickly pour out the remaining liquid before the crystals reach the center. Tear open the cone and observe the *prismatic, needle-like crystals.*



Rhombic Crystals of Sulphur

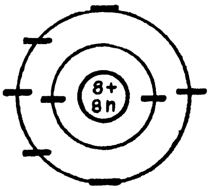
Prismatic Sulphur Crystals

Crystalline Allotropic Forms of Sulfur

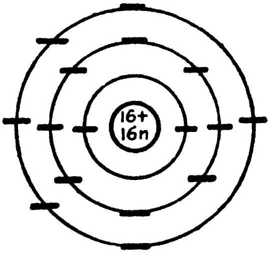
Now heat sulfur in a test tube until it boils. You observe that during the heating the color is gradually changing from amber to dark red and finally to dark brown. At the same time its viscosity is changing. At the beginning it flows like water, then it is more like honey, then so thick that it is almost solid and will not flow at all, and finally at the boiling point it is quite fluid again. While it is boiling, pour it quickly into some cold water. Examine the cooled *amorphous sulfur* you have made. It is not crystalline, but soft and *plastic,* like rubber.

Allotropism. Many other elements besides sulfur show such *allotropic forms;* that is, different modifications of the *same element* in the *same state.* We know that the differences are due to *differences in energy content;* X-ray examination shows that the *atoms are arranged differently* in each allotropic variety of the element. Phosphorus can exist either as a yellow or a red solid; oxygen, as O_2 or O_3 (ozone); carbon, as diamond or graphite; etc. Sulfur gradually changes back from both the amorphous and the prismatic form to the form having the least energy, the rhombic modification.

Chemical Properties of Sulfur. Study the diagrams below. Note that the valence shells are identical. This should lead us to expect both elements to exhibit similar chemical properties. They do. Just as oxygen combines with metals to form oxides, so sulfur supports the combustion of metals to form sulfides. Mix 6.5 grams of zinc powder with 3.2 grams of sulfur. Place the mixture on an asbestos pad. Keep at a safe distance, and touch the mixture with the hot end of a long, glass rod. You observe a sudden brilliant flare. A good deal of the zinc sulfide formed rises as a thick cloud, but enough remains on the asbestos for you to examine the white powder; thus



Oxygen
Atomic weight=16
Atomic number= 8



Sulfur
Atomic weight=32
Atomic number=16

$$Zn+S \rightarrow ZnS$$

Sulfur, however, also behaves as a metal. It burns in air to produce sulfur dioxide:

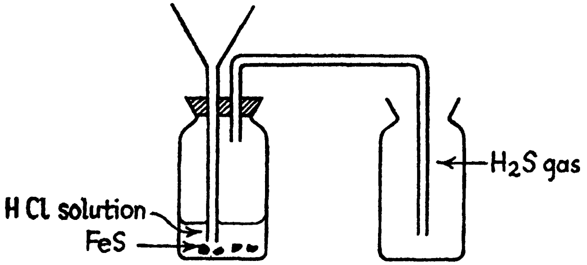
$$S+O_2 \rightarrow SO_2$$

Uses of Sulfur. *Gunpowder* is a mixture of sulfur, charcoal and potassium nitrate. The *match industry* uses large quantities of sulfur. Goodyear discovered that *rubber* acquires many useful properties when *vulcanized* by sulfur; thus starting the great rubber industry. *Sulfur sprays* and dusts are used to *kill fungi* on plants. Large amounts of sulfur are consumed by the *sulfite* paper making process. The chemical industries produce a host of useful sulfur compounds. But the quantity of sulfur that goes into the *manufacture of sulfuric acid* exceeds by far the amount used for all other purposes put together.

HYDROGEN SULFIDE

Occurrence of Hydrogen Sulfide. The distinctive odor of rotten eggs is due to the gas, hydrogen sulfide, which is given off whenever animal matter decays. It is present in volcanic gases, in "sulfur health spring" waters, in natural gas and in manufactured fuel gases. It is a nuisance, and must always be removed from fuel gases to prevent the corrosion of the stove in which they are burned.

Laboratory Preparation. The gas is prepared by adding dilute hydrochloric acid to ferrous sulfide:



Preparation and Collection of H_2S

$$2HCl+FeS\rightarrow FeCl_2+H_2S\uparrow$$

H_2S is collected as shown because it is *more dense than air* and *soluble in water.* It is *colorless* and *quite poisonous.* Its *unpleasant odor* which our nose detects when the gas is present to the extent of only one part in 10,000 parts of air warns us of the danger.

Chemical Properties of H_2S. Bring a flame to the mouth of a dry bottle of the gas. It burns with a blue flame. Water and a yellow solid deposit on the inside of the bottle:

$$2H_2S+O_2\rightarrow 2H_2O+2S \text{ (Incomplete combustion)}$$

Light the gas issuing from the delivery tube. You now detect a suffocating odor. Plenty of oxygen is available, so that complete combustion occurs:

$$2H_2S+3O_2\rightarrow 2H_2O+2SO_2 \text{ (Complete combustion)}$$

Bring a *silver* spoon near hydrogen sulfide, and see how quickly it *blackens:*

$$H_2S + 2Ag \rightarrow Ag_2S + H_2\uparrow$$

Bubble the gas through solutions of various metallic salts, and observe the results summarized in the following equations. These illustrate that H_2S has value to the chemist when he is called upon to analyze "unknown" solutions:

$$\begin{aligned}
H_2S + ZnSO_4 &\rightarrow ZnS\downarrow + H_2SO_4 \\
&\quad \text{white} \\
H_2S + Cd(NO_3)_2 &\rightarrow CdS\downarrow + 2HNO_3 \\
&\quad \text{yellow} \\
3H_2S + 2SbCl_3 &\rightarrow Sb_2S_3\downarrow + 6HCl \\
&\quad \text{orange} \\
H_2S + Pb(NO_3)_2 &\rightarrow PbS\downarrow + 2HNO_3 \\
&\quad \text{black}
\end{aligned}$$

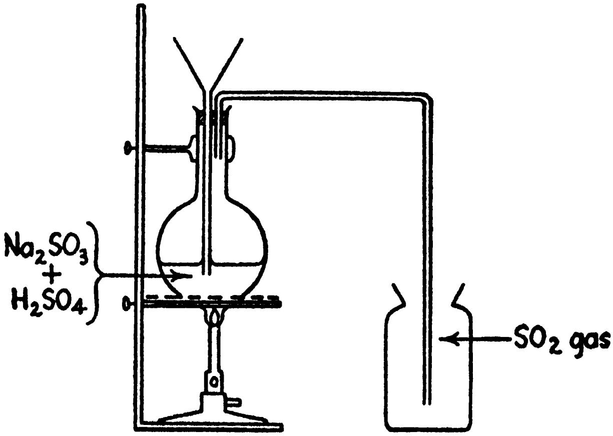
SULFUR DIOXIDE

Preparation of SO_2. Commercially, sulfur dioxide is prepared either by burning sulfur or by roasting sulfides:

$$S + O_2 \rightarrow SO_2$$
$$4FeS_2 + 11O_2 \rightarrow 2Fe_2O_3 + 8SO_2\uparrow$$

In the laboratory the gas is prepared, as shown in the diagram, by adding sulfuric acid to sodium sulfite:

$$Na_2SO_3 + H_2SO_4 \rightarrow Na_2SO_4 + H_2O + SO_2\uparrow$$



Preparation and Collection of SO_2

Like H_2S, it is soluble in water and denser than air. It is colorless and has a sharp, *suffocating odor.* Sulfur dioxide is easily liquefied.

Chemical Properties of Sulfur Dioxide. When tested, the solution of this gas in water will be found to turn litmus from blue to red. A substance that combines with water to form an acid is called an *acid anhydride:*

$$SO_2+H_2O \rightleftarrows H_2SO_3$$

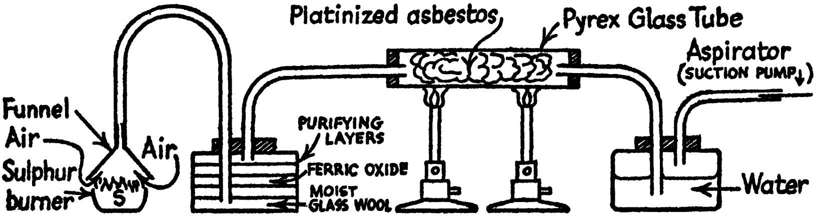
The two arrows indicate that the reaction is *reversible.* By heating the sulfurous acid solution, all of the SO_2 can be driven out of solution, leaving pure water behind. Sulfurous acid is a *reducing agent;* it is oxidized to sulfuric acid merely upon contact with air:

$$2H_2SO_3+O_2 \rightarrow 2H_2SO_4$$

Uses of Sulfur Dioxide. When half of a freshly cut red apple is placed in a bottle of SO_2, it will be noticed that the red skin is bleached, and that the meat of the apple does not turn brown as it would in air. Wet sulfur dioxide (which is really sulfurous acid) may therefore be used to ***bleach*** and ***preserve fruits*** and to ***bleach silk, wool, straw, flowers*** and other materials which would be injured by the use of chlorine. Dry liquefied SO_2 is used as the *refrigerant* in many household electric refrigerators. The most important use of sulfur dioxide is in the *manufacture of sulfuric acid.*

SULFURIC ACID

Preparation of Sulfuric Acid. H_2SO_4, *hydrogen sulfate,* or *oil of vitriol* as it is sometimes called, is manufactured by the *contact process.* As you already know, sulfur burns readily in air to form sulfur dioxide. But to get sulfur to release the two remaining electrons from its valence shell, a catalyst or contact agent must be used. Finely divided platinum serves for this purpose. When the sulfur in sulfur dioxide has changed its valence from +4 to +6, it can take another oxygen to form SO_3. This is the purpose of the contact process.



Contact Process for the Manufacture of Sulfuric Acid

In the laboratory we can duplicate the factory operation. Sulfur burns in a dish, while the SO_2 formed, plus additional air, is drawn through the train of apparatus. The mixture of gases passes through layers of moist glass wool and ferric oxide to free it from dust, arsenic and other impurities which would poison the catalyst. The SO_2 now combines with the extra oxygen when in contact with the heated platinum. The SO_3 formed is next drawn into water while the excess air escapes.

Note how the equations clearly indicate that the contact process consists essentially of two oxidation steps followed by the hydration of an acid anhydride:

$$S+O_2 \rightarrow SO_2$$
$$2SO_2+O_2 \rightarrow 2SO_3$$
$$SO_3+H_2O \rightarrow H_2SO_4$$

Properties of Sulfuric Acid. Sulfuric acid is a colorless, odorless, oily liquid, almost twice as heavy as water. *Pour* some concentrated H_2SO_4 *into water* in a test tube. Note that it dissolves, and that the test tube becomes too hot to hold. NEVER POUR WATER INTO SULFURIC ACID! Water floats on oil of vitriol; before it can mix thoroughly with the acid, the heat evolved changes the water to steam so suddenly that it spatters this dangerous acid. You will recall that *undiluted sulfuric acid* does not conduct the electric current because it is *unionized*. The *concentrated acid* is, surprisingly enough, shipped in steel drums and tank cars with perfect safety. However, *hot concentrated sulfuric acid* is a *powerful oxidizing agent*. It reacts with metals, copper for example, to liberate *not hydrogen* but sulfur dioxide:

$$Cu+2H_2SO_4 \rightarrow CuSO_4+2H_2O+SO_2\uparrow$$

Concentrated sulfuric acid absorbs water greedily. An unstoppered bottle of it soon overflows because it takes in the moisture from the air. It is so avid for water that it wrenches water out of dry compounds that merely contain hydrogen and oxygen as part of their composition. For example, it blackens wood (cellulose, $C_6H_{10}O_5$), and when poured on sugar ($C_{12}H_{22}O_{11}$), swells it up into a mass of steaming, porous, black carbon ten times its original volume. These properties characterize *concentrated* sulfuric acid as a *dehydrating agent.*

Dilute sulfuric acid, on the other hand, is a *typical acid.* This is to be expected from the fact that it ionizes to furnish H^+ions. The dilute acid cannot be safely shipped in iron containers; it would eat right through:

$$Fe+H_2SO_4 \rightarrow FeSO_4+H_2\uparrow$$

Uses of Sulfuric Acid. The millions of tons of sulfuric acid produced annually in the United States find use in practically every industry of the country. Among the chief users are the following: (1) The *fertilizer industry* converts practically insoluble phosphate rock, $Ca_3(PO_4)_2$, into more soluble calcium phosphate salts that plants can utilize. Almost thirty per cent of all the sulfuric acid made is used for this purpose alone. (2) The *petroleum refiners* get rid of the harmful, dark, gummy substances in crude oil with the aid of sulfuric acid which removes them in the form of a heavy mud. (3) The *chemical industry* finds hundreds of uses for sulfuric acid. One important use referred to earlier is in *making other acids.* Not only is it the cheapest acid, but its boiling point is the highest. Thus in heating the salt of the desired acid with H_2SO_4, there is no danger of the latter distilling over instead of the acid you want:

$$Ca_3(PO_4)_2+3H_2SO_4\rightarrow 3CaSO_4+2H_3PO_4\uparrow$$

When coke is made by heating soft coal, ammonia is driven off in the coal gas produced. Sulfuric acid recovers this valuable gas in the form of ammonium sulfate, another important fertilizer:

$$2NH_3+H_2SO_4\rightarrow (NH_4)_2SO_4$$

(4) The *steel industry* uses more than ten per cent of all the sulfuric acid produced to pickle steel. The rust on the surface must be cleaned off before coatings of tin, zinc or enamel will adhere properly to the steel:

$$Fe_2O_3+3H_2SO_4\rightarrow Fe_2(SO_4)_3+3H_2O$$

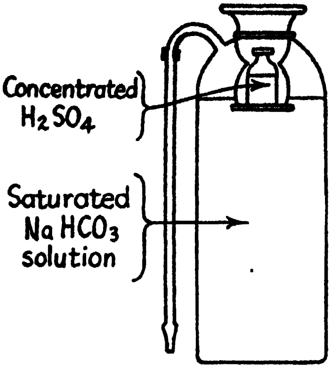
(5) The *storage battery manufacturer* must provide sulfuric acid as the electrolyte. It enables the lead and lead peroxide plates to produce the electric current needed in automobiles.

(6) The *explosives manufacturer* uses sulfuric acid as a dehydrating agent. Without its aid, nitric acid is unable to convert, for example, glycerine into nitroglycerine, or toluene into T.N.T. (trinitrotoluol):

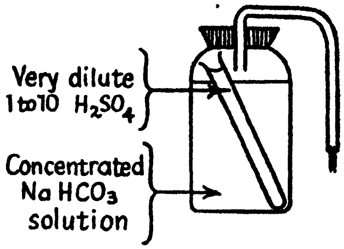
$$C_3H_5(OH)_3+3HNO_3\rightarrow C_3H_5(NO_3)_3+3HOH$$
$$CH_3\cdot C_6H_5+3HNO_3\rightarrow CH_3\cdot C_6H_2(NO_3)_3+3HOH$$

Water is one of the products of the reaction. It tends to make the reaction reverse itself, and therefore must be removed. Heating the mixture to boil the water off would be disastrous. Cold concentrated sulfuric acid effectively captures the interfering water. For this reason, nitration is accomplished by using a mixture of concentrated nitric and sulfuric acids.

The Chemical Fire Extinguisher. This consumes relatively little sulfuric acid, but the extinguisher itself is so valuable a safety device that we all should be familiar with it. It consists of a copper tank almost filled with concentrated sodium bicarbonate solution, as shown in the diagram. Supported near the top is a bottle two thirds filled with concentrated



Acid-Soda Fire Extinguisher



Working Model

sulfuric acid. The bottle stopper fits very loosely. To use this extinguisher, invert the tank. The stopper drops out, allowing the sulfuric acid to pour into the bicarbonate of soda solution. The reaction that follows releases a large volume of carbon dioxide gas which rises to the top and forces the solution out through the nozzle:

$$2NaHCO_3 + H_2SO_4 \rightarrow Na_2SO_4 + 2H_2O + 2CO_2\uparrow$$

The acid-soda fire extinguisher must be inspected at definite intervals and refilled if necessary even though it has not been used. As you will recall, sulfuric acid absorbs water from the air. In this device the stopper fits loosely; thus, after a time enough water may be absorbed to make the acid overflow, so that when you really need it, you get no pressure behind your stream. This extinguisher is not good for oil fires. It is not to be used on fires caused by electrical short circuits either, for you remember that acids and salts are strong electrolytes which conduct electricity well. The second diagram suggests how to construct a working model of the chemical fire extinguisher.

NITROGEN AND ITS DERIVATIVES

Occurrence of Nitrogen. Like oxygen, hydrogen and sulfur, nitrogen, too, is one of the elements making up protoplasm. The nitrate deposits in Chile, already mentioned, and those in South Africa; the ammonia that comes off when soft coal is converted into coke; plus the relatively small amount of various nitrogen compounds in fertile soil, account for most of the fixed or combined nitrogen available to man.

We live at the bottom of an ocean of air that extends upward for hundreds of miles. It is the weight of this air above us that barometers measure. At sea level the pressure exerted by the air is approximately

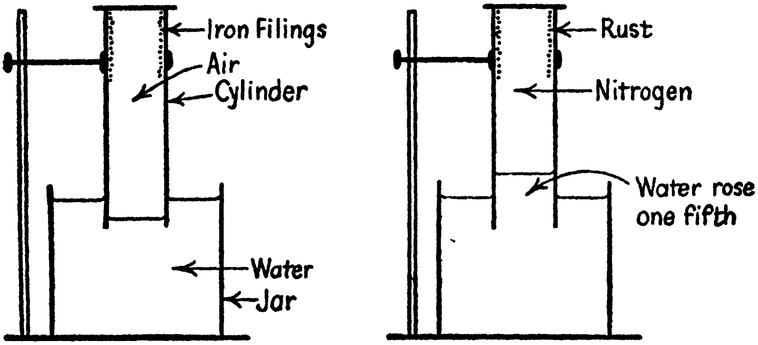
15 pounds on every square inch. Over three quarters of the weight of this tremendous reservoir of air is due to free, elementary nitrogen.

Composition of the Atmosphere. Though air is a mixture, its composition does not vary much. Green plants, by the process of photosynthesis, keep removing carbon dioxide from the air and returning oxygen to it. All living things, plants and animals alike, keep taking oxygen from the atmosphere and returning carbon dioxide to it during the respiration process. Fermentation, decay and the burning of carbon-containing materials also keep sending carbon dioxide into the air. Nitrogen, too, is being constantly removed from the air and being returned to it again by means of the "nitrogen cycle," which will be discussed later. These conflicting processes just about balance each other; and the winds help in the mixing.

Very careful analyses of *dry air* samples taken from many parts of the world show the *composition* to be roughly as follows:

	By Volume	*By Weight*
Nitrogen	approx. 78%	approx. 76%
Oxygen	" 21%	" 23%
Argon and other rare gases	" 1%	" 1%
Carbon dioxide	" 0.04%	" 0.05%
Water vapor	varies widely	
Dust	" "	

The following simple experiment enables us to make a rough analysis of the air. Wet the inside of a glass cylinder so that iron filings will adhere to the glass. Invert the cylinder and place it, as shown in the diagram below, so that its mouth is below the water level in a jar of water. Let it stand over night. The next day it will be found that the



First Day After 24 hours

Experiment Showing that Nitrogen Makes up About 80% of the Air water has risen about one fifth of the length of the cylinder. The iron filings have rusted. In doing so they have combined with the oxygen gas that made up one fifth of the trapped air to form rust, Fe_2O_3, which takes up hardly more space than the original filings:

$$4Fe+3O_2 \rightarrow 2Fe_2O_3$$

Originally the pressure inside the cylinder was the same as the atmospheric pressure outside. But as the oxygen inside the cylinder was being consumed, a partial vacuum formed. The air pressure outside, therefore, forced water up into the cylinder until inside and outside pressures were equal.

Properties of Nitrogen. Our familiarity with air tells us that nitrogen is a colorless, odorless, tasteless gas, only slightly soluble in water. It is slightly less dense than air. Upon testing the gas remaining in the cylinder with the aid of a burning splint, it will be discovered that nitrogen *neither burns nor supports combustion.* It is a very *inactive* element. Only very active metals when strongly heated can be made to combine with it.

The Nitrogen Cycle. In two distinct ways, nature *fixes* nitrogen; that is, converts free, elementary nitrogen into useful compounds. Fields in which leguminous plants (clover, beans, peas, alfalfa) have been growing are found to be rich in nitrogen compounds. The roots of such plants are the ideal environment under which a special type of bacteria, called nitrogen-fixing bacteria, can thrive. These bacteria fix the nitrogen of the air, converting it into nitrogen compounds that the plants use for growth. More than those plants can consume is produced by the bacteria. When living things die, a process of decomposition sets in, which results eventually in the restoration of free nitrogen back into the air. It is something of a shock to realize that the breath of air you have just inhaled contains nitrogen that made up part of the body of some animal that once, years ago, roamed the earth!

A flash of lightning contains a great deal of energy. Nitrogen, inactive as it is, when hit by a bolt of lightning, does combine with oxygen:

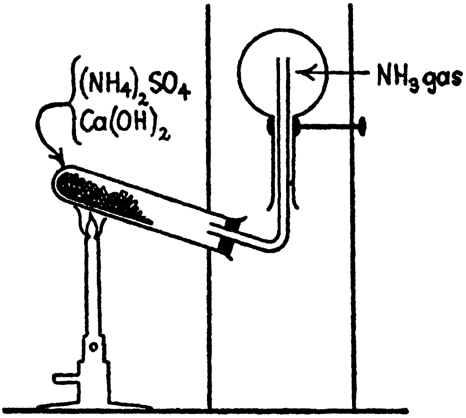
$$N_2+O_2 \rightarrow 2NO$$

The nitric oxide thus formed combines with additional oxygen and with water to form nitric and nitrous acids. In this way too, nitrogen from the air is fixed and supplied to plants by a natural process.

Man has also discovered ways of fixing nitrogen. How he does this we discover in the sections that follow.

AMMONIA

Laboratory Preparation of Ammonia. Set up the apparatus as shown in the diagram. Use any ammonium salt and any soluble base, for what you need are the ammonium ion $(NH_4)^+$ and the hydroxide ion $(OH)^-$. The materials shown here are the cheapest ammonium salt and the cheapest



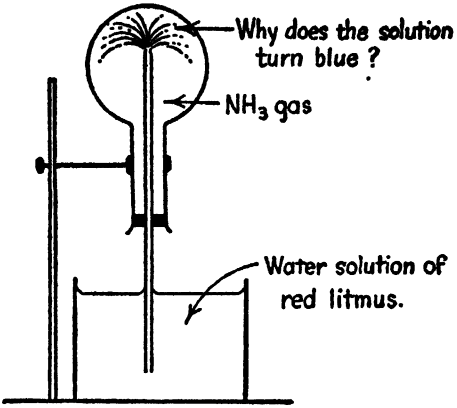
Preparation and Collection of Ammonia

base. Mix the ammonium sulfate and slaked lime on a piece of paper, and slide the mixture into the test tube. Heat the test tube and collect the ammonia liberated, with the delivery tube pointing up as shown. The reaction is a double replacement, followed by the decomposition of an unstable product:

$$\begin{array}{r} (NH_4)_2SO_4 + Ca(OH)_2 \rightarrow CaSO_4 + 2NH_4OH \\ \downarrow\quad \\ 2NH_3\uparrow + 2H_2O \end{array}$$

Properties of Ammonia. You know when the collecting bottle is filled with the *colorless* gas because you can detect its *characteristically pungent odor.* Be careful in smelling ammonia, for even though it is used in smelling salts to revive a person who has fainted, a good whiff of the concentrated gas can knock a person out. The method of collection indicates that ammonia is less dense than air, and soluble in water.

Ammonia Fountain. Ammonia's great solubility in water can be illustrated in a spectacular way as follows: Fill a flask with the gas, then fit into the flask a one-hole stopper carrying a long straight tube drawn to a small opening at the end that extends into the flask. Arrange the flask so that the lower end of the tube dips into a jar containing red litmus solution. As you watch, the water in the jar slowly creeps up the tube until it reaches the top, when it sprays out like a fountain. At the same time


Why does the solution turn blue ?
NH$_3$ gas
Water solution of red litmus.

Ammonia Fountain

the color changes from red to blue. Now we know that only $(OH)^-$ ions make litmus blue; let us see where they came from. Ammonia reacts with water as follows:

$$NH_3 + HOH \rightarrow NH_4OH$$

You will recall that our test on electrolytes and non-electrolytes showed NH_4OH to be a *weak base.*

Ammonia gas *reacts with acids to form* ammonium *salts.* To imitate smoke on the stage, use is made of this fact. By blowing one stream of air over cotton soaked with ammonia water, and another over cotton soaked with concentrated hydrochloric acid, so that the two streams meet, a dense white cloud of ammonium chloride can be made:

$$\underset{\text{gas}}{NH_3} + \underset{\text{gas}}{HCl} \rightarrow \underset{\text{white solid}}{NH_4Cl}$$

Commercial Preparation of Ammonia. Mention has already been made of the fact that ammonia is one of the by-products formed during the manufacture of coke. The most important method, however, for producing ammonia is the *Haber process.* A German chemist, Fritz Haber, was the first to work out the conditions necessary to *synthesize* ammonia *directly from its elements.* The equation, on paper, is simple enough:

$$N_2 + 3H_2 \leftrightarrows 2NH_3$$

However, many problems had to be solved before ammonia could be produced on a commercial scale. As we know from our discussion of oxygen, nitrogen is readily prepared from liquefied air. Hydrogen, you will also remember, can be made by the electrolysis of water, or more economically from water gas. So the raw materials are available in abundant amounts. But the reaction takes a long time unless a catalyst is used. Research, however, discloses that a mixture of finely divided iron and molybdenum is effective. Like most reactions, this one is reversible.

Controlling Reversible Reactions. Even with the aid of a catalyst, nitrogen and hydrogen react extremely slowly at ordinary temperatures. The chemist knows that the speed of a chemical reaction increases very rapidly as the temperature is raised; usually the reaction velocity doubles or triples for each twenty degree rise. From this it seems obvious that heating the mixture very hot would accomplish just what is desired. But now an extremely important law of nature must be considered.

Reversible reactions come to equilibrium, for practical purposes they may be said to stop, when the speed in one direction is equal to the speed in the opposite direction. Among the factors that influence the speed are the concentrations of the substances, and their pressures. Heat, too, must be considered just as if it were one of the materials involved, although we know that heat is energy, not matter. Each factor mentioned should be regarded as exerting its own stress on the system. *The system keeps adjusting itself to maintain equilibrium by going in the direction that relieves the strain.* To be more accurate, then, we should write the equation this way, since heat is one of the products of the reaction:

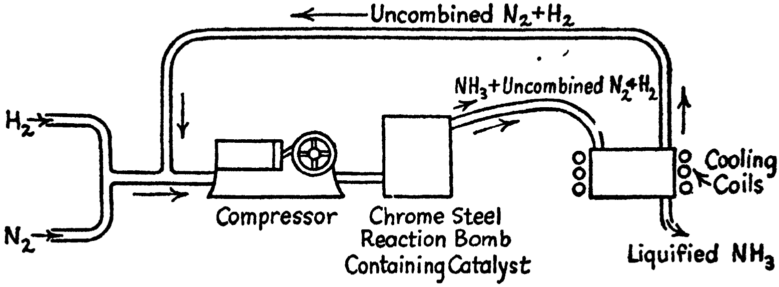
$$N_2+3H_2 \rightleftarrows 2NH_3+\text{Heat}$$

Additional heat (beyond enough to keep the substances sufficiently hot to react) therefore makes the reaction go towards the left; remove heat from the reaction chamber and you allow more nitrogen and hydrogen to react to synthesize more ammonia, i.e., the reaction tends to go to completion.

The *coefficients* in equations dealing with gases indicate *the relative volumes of the reacting gases.* In this equation we see that 1 cu. ft. of nitrogen combines with 3 cu. ft. of hydrogen to make 2 cu. ft. of ammonia. In other words, a volume shrinkage occurs (from 4 volumes to 2) when the reaction goes in the desired direction. Therefore, the higher the pressure, the faster the reaction goes towards the right; that is, more ammonia is made per hour. At this point a serious problem is encountered. When hydrogen reaches a pressure of 3000 pounds per square inch, it goes through ordinary steel the way water runs through filter paper. Until the metallurgists were able to invent a non-porous steel, development of the Haber process had to wait.

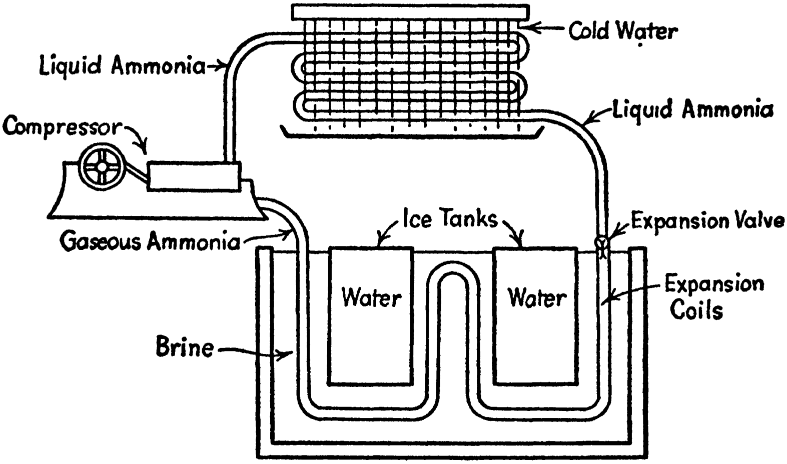
By continuously forcing in more nitrogen and hydrogen, and at the same time, removing the ammonia as it is formed, the speed of the reaction in the direction desired is further increased.

The diagram below shows the layout of a synthetic ammonia plant.



Flow Sheet of the Haber Synthetic Ammonia Process

Uses of Ammonia. Considerable amounts of ammonia are oxidized in the presence of a catalyst *into nitric acid*. In the Solvay process, you recall, ammonia is used in the *preparation of sodium bicarbonate*. Several important *ammonium salts are prepared* from it. $(NH_4)_2SO_4$ and calcium ammonium phosphate $Ca(NH_4)_4(PO_4)_2$ are important fertilizers. Sal ammoniac, NH_4Cl, is the electrolyte in dry cells. It is also used as a flux in soldering, because heat decomposes it into NH_3, which reduces the oxides on the metal surfaces, and HCl, which dissolves the oxides present. NH_4NO_3, mixed with TNT, or with dynamite, makes them still more powerful explosives. *Household ammonia*, really NH_4OH, is a weak base useful therefore as a cleansing agent.



Ice-Making Using Liquid NH_3 as the Refrigerant

As a *refrigerant* and to *manufacture ice,* liquefied ammonia is widely used. The principle involved is easy to understand. Evaporation is a cooling process; so is expansion. A nurse often brings down the temperature of a feverish patient with an alcohol rub. The evaporating alcohol takes heat from the surfaces it touches. Open your mouth wide and exhale against the back of your hand. You notice how warm your breath is. Now blow forcibly against the back of your hand. This time the exhaled breath feels cool. The expanding air takes heat from your skin. In refrigerators for ice making, and all others too, a liquefied gas is allowed to expand from the liquid state to the gaseous state. In so doing, it extracts heat from its surrroundings, thus chilling them. The refrigerant, NH_3 in this case, changes from liquid to gas. A compressor next changes the gas back to a liquid; and the cycle continues, as shown in the diagram.

NITRIC ACID

Preparation of Nitric Acid. The *arc process* produces nitric acid from air and water. It duplicates the natural process previously mentioned whereby the nitrogen of the air is energized by lightning, and combines with oxygen. Air is drawn between electrodes across which a hot electric arc flames. Two reactions occur, resulting in the production of nitrogen peroxide, NO_2. This gas reacts with water to form nitric and nitrous acids. The latter can readily be oxidized to nitric acid. The arc process is losing out in competition with other processes because it consumes so much electrical energy. If the cost of generating electricity can be greatly reduced, the arc process will become more important. The following equations summarize the *arc process:*

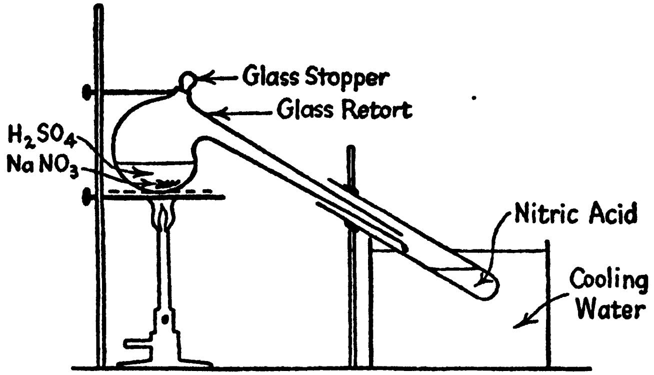
$$(1)\quad N_2+O_2 \rightarrow 2NO$$
$$(2)\quad 2NO+O_2 \rightarrow 2NO_2$$
$$(3)\quad 2NO_2+H_2O \rightarrow HNO_3+HNO_2$$

In discussing the uses of ammonia, the process of *oxidizing* NH_3 to make nitric acid was mentioned. At fairly high temperatures, when a mixture of air and ammonia is drawn over a platinum catalyst, the following reaction occurs:

$$4NH_3+5O_2 \rightarrow 4NO+6H_2O$$

The rest of the process is the same as the second and third steps of the arc process.

Both in the laboratory and on a commercial scale, nitric acid is produced by the action of concentrated sulfuric acid on the cheapest nitrate, sodium nitrate. This illustrates again the general method for making acids: use sulfuric acid plus a salt of the desired acid. Set up the apparatus as shown in the diagram. Carefully slide sodium nitrate through the mouth



Laboratory Preparation and Collection of Nitric Acid

of the retort, being careful not to get any of the salt into the delivery tube. With the aid of a long stemmed funnel add concentrated sulfuric acid. Replace the glass stopper. Heat gently. Soon you observe droplets condensing, and dripping into the receiving test tube which is kept cool by the jar of water in which it is partly immersed:

$$NaNO_3 + H_2SO_4 \rightarrow NaHSO_4 + HNO_3 \uparrow$$

Properties of Nitric Acid. Hydrogen nitrate is a colorless liquid with a sharp odor, denser than water, and soluble in it. The boiling point of nitric acid is 25 degrees below that of water, which boils at 212° F; this explains why nitric acid is so conveniently collected by distillation from the retort in which it is produced. Care must be taken not to overheat the acid as you prepare it, for *it is unstable.* Sunlight, too, decomposes it into water and N_2O_5, its anhydride. N_2O_5 in turn breaks up into O_2 and NO_2 which is a brown gas. This accounts for the brownish-yellow tinge of nitric acid that has not been carefully distilled.

In water solution, hydrogen nitrate forms a *strong acid.* It has the properties of a typical acid (turning litmus from blue to red, tasting sour, and neutralizing bases), but it *does not liberate hydrogen* when it reacts with metals. Nitric acid is a *powerful oxidizing agent,* so that any hydrogen that might be formed would be oxidized into water. Nitric acid attacks all metals except gold and platinum. For example, it changes relatively inactive copper and silver both into soluble nitrates. The equations below illustrate that instead of hydrogen we get nitric oxide or nitrogen peroxide depending on whether we use dilute or concentrated nitric acid:

$$3Cu + \underset{\text{dilute}}{8HNO_3} \rightarrow 3Cu(No_3)_2 + 4H_2O + 2NO \uparrow$$

$$Ag + \underset{\text{concentrated}}{2HNO_3} \rightarrow AgNO_3 + H_2O + NO_2 \uparrow$$

A glowing splint bursts into flame when held in the vapor above boiling nitric acid. We are compelled to use an all glass retort for the preparation of nitric acid because cork and rubber are quickly destroyed by it. Even cold dilute nitric acid attacks the skin immediately on touching it.

NOTE. *The first thing to do when nitric acid, or any other corrosive substance, gets on the skin or clothing, is to* flood *the area with water. Then, for acids, apply a thick paste of sodium bicarbonate. For strong bases, after the water, apply vinegar.*

Aqua regia is a mixture of concentrated nitric and hydrochloric acids. It will attack even gold and platinum, changing them to soluble compounds.

Uses of Nitric Acid. In addition to the amounts used to make *nitrate fertilizers,* over seventy-five per cent of the remaining 100,000 tons of our yearly production of nitric acid goes into the *manufacture of explosives.* Some explosives have already been discussed in connection with the dehydrating action of concentrated sulfuric acid. Guncotton is prepared in the same way as nitroglycerine:

$$\underset{\text{cotton}}{C_6H_7O_2(OH)_3}+3HNO_3 \rightarrow \underset{\text{guncotton}}{C_6H_7O_2(NO_3)_3}+3HOH$$

When slightly less concentrated nitric and sulfuric acids are used in the mixture for the reaction, or if a shorter time is allowed, then the nitration is not so complete. The product is, therefore, not guncotton but pyroxylin, which becomes *celluloid* when heated with camphor and alcohol in a closed vessel. Some pyroxylin is used to produce quick-drying *lacquers.* Nitric acid is also used in the preparation of dyes and other chemicals. A small amount is used to etch copper.

CALCIUM COMPOUNDS

Calcium Carbonate. The commonest compound of calcium is its carbonate. Limestone, marble, chalk, shells, coral, pearls and calcite are all merely different varieties of $CaCO_3$. Impurities account for many of the physical differences in naturally occurring calcium carbonate. Examination of the physical structure of the sample often discloses how nature formed it. Chalk, for example, is made up of the shells of tiny marine organisms.

Calcium carbonate *reacts with acids to liberate carbon dioxide.* In this way, using the same apparatus that was previously described for preparing hydrogen, we may prepare CO_2 in the laboratory:

$$CaCO_3+2HCl \rightarrow CaCl_2+H_2O+CO_2\uparrow$$

Water containing carbon dioxide dissolves calcium carbonate. It is in this way that limestone caves form. Some of the most exciting adventures of Tom Sawyer and Huckelberry Finn, you may recall, occurred in just such a cave. A trickle of water containing some CO_2 absorbed from the air may find its way through a fissure in a solid limestone formation. Gradually, during thousands of years, the trickle grows to the size of a river, as it dissolves away great masses of the limestone, leaving enormous caverns whose strange beauty leaves the spectator spellbound:

$$H_2O+CO_2+CaCO_3 \rightarrow Ca(HCO_3)_2$$

The stream, now carrying calcium bicarbonate in solution, may flow over another limestone cave in whose roof there may be some cracks. A few drops may get through to the inside and evaporate there, depositing their calcium carbonate. The drops that adhere to the ceiling leave their deposit sticking there. After years, a large icicle-like formation called a *stalactite* has developed. The drops that fall to the floor of the cavern, by their evaporation build up calcium carbonate columns, called *stalagmites.* Thus limestone caves may be filled up again with limestone carried in solution from a great distance:

$$Ca(HCO_3)_2 \rightarrow CaCO_3\downarrow+H_2O\uparrow+CO_2\uparrow$$

Temporary Hard Water. Water carrying calcium (or magnesium, or iron) bicarbonate in solution is called *temporary hard water.* It does not form a lather with soap. Instead, soap is wasted in reacting with the calcium ions in such water to form an insoluble, sticky calcium soap:

$$Ca(HCO_3)_2+Na\text{-}Soap \rightarrow Ca\text{-}Soap\downarrow+2HaHCO_3$$

When no more calcium ions are left in solution the water is soft. Temporary hard water can be softened by boiling, though this is an expensive method:

$$Ca(HCO_3)_2 \rightarrow CaCO_3\downarrow+H_2O+CO_2\uparrow$$

Unless hard water is softened before it is fed into boilers, the precipitated calcium carbonate causes a great deal of trouble. As *boiler scale,* it coats the inside of tanks, or lines the tubes of water tube boilers, thus reducing their capacity. It is a poor heat conductor and for this reason it causes wastage of fuel, and may even bring about boiler explosions. The formula $Ca(HCO_3)_2$ may also be written $CaCO_3 \cdot H_2CO_3$, suggesting that temporary hard water can most economically be softened by neutralizing the acid. In industry, therefore, such water is softened by the addition of calcium hydroxide:

$$CaCO_3 \cdot H_2CO_3+Ca(OH)_2 \rightarrow 2CaCO_3\downarrow+2H_2O$$

Both limestone and marble are used as building material and for statuary. They also are used in the Solvay Process and in the manufacture of lime, glass, cement and pig iron. These uses will be discussed in detail later.

Calcium Oxide. As shown in the diagram, crushed limestone or marble is converted into CaO, also called lime or quicklime: thus

$$CaCO_3 \rightarrow CaO + CO_2 \uparrow$$

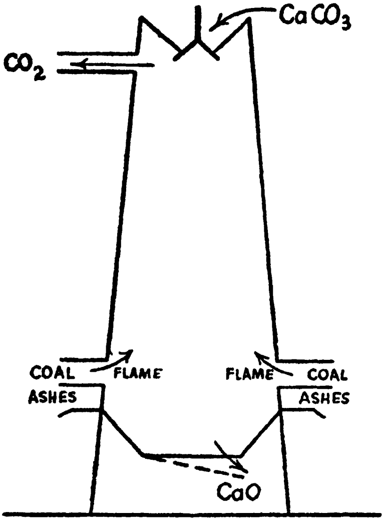
Calcium oxide is a white powdery solid that does not keep unless stored in air tight containers. When heated in an electric furnace with coke it is converted into calcium carbide (CaC_2), from which acetylene gas (C_2H_2) is prepared, as shown by the equations:

$$CaO + 3C \rightarrow CaC_2 + CO \uparrow$$

$$CaC_2 + 2HOH \rightarrow Ca(OH)_2 + C_2H_2 \uparrow$$

When quicklime is mixed with water to make *slaked lime,* it reacts so energetically that the heat evolved brings the water to a boil:

$$CaO + HOH \rightarrow Ca(OH)_2$$



Lime Kiln

The equation indicates that CaO is a *basic anhydride;* that is, it is a compound that combines with water to form a base.

Calcium Hydroxide. Hydrated or slaked lime is a white solid, only slightly soluble in water. The *solution* of $Ca(OH)_2$ is called *limewater.* It finds some use in medicine, and is used to *test for carbon dioxide gas.* If you blow through limewater it turns milky because of the precipitated chalk that forms:

$$CO_2 + Ca(OH)_2 \rightarrow CaCO_3 \downarrow + H_2O$$

A suspension of calcium hydroxide used as an inexpensive white paint is called *whitewash.* The *lime-sulfur spray* used by farmers for spraying fruit trees is made by boiling sulfur with water and slaked lime. The *tanner* uses hydrated lime to dehair hides. Its use in softening temporary hard water has already been mentioned. Slaked lime is used to *"sweeten" sour soils,* because as you know, it is a cheap base. For the same reason it is used to regenerate ammonia from the ammonium chloride by-product in the Solvay Process. *Bleaching powder* is also prepared from it.

Large amounts of calcium hydroxide are still used to make *lime mortar* even though the more expensive cement mortar is superior to it. Lime mortar is a mixture of slaked lime, water and sand, used to hold together the bricks and stones of buildings. It takes years for lime mortar to reach its maximum strength, for it sets or hardens as the result of two slow chemical reactions. It reacts with the CO_2 of the air to form $CaCO_3$; and with the sand, SiO_2, mixed with it, to form $CaSiO_3$, calcium silicate. Both the water originally added to the mixture and the water resulting from the two reactions must escape:

$$Ca(OH)_2 + CO_2 \rightarrow CaCO_3 + H_2O\uparrow$$
$$Ca(OH)_2 + SiO_2 \rightarrow CaSiO_3 + H_2O\uparrow$$

Calcium Sulfate. In Kansas, New York, Ohio, Michigan and Canada are found large deposits of a mineral called *gypsum*. It consists of calcium sulfate combined with water of crystallization, $CaSO_4 \cdot 2H_2O$. When gypsum is heated to a temperature of 250° F it loses three quarters of its water content, and crumbles into a white powder, known as plaster of Paris:

$$2CaSO_4 \cdot 2H_2O \rightarrow (CaSO_4) \cdot H_2O + 3H_2O\uparrow$$

When plaster of Paris is mixed with water, it recombines with it to form gypsum again. It expands slightly when it sets, a requisite for any material that is to find use as a *mold* or a casting:

$$(CaSO_4)_2 \cdot H_2O + 3H_2O \rightarrow 2CaSO_4 \cdot 2H_2O$$

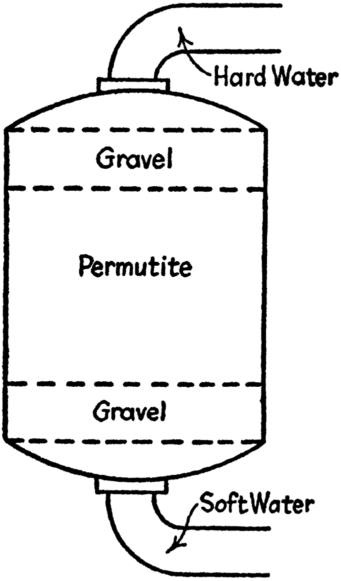
The doctor makes use of plaster of Paris when he sets broken bones. Dentists often use it to make impressions of the gums and teeth, to help in constructing bridgework to hold false teeth. Sculptors use plaster of Paris to make molds of their clay statues and then to make plaster casts in those molds. The building industries consume enormous quantities of plaster of Paris to make hollow gypsum blocks and wall-board, for stucco work, and for plastering (often mixed with lime).

Permanent Hard Water. Calcium sulfate is only slightly soluble in water, but even a small amount makes the water hard. Such water is called permanent hard water because boiling does not soften it. To soften hard water, the ions (usually iron, magnesium and calcium) causing the trouble must be removed. Washing soda, Na_2CO_3, is among the best of the water softeners. It rids the water of the ions that cause the hardness by precipitating them as insoluble carbonates:

$$CaSO_4 + Na_2CO_3 \rightarrow CaCO_3\downarrow + Na_2SO_4$$

Another efficient method for softening hard water is the *permutite* process. Permutite is a complex, crystalline, insoluble substance containing sodium combined with silicon and aluminum oxides. It has the ability to exchange its sodium for the ions that cause hardness without altering materially in structure or solubility. The diagram illustrates how hard water can be passed through a tank containing sand-sized crystals of permutite, and come out softened. Thus we have:

Hard(Ca^{++}) Water+2NaPermutite→Ca(Permutite)$_2$+Soft ($2Na^+$) Water



Softening Hard Water by the Permutite Process

The Na^+ ions in the softened water are harmless. After twelve hours' use, the permutite can be restored to its original efficiency by allowing a concentrated sodium chloride solution to soak in over night:

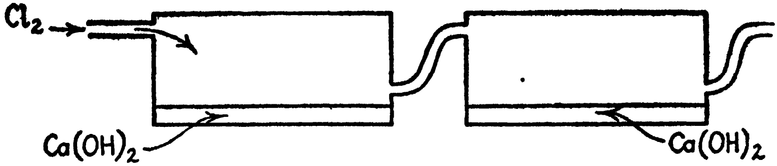
$$2NaCl + Ca(Permutite)_2 \rightarrow 2NaPermutite + CaCl_2$$

The calcium chloride is allowed to run into the sewer.

Bleaching Powder. When chlorine gas is passed over freshly prepared slaked lime, as shown in the diagram, bleaching powder forms: thus

$$Cl_2 + Ca(OH)_2 \rightarrow CaOCl_2 + H_2O$$

Chloride of lime, as it is frequently called, is not stable. It keeps releasing chlorine slowly. For this reason it is an excellent disinfectant. Bleaching powder should be regarded as chlorine in the form that can be most conveniently and safely handled by the layman. The theory of its behavior, therefore, is the same as that already given for chlorine itself. Decontami-



Manufacture of Bleaching Powder

nation squads of the army and civilian defense use bleaching powder to destroy *vesicant,* or blister "gases." Large quantities of chloride of lime are constantly used for purifying drinking water, to bleach cotton goods, and as a disinfectant.

TYPES OF CHEMICAL REACTIONS

Summary. A good way to review and clarify what you have already studied is to arrange all the equations encountered according to their type. Prepare four sheets of paper, heading each with the name of one type of reaction. Then go through the chapter, placing each equation you find on the proper sheet. Keep your review sheets complete by adding the equations to be given in the remaining sections as you come to them.

Practically *all chemical reactions* fall into one of *four types.* Occasionally, however, two or more reactions are represented in one single equation as if the reactions all take place at one time, instead of in separate steps, as they actually occur. When you encounter such a reaction it is necessary to use your knowledge and ingenuity so as to classify it properly. For example, the equation for the electrolysis of brine, as explained in the text, really represents two reactions: first, a simple decomposition; second, a simple replacement.

THE FOUR TYPES OF CHEMICAL REACTIONS

1. Direct Combination. *Synthesis* or *direct combination* are the names given to the type of reaction in which two or more elements or compounds combine directly to form a new compound. For example:

$$2H_2+O_2 \rightarrow 2H_2O$$
$$4Fe+3O_2 \rightarrow 2Fe_2O_3$$
$$H_2+Cl_2 \rightarrow 2HCl$$
$$Zn+S \rightarrow ZnS$$
$$S+O_2 \rightarrow SO_2$$
$$2H_2SO_3+O_2 \rightarrow 2H_2SO_4$$
$$SO_3+H_2O \rightarrow H_2SO_4$$
$$CaO+HOH \rightarrow Ca(OH)_2$$

You will discover additional examples to include in this group.

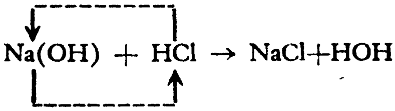
2. Simple Decomposition. The tearing apart of a compound by heat, by electrolysis, or by any other means, so that simpler compounds or elements are the products, is called *simple decomposition* or *analysis.* To those listed below you will be able to add other examples:

$$2H_2O \rightarrow 2H_2\uparrow + O_2\uparrow$$
$$2HgO \rightarrow 2Hg + O_2\uparrow$$
$$NH_4OH \rightarrow NH_3\uparrow + H_2O$$
$$NH_4Cl \rightarrow NH_3\uparrow + HCl\uparrow$$
$$CaCO_3 \rightarrow CaO + CO_2\uparrow$$
$$Ca(HCO_3)_2 \rightarrow CaCO_3 + H_2O + CO_2\uparrow$$
$$2CaSO_4 \cdot 2H_2O \rightarrow (CaSO_4)_2 \cdot H_2O + 3H_2O\uparrow$$

3. Single Replacement. Reactions in which a single element ousts another from a compound, and takes the place of the displaced element, are called *substitution, single* or *simple replacement* reactions.

$$Zn + 2HCl \rightarrow ZnCl_2 + H_2\uparrow$$
$$H_2 + CuO \rightarrow H_2O + Cu$$
$$Fe_2O_3 + 3C \rightarrow 2Fe + 3CO\uparrow$$
$$2Na + 2HOH \rightarrow 2NaOH + H_2\uparrow$$
$$Cu + 2AgNO_3 \rightarrow Cu(NO_3)_2 + 2Ag\downarrow$$
$$Fe + CuSO_4 \rightarrow FeSO_4 + Cu\downarrow$$

4. Double Replacement. When two compounds react by having their ions change partners, so to speak, we call the reaction a *double decomposition* or double replacement.



$$Na(OH) + HCl \rightarrow NaCl + HOH$$

$$2NaCl + H_2SO_4 \rightarrow Na_2SO_4 + 2HCl\uparrow$$
$$2HCl + FeS \rightarrow FeCl_2 + H_2S\uparrow$$
$$H_2S + ZnSO_4 \rightarrow ZnS\downarrow + H_2SO_4$$
$$Fe_2O_3 + 3H_2SO_4 \rightarrow Fe_2(SO_4)_3 + 3H_2O$$
$$C_3H_5(OH)_3 + 3HNO_3 \rightarrow C_3H_5(NO_3)_3 + 3HOH$$
$$CaC_2 + 2HOH \rightarrow Ca(OH)_2 + C_2H_2\uparrow$$

42. METALS: THEIR USES AND EXTRACTION

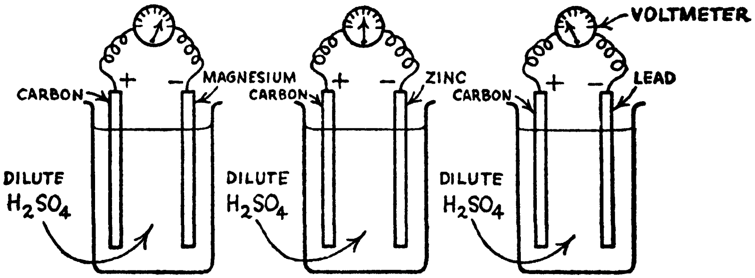
CHEMICAL ACTIVITIES OF METALS

Electrochemical Series of the Metals. A very useful list can be made by arranging the metals in the order of the ease with which they lose electrons. Potassium heads our list of some of the more common metals.

Replacement Series

Potassium
Sodium
Calcium
Magnesium
Aluminum
Zinc
Iron
Nickel
Tin
Lead
HYDROGEN
Copper
Mercury
Silver
Platinum
Gold

It is one of the very active metals which lose electrons on mere exposure to light. Potassium, therefore, finds use in the photoelectric cell, the well known "magic eye." In other words, the metals are arranged in *the order of their activities,* the most active metal first, and the least active last. The list is also called the *electromotive series* because the voltages shown on a voltmeter decrease in the same order when you use the different metals as the negative electrodes in a voltaic cell, where a graphite rod serves as the positive electrode. For example, a higher voltage is obtained between magnesium and carbon than between zinc and carbon. The voltmeter reading is still less if lead serves as the cathode.



Voltaic Cells

Replacement of Hydrogen in Water. Potassium, sodium, and calcium react with cold water to liberate hydrogen, and to form their respective hydroxides. Potassium reacts violently, calcium slowly:

$$Ca+2HOH \rightarrow Ca(OH)_2+H_2\uparrow$$

Magnesium, lower in the series, reacts with boiling water. Iron, still lower, must be red hot to react with steam to produce hydrogen, a process sometimes used as an industrial method of preparing the gas:

$$3Fe+4H_2O \rightarrow Fe_3O_4+4H_2\uparrow$$

The metals shown below hydrogen in the series do not replace hydrogen in water no matter how hot you heat them.

Replacement of Hydrogen in Acids. The inclusion of hydrogen in the replacement series might be justified by the fact that it lends electrons just as metals do when they react. But the real reason for including it is that it divides the metals into two classes. Those above hydrogen react with many acids to liberate hydrogen; those below hydrogen do not liberate hydrogen from acids:

$$Mg+2HCl \rightarrow MgCl_2+H_2\uparrow \text{ (Rapid reaction)}$$
$$Pb+H_2SO_4 \rightarrow PbSO_4+H_2\uparrow \text{ (Slow reaction)}$$
$$Cu+HCl \rightarrow \text{No reaction}$$

Nitric acid and hot concentrated sulfuric acid react with all the metals down to and including silver to produce the respective salts. These acids do not affect gold and platinum. Aqua regia (HNO_3+HCl) must be used to get them into solution.

The metals above hydrogen tarnish when exposed to air; those at the top rapidly, those further down more slowly. For this reason, these metals are never found free in nature; they must always be extracted from their ores.

The metals below hydrogen tarnish very slowly in air or not at all. These metals, too, occur in ores as compounds from which they must be extracted chemically. As you might expect from their position near the bottom of the series, they are also found in the native state, so that a mere mechanical process suffices to separate them from the impurities in their ores.

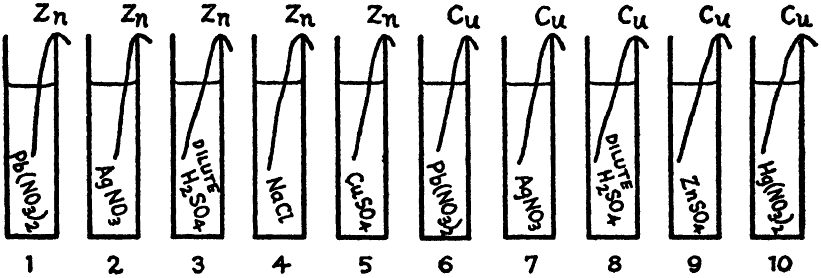
Combination with Non-Metals. Most of the metals burn, that is combine with oxygen to form oxides. The active metals burn vigorously giving off a great deal of heat and forming very stable oxides. It is difficult to burn mercury to make mercuric oxide. Mercuric oxide is easily decomposed. Silver will not oxidize in oxygen, ozone must be used. Gold and platinum are unaffected by oxygen no matter how hot they are.

The metals combine with other non-metals too. For example, they combine with chlorine to form chlorides, and with sulfur to form sulfides. As with oxygen, the reactions are vigorous, considerable heat is given off, and stable compounds are formed when the active metals react. As you go down in the series the heats of reaction decrease, and the stability of the resulting compounds goes down as well.

The metals low in the replacement series were known in ancient times because they were found native or could easily be obtained from their not very stable compounds. Only in modern times has man been able to tear the active metals free from the stable compounds in which they are found so tightly bound.

Replacement of Metals in Compounds by Free Metals. As you now realize, the electromotive series is based on experimental facts. Any metal in the series is more active than the metals below it, and less active than those above it. This means that any metal you select is a more powerful reducing agent than the metals that follow it; it releases electrons less readily than the metals that precede it.

We have the facts on the metals in the replacement series. Let us try to predict the results when we set up the conditions shown in the diagrams below. In jar #1 there is a solution of lead nitrate. Into the solution dips a strip of metallic zinc. Zinc is more active than lead; it should therefore replace the lead as indicated by the equation:



Experiment Showing the Replacement Power of Metals

$$(1)\ Zn + Pb(NO_3)_2 \rightarrow Zn(NO_3)_2 + Pb\downarrow$$

Let us check on our prediction. Crystals of metallic lead *are* adhering to what is left of the original zinc strip, just as predicted. The lead crystals drop off the zinc when you touch it. (This is *not electroplating* in the practical sense, because the zinc strip keeps getting thinner as part of it goes into solution. The deposit, therefore, has nothing to cling to.) In terms of the electron theory, the zinc has been *oxidized*, and the lead *reduced*. Zinc lost two electrons to the less active lead; metallic Zn° became the Zn^{++} ion, the Pb^{++} ion, by acquiring the two electrons from the zinc, became metallic Pb°:

$$Zn^\circ - 2\epsilon \rightarrow Zn^{++}$$
$$Pb^{++} + 2\epsilon \rightarrow Pb^\circ$$

You should now write your predictions for the other nine set ups. Then check against the actual results. Now refer to the equations and comments below, to make sure that you have not overlooked anything, and understand each result.

(2) $Zn + 2AgNO_3 \rightarrow Zn(NO_3)_2 + 2Ag\downarrow$
(3) $Zn + H_2SO_4 \rightarrow ZnSO_4 + H_2\uparrow$
(4) $Zn + NaCl \rightarrow$ no reaction (Why? See Replacement Series)

(5) $Zn+CuSO_4 \rightarrow ZnSO_4+Cu\downarrow$ (Original blue color gone)
(6) $Cu+Pb(NO_3)_2 \rightarrow$ no reaction
(7) $Cu+2Ag(NO_3) \rightarrow Cu(NO_3)_2+2Ag\downarrow$ (Solution now blue)
(8) $Cu+$dilute $H_2SO_4 \rightarrow$ no reaction
(9) $Cu+ZnSO_4 \rightarrow$ no reaction
(10) $Cu+Hg(NO_3)_2 \rightarrow Cu(NO_3)_2+Hg\downarrow$ (Solution now blue)

It is not necessary for the compound to be in solution in order to have its metal replaced. For example, the metal aluminum was first isolated by heating potassium with solid aluminum chloride; thus:

$$3K+AlCl_3 \rightarrow 3KCl+Al$$

The *thermite reaction,* to be discussed later, depends on the fact that aluminum replaces less active metals from their solid oxides:

$$Fe_2O_3+2Al \rightarrow Al_2O_3+2Fe$$

IRON AND STEEL

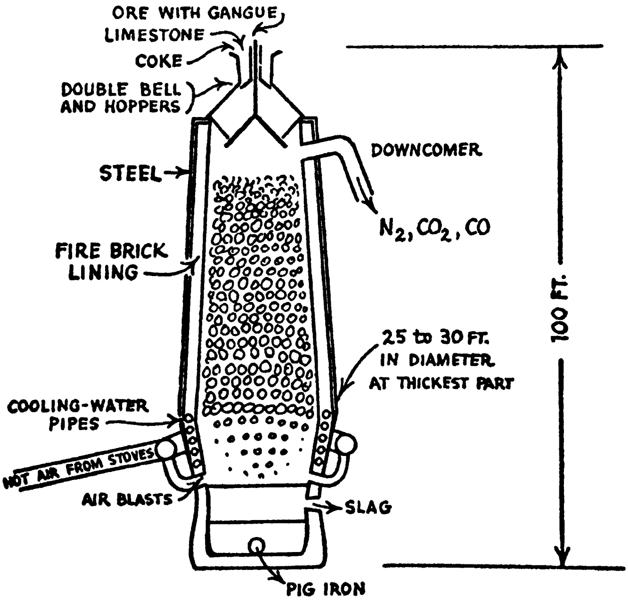
Iron Ores. Fortunately, iron is widely distributed in nature. The crust of the earth is approximately 5½% iron. The continents of Australia, Asia and Africa have potential iron ore reserves reckoned in hundreds of millions of tons each. In the State of Minas Geraes, Brazil, there is a mountain of iron containing between 12 and 15 billion tons of finest ore. Europe has over 40 billion tons, and America twice that amount. More than half our states have important iron ore deposits. Our greatest deposits are located near Lake Superior. The ore here is *hematite,* Fe_2O_3. Much of it is near the surface so that the inexpensive, quick method of open-pit mining can be used. The ore has relatively little *gangue* (impurities) mixed with it. Between 50% and 70% of the weight of the ore, as mined, is iron. Its location near the Great Lakes makes for low shipping costs to the iron and steel manufacturing cities. *Magnetite,* Fe_3O_4, is another important iron ore. In this country it is usually found mixed with too many impurities to make its smelting (i.e., treating to recover the metal) profitable. In Sweden the deposits of magnetite are the purest in the world. The famous Sheffield steels are manufactured from Swedish iron. Hitler's invasion of Norway was doubtless in part for the purpose of insuring his supply of Swedish iron ore. We need not discuss *siderite,* $FeCO_3$, because this iron ore is of no importance in this country.

Metallurgy of Iron. Metallurgy is the science of separating a metal from its ore. In the case of iron, this means first, getting rid of the gangue, and second, reducing the oxide ore to rid it of its oxygen and free the iron. Both these tasks are accomplished in the blast furnace. Coke is the fuel used to furnish the heat, and blasts of hot air support its combustion.

In some ore deposits the gangue consists of silicon dioxide (SiO_2) in the form of sand, gravel or quartz. To take care of it, a *flux,* in this case limestone, is added. Limestone on being heated decomposes into CO_2 and CaO, which is a basic anhydride. SiO_2 is an acid anhydride. The flux and gangue unite to form *slag* which melts in the furnace, trickles down to the molten iron on which it floats, and is drawn off at intervals. If the ore contains limestone as the gangue, then SiO_2 is added as the flux.

With the gangue thus disposed of, the only remaining problem is to reduce the Fe_2O_3. Coke, previously mentioned as the fuel, also acts as the reducing agent. Carbon monoxide, which forms in the furnace, plays an important part as well in reducing the ore.

The Blast Furnace. The diagram gives you an idea of the size and construction of a large blast furnace, capable of producing about 1000 tons of pig iron daily. A properly constructed blast furnace can be operated for twenty years without ever stopping for repairs. Only in the event of a breakdown or when business conditions are bad, do operations cease; otherwise blast furnaces run continuously, day and night, year in and year out.



Cross Section of Blast Furnace

Charging the Blast Furnace. The furnace is loaded through the top. Skip hoists dump the charge into the smaller upper hopper. A cone-shaped bell keeps the upper hopper closed until enough weight rests on the bell to exceed the upward pull exerted by a counterweight. Then the charge drops into the larger, lower hopper, which is also kept closed by a bell controlled by another, heavier counterweight. By this system, the hot gases are prevented from escaping into the air. A large pipe at the side of the top of the furnace, the downcomer, carries off the furnace gases.

To start a blast furnace, coke is first loaded in until the furnace is about half full. The coke is ignited and air is sent in. As the coke burns it settles, and the upper portion becomes wedged in, where the furnace begins to taper towards the crucible at the bottom. Next a layer of limestone is dropped in, followed by a layer of iron ore. From now on the loading is continual; coke, limestone and ore being added in that order for years until the furnace must be shut down. After the first fifteen hours of operation, enough slag has accumulated to be drawn off. In another fifteen hours or so the first iron is run out. From this time on, slag is withdrawn every three hours, and iron every six hours.

For *every ton of pig iron* withdrawn from the bottom of the blast furnace; *one half a ton of limestone, one ton of coke,* and *two tons* of ore must be dumped into the top; and *four to five tons of hot air* blown in through the blast pipes. A large furnace such as the one represented here, therefore requires *every day* 500 tons of limestone, 1000 tons of coke, 2000 tons of hematite ore, and 4000 to 5000 tons of air, to produce 1000 tons of pig iron.

The Hot Blast. If blasts of cold air were sent in to support the combustion of the coke, a large amount of heat in the furnace would be wasted in heating up this cold air. There is a tremendous amount of heat in the thousands of tons of furnace gas that are discharged through the downcomer. In fact, the latent and available heat is roughly equivalent to 50% of that of the fuel used. Part of this is utilized to preheat the fresh air needed. The hot furnace gases pass through a dust separator, and then through two pairs of stoves, or tall steel cylinders lined with special brick that absorbs the heat. While two stoves are being heated, the other two are giving up their heat to the fresh air being blown through them to become the hot blasts. The stoves are thus alternately heated and cooled. After passing through the stoves the furnace gas is piped to the power house where it is burned under boilers or in gas engines. Since it contains about 25% CO, it makes an excellent fuel gas.

Reactions in the Blast Furnace. The limestone added as flux decomposes into lime and carbon dioxide:

$$CaCO_3 \rightarrow CaO + CO_2$$

The lime formed combines with the gangue to form slag, calcium silicate:

$$CaO + SiO_2 \rightarrow CaSiO_3$$

Coke burns where the hot blasts of air enter the furnace:

$$C + O_2 \rightarrow CO_2$$

This CO_2 is reduced by the hot coke as the gas rises:

$$CO_2 + C \rightarrow 2CO$$

The iron oxide is reduced by the coke and by the carbon monoxide:

$$Fe_2O_3 + 3C \rightarrow 2Fe + 3CO$$
$$Fe_2O_3 + 3CO \rightarrow 2Fe + 3CO_2$$

Products of the Blast Furnace. So long as the slag was looked upon as a necessary evil, its disposal presented a problem; now it is regarded as a material to be put to use. Ford sends his slag to his cement plants, where, combined with the proper amount of shale, it produces a good cement. At another blast furnace, the molten slag is blown with air or steam to produce a fluffy fibrous material sold as rock wool to be used as a heat insulator in buildings.

The molten pig iron from the blast furnace may be run into molds to produce pigs of a size and shape convenient for handling, or it may be run into brick-lined ladles to be drawn away to the steel works. On the average, 25% of the pig iron produced is used in the form of castings without purification; the remaining 75% is purified and converted into some other form.

Composition and Properties of Pig Iron. Pig iron varies greatly in composition, as may be seen from a glance at the table below:

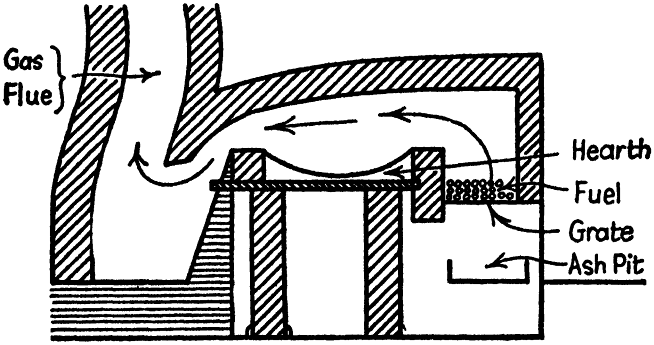
Fe	approx. 93%
C	3 to 4%
Si	1 to 3%
S	0.01 to 0.2%
P	0.04 to 1.5%
Mn	0.4 to 5%

It has some advantageous properties in spite of, and because of, its extremely high percentage of impurities. It melts readily, expands on solidifying and then shrinks relatively little as it continues to cool. These properties, considered also with its cheapness, explain its extensive use in making castings.

The difference between cast iron and steel is that cast iron contains less iron and more impurities than steel. The impurities in steel are usually the same ones as are found in cast iron. The carbon in both may be combined with the iron, but in cast iron it also appears as graphite. Graphite is crystalline and flaky, and splits easily into thin sheets like mica. It is the graphite that causes the peculiar behavior of cast iron as it solidifies and cools. It, too, explains the relatively low strength and lack of ductility of cast iron. Silicon tends to prevent the formation of oxides and blow-holes, and makes the iron more fluid; it also tends to keep the carbon uncombined as graphite. Sulfur makes the metal brittle at red heat, and causes it to solidify in the mold so rapidly that dirty iron with many blow-holes results. Phosphorus, besides reducing the melting point and making the molten iron more fluid, tends to keep the carbon combined but makes the casting very brittle when cold. Manganese counteracts the effect of sulfur but also increases the hardness of the cast iron.

Pig iron melts at around 2100°F. It is very brittle and cannot be welded, forged or tempered. At no temperature is it malleable. It is cast into stoves, pipes, radiators, and also into tubs and sinks that are later covered with porcelain. In the section on "Materials of Trade and Industry" cast iron is discussed further and in greater detail.

Wrought Iron. Though only about 3% of the pig iron produced ends up as wrought iron, the latter's usefulness and the principles of its manufacture justify our giving it some attention. To make wrought iron, the impurities that make cast iron brittle must be removed. The removal is accomplished by oxidation in a reverberatory furnace. The diagram shows such a furnace. The ashes drop through the grate and so do not contaminate the iron. The flames and smoke pass over the charge without touching it so as to avoid adding any carbon to the iron. The ceiling of the furnace reflects or *reverberates* the heat down to the charge.



Reverberatory Furnace

Reactions in the Reverberatory Furnace. The hearth has a basic lining consisting of a thick layer of iron oxide. Pigs are placed on the iron oxide layer and melt quickly. A man mixes the molten pig iron with additional iron oxide by means of a long iron rod. For this reason the furnace is also called a *puddling* furnace. As he continues the mixing, the iron oxide oxidizes out most of the impurities and the mass becomes more and more pasty until finally a lump of iron mixed with slag adheres to the mixing rod. The purer the iron the higher is its melting point. This explains the change in consistency of the melted iron from very fluid to pasty as it loses its impurities.

Note that the iron is purified by oxidizing out the impurities. An oxidizing atmosphere is employed, which contrasts with the reducing atmosphere maintained in the blast furnace. The carbon in the pig iron is oxidized to CO which burns with its characteristic blue flame on the surface of the charge:

$$Fe_2O_3 + 3C \rightarrow 2Fe + 3CO\uparrow$$
$$Fe_2O_3 + C \rightarrow 2FeO + CO\uparrow$$

The silicon is oxidized to SiO_2 which combines with iron oxide to form a slag:

$$FeO + SiO_2 \rightarrow FeSiO_3$$

Some of the phosphorus and sulfur, as oxides, escape with the CO, while the rest ends up in the slag as iron phosphate and sulfate. Practically all the manganese is oxidized to manganates that also form part of the slag.

The puddler works the now purified iron into 150-pound lumps containing slag. These lumps, called *blooms,* are placed under trip hammers and other squeezing devices which press out most of the admixed slag. They are then rolled into bars, cut, placed in stacks with alternate layers at right angles to the layers adjacent, welded together and then rolled again. In this way more slag is forced to ooze out and there results the purest commercial form of iron which still includes slag, but arranged now in the form of a network of fibers.

Properties and Uses of Wrought Iron. Containing about 1 to 2% slag running through it, wrought iron analyzes approximately as follows:

Fe		99.6 %
C	less than	0.12%
Si		0.05%
S		0.05%
P		0.13%
Mn	less than	0.05%

Because wrought iron is *malleable, tough* and *easily welded,* it is made into chains, anchors, horseshoes, nails and ornamental gates, andirons, etc. Its *ductility* allows it to be drawn into wires. Its *softness* makes it desirable for keys and threaded pipe. It is *magnetic* so long as current runs through insulated wires wound around it. When the current stops flowing, it loses its magnetism. In other words, unlike steel, wrought iron forms temporary magnets. This property makes it valuable as the cores of electromagnets for lifting iron, and for electrical motors, generators, bells, etc.

Wrought iron is defined as a malleable iron, which does not increase noticeably in hardness when suddenly cooled from a red heat. Except for the 1 to 2% of slag that remains mechanically mixed with it, wrought iron is almost identical with very low-carbon steels. For this reason, the latter, which are much cheaper, are being extensively used in place of wrought iron. Like wrought iron, steel too, is prepared from pig iron by oxidizing out the impurities. Steel contains at least 98% iron and from 1.5% to almost no carbon, plus varying amounts of other elements, present either unavoidably as impurities or added intentionally to give special properties to the steel. Various types of steel will be taken up in the section on "Materials of Trade and Industry."

COPPER

Occurrence of Copper. Copper, like iron, is found widely distributed in various parts of the world. The most important deposits are located in the United States, Chile, Africa, Canada and Spain. We produce about one and a half billion pounds of the metal annually, which is approximately sixty per cent of the world's production. Our leading copper-producing states are Arizona, Utah, Montana and Michigan. One of our greatest producing mines consists of enormous mountains of ore that are worked by open-pit methods. This is the famous copper mine at Bingham, Utah. However, copper ore is usually not at the surface. At one mine in northern Michigan, men must go down into the earth for over a mile to reach the ore. Occasionally copper is also found native. Lumps of free copper about six feet thick and weighing a quarter of a ton have been discovered. Because metallic copper is relatively inactive and for this reason can exist chemically uncombined for a long time, it was the first metal discovered and used by man. There is evidence to show that copper was used prior to 5000 B.C. The American Indians were making copper implements before the white man arrived. Most of the copper found, however, is in the form of compounds from which the metal must be chemically extracted. The chief copper ores are *chalcopyrite,* $CuFeS_2$; *chalcocite,* Cu_2S; *cuprite,* Cu_2O; and *malachite,* $CuCO_3 \cdot Cu(OH)_2$.

Ore Concentration. When copper ores are mined, considerable amounts of gangue, or worthless rock, come along with the ores. Unless the cost of separating the gangue from the desired ore is low, it does not pay to mine the ore. A remarkably efficient method called the *froth flotation process* is almost universally used for concentrating copper and other low grade ores. The ore, together with its associated gangue, is finely ground and mixed with water containing a little oil; air is then bubbled through the suspension. The ore is more easily wet by oil than by water, so that the metallic copper or copper sulfide in the mixture acquires a thin film of oil. The gangue is more easily wet by the water than it is by the oil. As a result of the agitation of the mixture by the air, a froth forms on the surface. Mixed with this froth there is usually about 90% of the copper in the original ore as mined. Practically all the gangue sinks to the bottom. The froth is easily separated from the water, and the now concentrated ore is sent to the smelter. The value of this method is evident when it is considered that over 80% of the original bulk of the ore, the worthless part, has been eliminated in this simple way.

Copper Smelting. When the ore consists of the oxide or carbonate, a *wet method* can be used. Dilute sulfuric acid is used, and a solution of copper sulfate results:

$$CuCO_3 \cdot Cu(OH)_2 + 2H_2SO_4 \rightarrow 2CuSO_4 + 3H_2O + CO_2 \uparrow$$

The copper is then precipitated by using scrap iron:

$$Fe + CuSO_4 \rightarrow FeSO_4 + Cu \downarrow$$

Most copper occurs in the form of its sulfide ores which frequently also contain iron sulfide, gold, silver, arsenic, nickel and many other elements. The problem of the copper metallurgist is to recover from the complex ore, the maximum yield of metal in as pure a state as possible.

In general, *two steps* are necessary in the *metallurgy of ores of metals* that are *less active than aluminum.* Carbonate and sulfide ores are first roasted. *Roasting* simply means heating the ore in the presence of air until all parts have been converted into oxides:

$$CuCO_3 \rightarrow CuO + CO_2 \uparrow$$
$$2ZnS + 3O_2 \rightarrow 2ZnO + 2SO_2 \uparrow$$

The second step is *reduction* by means of carbon. Exactly this is done in the blast furnace when pig iron is produced from hematite or magnetite:

$$2Cu_2O + C \rightarrow 4Cu + CO_2$$

In practice, for the purpose of accomplishing the same result most economically, modifications depending on the unique characteristics of each ore have been instituted. The froth flotation concentrates, for example, must first be *sintered,* that is, heated until they form large lumps. If this were not done the finely powdered concentrates would be blown out of the blast furnaces where they must later be treated. There is no *one* way of recovering copper from its ores. The metallurgist must examine each shipment of ore he receives, and base his procedure on the nature of the ore he is handling at the time. At one large eastern plant the sintered concentrates, ores from Arizona and from foreign countries, brass scrap, coke and flux (SiO_2) are fed into the *blast furnace.* (More usually, ores are given a *preliminary roasting* to rid them of some of their sulfur content.) In the blast furnace a number of reactions take place, the most important of which follow:

$$3Cu_2O + 3FeS \rightarrow 3Cu_2S + 3FeO$$
$$FeO + SiO_2 \rightarrow FeSiO_3$$

Matte, a mixture of molten copper sulfide and iron sulfide, settles to the bottom of the blast furnace, while above it floats the *slag* which consists mainly of iron silicate. The flue dust consists largely of zinc oxide which the paint manufacturers buy. The slag is sent to the dump. The matte, in addition to the iron impurity, also still contains the gold and silver originally in the ore plus, at times, arsenic, antimony, zinc, lead, tin and nickel in varying amounts. The fused matte is transferred to a *copper converter,* a device very much like the Bessemer converter used to make steel. Here a blast of air oxidizes the metals present, changing them from sulfides to the corresponding oxides. The conversion takes place in the order of activity of the metals concerned. Some of the oxides escape through the top with the waste gases; other oxides, like iron oxide, combine with the flux present and are gotten rid of as slag. When practically all the iron has been oxidized, the slag is skimmed off. The air blasts are continued in order to burn off the remaining sulfur and produce metallic copper:

$$Cu_2S + O_2 \rightarrow 2Cu + SO_2 \uparrow$$

The copper thus produced is called *blister copper* because when solid, its surface is covered with blisters formed by the escaping SO_2. It still contains many impurities, including some dissolved Cu_2O. When air is blown through to burn off the sulfur, some copper is unavoidably oxidized. To reduce the copper oxide, the molten copper is *poled* by using green wood in the form of long poles. The gases given off by the wood and the charcoal formed serve as the reducing agents:

$$2Cu_2O + C \rightarrow 4Cu + CO_2$$

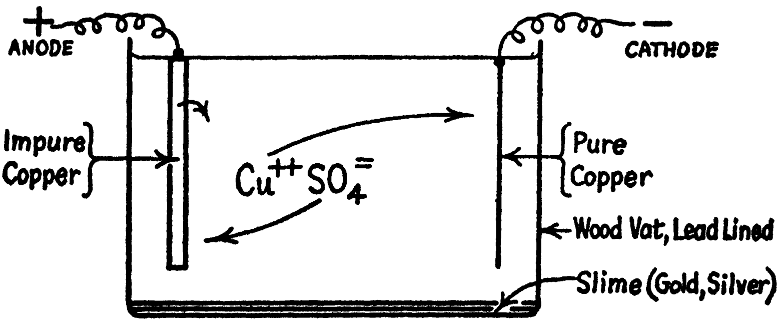
Refining of Copper. Even though the blister copper may be raised to a purity of 99% by poling, it must be still further refined. Most copper goes into electrical devices. Next to silver, copper is the best conductor of electricity we have. However, its conductivity is enormously reduced by impurities; the presence of only 0.4% of iron cuts down the conductivity of copper by more than 60%; 0.8% of arsenic reduces it by 70%; 0.5% of silicon by over 70%. If such impurities were not removed, much thicker wires would be required to conduct the same amount of current that thin, pure copper wires can safely handle. Thicker wires would require more material to be wrapped around them for insulation; the motors and generators would have to be larger; and the poles supporting such wires and transmission lines would have to be placed closer together. The harmful effect of impurities on the electrical conductivity of copper is so great that even a costly refining process would be fully warranted. But it so happens that the process is not expensive; in fact, it might be said to cost the refiner nothing, for the value of the gold and silver recovered is often greater than the cost of refining the copper.

The poled copper is cast into anodes of a size suitable for the 9′×3′× 3′ vats in which the *electrolytic refining* is to be carried out. The anodes are usually about an inch and a half thick, two feet wide and three feet long, although the dimensions vary. Thin sheets of pure copper serve as the cathodes. The electrolyte used consists of copper sulfate dissolved in water containing sulfuric acid. The anodes and cathodes are suspended alternately, close together in the vats, and properly connected to a source of direct current electricity. To insure a firm, uniform deposit of pure copper on the cathodes, the electrolyte is kept in slow circulation, the bath is maintained at a temperature of around 130°F, and the current density and the voltage between each anode and cathode pair is held so low (around 0.1 amp./sq. in. at 0.25 volt) that about three weeks are required for the process.

In the vat there are present ions furnished by the electrolytes in solution:

$$CuSO_4 \rightleftarrows Cu^{++} + SO_4^{=}$$
$$H_2SO_4 \rightleftarrows 2H^{+} + SO_4^{=}$$

The copper ion is attracted to the oppositely charged cathode which readily supplies it with the two electrons it needs to become metallic copper. In this way a Cu^{++} ion becomes a Cu^{0} atom as it plates on the cathode. At the same instant a copper atom from the impure copper anode goes into



Electrolytic Refining of Copper

solution as a copper ion. The cathode increases in thickness at the same rate as the anode gets thinner.

$$\text{At cathode: } Cu^{++} + 2\epsilon \rightarrow Cu^{\circ}$$
$$\text{At anode: } Cu^{\circ} - 2\epsilon \rightarrow Cu^{++}$$

In this way, the concentration of copper ions in solution never varies. The sulfate ion can be regarded as moving toward the anode and meeting a fresh copper ion which takes care of it. Thus, the sulfate ion concentration does not vary either. The voltage is controlled so that hydrogen ions are not discharged at the cathode. Iron, zinc, nickel and other metals present as impurities in the anode copper also go into solution, but they do not plate out on the cathode; being higher in the electromotive series than copper they require a higher voltage to discharge them from solution. Gold and silver being below copper in activity, cannot dissolve, for if they did they would immediately be replaced and precipitated by the metallic copper. As the anode plate dissolves, each particle of gold and silver dislodged drops to the bottom of the vat. The mud or slime that accumulates is carefully collected and treated to recover its precious metal content. Although the concentration of the copper sulfate does not vary, the electrolyte must be changed occasionally because it becomes contaminated with impurities. For example, iron, zinc and nickel impurities might have gone into solution from the impure anodes; their sulfates can build up to such high concentrations that even with the low voltage employed they would plate on the cathode.

Properties of Copper. Copper is a red metal which melts at 1981°F and has a specific gravity of 8.93. It is quite malleable and ductile. It can be drawn into wires so thin that a mile length of such wire weighs only slightly more than 11 grams (a nickel weighs 5 grams). In dry air copper becomes coated with a thin film of cuprous oxide, Cu_2O, which itself resembles copper in color. In ordinary air copper tarnishes to form a green basic carbonate. These tarnish coats act more or less as protective coverings. They are not like the rust on iron which not only does not protect iron from the action of the air but actually catalyzes further rusting.

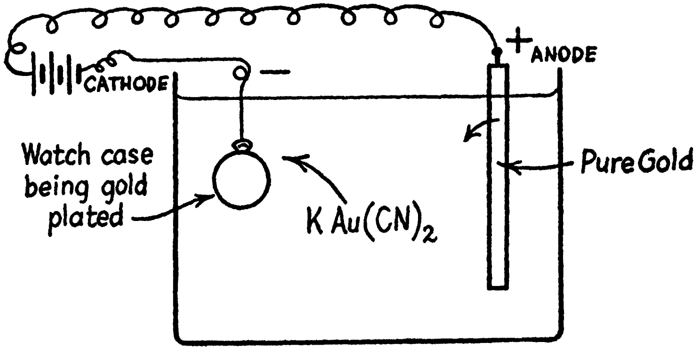
Reducing acids, like hydrochloric acid and dilute sulfuric acid, do not attack copper. In the presence of oxygen, however, these acids do attack copper. Two reactions occur; first the copper changes to cuprous oxide, and then the acid reacts with the oxide to form its salt:

$$4Cu + O_2 \rightarrow 2Cu_2O$$
$$Cu_2O + 4HCl \rightarrow 2CuCl_2 + 2H_2O$$

Oxidizing acids, like concentrated hot sulfuric and nitric acid, react with copper to produce their respective salts. The action here, too, is really first an oxidation followed by a type of neutralization, just as before.

Uses of Copper. For purposes where iron and steel lack the properties that copper and its alloys possess to a high degree, the latter are used. Because copper resists corrosion so well it is used for roofing, for covering ship bottoms, as linings for chemical apparatus, as tubing and condensers. Practically all electrotypes, like the ones used to print this book, are made of copper. Its outstanding use is for electrical conductors. Its many important alloys will be discussed in a latter chapter.

Electroplating. To make electrotypes, an impression of ordinary printing type is made in a suitable wax block. This negative mold is carefully brushed with graphite powder so that it will conduct electricity. The wax impression is next made the cathode in an arrangement very much like that used in the electrolytic refining of copper. Here, however,.pure electrolytic copper sheets serve as the anodes, while as before copper sulfate plus a little sulfuric acid serve as the electrolytes. When the coating of copper has become thick enough, the cathode is removed from the bath, the wax detached, and a low-melting lead alloy poured into the copper to give it the necessary firmness. Seamless copper tubes are also made electrolytically. Again the method is essentially the same: acidulated copper sulfate is the electrolyte; the cathode, a rod of the desired diameter; and the anode, a piece of cast copper. You have noticed that the plating always takes place on the cathode. This is to be expected, since metallic ions are always positively charged, and are therefore attracted to the negative electrode.



Electroplating an Object with Gold

In general, electroplating of any kind requires that the object to be plated be made the cathode. The bath must contain an electrolyte that furnishes ions of the metal that is to form the plating coat. The anode is made of the metal that keeps replenishing the ions that plate out. For zinc plating (wet galvanizing), a sheet of cleaned steel is placed in a bath of zinc sulfate. The steel is made the cathode, and a bar of zinc the anode. For silver plating, the electrolyte is usually prepared from silver chloride and potassium cyanide (a deadly poison!). The object to be plated is, of course, made the cathode, and silver bars form the anodes. The diagram below illustrates the arrangement necessary for gold plating. In actual practice, it must be remembered, the concentration and acidity of the electrolyte, the temperature, the current density and voltage, and the mechanical conditions are all very carefully regulated by the experienced plater to insure his obtaining a firm, adherent plating coat.

ZINC

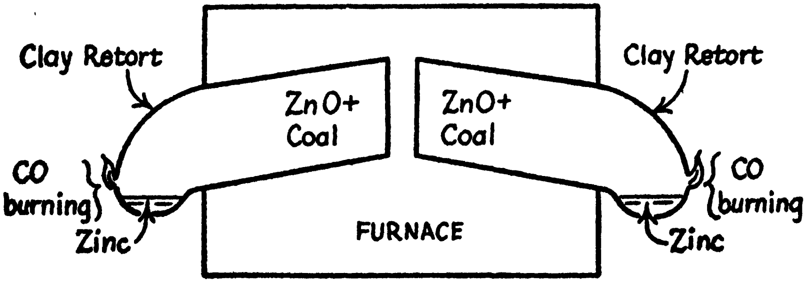
Occurrence of Zinc. Zinc ore deposits are widely distributed; North America, Europe, Australia and Asia have mines being worked to recover the metal. In the United States, from mines in Oklahoma, New Jersey, Kansas, Indiana and a few other states, we produce about 500,000 tons of zinc yearly, or half the world's annual supply. The most common zinc ores are: *zinc blend,* ZnS (large Missouri deposits); *smithsonite,* $ZnCO_3$; *calomine,* Zn_2SiO_4; *franklinite,* $Zn(FeO_2)_2$ usually associated with manganese oxides, and *zincite,* ZnO.

Metallurgy of Zinc. The method already given for producing copper describes fairly well how zinc is prepared. The carbonates and sulfides are roasted to yield the oxide:

$$ZnCO_3 \rightarrow ZnO + CO_2 \uparrow$$
$$2ZnS + 3O_2 \rightarrow 2ZnO + 2SO_2 \uparrow$$

The oxide is then treated with sulfuric acid, and the resulting zinc sulfate is electrolytically treated to obtain metallic zinc. Another method takes advantage of the fact that zinc is volatile, i.e., it changes readily to a vapor when heated. Accordingly, a mixture of zinc oxide and coal is placed in clay retorts, which are then heated in a furnace to bring about the desired reduction:

$$ZnO + C \rightarrow Zn + CO \uparrow$$



Zinc Oxide Being Reduced to Spelter

As the metallic zinc forms it boils off, but soon condenses in the cooler part of the retort from where it is ladled out. The CO burns at the mouth of the retort. Zinc produced in this way contains small amounts of impurities, and is called *spelter.*

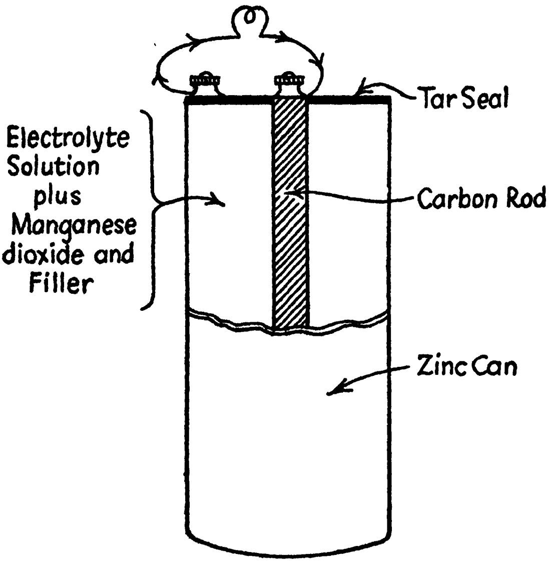
Properties and Uses of Zinc. Zinc is a blue-gray metal, distinctly crystalline in structure, melting at 787°F, and having a specific gravity of 4.32. It is brittle when cold unless it has first been given a mechanical rolling treatment while held at around 300°F. It is an active metal, reacting with acids to liberate hydrogen, and also reacting with boiling alkalies:

$$Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2\uparrow$$

It tarnishes readily in air to form a firm, adherent protective coating of a basic zinc carbonate. The coating prevents further tarnishing, and so explains the long lasting quality of zinc and galvanized ironware. Zinc burns readily in air. By allowing its vapor to burn, zinc oxide in a very finely divided condition is produced. Zinc oxide is a valuable white paint pigment; it is also used in enormous quantities as a filler for rubber used in automobile tires. The rest of the more than 100,000 tons of zinc oxide produced annually in this country goes into the manufacture of enamels, glazes and laboratory glassware. Zinc chloride is used for treating railroad ties to preserve them against rotting. Paper is converted into an effective electrical insulator by treatment with zinc chloride solution; the washed, pressed and dried product which results is called vulcanized fiber. Zinc sulfate solution is used to prepare cement and plaster surfaces preliminary to painting; when added to glue solutions, it inhibits decay.

Dry Cells. Zinc is the active metal used in so-called dry cells, used in flash lights, etc. They consist of a zinc can filled with solid ammonium chloride, manganese dioxide, graphite powder and a solution of ammonium chloride and zinc chloride to act as the electrolytes. In the center is a carbon rod that extends through the top which is sealed with a hard tar. Consider that zinc atoms tend to go into solution as zinc ions; while doing this, each atom leaves behind two electrons, i.e., $Zn^{\circ} - 2\epsilon \rightarrow Zn^{++}$. Within a short time, the zinc has accumulated many free electrons; it has, in other words, a negative charge. This negative charge attracts zinc ions and so counteracts the tendency of any additional zinc atoms to go into solution. An electric current may be regarded as the movement of electrons along a conductor. They move away from the end where there are many free electrons; that is, where the mutual repulsion is greatest (for like charges repel); and they travel towards the end having relatively few free electrons. If, then, a conductor (say the wire in an electric light bulb) is inserted between the terminals of the cell, the current flows from the negative zinc terminal through the wire to the positive carbon terminal. Once some electrons flow away from the zinc terminal, more zinc atoms can change to ions and go into solution. Within the cell itself, involved reactions are thought to take place. However, they probably can be understood if we regard ammonium chloride to be ammonia combined with hydrochloric acid:

$$2NH_3 \cdot HCl + Zn \rightarrow Zn(NH_3)_2Cl_2 + H_2\uparrow$$



Dry Cell

The hydrogen gas must be taken care of, otherwise it would cause the can to bulge. It would also deposit on the carbon rod, thus effectively insulating it and breaking the internal circuit. Any break in the circuit, whether internal or external, stops the current flow. The purpose of the manganese dioxide is to oxidize this hydrogen into water, and so render it harmless. The MnO_2 acts slowly. For this reason, dry cells are employed only where intermittent service is required.

Zinc itself is used for roofing and rain pipes. Sheet steel covered with zinc, called *galvanized iron,* is also used for roofing, as well as for wash boards, washtub covers, garbage and ash cans, etc. To prepare galvanized iron, the sheet steel is cleaned either with hydrochloric or sulfuric acid, or by sand blasting, and then dipped into molten zinc. The zinc may be applied by electroplating. Another way to get a layer of zinc on iron is to pack the iron object in zinc dust and then bake it; this product is called *sherardized iron.* Steel conduits for telephone cables are protected from rusting in this way.

Zinc protects the iron it coats by preventing contact with air. Because it is more active than iron, even if the iron is exposed, the zinc tarnishes away before the iron starts to rust. Zinc adheres well to iron because it forms an alloy with it. Zinc salts are poisonous, and it is for this reason that we have been forced to use less satisfactory and more expensive tin in our food canning industry.

Silver is frequently present in lead ores and it is, of course, desirable to recover it from the lead produced. Advantage is taken of four properties of zinc to accomplish this. Molten zinc is insoluble in molten lead; zinc dissolves silver much better than lead does; zinc is less dense than lead; zinc is volatile. Zinc is added to the lead containing the silver, the mixture is melted, whereupon the zinc holding most of the silver floats to the surface. It is skimmed off, and heated until it volatilizes, leaving the silver behind. Where gold and silver are recovered from their ores by the cyanide process, large quantities of zinc are used to precipitate these metals from their solutions. Zinc is a constituent of some important alloys (Babbitt metal, brass, bronze and German silver) which are discussed in the next chapter.

LEAD

Occurrence of Lead. Lead ores are found in most of the countries of the world. Canada and Mexico are important producers, but the United States heads the list with its annual production of a little less than 500,000 tons, or about one quarter of the world's production. Missouri, Idaho, Utah and Oklahoma are among the main lead-producing states. The most important lead ore is *galena*, PbS. *Anglesite*, $PbSO_4$, and *cerussite*, $PbCO_3$, are other ores, probably formed by the weathering of lead sulfide.

Metallurgy of Lead. The metallurgy of lead, though complicated by the large number of other metals usually found associated with it, follows in general the principles already laid down for recovering metals from their ores. The ore is usually concentrated by the flotation process already described. With rich ores, the necessary roasting and subsequent reduction are carried out in the same reverberatory furnace. By the proper control of temperature and by making use of the fact that lead sulfide is a reducing agent, the need for coke to reduce the oxide is avoided. Maintaining the ore at a dull red heat (approximately 1000°F) and seeing to it that an excess of air flows over the hearth, a *portion only* of the ore is roasted:

$$2PbS + 3O_2 \rightarrow 2PbO + 2SO_2\uparrow$$
$$PbS + 2O_2 \rightarrow PbSO_4$$

Next, the charge in the furnace is stirred. Sufficient fuel is added to the grate to make certain that the flames passing over the hearth contain no free oxygen, and to raise the temperature to about 1500°F. Now the reduction process goes on, freeing the lead, which collects in a basin at the side:

$$2PbO + PbS \rightarrow 3Pb + SO_2\uparrow$$
$$PbSO_4 + PbS \rightarrow 2Pb + 2SO_2\uparrow$$

How silver (and gold, too, if present) is separated from the lead has already been explained in connection with the uses of zinc. Lead is refined electrolytically just as copper is, using the impure lead as the anodes and thin sheets of pure lead as the cathodes, but instead of lead sulfate, lead fluosilicate, $PbSiF_6$, is used as the electrolyte.

Properties and Uses of Lead. Lead is a very soft, gray metal with a low melting point (621.3°F) and a high specific gravity (11.005). It tarnishes in air forming a protective basic carbonate coating. Hard waters cause a similar, insoluble coating on lead pipes, whereas soft water reacts with lead to produce lead hydroxide, $Pb(OH)_2$, which is sufficiently soluble to be harmful, since *all soluble lead compounds are poisons.* More than that, lead is a cumulative poison; that is, the body cannot easily eliminate it. A person who gets his drinking water through lead pipes may after several years find that he has developed a case of lead poisoning.

Dilute sulfuric acid has little effect on lead because of the insolubility of the lead sulfate that forms on its surface:

$$Pb+H_2SO_4 \rightarrow PbSO_4\downarrow+H_2\uparrow$$

Lead is therefore used for lining tanks used for holding sulfuric acid solutions, as for example, the tanks used in the electrolytic refining of copper. When lead is burned, an orange-yellow powder called *litharge,* PbO, results. It finds use in the manufacture of optical glass, enamels and glazes. A mixture of litharge and glycerine makes an excellent cement for glass and stone. Litharge changes to a bright red powder when it is heated in air for some time at around 950°F. This product, Pb_3O_4, is called *red lead* or *minium.* It is an oxidizing agent, and so hastens the hardening or "drying" of linseed oil with which it is mixed to make pipe joints leak proof. The same mixture is usually the first paint coat applied to iron work. Both litharge and red lead can be oxidized in various ways up to lead peroxide, PbO_2, a chocolate-brown powder. Lead peroxide acts as the positive electrode in the lead storage cell. Enormous quantities of lead are used in the manufacture of white lead, $Pb(OH)_2 \cdot 2PbCO_3$, a paint pigment that has excellent covering power. Many countries forbid its use, as does our Navy, on account of its poisonous nature. White lead paint is no good for painting interiors because hydrogen sulfide darkens it by changing it into black lead sulfide:

$$Pb(OH)_2 \cdot 2PbCO_3+3H_2S \rightarrow 3PbS+4H_2O+2CO_2\uparrow$$

Tetra-ethyl lead, $(C_2H_5)_4Pb$, is added in small quantities to gasoline to improve its anti-knock properties.

When lead is heated it softens and can easily be extruded through the properly shaped die to come forth as lead pipe. Lead is employed as a sheath for cables, as a substitute for tin foil, to make shot, and as a constituent of many important alloys like solder and antifriction metals which are described later. One of the greatest consumers of lead is the manufacturer of lead storage batteries.

TIN

Occurrence and Metallurgy of Tin. Tin ore deposits are not widely distributed. The world's yearly supply of approximately 150,000 tons of tin comes almost entirely from the Netherlands Indies, Malaya and Bolivia. Production has been restricted by those in control of the mines. The deposits in this country (Virginia, North and South Carolina, Dakota, Alaska) are not considered important. We import about 80,000 tons of tin a year. *Cassiterite,* SnO_2, is the only ore of tin. The metallurgy of tin is relatively simple, due to the high density of cassiterite and to the comparative inactivity of tin. The crushed ore is effectively concentrated (separated from its gangue) by washing it with water. Arsenic and sulfur are removed by roasting. A magnetic separator frees the ore from the Fe_3O_4 present. The resulting fairly pure SnO_2 is reduced to tin by means of coal in a reverberatory furnace. To further refine the tin, the molten metal is poured in a thin stream from ladles, thus allowing the oxyen of the air to oxidize out the remaining impurities.

Properties and Uses of Tin. Tin is a soft, silvery metal, with a specific gravity of 7.18, and a low melting point (449.4°F). It has low tensile strength but is very malleable. Tin has three allotropic forms, resembling sulfur in this respect. If kept very cold for a long time it crumbles into a gray powdery form, called gray tin. Above 338°F, tin changes over into a brittle rhombic form. *It is not tarnished by moist air,* nor much affected by other substances. This explains why the more than 16½ billion tin cans we make yearly are so satisfactory. Tin cans are 98½% steel with their surfaces protected by a 1/10,000 inch layer of tin. For this purpose about 27,000 tons of tin are consumed yearly. Sand blasted, pickled, carefully washed sheets of soft steel are run through a bath of molten tin to make tinware. Tin is used to make tin foil, and is a constituent of many important alloys (solder, bronze, type-metal, antifriction and fusible alloys) which are discussed in the next chapter.

Tin is converted into stannic chloride, $SnCl_4$, a fuming liquid, when it is treated with chlorine. Use is being made of this reaction in detinning tin scrap and discarded tin cans. Plans are under way now for substituting silver for tin as a lining for tin cans, as well as in solders, bronzes and bearing metals. This in spite of the fact that silver costs $5.00 a pound, or about ten times the price of tin. We are being forced into this because we cannot import enough tin to meet our needs, whereas we have plenty of silver.

NICKEL

Occurrence of Nickel. A little nickel (400 tons a year) is produced in this country, as the nickel sulfate by-product from copper refining. This is shipped to be processed in Canada, where Ontario's Sudbury district normally produces 85% of the world's nickel supply. Other producers are New Caledonia, the U.S.S.R., Norway, Burma and the Netherlands Indies. Since nickel is a strategic war material, its production has been enormously increased since 1938. Our imports more than doubled in 1939, rising to 52,000 tons. No reliable production figures have been made available since war preparations began. The ores, *pentlandite,* NiS with copper and iron sulfides mixed with it, and *garnierite,* a silicate of nickel and magnesium are the chief sources.

Metallurgy of Nickel. To obtain nickel from its ores is an extremely complex procedure. Nevertheless, the steps involved are merely combinations of the operations already described in connection with copper metallurgy—roasting, smelting, converting in a Bessemer furnace, and final reduction. Often the nickel is shipped as the oxide to the steel makers who can use it just as well in the compound form. For the manufacture of *monel metal,* it is not necessary to separate it from its associated copper. To obtain pure nickel, an electrolytic process is sometimes used. Pure nickel can also be obtained by the *Mond* process. This process is based on the fact that heated nickel combines with carbon monoxide to form volatile nickel carbonyl, $Ni(CO)_4$. Neither copper nor cobalt has this property, so that in this way nickel can be separated from them. When the nickel carbonyl is heated to about 330°F it decomposes, depositing pure nickel.

Properties and Uses of Nickel. This metal is silvery in color and has a faint yellow cast. Its melting point is 2651°F, and its specific gravity is 8.8. It is very hard and capable of taking a high polish. For these reasons, and because it tarnishes so slowly, nickel is frequently plated on other metals to protect them from corrosion. In recent years chromium has largely replaced nickel for this purpose because chromium plating does not tarnish at all, and is still harder. Nickel is used as the catalyst in the hydrogenation of oils, for spark plug points, and as the active metal in the Edison storage cell. Most of the nickel produced, however, finds its way into the host of extremely valuable nickel alloys. More than half of the world's nickel goes into nickel steel. Monel metal, German silver, and other nickel alloys will be taken up in the chapter on alloys. Because of the great need for nickel, the United States Congress, in March, 1942, passed a law taking this metal out of our nickel coin. Up to this time a U.S. nickel coin had 75% copper and 25% nickel. The new "nickel" is 50% silver and 50% copper.

ALUMINUM

Occurrence of Aluminum. In the form of rocks, clays and minerals, aluminum is the most abundant metallic element in the earth's crust. Oxygen makes up 50%, silicon 25%, aluminum 7%, iron 5%, while the rest of the 92 elements together comprise the remaining 13%. Stimulated by war needs, processes have been invented which may soon be put into operation for obtaining aluminum from its clays. However, at the present time, *bauxite,* hydrated Al_2O_3, is the one important aluminum ore. France and Hungary have large bauxite deposits, as have Surinam and British Guiana in South America. Our principal deposits are in Arkansas, Alabama, Georgia and Tennessee. In 1940 we mined only about 450,000 tons of bauxite a year, importing more than that amount from Surinam alone. In 1939 we produced 160,000 tons of metallic aluminum. The amount of aluminum being produced here and abroad since war preparations began has not been made public. The mineral, *cryolite,* $AlNa_3F_6$, is also important as an aid in the metallurgical process. It is found in commercial quantities only in Greenland.

The Metallurgy of Aluminum. In 1850, aluminum cost over $5.00 a pound; now its price is down to 15 cents a pound. You might agree that the drop in cost is remarkable but still consider the present cost high compared to the price of iron. Why should aluminum be about ten times as expensive as iron when it is so much more plentiful? The answer lies mainly in the fact that aluminum is so active chemically that it costs a lot to liberate it from its compounds. International cartel agreements and monopolistic control are believed by some members of Congress to be other factors, which, by keeping production down, have kept the price of aluminum up. Since aluminum is essential for airplane construction (a bomber takes 30 tons), our government now is seeing to it that much more will be produced.

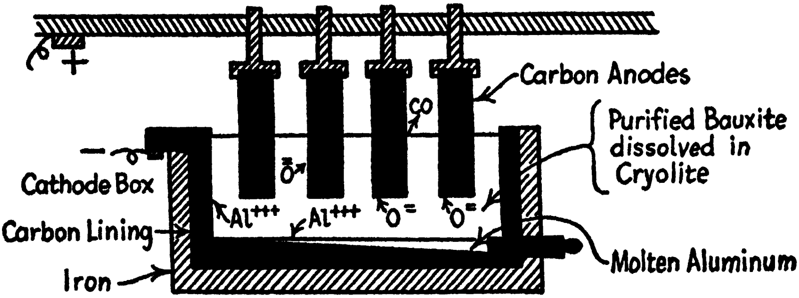
The Hall-Héroult Electrolytic Aluminum Process. Metallic aluminum was first produced by the reaction between metallic potassium and aluminum chloride:

$$AlCl_3+3K \rightarrow 3KCl+Al$$

This process is far from being commercially feasible, the necessary potassium itself being costly to produce. We cannot use carbon to reduce aluminum oxide, as we do in the case of the oxides of the other metals we have studied. Carbon is a less active reducing agent than aluminum is, so that no matter how hot you heat a mixture of coke and bauxite, you do not get aluminum. Two young men, each 22 years old at the time, Hall, an American, and Héroult, a Frenchman, in 1886, independently discovered the process now universally used. Their great discovery was that cryolite, which melts readily, is a good solvent for aluminum oxide, which alone does not melt below 3600°F.

Before aluminum can be made, pure Al_2O_3 must be prepared from the ore, because no satisfactory industrial method has been devised for purifying metallic aluminum. For this purpose, bauxite is treated with sodium hydroxide solution. The impurities, like Fe_2O_3, do not dissolve, while the Al_2O_3 goes into solution. The solution is filtered and $Al(OH)_3$ precipitated out. The aluminum hydroxide on being heated forms pure alumina, Al_2O_3.

The aluminum furnace consists of an iron box lined with a layer of baked-in carbon. It is usually about 8 feet long, 5 feet wide, and 2 feet deep. The box acts as the cathode in this process. Ground-up cryolite is loaded into the box and melted, either by external heat or by the heat developed as a result of the poor electrical contact made when the carbon anodes are first lowered to the bottom of the box. Once the cryolite has melted, it can then conduct the current. The anodes are raised from the bottom, and the alumina added. It dissolves and ionizes. The aluminum ions travel to the cathode box, pick up three electrons each and drop to the bottom as molten aluminum. The oxygen ions are attracted to the carbon anodes where they each give up their two electrons, but do not escape as oxygen gas. Remember that the whole apparatus is very hot. The oxygen, therefore, combines with the carbon anodes, and escapes as carbon monoxide which is allowed to burn on the surface of the bath, or may be collected to be used as a fuel gas. The electrolysis of alumina is represented by the following equation:



Aluminum Furnace

$$2Al_2O_3 \rightarrow 4Al + 3O_2 \uparrow$$

Except for mechanical losses, the cryolite is not used up. Fresh alumina is added from time to time. As the anodes are consumed by the secondary reaction with oxygen, they must be replaced with new ones. The molten aluminum is drawn off periodically. The operation is continuous once it has started; a new anode can be inserted, fresh alumina added, or the molten aluminum tapped without interrupting the process.

Properties and Uses of Aluminum. Aluminum is a silver-white metal which melts at 1220°F, has a very low specific gravity (2.43), less than one third that of iron, and great tensile strength. Aluminum wire is a better conductor of electricity than copper wire of the same weight per foot. For this reason, aluminum wires are now being widely used for high-tension lines, particularly where the distances between supports must be large. When warmed up to the temperature of boiling water, it becomes quite malleable. Aluminum foil is in common use. Many objects like cooking utensils and jar and bottle caps are pressed into shape from sheet aluminum. The metal or its alloys can be extruded, rolled, forged or cast. Pistons, tubing, railway cars, truck bodies and airplanes are just a few of the many objects made of aluminum.

Aluminum objects do not corrode away quickly as might be expected from a study of the electrochemical series, in which aluminum stands among the active metals. The fact is that aluminum does tarnish very quickly, but the thin, transparent, hard, firmly-adhering film of aluminum oxide that forms is well nigh the perfect protective coating. Aluminum's activity is evident in the familiar photoflash bulb used by the newspaper photographer. It contains a little oxygen gas and a piece of thin, crumpled aluminum foil, which on being ignited burns to give off a brilliant light. Aluminum reacts with both acids and bases to liberate hydrogen. Neither should, therefore, be stored in aluminum vessels.

$$2Al + 6HCl \rightarrow 2AlCl_3 + 3H_2\uparrow$$
$$2Al + 2NaOH + 2H_2O \rightarrow 2NaAlO_2 + 3H_2\uparrow$$

Aluminum sulfate is its most important salt. It is made by the reaction between bauxite and sulfuric acid:

$$Al_2O_3 + 3H_2SO_4 \rightarrow Al_2(SO_4)_3 + 3H_2O$$

Thousands of tons of it in the form of its crystals, $Al_2(SO_4)_3 \cdot 18H_2O$, are used each year in the clarifying of drinking water, as a mordant in dyeing, to size paper, and in the tanning industry. In water solution, it hydrolyzes (reacts with water to form an acid and a base). It is the jelly-like precipitate $Al(OH)_3$ that results, which makes it effective:

$$Al_2(SO_4)_3 + 6HOH \rightleftarrows 2Al(OH)_3\downarrow + 3H_2SO_4$$

Hydrolysis reactions are reversible and slow. For this reason, when aluminum sulfate is added to water supplies, milk of lime is also added to react with it:

$$Al_2(SO_4)_3 + 3Ca(CH)_2 \rightarrow 3CaSO_4 + 2Al(OH)_3\downarrow$$

The gelatinous aluminum hydroxide settles, carrying down with it suspended particles, including some bacteria. Because of its high water content, crystallized aluminum sulfate is an excellent fire retarder. Cloth, when dipped into its solution and dried, is protected by the many crystals of the salt left enmeshed in the fibers.

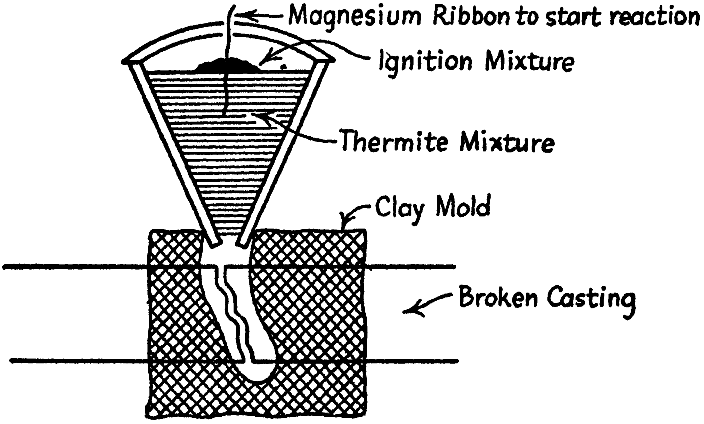
By treating a mixture of bauxite and carbon with chlorine at a high temperature, $AlCl_3$ is now being produced on a commercial scale. Large quantities are used to make gasoline by cracking heavier petroleum oils, and also in the manufacture of many synthetic organic chemicals.

The Thermite Process. War, unfortunately, has made the thermite incendiary bomb a familiar object to millions of people. The usual incendiary bomb consists of a carefully designed magnesium casing holding the thermite mixture. *Thermite* is a mixture of small aluminum flakes and scaly iron oxide (Fe_3O_4). Once started, the reaction goes to completion in spite of anything you do to try to stop it. It liberates so much heat that

$$8Al + 3Fe_3O_4 \rightarrow 4Al_2O_3 + 9Fe (+HEAT)$$

the molten iron formed, runs off *white hot.* Incidentally, there are ways of handling the burning magnesium shell of the bomb after the thermite reaction. Salt or sand helps smother it sufficiently to allow of its being carried outdoors. A spray of water causes the magnesium to be consumed in two minutes instead of twenty, thus permitting secondary fires to be attended to more promptly.

Thermite has been used for many years in thermite welding. Broken castings on ships far from shore are welded in this way. Rails can be welded in place; repairs can be made on heavy machines without moving them to distant shops. A clay mold is build around the break in such manner that the molten iron released from above runs down, fills in the crack, and solidifies, thus effectively welding the broken ends together.



The same reaction is used to prepare pure metals, like manganese, chromium, tungsten and molybdenum, from their oxides which, in other ways, are not easy to reduce:

$$3MnO_2+4Al\rightarrow 2Al_2O_3+3Mn$$

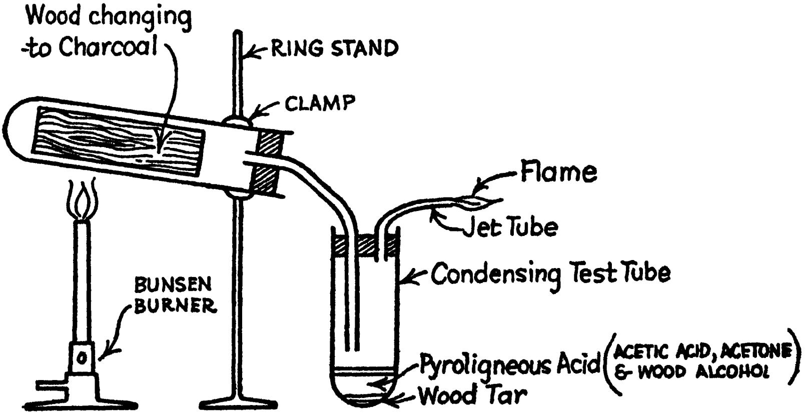
Aluminum is added in small amounts to steel to prevent blow-holes in castings. *Duralumin, magnalium* and *aluminum bronze* are outstanding aluminum alloys.

43. FUELS

Solid Fuels. Any material that is combustible might be used as a fuel. This does not mean that we consider diamonds, furs, linseed and olive oil, and ether to be fuels, even though they can all be made to burn. To be classed as a fuel, a material must be relatively cheap and available in large amounts. Its kindling temperature must not be too high (this requirement excludes graphite); it must give off sufficient heat on burning to warrant the labor of handling it and the ash that is usually left.

All solid fuels are vegetable in origin. The geologist can trace the changes that have occurred as the trees, shrubs, mosses and grasses were transformed by nature into the fuels of industry. Plants are made of a woody fiber that is mainly cellulose, $C_6H_{10}O_5$, and a complex mixture called sap. *Wood* itself is really an expensive fuel and it burns rapidly; it is used where local conditions make it difficult to obtain better fuels.

Destructive Distillation of Wood. When wood is heated in the absence of air, many chemical changes occur, indicating that wood is far from


Wood changing to Charcoal
RING STAND
CLAMP
Flame
Jet Tube
Condensing Test Tube
BUNSEN BURNER
Pyroligneous Acid (ACETIC ACID, ACETONE & WOOD ALCOHOL)
Wood Tar

Destructive Distillation of Wood

being a simple material. The decomposing by heat of a complex organic material, and condensing its volatile products, is called *destructive distillation*. When we carry out this process using wood, we find that the moisture in the wood is driven off first. This sweeps out the air in the apparatus. The wood begins to char, a yellowish fog-like smoke passes into the condensing tube where some of it remains as a liquid while the rest escapes through the jet-tube. A lighted match brought to the jet causes the escaping vapors to burn with a smoky yellow flame. After a time, further heating of the wood produces no additional change; it has been completely converted into *charcoal*. Examination of the liquid in the condensing tube shows it to be a mixture which separates into an oily, a watery and a tarry layer. The odor of the liquid mixture justifies its name of *liquid smoke*.

Analysis of the liquid smoke shows that it contains a large number of compounds including benzene, paraffins, carbolic acid, creosotes, acetic acid, acetone, wood alcohol and many others. The vapors of those, mixed with acetylene, carbon monoxide, methane and hydrogen, escape from the jet-tube and furnish the fuel for the yellow flame at the jet. The black, porous charcoal we have made is a fuel of minor importance. It does not burn with a flame because it itself is not volatile, nor does it contain any volatile matter. No material can burn with a flame unless it can produce volatile matter; for, you see, a flame is merely a burning gas or vapor.

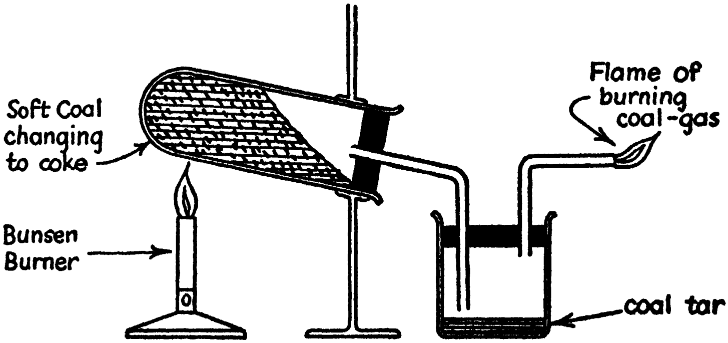
Origin of Fossil Fuels. What we have accomplished by heating wood in the absence of air is similar to what nature has been doing through the ages in converting vegetable matter into natural fuels. Bacteria probably play a part at the beginning, but it is the enormous pressure exerted by the heavy layers of earth and rock on the buried vegetable matter that brings about the major changes. Air is excluded, pressure brings about a rise in temperature, and volatile matter is driven off while the percentage of carbon keeps increasing in the remaining solid as time goes on. For this reason, the process is termed *carbonification*. The vapors condense in colder places, forming oil deposits; the gases may be trapped by impervious rock layers, to be discovered later and the natural gas utilized by man.

Peat. In swamps and around bodies of water, a low-grade fuel, called *peat*, is found. In point of time, compared to the other fuels, it is least removed from its vegetable origin. The original moss or grass structure is still visible. Its moisture content is high. On the dry basis its percentage of carbon is approximately the same as that of wood, about 50%. Its volatile matter is largely CO_2 and so has little heating value. The calorific value of peat varies between 1500 and 9000 B.T.U. per pound, depending on its moisture and ash content. (B.T.U. stands for British thermal unit, the amount of heat that will raise the temperature of one pound of water one degree Fahrenheit.)

Lignite. This fuel differs from peat in that it originated from trees rich in resins, like palms and cypress. Its volatile matter is also mainly CO_2. Its carbon content is slightly greater than that of peat, but its moisture content is also high. It is a better source of heat than all but the most exceptional peats; most lignites having a calorific value close to 8000 B.T.U. per pound.

Bituminous or Soft Coal. Included in this class are coals that vary widely in their characteristics. They come next to lignite in age. Some low-grade bituminous coals can hardly be distinguished from high-grade lignites, except for the fact that the volatile matter from bituminous coal has a high calorific value. The percentage of carbon in western U. S. bituminous coal runs from 50 to 65%, in eastern U. S. bituminous, from 60 to 75%. The calorific value is usually double that of lignite, ranging from 14,400 to about 17,000 B.T.U. per pound. The high percentage of hydrocarbons in its volatile matter contributes greatly to the heating value of bituminous coal. The production of bituminous coal in the U. S. varies with the demand, which is influenced by business conditions, competition from the oil producers, and other factors. In 1920, one of the peak years, we produced over 568,000,000 tons. Just prior to the war we mined approximately 400,000,000 tons, probably one third of the world's production. About half of our states have large coal deposits, with West Virginia, Pennsylvania, Illinois and Kentucky leading in the amounts of bituminous coal mined.

Destructive Distillation of Soft Coal. Approximately one-sixth of the bituminous coal we mine is not burned as such but is destructively distilled to produce *coke* and other valuable products. As you know, the blast furnaces use enormous quantities of coke for reducing iron ore to pig iron. Coke is an excellent fuel, clean burning and leaving little ash. The U. S. produces about 50,000,000 tons of coke yearly, about one-third



Destructive Distillation of Soft Coal

of the world's production. In the laboratory, using the same arrangement that we employed in making charcoal from wood, we can prepare coke from soft coal.

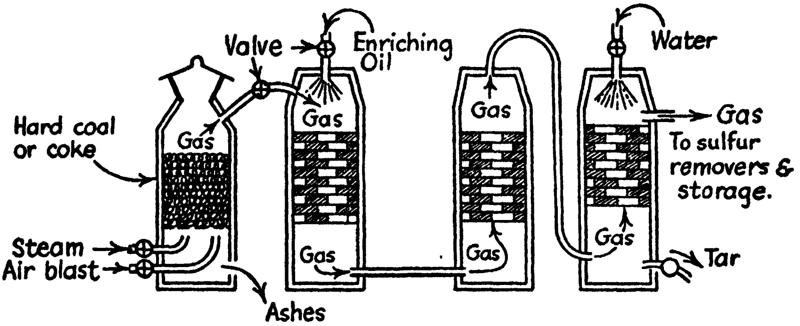
As before, a combustible gas comes off. It is known as *coal gas,* and is a valuable fuel gas.

In the condensing tube, coal tar is found. Coal tar is a mixture of liquids like benzene and aniline, solids like naphthalene, and asphaltic and resinous substances. Less than ninety years ago this very coal tar was just a messy nuisance. It clogged sewers and poisoned the streams into which it was dumped. Then a seventeen-year-old English boy, William Perkin, tried some experiments. This London chemist discovered how to make a beautiful purple dye from the troublesome tar. Thus the great aniline-dye industry was created! Now chemists synthesize dyes, drugs, perfumes, explosives, hormones and even vitamins from the lowly coal tar. Some of the compounds in coal tar can cause cancer of the skin. Doctors and chemists all over the world are studying coal tar, searching for and already finding clues that promise hope for victory over this dread disease.

Hard Coal, or Anthracite. It is difficult to draw a sharp distinction between bituminous coal and anthracite. On the basis of total carbon, coals running from 75% to 87.5% carbon are called semi-bituminous, while those having 87.5% to 92.5% are called semi-anthracite. Anthracite coals are 92.5% to over 97% carbon, with very little volatile matter, usually less than 4%, which is almost entirely methane. Anthracite is harder than bituminous coal and is cleaner burning, producing almost no soot. For this reason, even though it is more expensive than soft coal, it is used in large cities where the citizens insist on clear air through which health-giving sunshine can penetrate. It usually leaves more ash than soft coal but has approximately the same calorific value. Pennsylvania, our one important hard-coal state, has in recent years been mining about 50,000,000 tons of anthracite yearly. This is twice Russia's and a little less than half the world's production.

Gaseous Fuels. Gas is the ideal household fuel. It can be piped to just where you want it, turned on and off as desired, and its flame is easily regulated. There is no problem of ash-handling and disposal. Its convenience, however, is usually counterbalanced somewhat by its cost to the consumer. In most localities, the consumer pays more per B.T.U. for gas than for coal or oil.

Water Gas. For domestic use, where natural gas is not available, *water gas* is usually supplied. This gas is essentially a fifty-fifty mixture of hydrogen and carbon monoxide. (Do not confuse water gas with water vapor. Water vapor is H_2O in its gaseous state.) Water gas is prepared by an intermittent process. First, coke or hard coal is made to burn in a blast of hot air until the whole mass is white hot. While this is happening, the CO_2 formed is allowed to escape into the air through an opening at the top of the 10′ × 15′ cylindrical furnace.



Manufacture of Water Gas

Hard coal and coke are almost pure carbon, and the reaction, therefore, is simply:

$$C+O_2 \rightarrow CO_2$$

Next the air is turned off and the opening at the top of the furnace closed. Instead of air now steam is forced through the white hot coke, while a gas exit valve at the side near the top is opened:

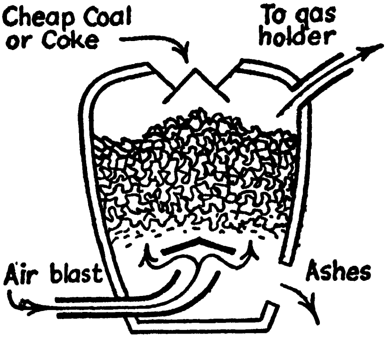
$$H_2O+C \rightarrow \underbrace{H_2+CO}_{\text{water gas}}$$

The reaction between the steam and hot coke absorbs heat. Within about four minutes the coke becomes too cool to react further. The steam is shut off and the gas exit valve closed. The air blast is again turned on and the roof hole opened. The coke again burns and within about two minutes becomes white hot, ready to react with steam as before. In this manner, air and steam are alternately sent into the furnace. Fresh fuel is fed into the furnace through the top, and ashes removed from the bottom when necessary.

Hydrogen and carbon monoxide both burn with blue, almost invisible flames which furnish very little light. Years ago, when water gas was used mainly as illuminating gas, it was necessary to add oil to the gas to make it burn with a luminous flame. Nowadays oil is added too, but the main purpose is to raise the calorific value of the gas (averaging 340 B.T.U./cu. ft. unenriched). Water gas is enriched with the oil after it leaves the furnace on its way to the large gas tank where it is stored.

Before it reaches the gas tank, however, it is passed through water to remove tar, and through trays of iron oxide and lime to remove H_2S and other sulfur compounds.

Producer Gas. For industrial purposes, where a gas with low calorific value (about 140 B.T.U./cu. ft.) is satisfactory, *producer gas* is manufactured. The process is simple.



Manufacture of Producer Gas

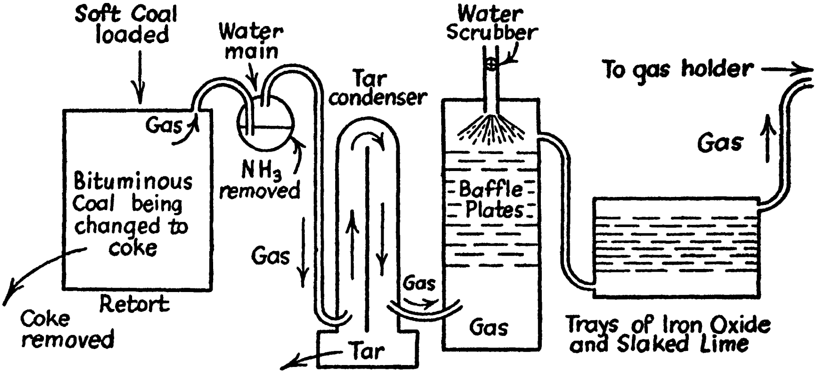
A blast of air is sent through cheap coal or coke in a furnace; but here only enough air is supplied to support a partial or incomplete combustion. The reaction is:

$$2C+O_2\rightarrow 2CO$$

However, the nitrogen originally in the air remains mixed with the CO generated. The result is a gas mixture containing about 60% nitrogen (which does not burn), about 10% hydrogen and hydrocarbons, and 30% carbon monoxide.

Natural Gas. We produce about two and a half trillion cubic feet of natural gas annually. The natural gas wells from which it is merely tapped are usually found near coal and petroleum deposits. Natural gas is frequently almost pure methane, CH_4. It is the same gas that is often found in coal mines, where it is called *fire damp,* and has been the cause of many frightful explosions. It bubbles up through marshes as the result of the decomposition of buried organic matter, and so is often referred to as marsh gas. Tremendous quantities are consumed in the production of carbon black, a filler for rubber. Over 100,000 tons of carbon black are made yearly by burning natural gas with insufficient air, so that a sooty flame results. Piped to homes and factories, natural gas accounts for half of the fuel gas used in this country to produce heat and light. It has a high calorific value, about 1000 B.T.U. per cu. ft., which is almost three times that of unenriched manufactured fuel gas.

Coal Gas. This valuable fuel gas is a by-product of the manufacture of coke, which is made by the destructive distillation of bituminous coal. The diagram below shows the arrangement of apparatus as used in industry.



Manufacture of Coal Gas

The volatile matter driven off from the soft coal in the retort contains valuable coal tar and ammonia. Both of these are recovered and sold. Hydrogen sulfide is removed just as it is from water gas. (Sulfur and sulfur compounds in a fuel can do a great deal of harm. When the fuel burns, these substances are oxidized to sulfuric acid. Sulfuric acid corrodes burners, cylinders and pistons, boilers, smoke stacks or any other metal parts it touches.) The coal gas that reaches the gas holders is a mixture consisting of about 50% hydrogen, 35% methane, 10% carbon monoxide, and the rest mainly hydrocarbons. These hydrocarbons, like ethylene and acetylene gas, and benzene vapor, give to coal gas its luminous flame. It is not necessary to add oils, as in the case of water gas, to make coal gas suitable for use as an illuminating gas. Its calorific value runs around 600 B.T.U. per cu. ft.

Acetylene. When quicklime, CaO, is heated with coke in an electric furnace to about 5000° F, calcium is not produced, but instead calcium carbide, CaC_2, results:

$$CaO + 3C \rightarrow CaC_2 + CO\uparrow$$

Water reacts with calcium carbide to liberate acetylene, C_2H_2.

$$CaC_2 + 2HOH \rightarrow Ca(OH)_2 + C_2H_2\uparrow$$

Over 500,000,000 cu. ft. of acetylene are manufactured in the United States annually. It is used mainly as the fuel in oxyacetylene welding and cutting, and as the starting material for many organic synthetic products. Acetylene has a higher calorific value than any of the common fuel gases, 1579 B.T.U. per cu. ft.

Liquid Fuels. Transported by pipe lines, tankers, railroads and automobile trucks, easily stored and handled, liquid fuels are our most important type of fuel. At present, benzine, wood alcohol and grain alcohol, although satisfactory for small stoves and for specially designed internal combustion engines, are of negligible importance compared to petroleum fuels. Gasoline, the main petroleum product, has a high calorific value, approximately 20,800 B.T.U. per pound as compared to 17,000 for high grade coal.

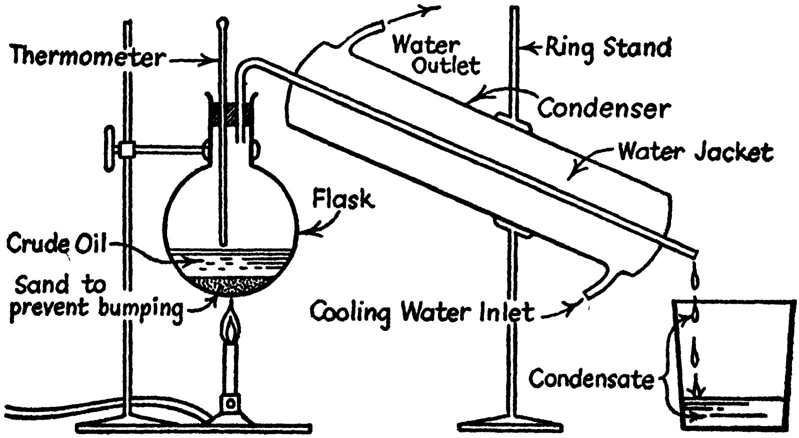
Petroleum. Deposits of petroleum oil are widely distributed all over the world. Many theories have been offered to explain their formation. One, already referred to, considers petroleum to be the condensed vapors driven off during coal formation by carbonification. Another theory is that petroleum is the end product from the bodies of prehistoric animals buried there ages ago; fossils of dinosaurs have been found in oil deposits. Because the regions of great magnetic needle deflection are the regions where oil is located, some geologists believe petroleum was formed as the result of a reaction between water and iron carbides.

Petroleum is a dark greenish-brown viscous oil. It consists of a mixture of many hydrocarbons. (A hydrocarbon is a compound composed of hydrogen and carbon only.) Crude oil from the Pennsylvania region is largely a mixture of *paraffin hydrocarbons.* California crudes are called *asphaltic base* oils, because the residue remaining after distillation is asphalt. The Texas and Oklahoma oils vary greatly but usually are intermediate in properties between the eastern and western oils.

In normal times the world produces about 2,000,000,000 barrels of crude petroleum annually. (1 barrel contains 42 gallons.) The United States furnishes five-eighths of the total; Russia and Venezuela coming next, each contributing a little over one-tenth of the world's supply. Texas, California, Oklahoma, Illinois and Louisiana are our leading oil states. We produce normally between $1,250,000,000 and $1,500,000,000 worth of petroleum yearly. (Our annual coal production is valued at $1,000,000,000.) There seems to be no valid reason for believing that we will soon exhaust our fuel supplies. We have enough coal to last two thousand years. If forced to, we can hydrogenate coal to make gasoline. The pessimist who calculates that we will use up all our oil in thirty years also ignores the fact that new deposits are still frequently being discovered. Besides this, the oil in shale rock which can be recovered at a slightly higher cost is sufficient to last us more than three thousand years at our present rate of consumption.

Petroleum Refining. Crude oil, after it comes from the well, is shipped or pumped to the refinery. Often the oil contains water mixed with it in the form of an emulsion. Such emulsions present a difficult problem, and as yet no completely satisfactory method has been found for breaking them up. The Cottrell process consists in flowing the crude oil between two highly charged electrodes. The tiny water droplets coalesce to form larger drops which then separate from the oil. Sulfur compounds also must be removed. One method consists in treating the oil with copper oxide. After agitation, the resulting copper sulfide is filtered off. It is roasted and so converted again into copper oxide, ready to do its sulfur-removing job again.

Distillation. The mixture of hydrocarbons called crude oil is a liquid solution consisting of many miscible liquids in which are dissolved many hydrocarbon gases and solids. The refiner's task is to separate these into useful commercial products. The process employed is called *fractional distillation*. Ordinary distillation is used to purify a single liquid. It consists in boiling the liquid and then condensing the vapor and collecting the condensate. For example, when salt water is placed in the distilling flask and boiled, steam passes into the condensing tube, changes back into water and runs down into the receiver. During the distillation the salt does not vaporize at all, so that at the end solid salt remains in the distilling flask. When crude oil is heated, however, the vapor above the liquid is not just one compound—it is a mixture of compounds. While the temperature is low the vapors that boil off come from the compounds having the lowest boiling points. Soon boiling ceases. Then when you raise the temperature, boiling begins again but this time the vapors come from those hydrocarbons having a higher boiling point



Petroleum Refining

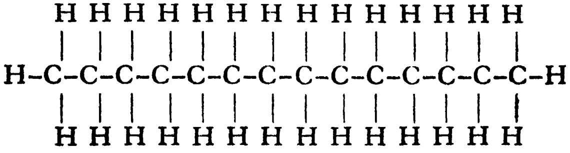
than those already boiled off. Using the apparatus illustrated below, we can duplicate the fractional distillation that is carried out at the large oil refinery.

When crude oil is first heated gases come off which do not condense. Only in recent years have the refiners bothered to conserve them; formerly they were wantonly wasted. The fraction that distills over between 100° F and 150° F is sent to one tank and sold as *petroleum ether*. *Naphtha* is the name given to the portion coming over between 175° F and 250° F. The fraction collected between 140° F and 400° F is *gasoline*. The mixture called *kerosene* is the fraction boiling between 300° F and 500° F. Later fractions are separated into fuel oil, lubricating oils, vaseline, paraffin and possibly petroleum coke or asphalt. These fractions differ not only because their components have different volatilities (ease with which they evaporate), but also different flash points. *Gasoline* takes fire readily. It consists mainly of hexane, C_6H_{14}; heptane, C_7H_{16}; octane, C_8H_{18}, and nonane, C_9H_{20}. These are all fairly volatile so that a lighted match held just above the surface of gasoline ignites it at once. Gasoline is not at all viscous; it flows with ease. Kerosene is a mixture of hydrocarbons with higher molecular weights than those in gasoline, that is, they are less volatile and more viscous. They do not take fire so readily. A lighted match thrust into kerosene is actually extinguished and the kerosene does not take fire. A wick is used in a kerosene lamp to help it evaporate.

Fuel oil containing heavier molecules is still harder to ignite and more viscous. *Lubricating oils* with still larger molecules are very viscous; and it is just this property that makes them valuable. The hydrocarbons from $C_{16}H_{34}$ and up are solids. They are what make up solid paraffin. After the lighter fractions have been distilled off the liquid remaining in the retort is cooled. The solid paraffin comes out of solution and is separated from the liquid by filtration. The filtrate which now has relatively little solid paraffin dissolved in it, is pumped to another still for further treatment.

You notice from the distillation temperatures given above, that the naphtha fraction could be included in the gasoline fraction. As a matter of fact the refiner knows that the names are somewhat arbitrary and actually describe mixtures whose specifications may vary. What is called kerosene one year might some years later be sold as gasoline. This is perfectly proper because gasoline engines are being improved constantly, so that they can operate well on less volatile fractions. The refiner tries to make the maximum amount of the fraction most in demand. Petroleum refining was first commercially practiced to make kerosene. At that time gasoline was considered a by-product and was allowed to drain off into the nearest river. Laws were soon passed to put an end to this dangerous practice. Around 1900 automobiles entered upon the scene. Then gasoline became the desired product, and kerosene the by-product. In 1899, less than 5½ gallons of gasoline and 24 gallons of kerosene were produced per barrel of crude oil; today, 19 gallons of gasoline and only 2 gallons of kerosene are made per barrel of crude oil in the modern refinery.

Cracking of Petroleum Oils. The increase in gasoline yield is secured not only by more careful fractionating to obtain more straight-run gasoline, but also by a decomposition reaction called *cracking;* moreover, the gasoline so obtained is of a type that is more efficient in automobile and airplane engines. The chemist knows that many of the hydrocarbons in the heavier fractions that sell for less than gasoline are straight chain compounds. He knows, for example, that the atoms in $C_{14}H_{30}$ are arranged in a molecular structure like this:



$$\begin{array}{cccccccccccccccc}
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 & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \\
\mathrm{H}- & \mathrm{C}- & \mathrm{C}- & \mathrm{C}- & \mathrm{C}- & \mathrm{C}- & \mathrm{C}- & \mathrm{C}- & \mathrm{C}- & \mathrm{C}- & \mathrm{C}- & \mathrm{C}- & \mathrm{C}- & \mathrm{C}- & \mathrm{C} & -\mathrm{H} \\
 & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \vert & \\
 & \mathrm{H} & \mathrm{H} & \mathrm{H} & \mathrm{H} & \mathrm{H} & \mathrm{H} & \mathrm{H} & \mathrm{H} & \mathrm{H} & \mathrm{H} & \mathrm{H} & \mathrm{H} & \mathrm{H} & \mathrm{H} &
\end{array}$$

But gasoline is a mixture of compounds of about half this size. Why not split the large molecule into two molecules each having seven carbon atoms, or into one six-atom and one eight-atom molecule? The petroleum chemist now uses many methods to accomplish just this. After the straight-run gasoline has been distilled off from the crude oil, the remaining heavy oil is pumped to the cracking unit. Here its temperature is raised to approximately 970° F and its pressure increased to 275 pounds per sq. in. In this way from 50% to 75% of the oil is cracked into gasoline, while the rest decomposes into solid carbon and compounds not suitable for motor car engines. Using silica, SiO_2, or alumina, Al_2O_3, as a catalyst, and a temperature of about 900° F, up to 85% of the oil can be catalytically cracked into gasoline.

It might have occurred to you that more gasoline would also be obtained from crude oil if the smaller molecules, containing from one to five carbon atoms, could be made to combine with one another; the oil chemist is accomplishing this too. He is not only making more gasoline from each barrel of crude oil, but he is producing a better grade of gasoline. He adds hydrogen to the molecules. He gets the atoms to arrange themselves differently as well. The new gasolines contain, in addition to straight chain molecules, some with side chains and some in the form of rings. The better gasolines give more power and do not knock as readily as low-grade gasolines do in high compression engines.

The "knock" in a gasoline engine is caused by too rapid burning or detonating of the gasoline. The gases resulting from the explosion, therefore, expand faster than the piston can move, and the engine knocks. Push an open door to close it. You can push slowly or quickly as you wish and the door closes. Open the door again; now try to close it with a *punch* and see what happens! This gives you some idea of a motor knock. The remedy for knocking is to make a gasoline which, when mixed with air and compressed in the cylinder, ignites only in the region of the spark and then passes the flame along from molecule to molecule relatively slowly. Straight chain hydrocarbons pass the flame too quickly. Side chain molecules are better because they pass the flame along more slowly. Arrange a set of dominoes in a straight line, each piece standing up and no space between. Push the first domino, and the last one falls at practically the same instant as the first. Now arrange the dominoes in a zigzag line. When you push the first, the last one falls over after a longer time interval than when the dominoes were arranged in a straight line.

Ethyl Gas. Using either arrangement, now stand up the dominoes with some space between each and push the first one. This time it takes much longer for the effect of the push to reach the last domino. Tetra ethyl lead, $(C_2H_5)_4Pb$, is a compound which, when added to gasoline in very small amounts (about a thimble-full to a gallon), slows down the rate at which the flame is propagated. Its effect in the gasoline engine is analogous to the effect we get by spacing our dominoes. Tetra ethyl lead, in this way, increases the anti-knock quality or "octane rating" of a gasoline.

Uses of Petroleum Products. Aside from its main use in internal combustion engines, *gasoline* is also used as the fuel in gasoline stoves and gasoline torches. *Kerosene,* too, is used in stoves and kerosene lamps. *Naphtha* is used as a solvent in dry cleaning, although better and less expensive solvents are displacing it. As its name implies, *fuel oil* is burned to furnish heat in factories and other buildings instead of coal. Many large ships are oil burners. *Lubricating oils* reduce the friction between moving machine parts, and thus not only increase the life of the machine by cutting down wear, but also reduce the power required to run the machine. *Petrolatum* is sold as vaseline to be used as an ointment, or as grease for lubricating special machine parts. *Paraffin* is used in sealing preserves and in making candles, wax paper and electrical insulators. The *asphalt* residue left after the distillation of some types of petroleum oils is used in road-making.

SELECTED REFERENCES FOR FURTHER STUDY

ALLCOTT, A. & BOLTON, H. S. Chemistry Today. Oxford University Press.
BLACK, N. H. & CONANT, J. B. New Practical Chemistry. Macmillan Co.
BRAUER, O. L. Chemistry and Its Wonders. American Book Co.
BROWNLEE, R. B., et al. Chemistry in Use. Allyn & Bacon.
BUNZELL, H. H. & NISENSON, S. Everyday with Chemistry. Grosset.
FOSTER, W. Elements of Chemistry. D. Van Nostrand & Co.
FRENCH, S. J. The Drama of Chemistry. The University Society.
HAYNES, W. This Chemical Age. Alfred Knopf.
HOLMES, H. N. Out of the Test Tube. Long & Smith Co.
JAFFE, B. New World of Chemistry. Silver Burdett Company.
MCCUTCHEON, T. P., SELTZ, H. & WARNER, J. C. General Chemistry. D. Van Nostrand Co.
MCPHERSON, W. et al. Chemistry for Today. Ginn & Co.
SCHLESINGER, H. I. General Chemistry. Longmans, Green & Co.
TIMM, J. A. An Introduction to Chemistry. McGraw-Hill Book Co.

CHAPTER IX

MATERIALS OF TRADE AND INDUSTRY

Monroe M. Offner

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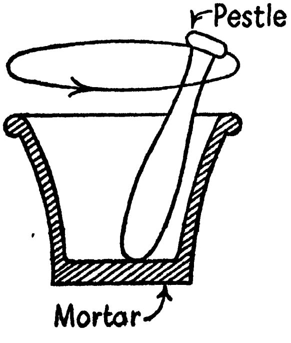
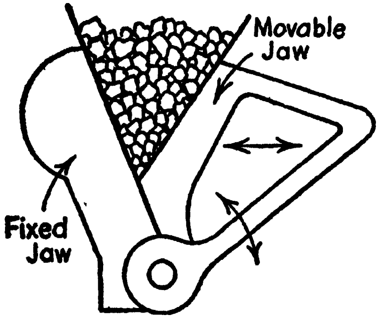
44. CERAMICS AND RELATED MATERIALS

Grinding. Many industries, particularly the cement and brick industries, depend for their success on the efficiency with which they accomplish their grinding operations. It is therefore worth our while to spend a little time on some of the important types of machines that are used. The raw materials that the metallurgist or chemical engineer receives are often delivered to him in the form of large lumps. He has to reduce their size in order to process them efficiently. If two materials are to react to form cement, let us say, they must be brought into close contact. Placing a large limestone rock next to a lump of shale obviously would not do, for the interior portions of each could not react. When, however, the rocks are broken into tiny fragments they can be well mixed, in the desired proportions, and brought into intimate contact. To carry out the flotation process, you will recall, the ore must first be powdered. Superphosphate fertilizer cannot be made unless the sulfuric acid can reach each bit of the phosphate rock, which, for this reason, must first be ground up.

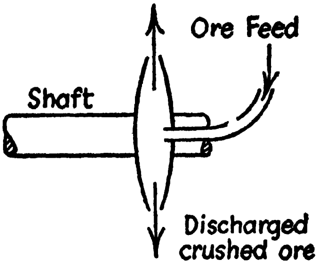


For breaking large pieces of ore, hand sledges or *steam hammers* are sometimes used. To crush lumps as large as 7′×5′ down to 16″ pieces, and 2′ pieces down to 1″ sizes, *jaw breakers* are usually employed. They resemble a huge nutcracker with one fixed jaw and the other, a pivoted movable jaw actuated by an eccentric. A *gyratory breaker* does the same job. It is really the chemist's mortar and pestle, much enlarged, revolved by machine instead of by hand.

Another type of coarse breaker is the *disc crusher*. This consists essentially of two rotating manganese-steel discs, with their circumferences closer together than their centers which are slightly dished. Centrifugal force throws out the crushed ore.

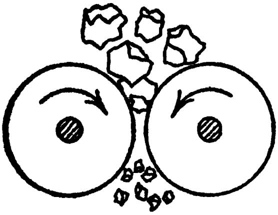


Jaw Breaker

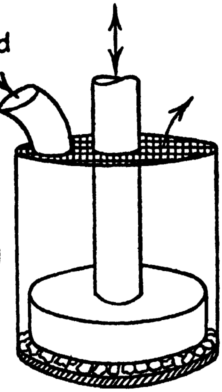


Disc Crusher

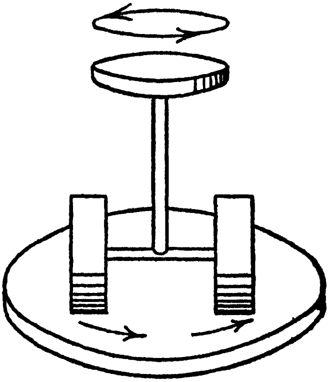
For finer grinding, *crushing rolls* are employed. Two horizontal cylinders of hard metal, parallel to each other and rotating towards each other do the crushing. Pieces no larger than 1½″ comprise the feed which is discharged ground up so that about 80% are between 3⁄25″ and 1⁄150″, in size. The *steam stamp,* the largest and most powerful of all finishing crushers, resembles the mortar and pestle. The stamp is forced up by steam pressure and drops down to crush the ore which is fed to the mortar mixed with water. The top of the mortar is covered by a screen through which the water overflows, carrying with it the suspended particles fine enough to pass through the mesh of the screen.



Crushing Rolls

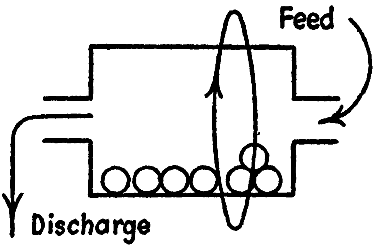


Steam Stamp

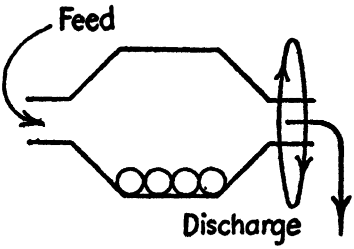


Chili Mill

For still finer crushing, a *Chili mill* is frequently used. It consists essentially of a flat-bottomed circular trough in which heavy rollers travel, crushing by their weight the material over which they run. The *tube-mill,* however, is the commonest device used for fine grinding. Cement clinker, for example, is ground in the cylindrical type. A steel cylinder is rotated on trunnions (hollow axles), while hard pebbles or steel balls inside crush the ore because of the centrifugal force with which they are thrown towards the shell.



Cylindrical Tube-mill



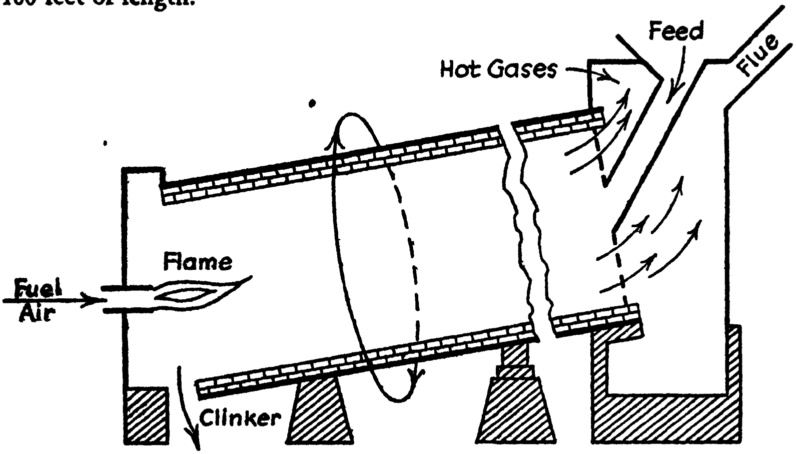
Conical Tube-mill

Cement and Concrete. A person driving along on our modern highways, visiting the enormous dams recently constructed, or inspecting the aqueducts which bring water to our great cities, might imagine that the concrete used in their construction is one of the newer materials of science. But he would be far from correct, for concrete is a mixture of sand, gravel and cement which the ancient Romans, more than two thousand years ago, used in the construction of many of their structures, parts of which are still standing. In the United States we produce in excess of 125,000,000 barrels of cement annually, worth approximately $184,000,000. The amounts produced have never been limited by lack of raw materials, for limestone and shale are widely distributed in this country. At least half of our states manufacture cement.

Raw Materials. Even impure limestone, $CaCO_3$, serves satisfactorily as one ingredient, while clay, or any clay-like rock, or blast furnace slag, serves well as the other necessary ingredient. The cement chemist sees to it that his raw materials are mixed in the proportions that will yield a cement analyzing approximately 64% CaO, 22% SiO_2, and 7% Al_2O_3, with only small amounts of Na_2O and K_2O, and no more than 5% MgO or 3% SO_3. What is of greatest importance is that the amounts of basic anhydrides (lime and magnesia) and the acid anhydrides (silica and alumina) must be almost exactly in the ratio of 2 to 1. This ratio is required to form the complex silicates, $(CaO)_3 \cdot SiO_2$, and aluminates $(CaO)_3 \cdot Al_2O_3$, that later react with water to give strength to the concrete when it sets.

Cement Manufacture. The raw materials are reduced to the necessary fineness in crushers and ball mills which have already been described. Until recently dry grinding was exclusively used. Now the newer plants carry out wet grinding because it is more efficient and greater accuracy in proportioning the ingredients can be attained by mixing the slurries (muds) than by mixing the dry powders. In either case the thoroughly mixed materials are fed into the upper end of a cement kiln.

Cement Kiln. The modern cement kiln consists of a rotary steel cylinder from 100 to 500 feet in length, and from 6 to 14 feet in diameter. Its fire-brick lining, which is approximately 8 inches thick, serves to protect the steel from the intense heat of the interior (about 2800° F). The diagram shows how the rotary cement kiln is set to slope from the feed end down towards the discharge end. The incline is usually about 6 feet for every 100 feet of length.



Rotary Cement Kiln

The effect of the slope and the slow rotation of the cylinder (about 30 rotations per hour) is to move the materials towards the hot end at the rate of approximately 50 feet per hour. A long flame produced by the burning of powdered coal, oil or gas, blasts into the kiln from the low end, protruding towards the upper end for a distance of 25 feet or so. It is in this region that the chemical changes occur which convert the lime, silica and alumina into the desired silicates and aluminates. The temperature is not high enough to convert the materials into liquids, yet it is sufficiently hot to soften them. They drop out at the bottom of the kiln as glowing clinker in pieces about ½ inch in diameter. The largest kilns have a daily capacity of about 1500 barrels of cement clinker. After cooling, the clinker is pulverized. The finer the powder, the stronger the concrete made from it. About 3% of gypsum, $CaSO_4 \cdot 2H_2O$, is added to the cooled clinker during the final grinding to prevent too rapid setting.

Concrete. Water reacts with cement in the same way as it does with plaster of Paris. Concrete, a mixture of cement, sand, gravel and water, therefore, sets by the formation of hydrated, intermeshing crystals. The chemical reactions that take place when concrete sets are shown by the following equations:

$$2(CaO)_3 \cdot SiO_2 + 9H_2O \rightarrow (CaSiO_3)_2 \cdot 5H_2O + 4Ca(OH)_2$$
$$(CaO)_3 \cdot Al_2O_3 + Ca(OH)_2 + 11H_2O \rightarrow (CaO)_4 \cdot Al_2O_3 \cdot 12H_2O$$

The fact that the fineness of the cement has so great an effect on the eventual strength of the final concrete structure, shows that a gelling action also takes place as concrete sets.

For structures requiring great strength, for road building and for waterproof construction, the concrete is usually made by mixing one part of cement, two parts of sand and three parts of coarse aggregate (gravel, stone, pebbles). *Reinforced concrete* is concrete in which steel rods have been embedded to add to its tensile strength. The concrete mix usually used for reinforced concrete work (floors, arches, tanks) is 1:2:4, that is, one more part of stone than is used for the rich mixture first mentioned. For massive foundations a lean mixture (1:3:6) has sufficient strength.

Cement manufacturers carry out research to improve their manufacturing methods, their product, and also ways of using concrete. For example, as a result of recent research, houses are being built, one complete room at a time, by spraying layers of concrete slurry on large inflated balloons. When the room has been completed and the concrete has set, the balloon is deflated and withdrawn through the space left for the doorway.

Bricks. The art of brick making is an ancient one. In the United States we produce about $100,000,000 worth of bricks and related products annually. The raw material for bricks and tiles is clay, which is the residue remaining after the potash of feldspar, $KAlSi_3O_8$, has been removed by weathering. (Feldspar is one of the constituents of granite and other rocks.) The purest clay, called *kaolin,* is described chemically as a hydrated aluminum silicate, with the approximate formula, $Al_2O_3 \cdot 2SiO_2 \cdot 2H_2O$. Ordinary clay usually contains many impurities including sand, calcium and magnesium carbonates, and small amounts of iron and manganese oxides which color it.

Tempering of Clay. It is from such common clay, which is widely distributed, that bricks are made. Shale, a rock that consists of clay, is also used after being pulverized in a pan grinder, which is merely a small Chili mill. Clay must be tempered before it can be molded. Little is understood about the tempering of clay except that it is accomplished by mixing it thoroughly. A small amount of organic matter must be present, for it is known that bacteria play an important part. To make bricks without straw means to work it into the plastic mass desired without the aid of helpful bacteria.

Shaping Bricks. After tempering, the clay is sticky and plastic and ready for molding into bricks. This may be done by hand by pressing it into molds. More frequently the soft clay is placed in a vertical steel cylinder into which a piston fits. The piston forces the clay out of an opening at the bottom end of the cylinder, into molds held below. Bricks made in this way have fairly smooth surfaces and sharp edges, and are suitable for the fronts of buildings. The commonest bricks are formed in two operations. First the tempered clay is fed into a device that looks like a large meat grinder. Instead of many openings, however, there is just one rectangular opening with its dimensions those of the height and width of the bricks to be made. Out comes a long rod of plastic clay, just as shaving cream issues from a squeezed tube. By belt conveyor the clay is carried along to the cutting table. Here wires slice the long column into bricks of the length desired.

Drying. Next the bricks must be thoroughly dried. Open air drying is practiced in some brick factories. Usually, however, the bricks are dried in large rooms through which hot air circulates. Another common drying method is to have the bricks, loaded on cars, enter a long drying tunnel. As one car of dried bricks is pulled out at one end, another car of fresh plastic bricks enters the other end. Whichever artificial drying method is used, it takes about four days for the bricks to become sufficiently dry to be sent to the kilns. Bricks that still retain too much moisture warp or crack when subjected to the high temperature of the kilns.

Firing. The thoroughly dried bricks must next be heated until only the more fusible parts melt to form a glass-like cement that binds the rest together. This process is frequently called "burning," but you know from your study of chemistry that the clay is not oxidized. Firing is carried out in large kilns some of which can hold more than a million bricks at one time. A typical kiln consists of a fire-brick lined structure with one or more furnaces built in the outer wall. The hot gases from the furnace are made to circulate among the bricks so as to heat all of them to the same degree. The common red bricks are heated to approximately 2000° F for about seven days, and another seven days are required to cool them down slowly enough to avoid producing cracked or fragile bricks. To give the bricks a glazed surface, a higher temperature is maintained in the kiln, sufficient to melt just the outer surfaces. Some bricks and sewer pipes are given a glazed surface by throwing salt into the kiln. At the high temperature, salt reacts with the clay to form a sodium-aluminum-silicate glass that fills the surface pores. Another method of glazing calls for a second firing. A clay wash or slip that fuses easily is applied to the once-fired bricks, which are then fired again.

Tiles. The kind of brick produced depends on the kind of clay used. Fire-bricks, for example, are made of fire-clay which is purer than ordinary clay. Tiles are made by mixing several types of clays with definite amounts of feldspar and flint. The wet mixture is pressed into molds, dried, and fired. Wall tiles are glazed by applying the proper slip and firing again. Floor tiles are fired once, but at a temperature high enough to melt the surfaces.

Glass. Excavations of ruins of ancient civilizations reveal the fact that man has made and known glass for about six thousand years. Yet it is only in fairly recent years that chemists have made important improvements in it. During the first World War, when supplies of chemical glassware from Europe were no longer available, American chemists started producing new and better types of glass. American inventors have also designed remarkable machines for automatically forming bottles, electric light bulbs, vials, tubing, sheet glass and even fiber glass. One such machine can turn out thousands of bulbs or bottles every hour.

Soda-lime Glass. The glass most frequently seen in inexpensive drinking glasses, electrical work, window panes and plate glass is made by melting a mixture of sand, SiO_2, soda ash, Na_2CO_3, and limestone, $CaCO_3$. The following reactions take place:

$$Na_2CO_3+SiO_2 \rightarrow Na_2SiO_3+CO_2\uparrow$$
$$CaCO_3+SiO_2 \rightarrow CaSiO_3+CO_2\uparrow$$

Three to four times as much silica as these equations call for, however, is actually added to the glass furnace. The resulting glass, therefore, is not merely a mixture of sodium and calcium silicates. It is a mixture of three mutually soluble compounds. The glass chemist would write the approximate formula of soda-lime glass in this way: $Na_2O \cdot CaO \cdot 6$ to $8SiO_2$.

Analysis of ordinary window glass shows it to be approximately 73% SiO_2, 12½% each of CaO and Na_2O, 1% of Al_2O_3, and 1% of other substances. The manufacture of lime-soda glass is one of our great industries; it produces most of the more than $400,000,000 worth of glass products made in this country annually. The large factories load the raw materials plus as much as 75% of old broken glass into tank furnaces. The tank furnace is a long rectangular trough about 150 feet long, 30 feet wide, and 5 feet deep, capable of holding about 2000 tons. The charge is melted by means of burning producer gas which enters through openings in the side of the furnace above the hearth. The arched roof of the furnace reflects the heat down towards the hearth, just as in the reverberatory furnace used for making wrought iron. The air and producer gas are preheated, using the same principle used for preheating the air blasts of the blast furnace. Instead of allowing the burned gases to escape directly up through the chimney, they are first passed through a brick checkerwork. When this checkerwork has become hot, the exhaust hot gases are switched to another flue and the fresh air and fuel gas are pumped through, thus becoming hot before they reach the furnace. The hot exhaust gases at the same time are flowing through another brick checkerwork to make it hot enough to preheat additional fresh air and fuel gas.

The temperature in the furnace is maintained at around 2500° F. The hearth is made of fire-clay blocks which must be of such composition that they neither melt at this high temperature nor react with the ingredients of the charge. Silica brick is used for the construction of the arched roof. As the glass is removed from one end of the furnace, after passing under a low bridge that skims off any floating impurities, an equivalent amount of fresh charge is added at the other end. These furnaces operate continuously, twenty-four hours a day, never shutting down except for repairs.

Annealing. After molten glass has been shaped as desired, it is allowed to cool and solidify. Right after being shaped, the glass object is still very hot compared to the temperature of an ordinary room. If the glass were allowed to remain in such a room its outer surface would radiate its heat much faster than the interior could cool off. In other words, the outer portions would contract faster than the inside. This would result in interior strains. Glass with interior strains is very brittle, shattering if scratched or tapped sharply. For this reason all glassware is *annealed;* that is, cooled so slowly that all portions, inside and outer, come down to room temperature at the same rate. To accomplish this, the hot glass is placed in an oven having the same temperature as the glass. The oven's temperature is then permitted to go down very slowly until room temperature has been reached. Only expensive glassware is annealed by means of these ovens. Ordinary glassware is placed on a moving platform that carries it through a long tunnel. This *lehr,* as it is called, is hot at the receiving end and cool at the discharging end, and the glass travels through slowly. For small glass objects the lehr may be only 100 feet long, and the annealing time or travel time only two hours. For large objects and plate glass the annealing time may be as much as six hours and the lehr 1000 feet in length.

Window Glass. Most of the window panes for our homes are now made by lowering an iron bar held horizontally, down to the surface of the molten glass, and then raising the bar. The glass adheres to the bar and follows it in the form of a sheet. The bar is pulled over a roller which spreads out the adhering glass train. Formerly window glass was formed by dipping the end of a metal tube into the molten glass. Then the tube was slowly raised as air was blown through it. The result was a long, vertical, cylindrical glass bubble. (A child, softly blowing into a clay pipe as he slowly raises it out of soapy water, achieves the same shaped soap bubble.) The glass cylinder was then cut, flattened out and annealed.

Plate Glass. This higher priced product is produced in smaller furnaces, one batch at a time. The entire molten batch is poured on a flat table. A heavy iron roller irons out the glass down to the desired thickness. The glass, still quite hot, must next be annealed. Rolling alone does not produce perfect surfaces on the glass. Objects viewed through inexpensive window glass often appear distorted. Plate glass must have surface irregularities removed by grinding, a process that is costly and wasteful, usually consuming more than one third of the original sheet.

Plate glass is ground down and polished by placing the sheet on a bed of plaster of Paris on a large, flat, horizontally rotating table. Flat iron discs, also rotating horizontally, but in the opposite direction, are made to press down on the upper surface of the glass. Coarse sand and water fed to the table get under the rotating iron discs. The sand particles are harder than the glass and therefore wear down the high spots. Finer and finer sand is added as the grinding progresses. To complete the grinding emery powder is used. The job must be repeated to make the other side flat and parallel to the first side. A similar machine is used to polish the plate glass surfaces. Here, felt covered discs are used to press down on the glass. Rouge, Fe_2O_3, is employed as the polishing abrasive. Curved glass is made by placing a polished flat glass plate on a mold of the desired curvature. Both are placed in an oven which is then heated very slowly to the temperature at which the glass softens sufficiently to sink into the form wanted. The oven then must be cooled slowly enough to allow the glass to become thoroughly annealed again.

Blown and Pressed Glassware. Ordinary bottles and similar objects are made by dipping a long brass tube into molten glass. The lump of soft glass that adheres to the tube is then held inside a metal, hinged mold which is closed around the glass. Air blown down through the tube inflates the glass until it fits the mold. Some glass objects are made by merely pressing the molten glass into the desired shape by the use of the proper mold.

Skilled glass workers blow chemical glassware into the forms called for without employing molds. They may take machine made tubing of suitable size, heat it until it is soft enough to work and then by blowing and shaping it, construct the desired apparatus. Cut-glass ware is first blown or pressed into the blank form, and then cut and polished by means of abrasive and polishing wheels to produce the design desired.

Lead Glass. For cut-glass ware, radio bulbs, art glass and optical glass, flint or lead glass is used. It contains no lime, but instead runs approximately 45% PbO, 45% SiO_2, and the rest Na_2O with ½% K_2O. Some of the new optical glasses may contain boric oxide, B_2O_3, in place of part or all of the silica, and other metallic oxides in place of the lead oxide. Flint glass is made in small batches; it is kept molten for a long period to insure the escape of gas bubbles, and mechanically stirred to insure uniformity of composition. It is more brilliant than ordinary glass, takes a higher polish, and has a lower coefficient of expansion (does not crack as readily when subjected to sudden temperature changes).

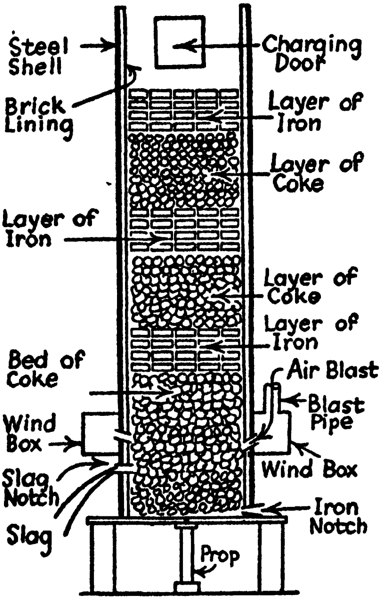
Borosilicate Glass. For laboratory apparatus and in the form of cooking and baking utensils, Pyrex glass is proving to be highly satisfactory. It is much more resistant than soda-lime or lead glass to mechanical shock, to cracking as the result of sudden temperature changes, and to the solvent action of water, dilute acids and alkalies. It requires a higher temperature before it softens. The laboratory worker, therefore, must use a special burner to bend, blow and fire-polish Pyrex glass, whereas the ordinary Bunsen burner flame is sufficiently hot to soften ordinary glass. Borosilicate glass is high in silica, usually over 80.5% SiO_2, 12% B_2O_3, 4% Na_2O, 2% Al_2O, and the rest K_2O, CaO, and MgO.

Colored Glass. All glass contains a trace of iron silicate which gives the glass the green color you see when observing the edge of a pane of glass. Many other metals also form colored silicates. Some metallic compounds decompose in molten glass to free the metal in the form of extremely tiny particles. These colloidal particles impart a color to the glass. Stop lights owe their red color to selenium; copper oxide also makes red glass, and colloidal gold imparts a ruby red. Go lights get their green color from chromium silicate. Uranium is used to make yellow glass. Small amounts of wood or coal give glass an amber color. Purplish shades are obtained by adding a manganese compound. Cobalt makes a blue glass. If large amounts of either of the latter two metals are used, the shade is so dark that the glass appears to be black. Milky glass is made by the addition of either calcium fluoride or tin oxide. Frosted glass is made by sand blasting or by etching the glass with hydrofluoric acid, HF.

45. IRON AND STEEL

Cast Iron. Pig iron and wrought iron have already been discussed in the preceding section (Chapter VIII). Frequently the terms "cast iron" and "pig iron" are used interchangeably. This is unfortunate because it is misleading. It is true that for a few uses the product of the blast furnace, pig iron, is merely remelted and then without any further treatment, cast into the desired forms. Where low cost, bulk, weight and only compression strength are the main requirements, such castings are satisfactory. More properly the term cast iron should be used to describe the product coming from the furnace used to remelt and improve the pig iron fed into it.

Cupola Furnace. In most foundries (buildings in which castings are made) the cupola furnace is used to manufacture cast irons. This furnace is a tall, brick lined, steel cylinder, open at the top and closed at the bottom, with doors that swing down when the prop supporting them is removed. At the side near the bottom is an opening through which the molten cast iron is run off. A little further up on the side is another opening for the removal of the slag. Above the slag notch are openings through which blasts of air are blown into the furnace. Near the top at the side is a door through which the charge is loaded into the furnace.



Cupola Furnace

The charge consists of (1) coke, which serves as fuel; (2) calcium carbonate in the form of limestone, marble or oyster shells, which acts as a flux to combine with the ash from the coke and also to remove dirt on the metal; and (3) pig irons of various types, scrap iron and scrap steel. By using calculated amounts of the many kinds of irons and steels the foundryman has on hand, he can produce a cast iron to suit his purpose.

The furnace is usually started up each day and run until enough cast iron has been prepared to make the day's quota of castings. While the furnace is in operation it is being charged continually. Slag runs out of the slag notch as it reaches the opening. The cast iron is tapped about every quarter hour. At the end of the day, the prop holding up the bottom doors is removed, in order that the material in the furnace can drop to the floor.

Cast iron is cheap, but brittle. Its use in making cast iron pipe, bathtubs, stoves and radiators has already been mentioned. The automobile industry uses cast iron for many small parts of the car, as well as for the engine cylinder block, cylinder head, manifold, water pump case, piston, piston ring, flywheel, and brake drum. In each case, it is necessary to machine the casting to bring it to the proper size.

Malleable Cast Iron. For machine parts requiring greater strength and yet of such complicated shape that casting is the most economical method of producing them, malleable cast iron is used. Cast iron of the proper composition is poured into the molds as before; but the castings after solidifying are given a special heat treatment, without melting them, which makes them completely different products. The castings are packed in iron oxide and heated to between 1300° to 1700° F, and kept at this high temperature for about 60 hours. The heating up and cooling down must be gradual, so that this whole annealing process may take from three days to a week. This treatment results in the removal of carbon from the outer portions of the castings. It also changes the percentage of combined carbon in the inner portions, and causes the carbon that remains in the form of graphite to be distributed as tiny crystals instead of large pieces. The resulting castings are therefore much stronger than ordinary cast iron.

They are used in railroad work for couplers and journal boxes; for parts of agricultural machinery; for pipe fittings and valves. Cheap hammers, hatchets, wood chisels, small bevel gears and skates are also made of malleable cast iron. The automobile industry makes its wheel hubs, differential parts, clutch and brake pedals, bearing caps and spring hangers of this same material.

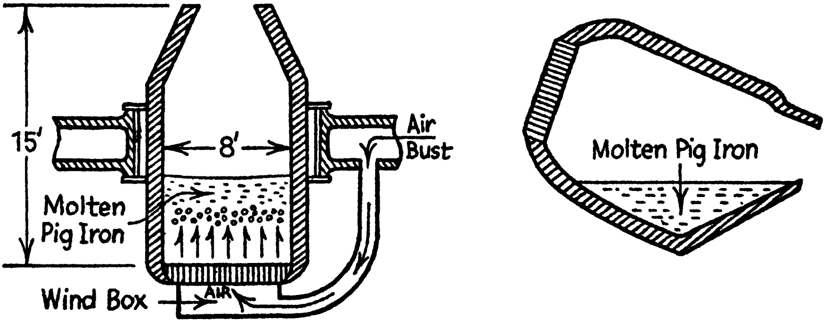
Terminology. *Cast iron* is iron containing too much carbon to allow it to be malleable at any temperature. (But see malleable cast iron.) It is distinguished from steel by its more than 2.20% carbon content. *Pig iron* is cast iron as it runs from the blast furnace or in its solid form as pigs. *Gray cast iron* is cast iron containing so much graphite that the fracture looks gray. *White cast iron* is cast iron having so little graphite that the fracture looks silvery. *Malleable cast iron* is cast iron which after casting has been so treated, without subsequent melting, as to become malleable.

Steel. Shortly before our production was stepped up for defense needs and later for the war effort itself, we were producing approximately 53,000,000 long tons of steel annually. That was less than 65% of our capacity. Since then there has been a tremendous increase in production, and our capacity also has been expanded.

There can be no precise definition of the word steel, due partly to its many varieties; the automobile industry alone has drawn up specifications for more than a hundred different steels, each to meet a definite need. Cast iron and wrought iron cannot be adequately distinguished from steel by chemical analysis alone. For example, to differentiate cast iron from steel by drawing the line at 2.2% carbon, ignores the important effects of mechanical and heat treatment, not to mention the effects of impurities and alloying materials. Steel is probably best defined in terms of its properties. Steel, then, is iron that is malleable either when cold or at some temperature below its melting point, *and also* meets *one* of the following requirements: (1) no further treatment is required to make it malleable after casting; or (2) its hardness increases greatly when it is suddenly chilled from a red heat; or (3) it is both malleable when cast and also capable of being hardened by sudden cooling. Aside from alloy steels, steels are classified as ***mild*** or ***soft steel*** (containing less than 0.25% carbon), ***medium steel*** (containing from 0.25% to 0.60% carbon) and ***hard steel,*** which usually contains from 0.60% to 1.7% carbon.

Uses of Steel. The largest single product, comprising over 30% of the steel mills' output, is *sheet steel.* Automobiles, railroad cars, kitchen cabinets, furniture and tin cans are some of the forms in which we see this product. The Golden Gate Bridge, the longest single-suspension span bridge in the world, has two 750 foot towers which contain 43,000 tons of *structural steel.* Its three-foot thick cables are made of 80,000 miles of 3/16″ galvanized cold-drawn steel wires weighing 21,000 tons. The Empire State Building has a structural steel skeleton that required 50,000 tons of steel. Automatic machines make nails, bolts, nuts and rivets from *steel wire* or *rods.* One such machine can turn out 600 nails per minute. *Steel pipe* and tubes can be made in the seamless variety, or as welded pipe. *Rails* are still being produced mainly for replacement purposes and in much smaller amounts than when our railroad systems were expanding.

Bessemer Steel. The steel age in which we now live began about eighty years ago, when Henry Bessemer in England and William Kelly in this country independently invented the first commercial method for producing steel quickly in large quantities at a low price. They found that almost all of the impurities in pig iron could be eliminated by bubbling air through the molten metal. The furnace "without fuel" in which this is accomplished is called the *Bessemer converter.* It consists of a riveted steel shell supported on two trunnions by means of which it can be tilted through an angle of 270°. One trunnion serves as a windpipe, connecting



Front View
(Upright)

Side View
(Loading Position)

Bessemer Converter

the blast from the blowing engine with the wind box at the bottom of the converter. The furnace has an acid lining; that is, a lining made of a material composed principally of silica, an acid anhydride. The nature of the pig iron produced is such that it forms a silicious slag in the Bessemer converter. This slag does not react with the lining which therefore lasts, with occasional repairs, for approximately 15,000 heats. A basic lining would act as a flux, combine with the slag and so be soon worn out.

The Bessemer Process. About fifteen tons of molten pig iron are poured into the converter tilted to its loading position. The air blast is then turned on and the converter quickly tilted back to its upright position. The air, entering by means of 250 half-inch openings in the bottom, bubbles through the foot and a half of liquid metal until the impurities have been oxidized. Then the converter is quickly tilted again to the horizontal position, and the air blast turned off. Finally, *spiegeleisen* (pig iron containing about 20% manganese and 5% carbon) or *ferromanganese* (an iron alloy containing about 60% manganese and 6% carbon) is added in an amount previously calculated to make a steel of the desired carbon content. The manganese is added to reduce the iron oxide and iron sulfide that are still present. The converter is now tilted still further so that its contents can be poured into the waiting ladle, from which the molten steel is then poured into ingot molds.

The Chemistry Involved. At the moment the loaded converter is tilted to the upright position and the air (at the rate of 20,000 cu. ft. per minute) gushes through the molten pig iron, a violent, rapid oxidation begins. A shower of brilliant sparks (mainly droplets of slag) shoots out of the mouth of the converter during the first three minutes of the blow. It is during this period that the silicon and manganese impurities are being oxidized:

$$\mathrm{Si+O_2 \rightarrow SiO_2(+HEAT)}$$
$$\mathrm{2Mn+O_2 \rightarrow 2MnO(+HEAT)}$$

Some iron, too, is oxidized, but the silicon and carbon quickly reduce it again:

$$\mathrm{2FeO+Si \rightarrow 2Fe+SiO_2(+HEAT)}$$

After the first three minutes an orange-colored flame appears at the converter's mouth. It grows in size and intensity until a roaring, white-hot tongue of flame protrudes thirty feet into the air, while at the same time the shower of slag sparks continues. Soon the flame begins to flicker and shorten. The operator knows that now the carbon has been burned out practically completely (to less than 0.1%). He quickly tilts the converter to the horizontal position and turns off the blast. The entire blow has taken just ten minutes. The carbon has been changed to CO which has burned to CO_2 at the mouth:

$$\mathrm{2C+O_2 \rightarrow 2CO(+HEAT)}$$

Each of the reactions mentioned gives off large quantities of heat. It is for this reason that no fuel has to be added (the impurities act as the fuel). The air for the blast need not be preheated. In fact, the operator sometimes sends in steam with the air in order to reduce the temperature of the furnace which might otherwise get too hot.

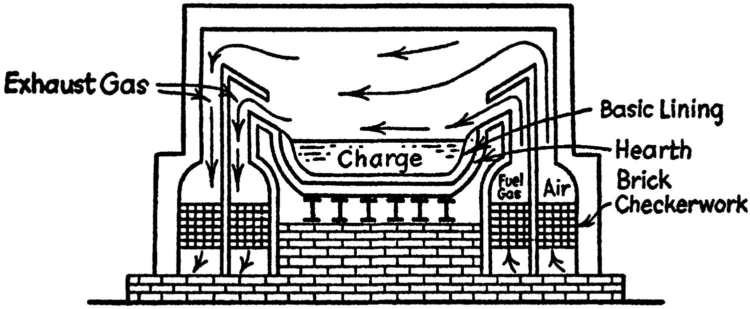
At the end of the ten minute blow, practically all of the carbon, silicon and manganese have been eliminated. The oxides of the latter two have combined to form manganese silicate slag floating on the molten metal:

$$MnO+SiO_2 \rightarrow MnSiO_3$$

The phosphorus and sulfur impurities, however, are still present. As you know, the former tends to make the steel brittle when cold, and the latter tends to make it brittle when hot. The iron ores that produce low phosphorus and sulfur pig irons are growing scarce. Bessemer steel is not very uniform in quality and frequently contains blow holes. For these reasons the Bessemer process has been replaced almost entirely by the open-hearth process. The change-over was slow at first; fifteen years ago 70% of our steel was still being made in the converter, while today only 7% of the steel produced is Bessemer steel.

Open-Hearth Steel. Over 90% of all the steel produced is now made by the open-hearth process. The process takes from ten to twelve hours as compared to the fifteen minutes or so for the Bessemer process. This longer time, however, permits of very careful control through chemical analyses; so that exacting specifications can be met. Besides this advantage of the open-hearth process, it also uses much larger batches, as much as 400 tons, though the usual furnace handles from 100 to 150 tons of steel, as compared to the 15-ton converter.

The Open-Hearth Furnace. The furnace consists of a long shallow hearth enclosed in fire-brick held together with steel. Through ports at the sides



Open-Hearth Furnace

above the hearth, preheated air and fuel enter to pass over the charge and thus heat it up. Using the principle that is now so common in industry, viz., of utilizing all available heat, the hot exhaust gases from the furnace and the cold air for the furnace are alternately passed through a checkerwork of fire-brick every quarter of an hour.

The Open-Hearth Process. The process is begun by placing limestone and scrap iron on the hearth. As soon as this has melted, molten pig iron is added. A usual charge would be 45% scrap iron, 45% pig iron, 10% limestone. The melter with his chemists keeps a constant check on the progress of the reactions. He sees that the temperature does not vary much from 2900° F. At a lower temperature the desired reactions cease; a higher temperature may melt the silica-brick roof. The slag must always be high in lime, CaO, otherwise the hearth lining itself would begin to be used up as the flux and so be lost. Additional limestone may therefore have to be added from time to time, sometimes as much as four times as much as was first loaded. Iron ore, Fe_2O_3, is also added at intervals to supply oxygen in addition to that absorbed from the gases above the hearth. Within 10 to 12 hours, steel of the specific composition, as shown by chemical analysis, has usually been produced.

The Chemistry Involved. Like the reverberatory furnace and the Bessemer converter, but unlike the blast furnace, the open-hearth furnace is an oxidizing furnace. Care must be taken to oxidize out the impurities without oxidizing the iron itself. The silicon, manganese and phosphorus oxidize into acid anhydrides which combine with the calcium oxide to form the slag of calcium silicate, manganate and phosphate. Sulfur may be removed in the form of gaseous sulfur dioxide, or absorbed by the slag as a sulfide or sulfate. The excess carbon is burned to carbon monoxide, which, of course, escapes as the gas. In the Bessemer process all the carbon must be burned out of the steel and then the desired amount added at the end. In the open-hearth process, too, this is sometimes necessary. If the ore happens to be high in phosphorus, then by the time it has been oxidized and taken into the slag, all of the carbon may have been eliminated. To recarburize the steel, the operator adds ferromanganese together with anthracite coal, coke or charcoal.

Steel Ingots. When the molten steel satisfies the melter, a huge ladle is carried by overhead crane to the tap hole at the rear of the furnace, and the molten steel is poured into the ladle. Waiting on flat cars are ingot molds. The molds are usually made of cast iron and are open at both ends. They rest on heavy, cast iron plates. The crane carries the ladle to a position above a mold and when the steel has cooled to the right temperature, a stopper in the bottom of the ladle is raised allowing the steel to flow down into the mold. When the mold is filled, the stopper is replaced and the operation repeated above the rest of the molds until the ladle has been drained. The steel solidifies in the mold in the shape of a large, slightly-tapering, rectangular block and weighs about five tons. Because of the tapered shape of the ingot, it is a simple matter to raise the mold and leave the ingot standing on the plate. The ingots are stripped, as it is called, as soon as they have solidified sufficiently. The outside portions are much cooler than the interiors and therefore, internal strains result.

Soaking and Rolling. For this reason among others, the ingots are carried by crane to large pit furnaces called soaking pits, where they are carefully reheated until each ingot has become uniformly hot throughout. A red-hot ingot requires approximately an hour in the soaking pit which is maintained at 2000° F. An initially cold ingot needs six to eight hours. Huge tongs lift the sufficiently soaked ingots from the pits. During the soaking, impurities have risen to the top portion of the ingot. Sometimes as much as half a ton of steel must be cut from the top of the ingot to make sure that only sound steel goes into the finished product. The faulty steel is returned to the open-hearth furnace while the rest of the orange-hot ingot goes to the *blooming mill.*

Here it is squeezed between heavy steel rolls that operate as do the rolls of a clothes wringer. The ingot passes back and forth as the direction of rotation of the rolls is changed and their distance apart gradually decreased. By turning the metal on its side after each trip, a long rod with a square cross-section is produced. This is then cut up into blooms of convenient lengths. Blooms can be rolled into rails or structural shapes by using specially shaped rolls. They are also rolled into rods which can then be drawn into wire. Slabs instead of blooms are rolled when plate or sheet steel is the desired product.

Forging and Pressing. It is obvious that rolling cannot be used to produce crankshafts, connecting rods or even gear blanks. Odd-shaped objects like these, or heavy axles and battleship guns, are forged. *Forging* means the hammering of red-hot or white-hot metal into shape. (Usually the steel must be machined subsequently to bring it to the finished form.) The blacksmith hammers his wrought iron or soft steel rod on the anvil to make a horseshoe. For larger objects, like a seventy-foot gun forging, various types of machine hammers are necessary. *Drop forging* is used when many duplicates of standardized parts are desired. Drop forging consists in hammering the white-hot steel into dies. Presses of various sizes instead of hammers are used to shape armor plate and automobile parts like crankcases, axle housings, body panels and fenders. One of the largest steel castings ever produced is the top cap for a 12,000 ton armor-plate press. This top cap alone required the pouring of six hundred thousand pounds of steel!

Crucible Steel. Steels of special composition for high-grade tools and surgical instruments that are needed in relatively small quantities, are still being made by the almost obsolete crucible process. The furnace proper is below the floor in which there are holes through which crucibles can be lowered. Made of refractory materials (fire-clay, graphite, mixtures of these, or other infusible substances), the crucibles are about a foot in diameter and a foot and a half high. They hold approximately 100 pounds of metal. The crucible is loaded with wrought iron, scrap steel, charcoal and other materials in amounts calculated on the basis of chemical analyses to yield the desired steel. Experience has taught the steel maker just how much carbon the steel will absorb from the crucible itself, and he naturally takes this into account as well. The covered crucible remains in the furnace for about four hours. It takes three-quarters of this time for the metal to melt, with the refining proper occurring during the last hour. It is during this time that the slag originally present in the wrought iron is rising to the surface, and the carbon, manganese, silicon and other added elements are being absorbed by the steel. When the steel is finished, the crucible is pulled up out of the furnace by means of long-handled tongs. The steel is carefully poured into molds, and solidifies into ingots usually 3″×3″×2′ in size.

Electric Steel. Approximately 1½% of the steel being made just before the war was produced in electric furnaces. *Electric steel* has now almost completely replaced crucible steel because the electric process of producing high quality steels, including stainless and alloy steels, has the following advantages:

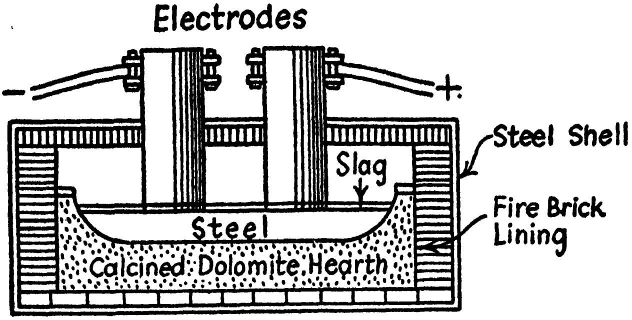
(1) Over a very wide range, the temperature can be maintained quite closely at any desired point. In this way sulfur can be reduced to a lower percentage than by the older methods. Enmeshed slag and dissolved gases are eliminated by the heat alone, so that it is not necessary to add manganese or other elements to accomplish the same task. The result is a steel stronger by almost 10,000 lb. per sq. in. than ordinary Bessemer or open-hearth steels.

(2) No impurities enter the steel from the fuel, since none is used; all the heat is supplied by electrical means.

(3) Oxidation can be avoided since no air is introduced into the furnace. If desired, a reducing atmosphere can be maintained.

(4) Instead of a 100-pound batch, as in the crucible process, the electric furnace can be designed in sizes to handle from 2 to 75 tons of steel, though the usual furnace has a capacity of approximately 10 tons.

The Electric Refining Process. The type of electric furnace most commonly used is the Héroult. It consists of a steel cylindrical shell, lined with refractory bricks (silica for the roof and magnesite for the rest). The



Heroult Electric Furnace

hearth is made of calcined dolomite, a rock composed of calcium and magnesium carbonates. Two huge (15″ diameter) carbon electrodes extend through the roof into the slag. The current (more than 10,000 amperes) enters through one electrode, arcs through the slag, then passes through the steel in the hearth and arcs up through the slag to the second electrode. The electricity is converted into heat as it flows through the steel, just as it is in the resistance wires of an electric toaster. The steel to be refined is usually poured into the furnace as a liquid. Materials to make a slag rich in iron oxide are then added; this slag serves to oxidize and remove the phosphorus, and, having done this, is skimmed off. Now a new slag running high in lime is formed and carbon is added; this floats on the slag and reduces it. The reduced slag in turn reduces remaining oxides in the steel. It is only after this has been done that the sulfur in the steel is absorbed by the slag. If an alloy steel is being prepared, the alloying elements are now added. When the steel has been brought up to its specifications, usually after about four hours, the current is turned off, and the steel poured by tilting the furnace.

Cast Steel. Originally, cast steel was the trade name for crucible steel. Now it means steel which has been poured into molds of the desired forms instead of into ingot molds. The only important operating difference between the two pourings is that a higher temperature is employed when castings are made than when ingots are prepared. For large castings, the metal is used just as it comes from the Bessemer or open-hearth furnace. For smaller castings, electric furnaces or small melting furnaces are used in the foundry. The melting furnaces resemble small editions of the large steel furnaces. Machine parts that will be subjected to strains, shocks and vibrations too great for cast iron to withstand are made of cast steel.

46. ALLOYS

Nature of an Alloy. When a *metal* is melted and then *mixed* with one or more *other metals that dissolve in it,* an *alloy* is formed. However, not all metals are miscible (mutually soluble). Some metallic mixtures separate into distinct layers when the molten mixture is allowed to solidify. Such mixtures are not alloys. An alloy should be regarded as a solid solution; like a true solution, it must be a *uniform mixture,* the parts of which do not separate on standing. Instead of a second metal, a non-metal like carbon or silicon can be the alloying substance. Even a compound like iron carbide, Fe_3C, can be one of the constituents of an alloy. An alloy, therefore, is a uniform mixture of one or more metals which may also contain one or more non-metals and one or more compounds. Since steel always contains carbon, it, too, is an alloy. Alloys are manufactured because they have desirable properties not possessed by the individual constituents. For example, alloys can be made that melt more easily than their constituent metals; or that are harder, or more ductile, or more flexible.

Alloy Steels. A steel which owes its improved properties chiefly to some element other than carbon is called an alloy steel. A small amount of silicon (1/10% to 2/10%) is sometimes added to steel to rid it of blowholes. Manganese is always added to Bessemer and open-hearth steels in amounts up to 1½% to get rid of oxygen and to counteract the effect of sulfur. But in neither case do we speak of the product as an alloy steel. Only if the added element itself gives a new property to the product is the latter called an alloy steel. There are many alloy steels, but lack of space will permit a discussion of only the more important ones.

Manganese Steel. This alloy steel usually contains from 10% to 11% manganese. It is non-magnetic and so hard that it cannot be drilled or machined, but must be ground. It can be forged only with great difficulty. For these reasons, manganese steel must be cast into the form required. It is used for burglar-proof safes, jaws of rock-crushing machinery, steam shovel buckets, rails for crossings, and wherever else a hard, tough steel is called for.

Nickel Steels. There are a number of nickel steel alloys whose properties differ depending on the percentage of nickel they contain. Approximately 3½% nickel in steel increases its strength considerably; its hardness goes up also, but not enough to make machining too difficult. This alloy is used for structural work on bridges, for rails, marine shafting and steel castings, and is one of the principal alloys used in the automobile for gears in the transmission and rear axle. The 36% nickel alloy is known as *invar.* Its coefficient of expansion is practically zero. In other words, an invar steel measuring tape is no longer on a hot day than on a cold day; its length is *invar*iable. Other scientific measuring devices and clock and watch parts are made of this alloy. A wire that does not expand and contract at the same rate as glass cannot be sealed firmly in glass. The nickel steel alloy called *platinite,* which contains from 42% to 45% nickel, has the same coefficient of expansion as glass. For this reason it is used in so-called armored glass, the glass into which a network of nickel steel wire has been embedded. It has also replaced platinum for use as the lead-in wires that must be fused in through the glass at the base of electric light bulbs.

Silicon Steel. Approximately 3% silicon gives to steel remarkable magnetic properties. Silicon steel makes excellent electromagnets, and the electric generators and motors in which it is used have very high efficiencies. *Duriron,* the trade name for a silicon steel containing about 13% silicon, is used for pipes, valves and elbows in industries handling acids that would quickly corrode ordinary steel. It can be cast, but is so hard that it must be ground instead of machined. Automobile leaf springs are sometimes made of a 2% silicon steel.

Chromium Steel. A very hard, tough alloy steel, suitable for armor plate and armor-piercing projectiles, for crushing machinery, ball bearings, files and cutting tools, can be made by the addition of 1% to 2% of chromium to steel. When nickel is also added, a strong alloy suitable for automobile gears and axles is produced. With silicon added the chromium steel formed resists heat and corrosion to a high degree, so that it serves well in exhaust and intake valves that must operate while red hot. *Stainless steels* used in cutlery and surgical instruments, laundry and milk bottling machinery, may have chromium as the chief alloying element. The usual stainless steel, however, contains 18% chromium and 8% nickel. The glistening new stream-lined trains, stainless steel pots and pans, ornamental trim on buildings and many other objects that stay bright in spite of heat and the many corrosive agents in the air are all made of this alloy. It is sold under many trade names, including "Allegheny metal" and USS18-8.

Molybdenum and Vanadium Steels. Both vanadium and molybdenum impart additional strength to steel. Molybdenum improves the nickel steel used in automobile gears. Chrome-vanadium steels are also used for transmission and rear axle gears, and for leaf springs as well.

High Speed Steel. When ordinary steel is used to machine metals at a high cutting speed, the tool edge becomes dull very quickly. This is due to the fact that carbon-steel softens as a result of the high temperature caused by rapid cutting. The invention of high speed steels started a new epoch in the tool-steel industry. Work could be turned out much more quickly and the cost of machining was greatly reduced. A number of metals alloy with steel to produce a steel which is naturally hard. It does not require heat treatment to make it so, nor can it be made soft by any treatment. It therefore "keeps its temper" even at red heat; it retains its cutting edge for a long time, which means that time consumed in resharpening is reduced to a minimum. When added in the proper amounts, chromium, tungsten, vanadium, molybdenum or manganese converts steel into this self-hardening, high speed steel, which has played one of the most important roles in making large scale automobile production possible.

Non-ferrous Alloys. Besides iron alloys, the carbon and alloy steels, there are a vast number of alloys that contain no iron, the non-ferrous alloys. These, too, help the wheels of industry go round, and while somewhat less important than steel in our present civilization, they are nevertheless worth knowing about. Including the alloy steels as well as the non-ferrous alloys there are something like two thousand alloys available. Many of these are used for very special purposes. Brasses, bronzes, and aluminum alloys are all used in engineering work. Only some of the more common non-ferrous alloys will be discussed here.

Bronzes and Brasses. Copper is the main constituent of a large number of useful alloys. The bronzes all contain copper and tin, and sometimes zinc or other elements. Bronze is harder and has greater tensile strength than copper. Its resistance to corrosion is much greater than that of steel. Bronze castings are therefore used in place of steel castings for ship propellers and for parts of machines used in paper making and in other industries where rust spots would damage the products.

Brasses are copper alloys containing zinc and sometimes other elements to modify some property. They are not as strong nor as flexible as copper, but have a lower melting point and can be more easily cast and machined. The table below lists some of the hundreds of copper alloys. It should be remembered that the commercial names are by no means scientific. For example, there is another alloy of copper, containing 1% P, 10% Pb, and 10% Sn, used for bearings and pumps, which, though very different from the alloy called phosphor bronze in the table, is also designated by the same commercial name.

German Silver. This is the name given to a number of alloys which can be regarded as brasses in which part of the copper has been replaced by nickel. It is also known as *nickel-silver.* The alloy has a silvery color, is highly resistant to corrosion, has high electrical resistance, gives strong, tough plates and wires, and sound castings. Due to its nickel content it is more expensive than brass. It is used for tableware, even when not silver plated. It is used as a resistance wire in rheostats, but not as a heating element, for at high temperature it deteriorates.

BRASSES AND BRONZES

Commercial Name	Percentage Composition				Typical Uses
	Cu	*Zn*	*Lead*	*Tin*	
Commercial Bronze	90	—	—	10	Automobile radiators; wire screens; wherever exposed to atmospheric corrosion.
Red Brass	80	20	—	—	Machinery; highly resistant to corrosive action.
Brazing Brass	75	25	—	—	Fabricated articles requiring brazing.
Commercial Brass	65	35	—	—	Household gadgets; lamp fixtures; ornamental objects; indoor work.
Muntz Metal	60	40	—	—	Bolts; nuts; wire; etc.
Yellow Brass	62	35	3	—	Automatic machine work.
Tobin Bronze	60	39.25	—	0.75	Bolts; nuts; plates; piston rods; propeller shafts.
Phosphor Bronze	92 (+small % P)	—	—	8	Radio parts; electric switches and contact fingers.
10% Aluminum Bronze	90	—	—	—	Parts requiring general strength and resistance to corrosion and wear.
Beryllium Copper	97.4	Ni 0.35	Be 2.25	—	Springs and diaphragms; wherever high resistance to impact and fatigue is desirable.

GERMAN SILVER

(Also called Nickel-Silver)

Typical Commercial Alloys:			
Cu	*Ni*	*Zn*	*Uses*
47%	30%	23%	Tableware
75	20	5	Strong castings (steam fittings)
54	16	30	Small screws; bolts; small mechanical parts.
52	25	18 (5% Fe)	Resistance wire for rheostats

Bearing Metals. These alloys have been devised to provide in one bearing a hard constituent to resist wear and to present a surface with a low coefficient of friction, and a soft constituent to distribute the load uniformly in order to prevent over-heating and binding. The axle that runs in these bearings suffers very little wear, whereas the bearings must be renewed occasionally. Below are given typical bronze bearing alloys:

BRONZE BEARING ALLOYS

Type	Percentage Composition *Cu*	*Al*	*Sn*	*Pb*	*Ni*	*Zn*	Spec. Grav.	Brinell Hardness	Typical Uses
A	87.5	—	11.0	1.5	—	—	8.8	68	Gears; spur gears; worm gears; feed nuts for working tools.
B	84.5	—	10.0	2.5	3.5	—	8.8	87	Heavy duty services; valve-stem guides; tools.
C	80.0	—	10.0	10.0	—	—	8.9	63	Lathes; grinders; cross-head and crank pin bearings in general machines; high speed tools.
D (Also called Lumen Bronze)	10.0	4.0	—	—	—	86.0	6.9	115	Pivots, bearings and bushings on bridges and cranes.

Below are given typical Babbitt alloys used for bearings and journals:

BABBITT METALS (*White Bearing Alloys*)

Type	Percentage Composition *Cu*	*Sn*	*Sb*	*Pb*	Spec. Grav.	Brinell Hardness	Melting Point (F°)	Typical Uses
A	4.5	91.0	4.5	—	7.3	17	430°	Crank-pin bearings in steam and gas engines.
B	8.0	84.0	8.0	—	7.5	27	465°	Where heavy duty bearing service is required.
C	3.0	75.0	12.0	10.0	7.5	24	365°	Machine bearings, machine tools, main engine bearings.
D	—	—	15.0	85.0	10.3	15	470°	Light service bearings; line shafting; against soft steels.

Fusible Alloys. To meet the need for a metal that will melt at a low temperature and yet have sufficient mechanical strength to meet its other requirements, a number of interesting alloys have been devised. The two most common of those that melt below the boiling point of water are *Wood's metal* and *Rose's metal.* They are used in automatic fire alarms, sprinkler systems, fire door hooks, and as electric fuse wires. Wood's metal melts at 149°F and is composed of 50% Bi, 12½% Sn, 12½% Cd, 25% Pb. Rose's metal melts at 203°F, and is composed of 48.9% Bi, 23.6% Sn, 27½% Pb. The tables below give the typical compositions and melting points of some of the commonest types of *solders:*

LEAD AND TIN SOLDERS

Type	Percentage Composition *Sn*	*Pb*	*Sb*	Melting Point (F°)
A	100	—	—	450°
B	63	37	0.12	358°
C	40	60	0.12	358°
D	31	67	2	370°
E	—	100	—	620°

SILVER SOLDERS

TYPE	PERCENTAGE COMPOSITION *Ag*	*Cu*	*Zn*	*Cd*	MELTING POINT (F°)
A	10	52	38	(trace)	1510°
B	20	45	30	5	1430°
C	50	34	16	—	1280°
D	80	16	4	—	1360°

Monel Metal. This alloy is meeting with competition from the newer stainless steels. Like them, it is strong and tough and resistant to corrosion. It is used in the construction of tanks, pumps, valves, etc. in industries handling acids and alkalies. Equipment for hospitals, restaurants and household kitchens are built of this attractive, easily cleaned alloy. Monel metal usually contains from 65% to 70% Ni, 26% to 30% Cu, up to 3% Fe, up to 1½% Mn, and up to ¼% each of Si and C. (*Mond "Seventy" Alloy* is one of the trade names for monel metal.)

Lightweight Alloys. Almost as strong as steel but with less than half its density, resistant to corrosion and easily worked, *duralumin* is used extensively in airplane and dirigible construction. It is composed of 95% Al, 4% Cu, ½% Mn, and ½% Mg. *Magnalium* is another important aluminum alloy that contains from 5% to 30% magnesium. It, too, is light, tough and strong. Unlike pure aluminum, magnalium is easily machined. It is used in airships, chemical balances and other scientific instruments. Only in comparatively recent years have we begun to produce much magnesium in this country; it is now being made from the magnesium chloride in sea water. With increased production its price is certain to go down. The price of *Dow metal* (the trade name for magnesium alloys) will go down too, and it will find more extensive use in airplane, automobile and household-appliance construction. One Dow metal alloy (91.7% Mg, 8% Al, 3/10% Mn) used for machine parts after the sand casting has been heat-treated is typical of the group. They all contain magnesium (85% to 98.5%) with varying amounts of Ni, Cu, Cd, Mn and Al. Dow metal can be easily worked, has fair strength, and is the ***lightest structural alloy*** on the market. Its density is less than one-quarter that of steel.

The Hardest Alloys. Harder than the self-hardening steels already mentioned, there are alloys of cobalt that are ideal for use as the cutting tips for high speed machine tools. They find use also as die facings. *Stellite* is the trade name for a group of such alloys. One stellite contains 75% Co, 16.5% Cr and small amounts of W, Mo, Ni, Fe, C or Si. Another typical stellite is composed of 45% Co, 30% Cr, 15% W and small amounts of

Mn, Fe, C or Si. Another significant alloy is *carboloy*, the hardest of all known alloys. It is a mixture containing from 80% to 97% of tungsten carbide held together by cobalt. The two ingredients, tungsten carbide crystals and powdered cobalt, are subjected to tremendous pressure while heated to just below the melting point of cobalt. For this reason the product is called *cemented tungsten carbide*, its trade name in this country being carboloy. Cemented tungsten carbide is close to diamonds in hardness and is used for the tips of high speed cutting tools, and for wear-resistant dies.

47. MISCELLANEOUS MATERIALS

Abrasives. An abrasive is a substance used for wearing down surfaces by cutting, grinding or polishing. The sand on sandpaper is an example of a familiar abrasive. There are many kinds of abrasives, some that are found in the earth, or natural abrasives, and others that are manufactured, or synthetic abrasives. Before discussing them, however, it is worth noting the extent to which abrasives are used in industry and the kind of work they accomplish.

Abrasives are used in the wood working industries for finishing furniture, etc.; in the stone industries for cutting, carving and surfacing marble, granite, etc.; in the leather industry, for finishing shoes; in the paper industry, for grinding pulp; and in the glass industry, for surfacing and bevelling plate glass. Gems, pearl buttons, and plastic objects are shaped and polished by abrasives. Cutting tools and knives are sharpened on abrasive stones. Precision grinding machines and grinding wheels make possible the mass production of interchangeable machine parts which must not differ from each other by more than a fraction of a thousandth of an inch; by this means the use of hardened steel crank pins and other wearing machine parts is made possible. To illustrate the speed and accuracy with which precision grinding can be carried out, consider the following example: in ten hours, with one Carborundum wheel, it is possible to turn out 50,000 quarter-inch steel balls for ball bearings, with a variation in size of only one-fourth of one ten-thousandth of an inch! Abrasives incorporated in floor surfaces such as tile or concrete and even in steel treads have done much to reduce the number of accidents caused by slippery floors and steps.

Diamond. The hardest substance known is diamond. That is to say that only diamond can make a scratch on diamond, but it can cut into the surface of any other substance. Diamond is crystallized carbon. You may recall that sulfur has three allotropic forms: rhombic, prismatic and amorphous. So, too, carbon has three allotropic forms: graphite (the "lead" of a pencil), diamond and amorphous (charcoal, coke, lamp black, etc.)

Diamonds are usually prized for their ornamental value, but their greatest worth lies not in their beauty but in their hardness. By means of diamond drills (drills with diamonds set in their cutting edges), wells can be bored through the hardest rocks to great depths in the earth. Because of the high prices commanded by colorless or faintly bluish diamonds, and even "off-colors" tinged with yellow, brown or green, only tiny chips or dust from these are used as abrasives. Diamonds whose crystal structure is such that no gems can be cut from them, or whose color is unattractive, are called *borts* and are used for polishing precious stones. Most diamonds come from South Africa, although they have been found in small amounts in other places including our own country. From Brazil comes the black diamond called *carbonado.* Its color bars it from the jewelry field, so that it is used in drill points, for grinding-wheel dressers and saw teeth for cutting stone.

A number of chemists have tried to produce artificial diamonds and several have succeeded. Moissan, a French chemist, made diamonds by thrusting charcoal into the center of a crucible of molten iron, and then dropping the mass into a barrel of water. The sudden solidifying and contracting of the outer surface caused tremendous pressure to be exerted on the interior. The iron was then dissolved away in acid, and extremely tiny diamonds were found. Others have tried, but they too found that the diamonds produced were hardly worth the price of the acid consumed.

Corundum. With the exception of diamond, *corundum* is the hardest of all minerals. It is crystallized aluminum oxide, Al_2O_3. When clear or translucent and colorless, it is the gem called white sapphire; when blue, it is known as sapphire, and when red, as ruby. When not of gem quality, corundum is used as an abrasive. In this country we have some important deposits which, however, are not being worked because artificial corundum is more easily prepared. *Adamantine spar,* which occurs in coarse crystals or masses and in granular form, is slightly translucent corundum varying from blue to black in color. *Emery* is opaque, granular corundum, always containing hematite or magnetite, and is usually dark gray in color. Both of these latter forms of corundum were formerly important abrasives, thousands of tons having been used in grinding and polishing glass, gems and metals; they have now been displaced, however, by artificial abrasives.

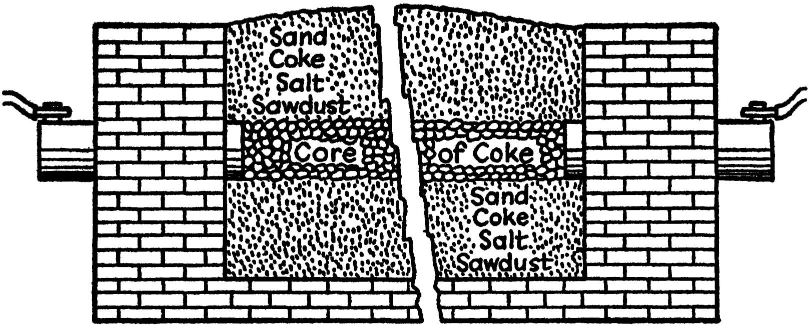
Garnet. This mineral also is found in beautifully colored crystalline forms. In such forms (usually red, but appearing in all colors except blue) it is used as a gem. The common garnet that is used as an abrasive is a complex silicate whose formula might be written $Ca_3Al_2(SiO_4)_3 \cdot Ca_3Fe_2(SiO_4)_3 \cdot Fe_3Al_2(SiO_4)_3$. Important deposits are located in New York, Georgia and North Carolina and in Central Europe. Garnet is considered a relatively soft abrasive, being unfit for grinding iron and steel, but satisfactory on paper or cloth for finishing hardwoods.

Silica Abrasives. Silica, SiO_2, is widely distributed all over the earth. Sand is practically pure silica. The large crystals are called *quartz.* An impure variety of quartz, gray in color and occasionally containing water of hydration, is called *flint.* It is found as translucent nodules in chalk-beds. When ground and graded it is the abrasive used on good quality sandpaper. It is usually almost as hard as garnet. *Sandstone* is a rock formed under pressure by nature, composed of quartz particles cemented together by feldspars, clays, carbonates and other minerals. The quartz grains are the abrasive in the grindstones and other sharpening stones cut from it.

Silicon Carbide. The story of manufactured abrasives begins in 1891 with the experiments of Acheson. He had worked with Edison in perfecting the carbon filament in the original incandescent lamp. This work suggested to him the possibility of using electricity to produce a crystalline form of carbon like diamond. He placed a mixture of clay and powdered coke in an iron pot such as plumbers use for melting solder. This pot he attached to one lead from a dynamo. He connected the other lead to a carbon rod which he inserted into the clay-coke mixture in the pot. He started his dynamo which sent enough current through the mixture to melt it and raise it to a very high temperature. After a time the mass was allowed to cool. On examination it was found to contain a few bright specks. Acheson tested these tiny crystals. They cut glass like a diamond and were, in fact, almost as hard as diamond. He guessed that the crystals were composed of carbon and corundum and coined the name *carborundum* for them.

He was wrong about the composition of his newly created abrasive. Carborundum is a compound of silicon and carbon, properly called silicon carbide, SiC. The Carborundum Company located at Niagara Falls where electric power is cheap, now manufactures silicon carbide in large electric furnaces. Acheson was able to get $880 a pound for it from jewelers and lapidaries in 1891, but due to large scale production, carborundum now sells for about 15 cents a pound.

The electric furnace is of the resistance type. It consists of a rectangular brick structure 45 feet long, 10 feet wide, and 10 feet high. The ends of the furnace which support the electrodes are permanent. To start a run, the furnace is half filled with a mixture consisting of sand 56%, coke 35%, sawdust 7%, and salt 2%. Then a three foot wide, one foot deep trench is formed from end to end in the mixture. Next, the trench is filled with granular coke which acts as a high resistance conductor between the electrodes. Finally, the other half of the charge is loaded into the furnace, completely surrounding the coke core, and the current is turned on.



Carborundum Furnace

The furnace consumes 3000 horse power and produces 16,000 pounds of silicon carbide during its 36-hour run. The current of 10,000 amperes passing through the core, heats it to incandescence and so raises the temperature of the charge to between 3000°F and 4000°F, within which temperature range the reaction occurs. The reaction is made clear by the following equation:

$$SiO_2+3C \rightarrow SiC+2CO\uparrow$$

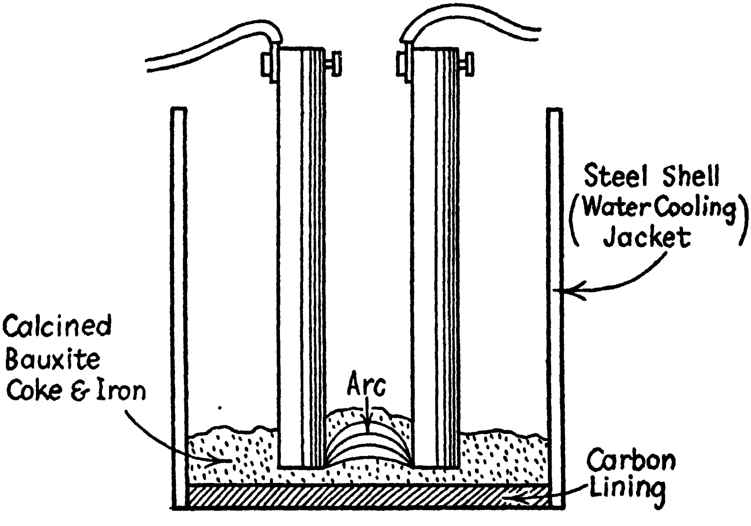
If heated above 4060°F, the silicon carbide decomposes, the silicon going off as vapor, and the carbon remaining behind in the form of artificial graphite. The sawdust is used to make the mixture porous enough for the carbon monoxide to escape easily. The salt converts impurities present in the sand and coke into volatile (easily evaporated) chlorides. After the reaction, the furnace is allowed to cool, the side walls are torn down, and the outer layer of uncombined mixture is removed. Clustered around the core are found the sharp, iridescent, purplish crystals of silicon carbide. The lumps of loosely adhering crystals are unloaded and shipped to the crushing plants.

Here it is crushed through heavy rolls and refined by washing, magnetic concentration and chemical treatment with boiling sulfuric acid and then with caustic soda solution. This chemical treatment removes graphite, silicon and silicides of iron and aluminum. After another washing, the grains are graded by means of classifiers, screens, water flotation and other methods into sizes from No. 6 mesh down to the finest powders. Now it is ready for use for metal or stone polishing, or for further fabrication into abrasive paper and cloth, grinding wheels and sharpening stones.

Silicon carbide abrasives are hard and sharp, but somewhat brittle. They serve well for the grinding of materials of low tensile strength like cast iron, brass, bronze and aluminum, for the finishing of leathers, and for glass beveling. Silicon carbide abrasives appear on the market under many trade names: Carborundum, Electron, Staralon, Carbolon, Carbonite, Carbolite and Crystolon, etc.

Crystallized Alumina. Acheson's success stimulated further research, and in 1899 Jacobs produced from bauxite the first manufactured corundum. Bauxite, you remember, is the ore from which aluminum is prepared. It is a very soft rock composed of alumina, Al_2O_3, with from approximately 15% to 35% of water of hydration, and contains as its principal impurities silica and oxides of titanium, iron and sometimes vanadium.

In the manufacture of crystalline alumina, the bauxite is first calcined to drive off its water of hydration, and then mixed with coke and iron borings. The usual proportions of the mixture are, roughly, 20 parts of calcined bauxite to one part each of coke and iron. This mixture is fed into a cylindrical arc-type electric furnace which consists of a steel shell 7½ feet in diameter and 6 feet deep. The bottom is carbon-lined and the sides are water-cooled. Through the open top two electrodes extend into the furnace. The current arcs across the electrodes, in this way supplying the heat to melt the mixture which is dumped in, a small amount at a time. As soon as the mixture in the furnace has melted, more raw material is added, and so in steps the entire charge is finally converted into a liquid mass. During the run which takes about the same time as a carborundum run. the iron, titanium and silica impurities have been converted into silicides which settle to the bottom. After the current has been turned off,



Artificial Corundum Furnace

it takes the ingot about 7 hours to cool and solidify. The alumina is now in the form of a tough, hard crystalline material analyzing about 96% alumina. The impurities at the bottom are easily separated and the alumina itself is crushed and graded.

Though not as hard as carborundum, artificial corundum is less brittle and tougher. For this reason it is used for grinding materials of high tensile strength like hard steels. Crystalline alumina is marketed under a number of trade names, including Federal Aluminous Abrasives, Aloxite and Alundum.

Grinding Wheels and Stones. Abrasives are used as separate loose grains for some types of grinding and polishing; attached to paper or cloth for other jobs; and bonded into porcelain, sodium silicate, shellac, rubber or one of the synthetic plastics for still other operations. By varying the amount of bond used, a grinding wheel suitable for grinding steel castings or one suitable for grinding tempered steel can be made. To make a vitrified grinding wheel, abrasive grains are first mixed with clay, feldspar, flint or other ceramic materials and then molded into shape by pressure. The wheel is next heated for about five days in a kiln held at about 2400°F, at which high temperature the clay bonding material becomes vitrified (like tile). Sharpening stones are made in the same way. After removal from the kiln, the wheel is trued on a lathe, bushed and subjected to speed tests. When in use a grinding wheel is subjected to high centrifugal impact and torsional stresses, for which reason each step in its manufacture must be carried out with great care. The sawing of granite, marble and carbon, and the cutting of tool steel, stellite and other hard materials, calls for a thin, tough, elastic abrasive wheel. A grinding wheel bonded with bakelite, for example, can be run at twice the speed of a vitrified wheel, and, like a rubber-bonded wheel, can be made quite thin.

Some Other Synthetic Abrasives. It has been known for many years that various carbides such as boron carbide and tungsten carbide are superior to silicon carbide in hardness and other properties. Unfortunately, they are costly to manufacture. Carboloy, cemented tungsten carbide, was discussed under non-ferrous alloys. It can, however, be regarded as abrasive grains of tungsten carbide bonded in cobalt.

Boron carbide, B_6C, marketed as Norbide, is made from petroleum coke and boric acid. (Boric acid, H_3BO_3, also known as boracic acid, is the familiar antiseptic frequently used as an eye-wash.) The boric acid is first heated until it decomposes into water and glass-like boron oxide:

$$2H_3BO_3 \rightarrow 3H_2O\uparrow + B_2O_3$$

The boron oxide then is reduced by the coke in an electric furnace heated to approximately 5000°F.

$$3B_2O_3 + 10C \rightarrow B_6C + 9CO\uparrow$$

Norbide is being used where other abrasives have been found unsatisfactory, for grinding dies, and for cutting and sharpening high speed alloy steel and carboloy tools.

Asbestos. There is a group of minerals with a fibrous structure which are valuable for their heat-resisting properties, and because they can be woven into fireproof cloth. The *asbestos* minerals are silicates of Ca, Mg, Fe, Al, Na and K. The long fiber variety found in Canada is most suitable for spinning and weaving. It is not as infusible nor as acid-resistant as the short fiber asbestos found in this country, and in the U.S.S.R., Italy and South Africa. Rocks in which the veins of asbestos lie occur near the surface so that open pit methods can be used to mine them. Both by hand and machine the rocks are crushed and the fibers recovered. The long fibers are woven into the fireproof cloth used in theater curtains and in blankets, clothing, shoes and gloves for welders, army flame-throwers and firemen. Shorter fibers go into asbestos paper and board, while the shortest fibers go into pipe, boiler and furnace coverings, and are used in the chemical industries for filtering acids.

Mica. This familiar transparent mineral, used in electrical condensers, is a hydrated potassium aluminum silicate. There are large mica deposits in South Carolina, New Hampshire and New Mexico. Mica can be split into sheets 1/2500″ thick. The brownish non-hydrated variety of mica found in Canada, New York and New Jersey is also used by the electrical industries. The resistance wires in electric toasters and irons are wound around it. Because it has approximately the same hardness as copper, and is at the same time a good insulator, this mica is used to separate the copper segments in the commutators of electrical generators.

Plastics. The word "plastic" originally described any material, like dough or soft clay, that was capable of being pressed into shape. Nowadays when we speak of plastics we include such a large number of different synthetic materials that no definition is possible. No definition is necessary, however, for you are familiar with them, having handled modern synthetic plastics in many forms. Fountain pens, toothbrush handles, light switches, eyeglass frames and telephone receivers are practically all now made of plastics. Your small radio cabinet, parts of the radio tubes, the timing gears of your car, cigarette holders, beads and buttons are just a few examples of common objects made of plastics.

Celluloid. The story of modern plastics begins strangely enough with the growing scarcity of elephant tusks. About seventy-five years ago the billiard ball manufacturers were worried because their raw material was becoming unobtainable. They therefore offered a prize for a substitute for elephant ivory. Hyatt, the man who later also invented the roller bearing, went after the prize. Cotton treated with a mixture of nitric and sulfuric acids changes into nitro-cellulose. Prolonged treatment gives an explosive, gun cotton; shorter treatment gives pyroxylin. Hyatt found that pyroxylin when heated with a little camphor and alcohol becomes a soft, putty-like mass.

When hot, this material, now commonly called *celluloid,* can easily be molded into almost any desired shape or rolled into sheets. The sheets placed in molds can, by means of steam, be blown to form hollow objects. When cool, celluloid can be cut, drilled and machined like wood. Almost any color can be imparted to it, so that it makes a good imitation of ivory, amber, tortoise-shell or marble. Much of the celluloid produced is used in its transparent form for motion picture and other photographic films; and in the manufacture of some safety glass. Safety glass, as you know, is used in automobile windshields and windows, and consists of a celluloid sheet sandwiched between two panes of thin glass. Celluloid is compounded to produce lacquers like Duco.

Celluloid is very inflammable, though now ammonium phosphate and other fire-retarding materials are mixed with it to counteract this dangerous property. It has a fair degree of mechanical strength but softens readily, even at so low a temperature as that of hot water. Its inflammability and its tendency to soften restrict the fields of usefulness to which celluloid can be applied. Nevertheless we use about 12,000,000 pounds of pyroxylin yearly.

Galalith, Aralac. Almost a quarter of a century passed after the invention of celluloid before another plastic was hit upon. When skimmed milk sours, it separates into two parts, curds and whey. It is not necessary to wait until the milk sours naturally to bring about this separation. Any dilute acid added to milk causes this solid, white, protein substance, called *casein,* to coagulate. Formaldehyde, $HCHO$, can be made by the partial oxidation of wood alcohol, CH_3OH, by passing the alcohol vapor mixed with air over a copper catalyst:

$$2CH_3OH + O_2 \rightarrow 2CH_2O + 2H_2O$$

Casein treated with formaldehyde hardens into a plastic that is not inflammable and is firmer than celluloid. This, the second of the modern plastics, is marketed under the trade names Galalith (meaning milk-stone) and Aralac. Although originally invented as a material to be used as a white blackboard in classrooms, it appears on the market almost exclusively in the form of buttons and buckles.

The Japanese about twenty-five years ago began producing a plastic made by hardening the protein in soy bean juice with formaldehyde. Henry Ford, just before the war, announced that his company had perfected a product made from soy beans which is superior to sheet steel for automobile bodies.

Bakelite. A brilliant Belgian-born chemist, Leo Baekeland, after selling his invention of Velox (daylight, photographic printing paper) to the Eastman-Kodak company for a million dollars, looked around for new chemical worlds to conquer. He decided to try to discover some chemical method for manufacturing a resin suitable for use in varnishes and lacquers. The lac insect produces shellac; Canada balsam is the resin from an American evergreen tree called the balsam fir; copal is a resin from various tropical trees. It was a product such as these that Baekeland sought for in his test tubes. Like all well-trained scientists he first searched the chemical literature to find out what knowledge was already available on this problem. His studies led him to experiment with phenol and formaldehyde. Phenol, C_6H_5OH, also known as carbolic acid, is one of the constituents of coal tar.

He did not discover what he was after at first. Instead, he found that phenol and formaldehyde catalyzed by an acid or a base and under suitable heat treatment reacted to form a hard, infusible solid which was not soluble in any of the common solvents. Baekeland had the vision to see the possibilities of this new material. He patented it in 1909 and named it Bakelite. It is the plastic that the electrical industries use in the form of molded products. Bakelite is a poor conductor of both heat and electricity; in other words, it is an excellent material for insulators. It is tough, it takes the form of the mold practically perfectly, and does not warp when hot. Baekeland made and patented many other types of Bakelite, even some that met his original specifications and could be used as lacquers.

Other Plastics. It is not surprising that other chemists tried their hands at inventing new plastics. Hundreds of patents have been granted by the U.S. Patent Office for methods of manufacturing plastics from all sorts of raw materials. Plastics now on the market are known by over a thousand different copyrighted trade names. Chemically they can be grouped into about fifteen types on the basis of the raw materials used. On the basis of the reactions involved in their production synthetic plastics can be grouped into two main classes.

You recall the reaction between a base and an acid to produce a salt and water.

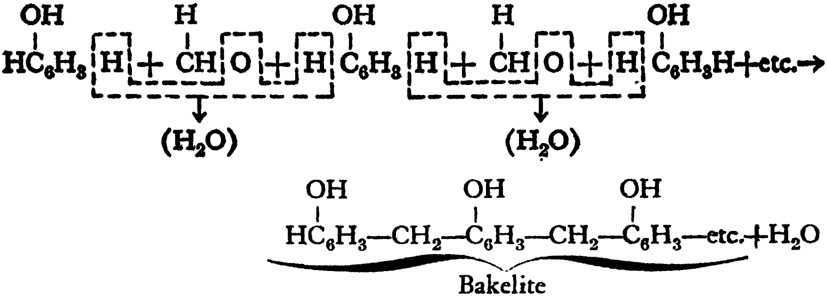
$$NaOH+HCl\rightarrow NaCl+HOH$$

In an analogous way cellulose, $C_6H_7O_2(OH)_3$, and nitric acid, HNO_3, can be made to react to form pyroxylin plus water. If acetic acid, $H(C_2H_3O_2)$, is used instead of nitric acid, cellulose acetate is the product.

$$C_6H_7O_2(OH)_3+3H(C_2H_3O_2)\rightarrow C_6H_7O_2(C_2H_3O_2)_3+3HOH$$

Cellulose acetate can be formed into threads and woven into cloth; as such it is sold as acetate rayon (Celanese). As a molded product it is called Tenite. Safety films for school and home movies are made of cellulose acetate, which, while it can be made to burn, is not nearly so inflammable as the celluloid it replaces. In sheets, as a transparent wrapping, it is called Kodapak and Vue Pak.

Condensation Process Plastics. Bakelite is made by a reaction similar in one respect to the reaction shown above for the preparation of cellulose acetate. Water is one product of the reaction. But here is the big difference: one molecule of formaldehyde, CH_2O, reacts with two molecules of phenol, C_6H_5OH, and links them together, while another molecule of formaldehyde in the same way links each of these phenols to another phenol molecule:



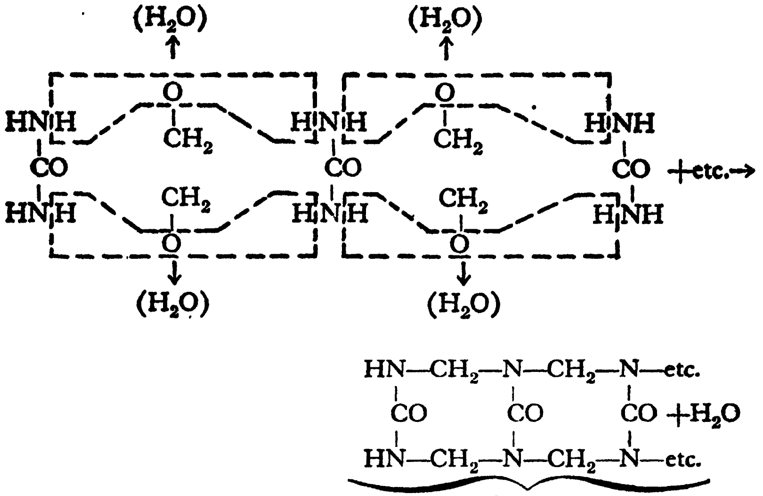
$$\overset{OH}{HC_6H_8}\underbrace{\boxed{H} + \overset{H}{CH}\boxed{O} + \boxed{H}}_{\downarrow\,(H_2O)}\overset{OH}{C_6H_8}\underbrace{\boxed{H} + \overset{H}{CH}\boxed{O} + \boxed{H}}_{\downarrow\,(H_2O)}\overset{OH}{C_6H_8}H + etc. \rightarrow$$

$$\underbrace{\overset{OH}{HC_6H_3}-CH_2-\overset{OH}{C_6H_3}-CH_2-\overset{OH}{C_6H_3}-etc.}_{Bakelite} + H_2O$$

In this way the formaldehyde furnishes a linkage group, CH_2, to connect many phenols into one large synthetic resin molecule.

Urea, $(NH_2)_2CO$ and formaldehyde also react in a similar way to form a synthetic plastic of which Beetleware and Plaskon are examples. After the phenolic and urea resins have once been heated they become permanently hard and infusible. They are termed thermo-setting.

Polymerization Process Plastics. The plastics belonging to this division are formed, as are those just described, by many molecules joining together, but in these polymerization reactions no water separates out to make the linkages. The compounds used have structures that only students of organic chemistry can understand. However, some understanding of what occurs during polymerization may be gleaned from a simple analogy. You probably know that wool fibers under the microscope appear as scale-covered hairs. Wool felt is made by beating and pressing wool fibers together. The wool fibers stick together because the scales get hooked into each other. Now methyl methacrylate, which polymerizes to form Lucite and Plexiglas, while having no "scales," does have what the chemist calls an *unsaturated linkage.* Instead of beating or pressing the molecules, the chemist heats the compound. Many molecules "hook-up" at the points where the unsaturated linkages are located in the molecules to form the synthetic plastic.



$$\begin{array}{ccccc} HN-CH_2-N-CH_2-N-\text{etc.} \\ \quad | \qquad\quad | \qquad\quad | \\ \quad CO \qquad CO \qquad CO + H_2O \\ \quad | \qquad\quad | \qquad\quad | \\ HN-CH_2-N-CH_2-N-\text{etc.} \end{array}$$

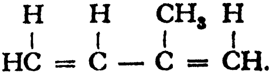
Beetleware, etc.

The plastics of this class are finding many new uses. They can be molded and cast. They are used in bonding plywood. Airplane windows are made from them. They are used, too, in safety glass. Elastic belts, suspenders and wrist-watch straps are some other of the many forms in which this type of plastic reaches the public. Like celluloid, these plastics are thermoplastic; that is, they soften up and melt when heated.

Rubber. Prior to the war the world produced approximately one million tons of natural rubber annually. The bulk of it came from the Middle East, while the Amazon Valley region of South America, and Africa each contributed 1½% to the total; and Mexico furnished 9⁄10% in the form of guayule rubber. From 1935 to 1940 the United States absorbed between 50% and 60% of all the natural rubber produced. In 1941 we imported a million tons. Rubber is used for tank treads, pontoon bridge supports, gas masks, barrage balloons, airplane and jeep tires and many other objects. In 1941, 75% of our rubber imports went into tires and inner tubes (more than 61 million tires were made). The remaining 25% was consumed by the manufacturers of a variety of products including suspenders, garters, rubber toys, erasers, raincoats, rubber heels, boots and overshoes, rubber belting for industry, hose, covering for electric wiring, hard rubber for combs, fountain pens and electric insulators.

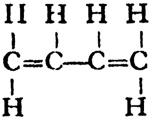
Natural Rubber. Many tropical plants yield a milky, sticky liquid called *latex,* which coagulates into a plastic solid when subjected to smoke or treated with acetic acid (vinegar). This solid is the raw natural rubber of commerce. The latex is obtained mainly from rubber trees scientifically grown on rubber plantations. A spiral cut is made through the tree's bark half way around, without harming the tree. The latex slowly oozes out of the tree, travels along the spiral path and drips into a cup hanging below.

Chemists analyzed rubber years ago, and found it to be made of groups of isoprene (C_5H_8) molecules. The organic chemist prefers to write structural formulas because they suggest properties and possible reactions not indicated by ordinary formulas. Isoprene is written $\overset{\displaystyle H}{\overset{|}{HC}} = \overset{\displaystyle H}{\overset{|}{C}} - \overset{\displaystyle CH_3}{\overset{|}{C}} = \overset{\displaystyle H}{\overset{|}{CH}}$. Rubber is known to be a polymer of isoprene; that is, it consists of many isoprene molecules hooked together at the double linkages.



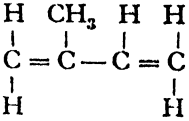
Natural rubber, though used to make waterproof clothing, shoes and bottles, was not satisfactory because it was too sticky and of unpleasant odor in hot weather, while in winter it was hard and brittle. Then Goodyear, after ten years of work, found that sulfur, when mixed and heated with rubber, converted it into "vulcanized" rubber which maintained its flexibility when cold and did not get sticky in summer. A larger percentage of sulfur resulted in hard rubber. Other workers found that a variety of materials compounded with rubber improve many of its properties. At present, thousands of different substances are mixed with the basic rubber and sulfur, each combination being designed to give a product that will best suit its specific purpose. Some act as catalysts to hasten the vulcanization process; others tend to counteract the deteriorating effects of slow oxidation. Carbon black, made, as you recall, by the incomplete combustion of natural gas, and zinc oxide are added to rubber to increase its resiliency and its resistance to abrasion. Thanks to scientific rubber compounding, the automobile tire's mileage has been increased more than fivefold while its price has been reduced by almost two-thirds.

Artificial Rubber. As mentioned before, isoprene can be obtained from natural rubber. However, isoprene polymerizes with difficulty to form rubber again; and isoprene itself is expensive to synthesize. ***Butadiene***, C_4H_6, is a compound that resembles isoprene. In fact a glance at the structural formulas shows the only difference to be a CH_3 group in place of one of the hydrogens of butadiene.



$$\begin{array}{cccc} H & H & H & H \\ | & | & | & | \\ C & = C - & C & = C \\ | & & & | \\ H & & & H \end{array}$$

Butadiene



$$\begin{array}{cccc} H & CH_3 & H & H \\ | & | & | & | \\ C & = C - & C & = C \\ | & & & | \\ H & & & H \end{array}$$

Isoprene

Butadiene is not as difficult to make as isoprene, so chemists selected it as a promising starting substance in the search for artificial rubber. Any plant containing starch or sugar furnishes the raw material needed to make alcohol, from which butadiene can be made. The by-product gases from petroleum refining can also be converted into butadiene. Acetylene, C_2H_2, too, can be built up to make butadiene. Finally, chemists succeeded in producing *chloroprene*. It differs from isoprene in one respect: it has a chlorine atom in place of the CH_3 group. Chloroprene polymerizes readily and forms an artificial rubber sold under the trade name, Neoprene. Neoprene is superior to natural rubber in some respects; for example, in its resistance to the action of heat and oils. It is not so good, however, as rubber for tires. Subsequently it was found that sodium catalyzed the polymerization of butadiene. Using the first two letters of butadiene combined with the symbol for sodium, the rubberlike product was named Buna; it was not, however, a satisfactory rubber substitute. Then chemists worked out methods for polymerizing a mixture of butadiene and styrene to form Buna S, a rubber substitute that seems excellent for tire treads. Buna N, the polymer made from a mixture of butadiene and vinyl cyanide, is more costly to prepare but has greater elasticity. Besides the artificial rubbers mentioned above, many others are being perfected at the present time.

SELECTED REFERENCES FOR FURTHER STUDY

BERRY, PAULINE. Stuff; the Story of Materials in the Service of Man. D. Appleton-Century Co.

FINCH, V. C., TREWARTHA, G. T. & SHEARER, M. H. The Earth and Its Resources. McGraw-Hill Book Co.

FURNAS, C. C. The Storehouse of Civilization. Columbia University.

LEIGHOU, R. B. Chemistry of Engineering Materials. McGraw-Hill Book Co.

MATHEWSON, C. H. (ed.). Modern Uses of Nonferrous Metals. Amer. Institute Mining & Metallurgical Engineers.

MERSEREAU, S. F. Materials of Industry. McGraw-Hill Book Co.

MORRISON, A. C. Man in a Chemical World. Scribner.

READ, W. T. Industrial Chemistry. J. Wiley & Sons.

RIEGEL, E. R. Industrial Chemistry. Reinhold Co.

SADTLER, S. S. Chemistry of Familiar Things. Lippincott.

SMITH, J. RUSSEL. Men and Resources. Harcourt Brace.

CHAPTER X

FUNDAMENTAL MACHINE ELEMENTS

Edward N. Wallen

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48. TAPERS

Machine Parts. All matter which is known to man is reducible to a relatively small number of chemical elements. This statement, with slight change of words, can be applied to the field of machinery. All machines are composed of a combination of certain machine elements. The physicist has reduced these machine elements to two simple machines—the lever and the inclined plane. Some authorities would add the push or pull of the force transmitted through hammer blows and connecting links. But such discussions are largely academic, since the designer and engineer solve the problem of transmitting power by utilizing machine elements such as gears, pulleys, cams, threads, machine bearings, etc.

The workman of today is either the slave of his machine or the master of his machine, depending upon his knowledge of the principles that are employed in its design. If he understands the reasons for using the particular elements that go to make up the machine he will have a better understanding of its operation, and will prove to be a more efficient worker. As a result of the present rush to train workers for our war industries, many machine operators are being employed who have little knowledge of the fundamental principles involved in their machines; they cannot, however, become really skilled men until they have acquired such understanding. As it is impractical to deal here with all of the more technical phases of machine design, we shall confine our discussion simply to the basic machine parts. Thus, stresses and strains are left to a more technical work; so are complicated and unusual mechanisms.

Tapers. One of the most valuable of all machine elements is the taper. Practically every revolving spindle in a machine that is designed to hold a tool is equipped with a taper seat. A *taper* may be defined as a gradual and uniform change in a dimension. This definition covers rectangular objects as well as those with a round cross-section. Lathe centers, drill chuck shanks, and milling-machine arbor shanks are examples of the more common tapers.

Tapers possess two important characteristics that make them valuable in machine work. First, a gradual or slow taper possesses holding power; a tapered shank that is a good fit in a tapered seat will hold itself in place unless an extremely heavy twisting force (torque) is applied to it. Secondly, despite its ability to hold itself in place, the tapered shank may easily be removed by a relatively slight blow against its smaller end; thus a tapered shank may be removed from a seat with but little effort.

Tapers will always reseat themselves. That is, they may be removed from the seat and replaced any number of times with the assurance that they will always be located in exactly the same position. This is true even though constant removal might wear the shank to a smaller size. The effect would simply be a deeper reseating. In actual practice the wear is so slight that this is of little consequence. In machine assembly, taper pins are used to ensure correct alignment of machine parts in the event that it becomes necessary to dis-assemble the parts for repair or cleaning. The taper pins will force the machine parts into exact location so that they can then be bolted with the assurance that the alignment is correct.

Tapers are designated according to the difference in diameters per unit of length. If a piece has a difference in diameter of 0.500 of an inch in one foot it is known as a five hundred thousandth of an inch per foot taper. The machinist is usually concerned with the taper per inch, since most of the tapers that he must produce are short in length and his calculations will deal with inches rather than feet. The taper per inch is found by dividing the taper per foot by twelve.

Taper Standards. Taper standards have been adopted by a number of manufacturers. These standards differ from each other in the amount of taper per foot. The most commonly used tapers are the Morse, the Brown and Sharpe, and the Jarno. The Morse taper is used extensively in drill press work and also in lathe spindles. The Brown and Sharpe standard is used in milling machines. As the name of these two standards indicate, they were developed by the Morse Twist Drill Company and by the Brown and Sharpe Manufacturing Company.

The *Morse* taper varies from .600 of an inch per foot to .625 of an inch per foot. The numbers of the tapers range from size 0, which is the smallest, to size 7, which is the largest. To understand what is meant by the numbers of the tapers, imagine a long rod which is tapered from one end to the other; if the rod were cut into short lengths each piece would differ in its diameters from the other pieces. If the largest piece were given a number, and the next in size were given the next number, and so on, the pieces could be referred to by these numbers. This is the basis for the number system of designating taper sizes. All of the dimensions for any listed taper may be found by locating the number of the taper in a table of standard tapers and reading across the column.

The *Brown and Sharpe* taper system uses a taper per foot of .500 of an inch. There is one exception to this: the Brown and Sharpe number 10 taper has a taper per foot of .516 of an inch. It is rather difficult to understand why these two taper standards have any variation in their series. It might be expected that the companies would adopt a taper per foot and use it for all numbers of the tapers. The answer is probably that specific problems were encountered by the engineering departments of the companies and were solved by using the variations.

The *Jarno* taper has the advantage of being consistent. There is no need to look up the dimensions in a table. The taper per foot for all numbers is .600 of an inch. The number of the taper is a key to all other important dimensions of the taper. It is only necessary to remember 2–8–10, and all other parts of the taper may be found by using the number of the taper as the numerator of a fraction for each of the numbers. For example, suppose we are considering a No. 8 Jarno taper; then $8 \div 2$ will give the number of half inches of length, or 4 inches; $8 \div 8$ gives one inch as the large diameter; and $8 \div 10$ gives .800 of an inch as the other diameter.

$$\frac{\text{Number of the taper}}{2} = \text{the length of the taper in half-inches.}$$

$$\frac{\text{Number of the taper}}{8} = \text{the large diameter of the taper.}$$

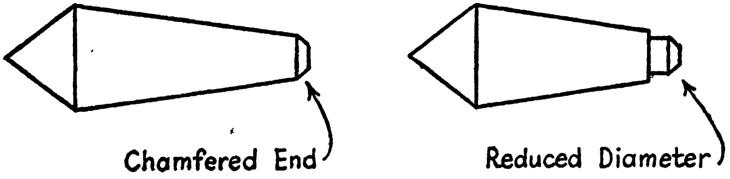
$$\frac{\text{Number of the taper}}{10} = \text{the small diameter of the taper.}$$

Despite the obvious advantages of such a simple standard, the Jarno taper has not yet come into general use. This is probably due to the expense entailed in changing over from the present systems to the Jarno. A few companies, however, have adopted this standard for use in their own plants; the Ford Motors company is the outstanding example.

The tapers which have been mentioned are sometimes referred to as the self-holding tapers. They are also called *slow tapers* because the change in dimension is gradual. Self-holding tapers are valuable where their holding power is needed. In milling machines the self-holding taper has at times proved to be a handicap. Milling machine arbors often became "frozen" into their sockets and removal of the arbor from the seat was difficult. Many of the older mechanics have seen sledge-hammer blows used to start a milling machine arbor from its seat. This freezing happened only when the arbor was left in place for a considerable period of time. The new standardized milling machine spindle is made with a taper of 3½ inches per foot in contrast to the former taper of .500 of an inch. The new taper serves to locate the arbors in the spindles. Driving is accomplished by means of keys which fit into the flanges of the arbor.

In 1937 the American Standards Association proposed and published an American Standard Taper series in an attempt to reduce the existing number of taper standards. Other standards in use besides the ones mentioned in the preceding paragraphs were the Sellers, the Reed, the Cleveland, the Hendey and the Standard Tool Co. All of these standards were of the self-holding type, and doubtless some of them were unnecessary duplications. The American Standards committee advocated the use of the three smallest of the Brown and Sharpe series, then the next six largest sizes were adopted from the Morse series, and the ten largest sizes were based on a taper of .750 of an inch per foot. In short, the new standard tapers are a compromise between the then existing systems.

The mechanic's job is to produce and use tapers regardless of the standards used. No matter what the taper per foot may be, if the taper shank is not an exact fit in the taper socket or seat, the taper is not satisfactory. The holding power and the self-locating power of the taper will not exist unless the taper is properly fitted, clean, and free from burrs or dents. A tapered shank obviously can not fit properly if a chip is allowed to remain on its surface when it is put into the taper socket. Tapers must be protected against careless handling. All tapered shanks are made with a reduced section at the small end so that, if it is necessary to remove the shank by striking, the taper will not be affected.



Mathematics of the Taper. The mathematics involved in computing taper problems is not particularly difficult. An understanding of the terms used, together with a clear mental picture of the shapes, should enable one to make all needed calculations with little difficulty. The following are the important principles:

(1) Knowing the taper per foot (as obtained from a table of tapers), the taper per inch is found by dividing the T.P.F. by 12. (T.P.F.=taper per foot).

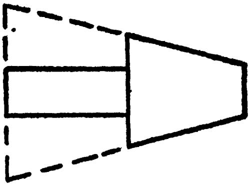
(2) Knowing the taper per inch, the taper per foot is found by multiplying the T.P.I. by 12. (T.P.I.=taper per inch).

When a taper is to be turned on a piece that is tapered for only a *part* of its length, the calculations for set-over of tail stock or of taper attachment must be made as though the piece was tapered for its full length.

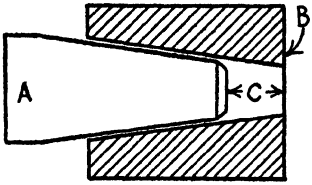
(3) To find the total taper of a piece, multiply the T.P.I. by the total length of the piece in inches.

(4) To find the amount of offset for the dead center when turning a taper, divide the total taper of the piece by two.

(5) When the diameters of the large and the small ends of the taper are given, the taper per inch is found by subtracting the small diameter from the large diameter and dividing the result by the length of the taper (the distance between the two diameters).



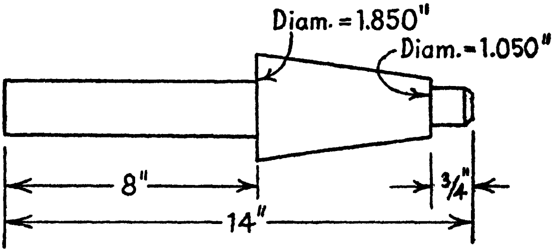
(6) When fitting a taper, the amount to be removed is found by multiplying the T.P.I. by the distance the taper has to go.



A = The piece which is being fitted.
B = The taper gage.
C = The distance the taper has to go.

Exercises

1. What is the taper per inch of a number 7 Brown and Sharpe taper?
2. What is the total taper of the number 7 B & S, if the length of the piece is 6 inches?
3. What is the taper per inch of a piece that has a small diameter of 0.623″ and a large diameter of 1.500″, if the length of the tapered portion is 7 inches?
4. For the piece sketched below, find (a) the taper per inch, and (b) the total taper for set-over purposes.



5. What is the amount of set-over for the piece shown above?
6. If the diameter at the small end is 0.778 and the T.P.I. is 0.051, what will be the diameter at the large end if the piece is 7 inches in length?
7. You are cutting a taper and you try it in the gage. You find that the end of the piece is ⅝ of an inch from the small end of the gage. If the taper per inch is 0.050″, how much stock must be removed from the piece to have it fit?

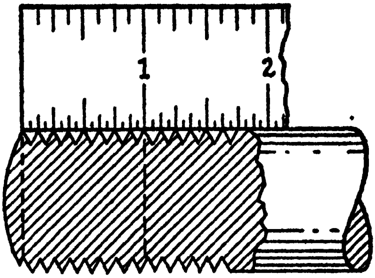
Exercises

1. Why are tapers important in machine construction?
2. What does the value of a taper depend upon?
3. How do the different standard tapers differ from each other?
4. What geometric shape does a tapered piece have?
5. What is the principal difference between a self-holding taper and a quick-release taper?
6. What are the "key numbers" of the Jarno taper system? What use is made of each?
7. If you are cutting a taper, using the offset tailstock method, and you find that the small end is smaller than it should be, which way should you move the tailstock to correct the setting error?
8. Suppose the tailstock is set correctly, but it is moved back along the ways of the lathe for a longer piece, will the taper per inch be the same?

49. SCREW THREADS

Screw Threads. Screw threads are formed by cutting or otherwise producing a helical groove in the surface of a round bar. The ridge which is left on the bar is known as the thread. The shape of the groove and the shape of the ridge are the same on practically all threads. External or *male* threads are formed on rods; internal or *female* threads are formed in nuts, etc. to fit the external threads. There are two primary functions of screw threads. They are used for holding parts of machines together, or they are used for moving machine elements for feeding purposes or adjustment purposes. The machine screw, which is used for fastening two sections of a machine together, and the nut and bolt, are examples of screw threads whose purpose is to fasten and hold. The feed screw of a milling machine and the lead screw of a lathe are examples of threads used for moving. Screw threads permit a fine control of movement and are therefore used in some measuring instruments; the micrometer and the graduated feed handles of machines are examples of threads applied to movement and measurements.

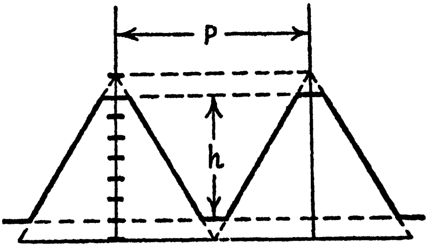
Screw Thread Standards. The shape of the screw thread and the number of turns of the thread in one inch determine the standard to which the thread belongs. Before the standards were accepted by most manufacturers, the number of threads per inch for a diameter was fixed by the individual concern. This created considerable confusion, since it was impossible to secure replacement threads economically. A threaded piece with a half-inch diameter might have had any number of threads per inch that the maker of the machine desired to put on it. The result was that the user could not replace a nut or screw without having one made up specially or securing one from the maker of the machine. Today the situation is



Measuring Screw Threads

quite different; comparatively few manufactured articles are made with special screws, and these only for some very definite reason.

Screw threads intended for holding purposes were formerly made with the Vee thread shape. The Vee thread had a theoretically sharp top and bottom, the sides were cut with an included angle of 60°, and the profile, or cross section, of the thread was an equilateral triangle. In actual practice it is impossible to make a truly sharp bottom in the thread; moreover, the sharp top is so weak and easily damaged that it is highly impractical. Thread makers accept orders for Vee thread tops today, but advise against them because of expense. The problem of the weak top and difficult-to-make bottom was solved by retaining the angles of the Vee thread with a flattened top and bottom. The formulas and constants for the thread calculations, however, are still based on those of the Vee thread.

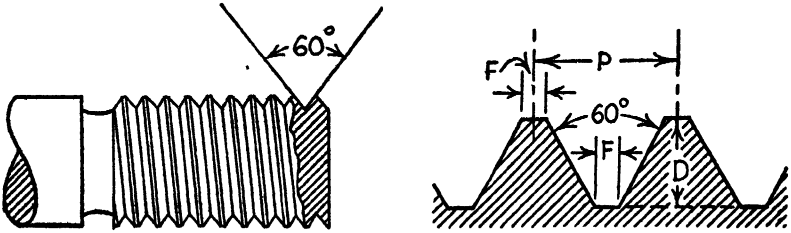


Origin and Development of American National Screw Thread Form

(Showing the modification of the Sharp-Vee thread into the more practical shape known as the American National Form).

Technically, one-eighth of the height of the Vee thread has been removed and one-eighth of the height at the bottom of the thread has been filled in. The American National thread is three-fourths as high as a Vee thread of the same number of threads per inch would be.

The American National Form thread has had several different names in the past. It was originally known as the Seller's thread; then it was called the Franklin Institute thread. It was later and for many years called the United States Standard thread; many mechanics still refer to the American National as the U.S.S. thread. In everyday practice the American National is simply called the National thread.



AMERICAN NATIONAL SCREW THREAD
(Formerly U.S. Standard Screw Thread)

$$P = \text{Pitch} = \frac{1}{\text{No. Th'ds. Per In.}}$$

$$D = \text{Depth} = \text{P} \times .64952$$

$$F = \text{Flat} = \frac{P}{8}$$

Society of Automotive Engineers Thread. The National Form thread is made in two series. The older U.S.S. thread was standardized into definite numbers of threads per inch for each diameter. A half-inch diameter thread, for example, was standardized with 13 threads per inch. The automobile industry found that while the shape of the U.S.S. thread was satisfactory, the number of threads per inch was not. The particular problem of vibration in the automobile necessitates safeguarding against loosening of bolts and nuts. As an aid to the resistance against loosening a greater number of threads per inch was found to be a help. The S.A.E. thread was adopted by the industry and is now known as the National Fine series. The old U.S.S. series was retained and is now called the National Coarse series. The use of the two different series makes it necessary for the mechanic to specify which particular series is called for when a job is to be turned out. A half-inch diameter nut of the National Fine series has 20 threads per inch as contrasted with the 13 T.P.I. of the Coarse series.

Screw Thread Terminology. The following definitions are essential to an understanding of screw threads.

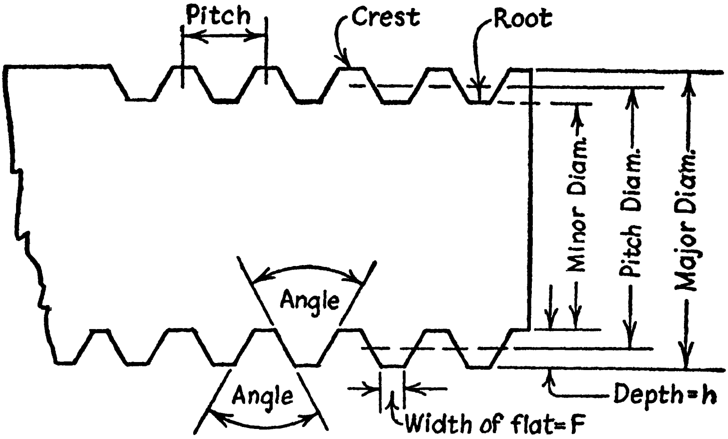
A *screw thread* is a ridge of a definite and regular cross-section formed as a helix on the surface of a round bar.

An *external thread* is formed on the outside of a cylinder.

An *internal thread* is formed on the inside of a cylindrical hole.

The *major diameter* is the diameter of the external thread measured over the tops of the threads. The major diameter was formerly known as the outside diameter.

The *minor diameter* is the diameter of the screw measured at the bottom of the threads. This was formerly called the root diameter.



Showing Terms to Designate Parts of a Screw Thread.

The *pitch diameter* is the diameter of the thread measured at a point on the thread where the width of the thread is equal to the width of the space between the threads. It is practically equal to the outside diameter minus the depth of one thread. Pitch diameter is of importance in measuring threads where greater accuracy is required.

The *pitch* of a thread is the distance from a point on one thread to a corresponding point on the next adjacent thread measured along the axis of the screw. The pitch also equals 1÷no. of threads per inch (T.P.I.).

The *lead* of a thread is the distance that the thread advances in one complete turn or revolution. The lead is equal to the pitch of the thread in a single thread screw. In multiple-threaded screws the lead is equal to the pitch times the number of starts of the thread. A fountain-pen cap is a good example of a multiple-threaded screw. Fountain pens are made with multiple threads in order to gain speed when tightening or replacing the cap. There may be four or five threads in the cap. The pitch is found as stated above, but the lead is found by multiplying the pitch by four or five, as the case may be.

The *depth of thread* is the vertical distance from the top of the thread to the bottom of the thread (crest to root).

The *double depth* is the depth of the thread on one side added to the depth of the thread on the other side of the screw. Double depth is used when selecting drills or making holes into which threads are to be cut.

The *crest* is the top of the threads.

The *root* is the surface at the bottom of the thread between the sides of adjacent threads.

The *angle of thread* is the included angle between the sides of the thread measured in an axial plane.

The *side* of a thread is the surface of the thread between the crest and the root.

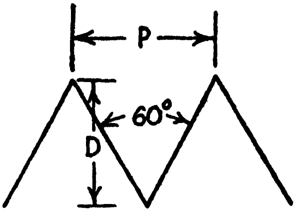
The *fit* of a thread is the relation between the internal and external threads with reference to ease of assembly. There are four classes of fits in the American Standard, ranging from the loosest, Class 1, to the closest, Class 4. Fits are designated on blueprints by the number of the fit which is called for; thus, ½″ N.C.—1 means that a National Form thread, coarse series, is to be made with a Class 1 fit. The fit of a thread is determined by the side contact between the external and the internal threads. The major and minor diameter do not determine the fit; in fact, there is usually some clearance between the crest of the screw and the nut.

Methods of Making Screw Threads. Screw threads are made in a number of ways. The cheaper threads, i.e., those made in great quantities on screws, nuts, and bolts, are cut with automatic dies or special taps. They may also be rolled. The machinist makes threads with the aid of hand taps and dies, or he cuts them in a lathe. Threads are also milled in thread milling machines and ground in special thread grinding machines.

Thread rolls are rectangular blocks of tool steel which have a series of grooves formed in one surface. The grooves have a cross-section which is shaped in the form of the thread to be produced. The grooves are cut into the block at an angle equal to the helix angle of the thread. The thread blank is made to the pitch diameter of the thread. When the blank is rolled between the blocks the metal of the blank is displaced so that the thread is formed. One pass of the rolls produces a thread.

Left hand threads are used where the rotation of the machine would have a tendency to loosen the nut or machine element that is held by the thread. A left hand thread is tightened by turning it in a counter-clockwise direction.

Finding the Depth of a Thread. The Vee thread forms an equilateral triangle. The depth of a Vee thread is found by multiplying the pitch by 0.866.



N=Number of threads per inch
P=Pitch
D=Depth of thread
Pitch=1/N

$$D = P \times \cos 30° = 0.866\,P = \frac{0.866}{N}.$$

The depth of the thread must be found when a thread is to be cut on a lathe. If a hole is to be made so that a tap can be used it is necessary to find the double depth. The double depth is the depth on one side added to the depth on the other side. The double depth may be found by multiplying the depth by 2, or by using the formula $\frac{1.732}{N}$, which will give the same result (1.732 is obtained by multiplying 0.866 by 2).

As previously stated, the National thread was developed from the Vee thread by theoretically flattening the top and filling in the root; this makes the National form only ¾ the height or depth of the Vee thread. The formula for the depth of a National Form thread thus becomes:

$$D = \frac{0.6495}{N}.$$

(The factor 0.6495 is obtained by taking ¾ of 0.866, since the National is only ¾ as high as the Vee.)

The formula for the double depth of a National thread is $\frac{1.299}{N}$.

The minor diameter is found by the formula:

$$D - 2h = D - \frac{1.299}{N},$$

where D=major diameter, h=depth, and N=number of T.P.I.

The pitch diameter is found simply by the formula $D - h$.

Exercises

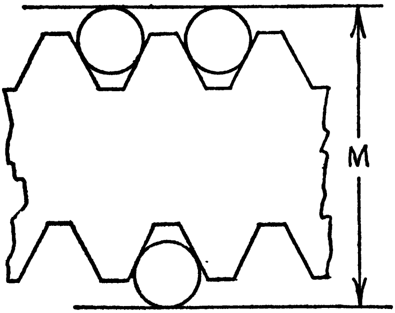
1. Find the depth and the minor diameter for each of the following sharp Vee threads:

(a) ½″—13
(b) 7/16″—14
(c) ⅜″—16
(d) 5/16″—18
(e) ¼″—20

NOTE: Remember that the first number given in reference to a thread is the major diameter, the second number is the threads per inch.

2. Find the depth and the minor diameter for each of the threads given above but make the calculations for the National Form thread.
3. A mechanic is required to cut a ¾″—10 N.C. thread in a lathe. What will the depth of the thread be?
4. What is the lead of a screw that has 10 threads per inch?
5. What is the lead of a double thread (multiple) if the pitch is 0.125″?
6. The lead of a triple thread is 0.300″. What is the pitch?
7. Find the pitch diameter of a 1″—8 National Form thread.

Measuring Screw Threads. The correct fit of a thread depends on the contact between the sides of the threads. When threads are made the size may be tested by trying the thread in a thread gage. The thread gage contains a thread that is presumed to be correct. The objection to this method of measuring or testing the fit is that it gives no indication as to how much material remains to be removed from the thread in order to make it fit. The thread either enters the gage or it does not. Thread micrometers are also sometimes used for measuring the thread; they have the advantage of giving an indication of the amount of material that must be removed in order to make the thread fit. Another method of accurately measuring the size of the thread is known as the *three-wire system* of screw thread measurement. In this method three small wires are held in the threads and a measurement is taken over the wires. An ordinary micrometer is used for this purpose. The wires used must all be of the same diameter, and must be selected for the pitch that is being measured. Some leeway in the diameter of the wires is permissible and wires are known as the "best diameter," the smallest diameter and the largest diameter that may be used.



Let M = the measurement over the wires,
D = the major diameter,
N = the number of threads per inch,
G = diameter of the wires.

The diameter of the wires to be used is then found as follows: The average or "best" diameter wire is 0.6 pitch; the maximum diameter is 1.010 pitch; and the minimum diameter is 0.505 pitch. Since the wires make contact on the sides of the threads they furnish a real measure of the correct size.

To find the measurement over the wires for a given thread the following formula is used:

$$M=D+3\,G-\frac{1.5155}{N}.$$

Harking back to the formula for double depth of a Vee thread, we find that 1.732 was used as a constant in finding the double depth. If the thread that is being measured by the three-wire system is a Vee thread, the formula for M would involve $\frac{1.732}{N}$ instead of $\frac{1.5155}{N}$. Remember the National Form thread was developed by flattening the top of the equilateral triangle that forms a Vee thread. The same triangle is the basis for the formula of the three-wire system. In the three-wire measurement system the flat crest is considered, but it is not necessary to consider the flat at the root because the triangle theoretically extends to the full depth. The outside diameter must be considered, however; therefore instead of using ¾ of the constant 1.732 (as we did in the formula for double depth) we use ⅞ of 1.732, which gives 1.5155.

EXAMPLE: Find the measurement over the wires for a ¾″—10 thread N.C.
SOLUTION: The pitch is $1/N$, or 0.100.
The wire will be: 0.6 of 0.100″, or 0.060″.

$$M=D+3G-\frac{1.5155}{N}=0.750+(3\times0.060'')-\frac{1.5155}{10}$$
$$=0.750+0.180-0.1515=0.7785,\ \textit{Ans.}$$

Exercises

1. Find the "Best" wire to use for the following threads:
   (a) 8 T.P.I. (b) 12 T.P.I. (c) 14 T.P.I.
2. Find M for the following:
   (a) 1″—8 National Form (b) ⅝″—11 N.C.

Find the measurement over the wires for the following Vee threads:
3. (a) 1″—8 (b) ¾″—10

Metric Standard Thread. The metric standard thread has the same profile as the American National thread. The angles are the same and the terminology is the same. The only difference between the two standards is in the manner of indicating the size of the thread. In the American system the number of threads per inch is stated; if the pitch is needed, it is calculated by dividing one inch by the number of threads per inch. This gives an idea of the size of the thread. In the metric system the pitch is given. Of course, all measurements in the metric system are given in terms of millimeters. Remember that there are 25.4 millimeters in one inch, and that one millimeter equals .03937″. For practical purposes the millimeter may be considered to be .0394″. Where the pitch is expressed in millimeters, the depth of the thread in terms of the inch may be found as follows:

$$D = .6495\,P \times .0394, \text{ or } D = .0256\,P.$$

In this formula, the constant .6495 is recognized as the same constant that is used when calculating the American National thread depth. Since .6495 and .0394 are always used in converting millimeter pitch into depth in inches, the formula is simplified by using .0256 to save the repetition of the multiplying process.

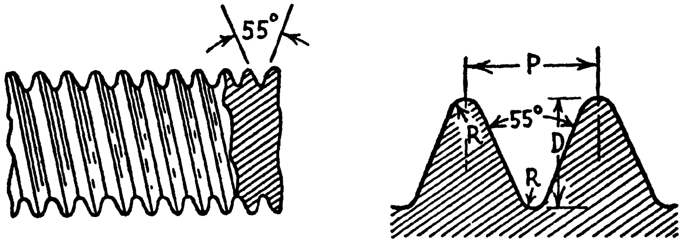
To change the outside diameter of a metric thread to inches, multiply the diameter in millimeters by .0394.

Exercises

Find the depth, the double depth, and the minor diameter in inches of each of the following Metric Standard threads:

1. Major diameter = 6 mm., pitch = .9 mm.
2. Major diameter = 20 mm., pitch = 2.5 mm.
3. Major diameter = 12 mm., pitch = 1.5 mm.

British Standard Whitworth Thread. While British Whitworth thread is used principally in Great Britain, it is also used to some extent in this country on boiler staybolts. The top and bottom of the thread are rounded, and the depth of the thread is about ⅔ that of a corresponding Vee thread. The angle is 55° instead of 60°.



Whitworth Screw Thread

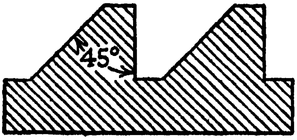
The depth of a Whitworth thread is found by the formula:

$$0.6403\,P, \text{ or } \frac{0.6403}{N}.$$

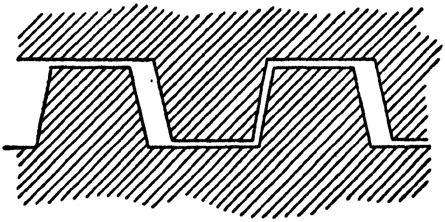
The radius at the top and the bottom is found by:

$$0.1373\,P, \text{ or } \frac{0.1373}{N}.$$

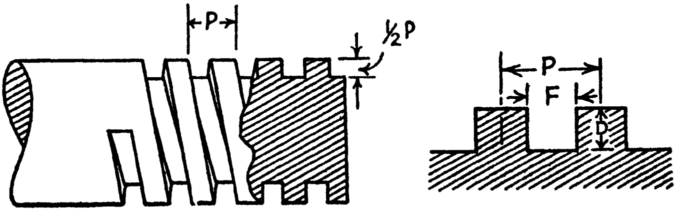
Special Threads. *Buttress threads* have not been standardized, but are usually made to the form shown in the sketch. Buttress threads are used to oppose pressure which is expected to come from one direction only. The screw blocks in the breech of cannon are made with buttress threads. They are also used in airplane engine building.



The *Dardalet self-locking thread* utilizes the holding power of tapered surfaces to resist loosening under vibration. The nut thread crest makes contact with the root of the opposing thread space. Both have a slight taper, and the result is a locking action similar to the action of a tapered shank in a tapered socket. The threads are made with Dardalet taps and dies. It is claimed that the use of other locking devices such as lock washers, etc., is unnecessary when the Dardalet thread is used. The thread is easily unlocked by turning it with a wrench.



Square threads are used for transmitting motion. The thread is, as its name implies, square. The sides of the thread are parallel. The depth is equal to the width. In practice, it is necessary to make the space in the nut slightly larger than the thread in order to allow a sliding fit. The depth of a square thread is one half of the pitch, and the thickness of the thread is one half of the pitch.



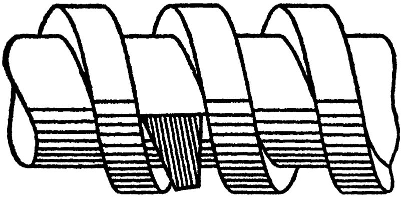
Square Screw Thread

$$P = \text{Pitch} = \frac{1}{\text{No. Th'ds. Per In.}}$$

$$D = \text{Depth} = P \times .500$$

$$F = \text{Space} = P \times .500$$

The square thread cannot be milled as readily as the slant-sided threads. This is because of the interference between the cutter and the sides of the thread. When cutting a square thread in a lathe it is necessary to grind the tool so that the sides of the tool will clear the sides of the thread. The tool used for cutting a square thread resembles a small parting tool. Unlike the parting tool, which has equal clearance on both sides, the square thread tool must have a greater angle of clearance on the leading edge of the tool.

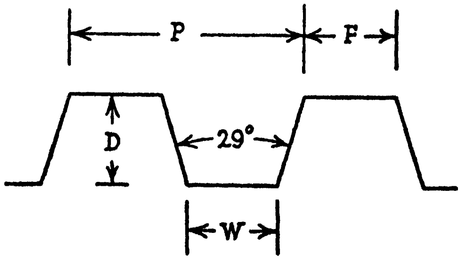


TOOL MUST CLEAR SIDES

The amount of clearance needed can be computed in several ways. On a piece of paper draw a right triangle with the pitch of the thread representing one side and the major diameter circumference of the thread representing the other right angle side. Draw the hypotenuse and measure the angle between the circumference line and the hypotenuse. This angle represents the angle of the following side of the thread. Draw the same type of triangle, but use the circumference of the minor diameter; the angle of the hypotenuse will be the angle of the leading side of the thread.

The angle may also be found by using a table of trigonometric functions. The lead represents the side opposite the angle, the circumference represents the side adjacent to the angle. From trigonometry we know that the side opposite divided by the side adjacent will give the tangent of the angle. Having the tangent of an angle it is only necessary to look in a table of tangents to find the angle. It must be remembered that the angles as found are the angles of the sides of the threads. In grinding the tool an additional degree of angle is given to it so that real clearance is obtained.

The *Acme thread* is another power-transmitting thread. It is a modification of the worm thread, and was designed to take the place of the square thread. The sides of the Acme thread have an included angle of 29°. The depth is the same as for a square thread of the same pitch. The Acme is a much stronger thread than the square thread.



The depth of the Acme thread is found by taking one half of the pitch and adding .010″ to it; the additional .010″ is to provide clearance at the bottom of the threads:

$$D=\frac{1}{2\times N}+.010 \text{ inches.}$$

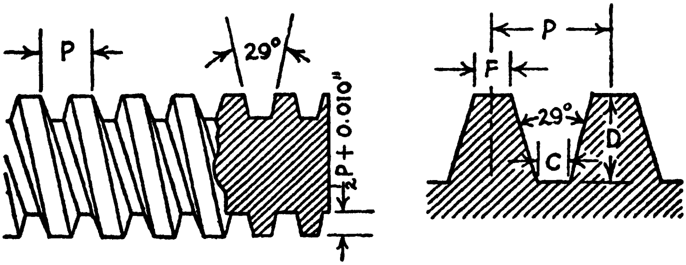
The minor diameter is found by subtracting the pitch plus .020″ from the major diameter:

$$\textit{Minor } D=\textit{Major } D-\left(\frac{1}{N}+.020\right)\text{inches.}$$

The width of the tool point is found by the formula:

$$\frac{0.03707}{N}-0.0052 \text{ inches.}$$

Note: In all of the thread problems, N represents the number of threads per inch.



Acme Screw Thread

$$P=\text{Pitch}=\frac{1}{\text{No. Th'ds. Per In.}}$$
$$D=\text{Depth}=\tfrac{1}{2}P+.010 \text{ In.}$$
$$F=\text{Flat}=.3707\ P$$
$$C=\text{Flat}=.3707\ P-.0052 \text{ In.}$$

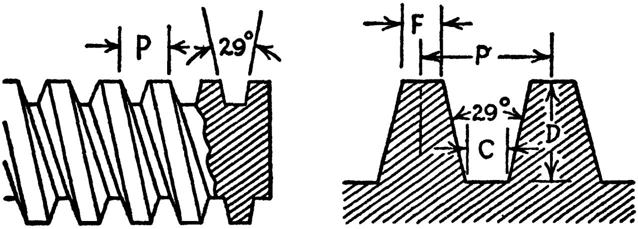
The *worm thread* is designed to mesh with a wormgear. It has a 20°-included angle, as has the Acme. The 29°-angle is used because of the pressure angle of the involute gear system, which is 14½°. Since the thread was to engage the wormgear it was given an angle of 14½° on each side, or an included angle of 29° between the sides. The worm thread is deeper than the Acme. The following formulas apply to the worm thread:

$$\text{Pitch}=\frac{1}{N}$$

$$\text{Depth} = \frac{.6866}{N}$$

$$\text{Width of top of thread} = \frac{0.335}{N}$$

$$\text{Width at bottom of thread} = \frac{0.310}{N}$$



29° Worm Thread Form

$$P = \text{Pitch} = \frac{1}{\text{No. Th'ds. Per In.}}$$

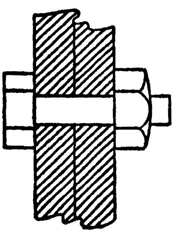
$D = \text{Depth} = .6866\ P$
$F = \text{Flat} = .31\ P$
$C = \text{Flat} = .335\ P$

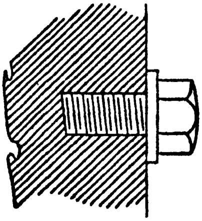
Exercises

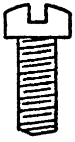
1. Find the minor diameter of a square thread that is 1-in. major diameter and has 8 T.P.I.
2. Find the double depth of a 6 T.P.I. square thread.
3. Find the clearance angles for the threading tool that is to cut a square thread in a screw of 2-in. major diameter with 4 T.P.I.
4. What is the width of the tool point in problem 3?
5. Find the depth of an Acme thread that has 6 T.P.I.
6. Find the minor diameter of an Acme thread of 1-in. major diameter with 6 T.P.I.
7. How wide should the point of the tool be if it is to cut an Acme thread with a 1½-in. major diameter with 6 T.P.I.?

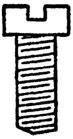
American Standard pipe threads are also known as the Briggs standard pipe thread. The thread is cut on a taper of three-quarters of an inch to a foot in order to secure better contact between the mating threads when "making" up a pipe line. The angle between the sides of the threads is 60°. The top of the thread is truncated by an amount equal to 0.033 × pitch. The depth of the thread is 0.8 × pitch.

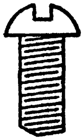
Bolts, Nuts and Screws. Screw threads are used to a great extent in fastening machine parts together. The threads appear on devices known as bolts or screws. A bolt is a piece of stock with a distinctive shaped head on one end and a screw thread on the other, onto which a nut is assembled. Machine screws are inserted into holes that have been made in a machine element and threaded to receive the screw. Much of the work of a machinist has to do with the selection of drills for the holes that must be tapped in the machine part or in a piece that is to be made into a type of nut. It is essential that he be able to recognize the various types of bolt heads and know why they are used.


A
Bolt


B
Cap Screw

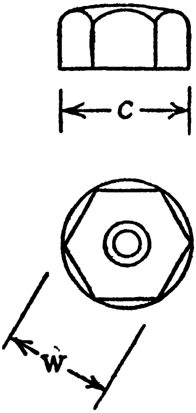

C
Fillister Head Screw

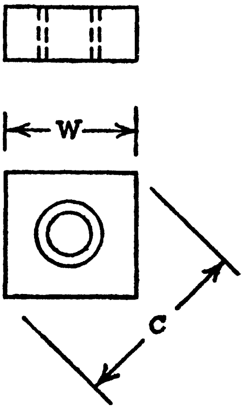

D
Flat Fillister Head Screw


E
Round Head Screw

In designating the length of a machine screw the head is not included if the screw is of the fillister head, round head, cap screw type. The length of a flat head screw and of a stove-bolt is considered as including the head.

Fillister head screws are used in machine construction where it is desired to counterbore the hole so that the head of the screw will be flush with the surface of the machine part.

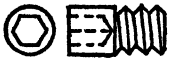


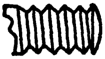


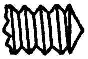
The regular American Standard bolt heads and nuts have proportions which are found by the formulas given below, where $D =$ major diameter of the thread:

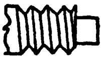
To Find	Hexagonal Nuts and Bolts	Square Nuts and Bolts	
W	$W = 1\frac{1}{2} \times D$	$W = 1\frac{1}{2} \times D$	to nearest $\frac{1}{16}''$
T	$T = \frac{7}{8} \times D$	$T = \frac{7}{8} \times D$	to nearest $\frac{1}{64}''$
C	$C = 1.155 \times W$	$C = 1.414 \times W$	to nearest $\frac{1}{16}''$

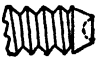
Set screws are used for fastening two machine parts together by pressure of the screw point against one member. Pulleys and other wheels are often secured by means of set screws. The hub of the pulley contains the thread; there may be a seat for the point of the set screw in the shaft. Because of the danger from revolving shafts with projecting set screw heads, the headless or safety set screw is used. This screw is made with a small socket at one end instead of a head. The socket may be hexagonal, square or fluted. Each type requires a special wrench.


Flat Point


Oval Point


Cone Point


Dog Point


Cup Point

Lag screws are made with a square head and have a cone point or a gimlet point. The thread on a lag screw is the same as the thread on a wood screw. Lag screws are used for fastening heavy objects to wooden beams. Machines are bolted to the floor with lag screws.

Self-tapping screws are a comparatively new development. They are used for fastening thin metal parts. Sheet parts may be fastened without the necessity of cutting a thread for the screw. The self-tapping screw has a sharp Vee thread and has been hardened.

Studs are screws which are threaded at both ends. The stud is screwed into one part of the machine, and a nut is used on the other threaded end to fasten the other machine part in place.

Tap Drills. Every screw thread that is used requires a corresponding thread into which it will fit. The selection of the correct size tap drill is of great importance. If a hole is made too large the thread will not have the expected strength. Careless workmen often "get away" with an insufficient thread in a hole unless it is inspected. The thread will "strip" quickly unless there is at least 23% of the needed thread height in contact. Experiments have shown that a thread that has 75% of the full thread depth is more economical to produce and will give complete satisfaction. Commercial holes are usually made with a 75% thread and tap drill tables are published which give this amount of thread.

The full depth of an American National thread will be obtained if the tap drill is selected by the formula:

$D-\frac{1.3}{N}$, where D represents the major or outside diameter. Now ¾ of 1.3 is .97, practically; hence the formula for a commercial tap drill becomes: $D-\frac{.97}{N}$.

Exercises

1. If the outside diameter (major diameter) of a screw is .875″ and the T.P.I. is 16, what size tap drill should be used to give a commercial thread?
2. Find the minor diameter of a ¾″—10 N.C. thread.
3. Find the tap drill size for a ¾″—10 N.C. thread.
4. Find the minor diameter of a ¾″—10 Sharp-Vee thread.
5. Find the thickness of head, the width across the flats, and the distance across the corners of a ⅝″ standard hexagonal nut.
6. Name some of the methods by which threads are made.
7. What were the old names of the National Coarse thread series?
8. Why are multiple threads used?
9. Why are left-hand threads used?
10. Why was the old U.S.S. system unsatisfactory to the automotive industry? What thread series developed from this dissatisfaction?

50. PULLEYS AND BELTS

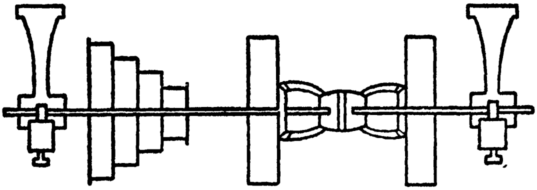
Transmission of Power. In every manufacturing plant there is a primary source of power. This source may be self-contained within the plant, in which case it will consist of engines, generators, etc. Or the power may be purchased from outside the plant, in which case it comes in over the transmission lines to the motors. The transmission of power from the point of origin within the plant to the machines is accomplished by means of belting, shafting, pulleys and countershafts. This, of course, relates to the mechanical transmission of power.

Mechanical transmission of power for short distances is obtained by means of gearing. Longer distances require belts or ropes; sprocket chain drives are used for positive driving over medium distances.

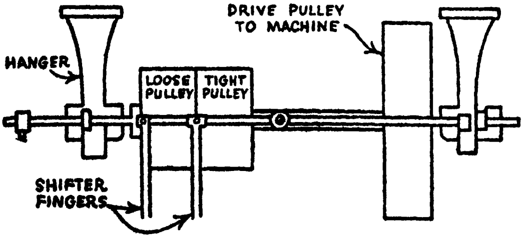
In shops where the machines are driven by belting, it will be noticed that a long shaft runs down the middle of the room; this shaft is known as the main shaft or *line shaft.* It is driven by a motor. If the width of the room or the placement of the machines makes it necessary, there may be another shaft which is driven from the main shaft, and which in turn is driving a group of machines. This is known as a *Jack shaft.* Over each machine there will be some form of countershaft. A countershaft exists for the purpose of providing a means of turning the power onto the machine or off, as required.

Countershafts have a variety of design, but in general they are of the friction clutch type or the loose pulley type. The *loose pulley countershaft* has two pulleys side by side on one section of the shaft. One of the pulleys is keyed or otherwise fastened to the shaft so that the shaft must turn if the pulley turns; the other pulley is loose on the shaft and turns without affecting the shaft. A shifter handle extends down to the reach of the operator. When the handle is moved it causes the belt to be shifted from the loose pulley to the tight pulley, thus sending the power to the machine.

In the *clutch type of countershaft* the shifter rod moves a sleeve or spool which is tapered on the ends. The sleeve forces friction fingers to lift; the fingers press a plate against the inside of the pulley, and power flows down to the machine. The clutch type of countershaft is used where a reversal of direction may be needed. The countershaft has two pulleys which are belted to the main shaft. One belt is open and the other is crossed; this produces a change of direction. Where it is not necessary to reverse the machine, the two pulleys may be of different diameters tc secure different speeds.

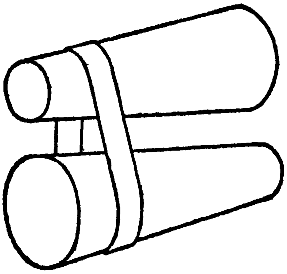


Friction Clutch Countershaft

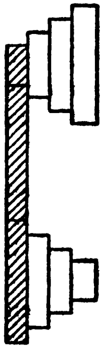


Loose Pulley Countershaft

Pulleys on the main shaft are usually of the split type. Split pulleys may be removed from or added to the shaft as needed. Examination of any pulley that is used with belting will show that it is not flat on the rim or face. The center is slightly higher; this is called a crown. The purpose of a crown is to keep a belt on the pulley. Belts will always tend to travel to the highest part of a pulley because the tension of the belt is greatest on that side. Since the highest part of the crowned pulley is in the center, the belt stays on unless there is a defect in the belt or in the alignment. Cone pulleys are used to obtain an unlimited number of speeds. The headstock of a lathe is usually referred to as a cone head. This is a misnomer, however, since the so-called cone of pulleys is in reality a step cone pulley.



Cone Pulleys



Step Cone Pulleys

Belting. Belts are made by cementing strips of tanned leather into long pieces. The leather used for belting is taken from the center of the hide along the backbone area and out to about twelve inches. Belts are made in single or double thickness. The majority of belts are single thickness. Belts are run with the smooth or hair side to the pulley in order to secure better contact than would be obtained from the flesh or rough side.

The horsepower transmitted by a belt depends upon several factors. The velocity of the belt, the width of the belt and the pull per inch of the belt together determine the horsepower transmitted. The arc of contact of the belt also plays an important role in the pull of the belt. The formula below will give a practical answer to a horsepower problem; for more accurate results handbooks should be consulted.

W=width of the belt in inches,
V=velocity or speed of the belt in feet per minute,
S=stress or pull in pounds per inch of width,
D=diameter of pulley in inches.

Since the diameters of pulleys are given in inches and the speed or velocity is given in feet, it is necessary to divide the circumference in inches by 12 in order to obtain feet. Since the formula for circumference is $3.1416 \times D$ and the result must be divided by 12, the constant .2618 is used as a multiplier of D (.2618 is obtained by dividing 3.1416 by 12).

$$\text{Horsepower} = \frac{.2618 \times D \times N \times W}{33{,}000}.$$

Exercises

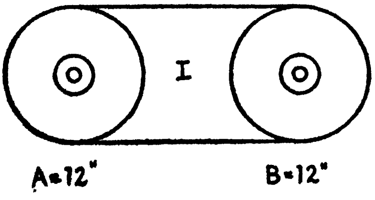
1. What horsepower will be transmitted by a belt which is 4″ wide, if the driving pulley is 12″ in diameter and the R.P.M. is 100? Assume a working pull of 48 lb. per inch of width.
2. A 10 H.P. motor runs at 1750 R.P.M. The pulley is 8″ in diameter and is 6″ wide. Assuming a working pull of 38 lb. per inch width of belt, what H.P. will be transmitted?
3. A belt is running over a 10-inch pulley and is transmitting 28 horsepower at 2100 R.P.M. What is the width of the belt, if the tension is 40 lb. per inch?

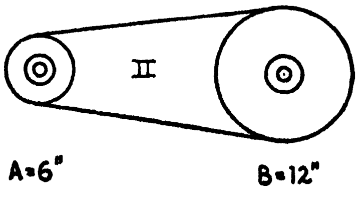
Pulley speed is the number of feet that a point on the rim of the pulley travels in one minute. It is equal to the circumference of the pulley multiplied by the R.P.M. If speed is represented by V, the circumference by C, and the R.P.M. by N, then the speed of a pulley is given by: $V = C \times N$. The diameter is given in inches, as a rule, so the constant .2618 may be used to find circumference in feet: thus, $V = .2618 \times D \times N$.

Exercises

1. What is the speed of a pulley that is 10″ in diameter when it is turning 120 times a minute?
2. A 12-inch pulley runs at 100 R.P.M. What is the surface speed?
3. A pulley has a peripheral speed of 312 feet per minute, and its diameter is 12 inches. How many R.P.M. is it making?

Speed of Pulleys Belted Together. If two pulleys are connected by a belt, the one in which the power starts is called the *driver* and the other is called the *driven pulley*. Neglecting the possibility of the belt slipping, the surface speed of both pulleys must be the same, since they are connected by the belt. If two pulleys of the same diameter are belted together, the driven pulley will make as many revolutions per minute as the driver





pulley. In Fig. I, both pulleys are assumed to be 12 inches in diameter. Each time that pulley A turns, pulley B must also turn. If pulley A makes 100 revolutions, pulley B must also make 100 revolutions. The speed of both pulleys is the same. The revolutions are the same.

The terms Speed and R.P.M. are often confused. Remember that two pulleys that are connected by belts will travel at the same rate of speed, although they may be turning at a different number of revolutions per minute. Speed relates to the *distance* traveled in one minute; revolutions refer to the *number of turns* made in a minute.

Consider Fig. II. Pulley A is driving pulley B. Pulley A is only one-half the size of pulley B. Pulley A will have to turn *twice* to make pulley B turn *once*. The distance traveled by a point on the rim of A will be equal to the distance traveled by a point on pulley B; but A must turn twice, i.e., make two revolutions, in order to do so. Let D represent the diameter of the driver pulley; N, the number of revolutions per minute of the driver; and S, its surface speed. The driven pulley diameter can be represented by d, its revolutions by n, and its surface speed by s and

The driver speed, $S = .2618 \times D \times N$

The driven speed, $s = .2618 \times d \times n$

Since both pulleys (the driver and the driven) are connected by a belt, the surface speeds are the same. Therefore, since $S = s$, the formula can be simplified by dropping out the two elements that are equal, and it becomes $.2618 \times D \times N = .2618 \times d \times n$. Since .2618 appears in both parts of the equation it, too, can be dropped by dividing both sides of the equation by .2618. The formula then becomes $D \times N = d \times n$.

In other words the relation between the driver and the driven pulley may be stated as follows: when two pulleys are connected by a belt, the product of the diameter and revolutions per minute of one equals the product of the diameter and revolutions per minute of the other.

Going back to Fig. I, the diameters are each given as 12″; assuming the R.P.M. of the driver to be 100, we have:

$$D \times N = d \times n$$
$$12 \times 100 = 12 \times n$$
$$n = \frac{1200}{12} = 100, \text{ Ans.}$$

Assuming that the driver in Fig. II is the 6 in. pulley, and that it is making 100 R.P.M., we have:

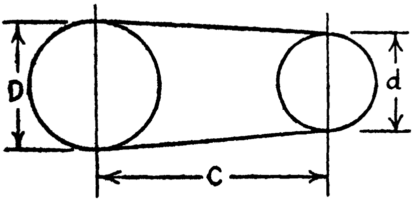
$$D \times N = d \times n$$
$$6 \times 100 = 12 \times n$$
$$n = \frac{600}{12} = 50, \text{ Ans.}$$

Always study the problem carefully to be sure that you understand which is the driver and which is the driven pulley.

Exercises

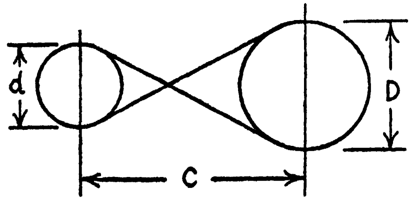
1. A motor pulley of 5-inch diameter is running at 1750 r.p.m. It is belted to a pulley on the main shaft, which is 30″ in diameter. How many r.p.m. is the main shaft pulley making?
2. It is desired to run a countershaft pulley at 180 r.p.m. The countershaft pulley has a diameter of 10 inches. The main line is running at 200 r.p.m. How large a pulley should be used on the main shaft?
3. The countershaft of a lathe is turning at 240 r.p.m. The steps on the lathe headstock are 4, 5, 6, and 7, and the steps on the countershaft's corresponding pulley are 9, 8, 7, and 6. Find the spindle speeds for each step.

Length of Belts. A common maintenance problem in the shop is the replacement of belts that have become unserviceable. The "rule-of-thumb" mechanic would probably measure the piece of new belting by matching it with the old belt. There is a good working formula for obtaining the length of a belt if the old belt is not available, or if a new pulley has to be belted up to the main shaft. If D is the diameter of the large pulley, and d is the diameter of the other, with C representing the distance between centers of the pulleys, the approximate length of the belt around the pulleys is found by:



$$L = 1.57(D+d)+2C.$$

If the belt is *crossed,* the length can be found by:



$$L = 1.625(D+d)+2C.$$

Example: Find the length of belt for an open belt between two pulleys whose center distance is 10 feet and whose diameters are 14″ and 16″, respectively.

$L = 1.57(D+d)+2C$

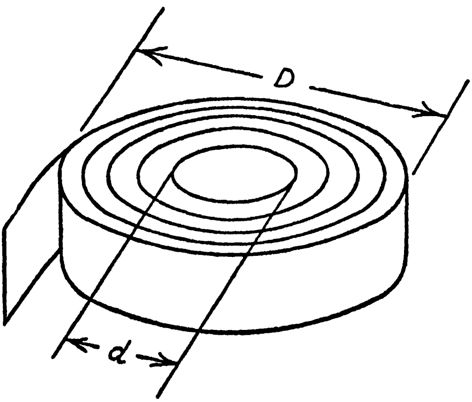
$L = 1.57(14+16)+240$

$L = 1.57 \times 30+240 = 287.1$ inches, *Ans.*

Exercises

1. Find the length of an open belt to connect two pulleys, 9″ and 15″ in diameter, respectively, with a center distance of 9½ feet.
2. Find the length of a crossed belt which connects pulleys 15″ and 20″ in diameter, respectively, with a center distance of 11′ 8″.

Belt Coil. It may be necessary to calculate the length of belt in a coil to save the labor of rewinding it if it should not be long enough for a replacement job. The length may be estimated by taking the average circumference and multiplying it by the number of turns in the coil; the outside diameter of the coil plus the inside diameter, divided by 2 will give the average diameter. The diameters are given in inches, and the length is expressed in feet.



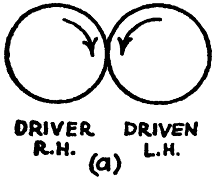
$$L=\frac{3.1416(D+d)\times N}{12\times 2}=.131(D+d)\times N.$$

Speed of Gears. The formula for pulley ratios is used when working out gear speed problems. Instead of the number of inches diameter, which was used in the pulley formula, the number of teeth in the gear is used. A gear with 40 teeth that is in mesh with another gear of 40 teeth will travel at the same surface speed and the same r.p.m. Let S=the r.p.m. of the gear, T=the number of teeth in the driver, and t=the number of teeth in the driven gear; then $T\times S=t\times s$.

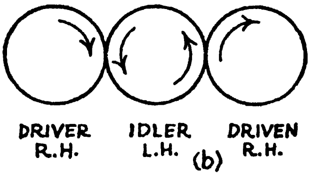
EXAMPLE: If a 24-tooth gear is in mesh with a 36-tooth gear, and the smaller gear is rotating at 120 r.p.m., how fast is the larger gear going?

$$T\times S=t\times s$$
$$36\times S=24\times 120$$
$$S=192$$

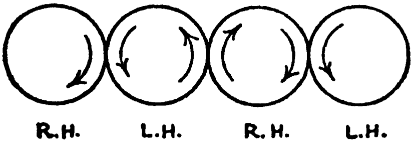
Gear Transmission Terms. In a simple train of gears, two or more gears are in mesh, but each gear is on a separate shaft. The first gear is called the driver, the last gear of the train is called the driven. The gears (in a simple train) between the driven and the driver gears are known as *intermediate gears*. In a train of three gears the middle one is commonly called an *idler* gear. Two gears in mesh always run in opposite directions; idler gears are introduced to change the direction of rotation of the driven gear. It is sometimes necessary to bridge a space between the driver and the driven gears and also have the driven gear run in the opposite direction from the driver gear; in this case two intermediate gears would be used.



(a)



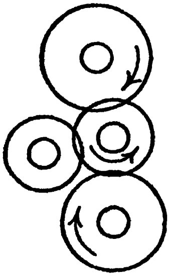
(b)



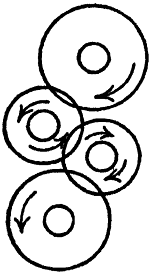
(c)

In (a) above it can be seen that the driver causes the driven to rotate in the opposite direction from that of the driver. The introduction of the idler in (b) causes both the driver and the driven to rotate in a right-hand direction. In (c) two gears are used to bridge a space and also rotate the driven gear in a left-hand direction.

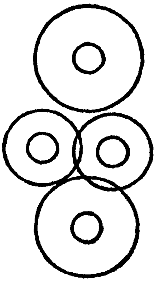
An interesting application of the effect of one or two intermediate gears is found in the reversing gear mechanism of a lathe. The mechanism is placed between the spindle gear and the stud gear. The gears are placed in neutral when no feed is wanted; one idler is used to cause the feed screw to rotate, and two idlers are used to rotate the feed screw in the opposite direction.



FORWARD



REVERSE



NEUTRAL

Remember that in a simple train there are only two gears that affect the r.p.m.; they are the first and the last, the driver and the driven. All the others, the intermediates, have no effect on speed; they only change direction and fill up space.

EXAMPLE: Referring to Fig. (a), suppose that the driver gear has 24 teeth and the driven gear has 36 teeth. When the driver gear makes one revolution, 24 teeth will have engaged and turned 24 teeth of the driven gear. If the driver turns 150 times, how many will the driven gear turn? By the formula:

$$T \times S = t \times s$$
$$24 \times 150 = 36 \times s$$
$$S = \frac{24 \times 150}{36} = 100, \textit{ Ans.}$$

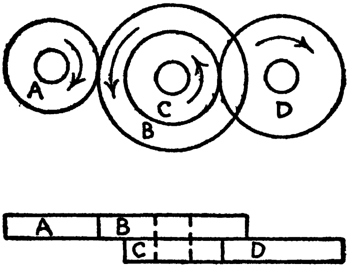
If there were another gear between the driver and the driven, or if there were several interposed gears, the result would still be the same; the driver would move 24 teeth on each gear down to the driven.

Exercises

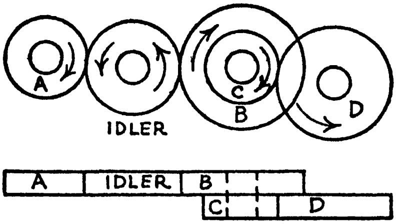
1. If the driver is turning 80 r.p.m. and has 40 teeth, how many r.p.m. will the driven go, if it has 30 teeth?
2. A gear train has the following gears in order: 32T, 28T, 40T, and 60T. If the first gear makes 120 r.p.m., how many r.p.m. will the last gear make?
3. The spindle gear of a lathe has 28 teeth, the tumbler reverse gears have 40 and 46 teeth, and the stud gear has 56 teeth. If the spindle turns 90 times, how many revolutions will the stud gear make?

Compound Gears. When the ratio between the driver and the driven gears is too great for simple gearing, another type, known as compound gearing, is used. If for example a fine thread were being cut on a lathe that had a lead screw of four threads per inch, the gear required for the stud might be too big for the machine. Suppose that the thread to be cut is a 56 T.P.I. screw and the lead screw has 4 threads per inch; the ratio is 1 to 8. The gear on the stud would be 24 and the gear on the lead screw would have to have 192 teeth. Such a gear could not be placed on the shaft without removing safety guards. Another objection to a high ratio is the unusual strain that is placed on the gear train of the machine. Summing up, compound gear trains are advisable where the ratio is greater than 1 to 3. They are used to save space and to reduce turning strain on the gear teeth and shafts.

In a compound train of gears two gears of the train are on the same shaft and are keyed together. One of the pair is a driven gear, the other is a driver gear. If an idler is used in the compound gear train, it is neglected in calculations just as it is in a simple gear train. Speed calculations for compound gear trains use the same general formula that is used for a pair of pulleys or a single pair of gears, the only difference is that there will be more than one driver and more than one driven gear in the problem. Let T=the number of teeth in the first driver, T_1=the teeth in the second driver, T_2=the teeth in the third driver, etc., and the same for the driven, i.e., t=the first, t_1=the second, etc. Let S=the number of revolutions of the first driver gear and s the number of revolutions of the final driven gear. Then



Compound gear train



Compound gear train with idler

$$S \times T \times T_1 \times T_2 \times \text{etc.} = S \times t \times t_1 \times t_2 \times \text{etc.};$$

$$\text{or } S = \frac{S \times T \times T_1 \times T_2 \times \text{etc.}}{t \times t_1 \times t_2 \times \text{etc.}}.$$

Example: In the compound gear train shown in the sketch, assume that gear A has 24 teeth, gear B 56, gear C 32, and gear D 64. If the first driver is rotating at 98 revolutions per minute, how many r.p.m. will the final driven gear make?

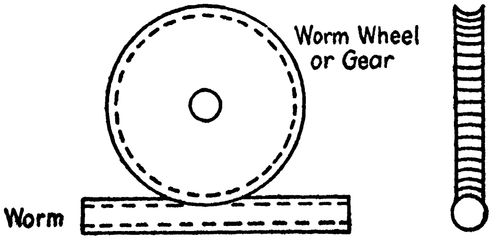
In all compound gear problems it is essential that the drivers and the driven gears are not confused. In the present instance, A is a driver, since it is the starting point for the power in the train. A is meshing with B, so B must be a driven gear. On the same shaft with B is C; the number of teeth on B has no direct effect on C, since they do not mesh with C. C is a driver of D, which is the final driven gear. From the formula:

$$S = \frac{98 \times 24 \times 32}{56 \times 64} = 21 \text{ r.p.m. for the final driven gear}$$

Exercises

1. Gear A has 48 teeth, gear B has 56 teeth, gear C has 28 teeth, and gear D has 32 teeth. If gear D is turning at 96 r.p.m., how many r.p.m. is gear A making?
2. Gear A has 32 teeth, gear B has 28 teeth, gear C has 56 and gear D has 48. How many r.p.m. must gear A make in order to cause gear D to turn at 30 r.p.m.?
3. A compound gear train has one idler between the first driver and the first driven gear. If the first driver (A) is turning right hand (clockwise), in what direction will the final driven gear turn?
4. A ratio of 3 to 5 between the driver and the driven shafts is required. Assuming that gear A has 40 teeth, gear B, 32 teeth, and gear D, 36 teeth, find the number of teeth that C should have to get the required ratio.
5. How many turns will the driver and final driven gears make in the following gear combination? Gear A has 28 T, gear B has 40 T, gear C has 72 T, and gear D has 64 T.

Some mechanisms require a very high ratio between the driver and final driven shafts and the space limitations of the machine will not permit a train of gears to be used. Under such conditions worm gears and worm wheels are used. A *worm* is in effect a threaded rod. The worm gear is cut so that the circumference of the worm wheel will fit into the teeth of the gear. A worm will turn the gear one tooth for each revolution of the worm unless it is a double-threaded, triple-threaded, etc. worm. In the index head used on milling machines, a single-threaded worm meshes with a 40-tooth worm gear. This produces a ratio of 40 to 1. It would be difficult to secure such a high ratio with ordinary gears even if they were compounded. Another advantage of a worm and worm wheel drive is in the control. In the index or dividing head, any part of a revolution may be easily secured simply by turning the worm shaft a small amount.

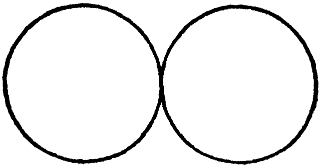


Sprocket and chain drives are used for the transmission of power over short distances. They furnish a positive transmission, i.e., there is no possibility of slippage. The speed of a sprocket and chain drive is computed with the same formula used in simple gear train calculations.

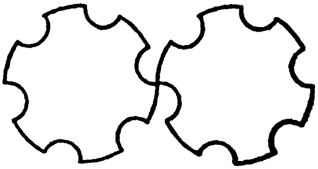
51. GEARS AND GEAR CUTTING

Basic Concepts. If two cylinders touch each other so that their surfaces are parallel, and if they are both mounted on shafts, then when one of them is turned the other will turn also. If the two cylinders are of equal diameters, one revolution of the driver will cause the other to turn one revolution. Under a very light load this type of drive might be satisfactory, but slipping between the cylinders would almost certainly occur if the load were increased even moderately.

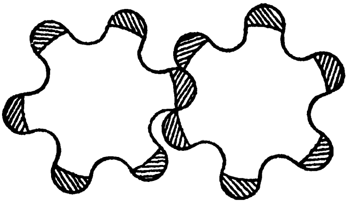
If instead of the smooth cylinders we use cylinders that have projections on their periphery, and so arranged that there is an interlacing between them, there will be no slipping.



(a)



(b)



(c)

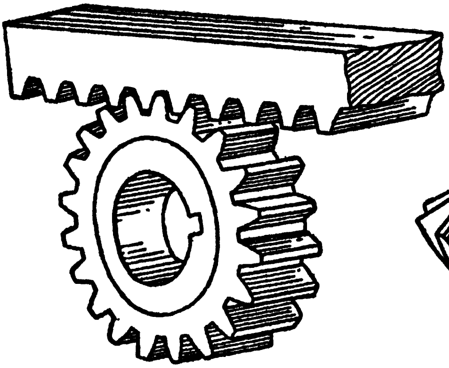
Let (a) represent two cylinders in contact; observe the diameters of the two cylinders. In (b) a portion of the surface has been removed to form grooves; the original diameters are still evident. In (c) a projection has been added to the cylinders in such a way as to engage the grooves of the other roll. The space between the original diameters and the top of the projections is called the *addendum*. The original diameters are not marked or visible on the completed toothed rolls, but they still exist technically. The diameters of the original rolls are called the *pitch diameters,* and the circle which marks the point of contact between the circumference of the rolls is called the *pitch circle*. The two toothed wheels or gears shown in (c) would clearly turn each other, but further refinements in tooth profile and dimensions are needed to secure smooth operation and greater efficiency. The development of the toothed wheel should be understood as a basis for the further study of gear teeth.

Classes of Gears. The most common type of gear is known as the *spur gear*. The teeth of spur gears are parallel to the axis of the shaft. They are used to transmit power between shafts that are parallel.

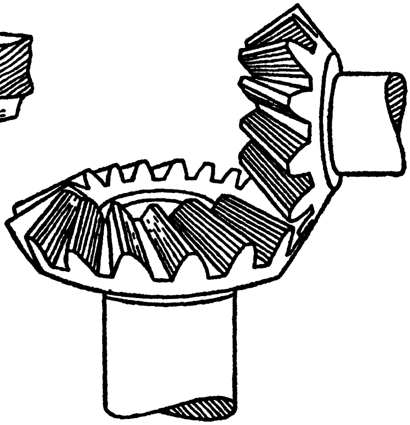
Bevel gears have their teeth cut on a cone, or parts of a cone. Bevel gears are used to transmit power between shafts that are not parallel, but whose centers would meet if they were extended sufficiently. A pair of bevel gears whose diameters, angles and number of teeth are the same, is called a pair of *miter gears.*

Racks are straight bars with teeth cut so as to mesh with gears and pinions to produce a straight line motion. The lathe carriage is moved along the ways by means of a rack and pinion.

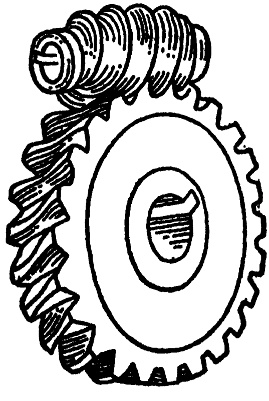
Worm gears are used to transmit motion between shafts that are not parallel and that could not meet. The elements of worm gears are screwlike.



Rack and Pinion



Bevel Gear and Pinion

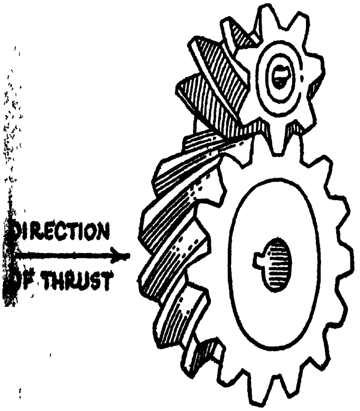


Worm and Worm Gear

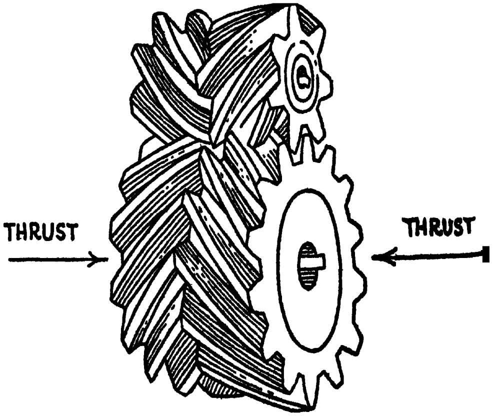
With the exception of the worm and worm gear, the gears mentioned above have straight line elements in the tooth shape. Unless a straight line tooth is very accurately made, the operation at high speeds will be noisy. This is due to the instant engagement of one tooth with the opposite tooth for its full length; a slap or knock may occur. This also affects vibration and chattering of machine parts. To overcome this noise and vibration, gears are made with teeth that will develop a *gradual meshing* of each tooth along its length.

To reduce the noise and vibration in spur gears the helical tooth and the herringbone tooth are used. The teeth of the helical spur gear are not parallel to the axis of the shaft. They are cut on an angle. The proper name for this type of gear is the *helical gear,* but mechanics sometimes call it the spiral gear. Since the teeth are cut on an angle the engagement of the teeth is continuous; one tooth and parts of the adjacent teeth are in mesh at all times. As the gear revolves, the relationship changes for each tooth, but there is always a gradual engagement of the teeth instead of the immediate engagement that a straight line tooth would have.

The *herringbone gear* consists in effect of two helical gears which are joined together. Actually, the gears may be made in sections, or the teeth may be cut on the one blank. The name herringbone is applied because of the appearance of the finished gear. In this gear the engagement is also continuous and gradual. Herringbone gears are used on heavy rolling mill machinery because of their strength and freedom from vibration. Another advantage of the herringbone gear is the absence of *end thrust.* In a helical gear the tooth angles produce a pressure toward the side, and the gear tends to be forced out of engagement as a result of this end thrust. This introduces problems of thrust bearing to oppose the pressure. Herringbone gears equalize the end thrust because the side pressure of one side is offset by the opposing side pressure of the other side.



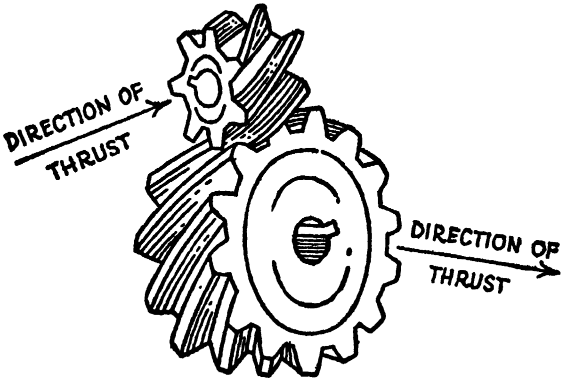
Helical Gears



Herringbone Gears

Bevel gears are subject to the same type of noise condition and vibration as spur gears. Designers have reduced the problem in bevel gears as they did in spur gears, viz., by developing gear teeth that engage gradually and are in continuous mesh. Spiral tooth bevel gears do not have the disadvantages that a straight line bevel gear tooth would have. They are used in rear axles of automobiles extensively.

Spiral gear drives are used to drive shafts that are not parallel and are not intersecting. The teeth of the spiral gear have a more pronounced helical angle than in the case of the helical spur gear tooth. Spiral gear drives are also called screw gears, because the action of the teeth is a sliding contact similar to that of a screw thread. The end thrust of screw gears is very much greater than that of the helical spur because of the greater helical angle.

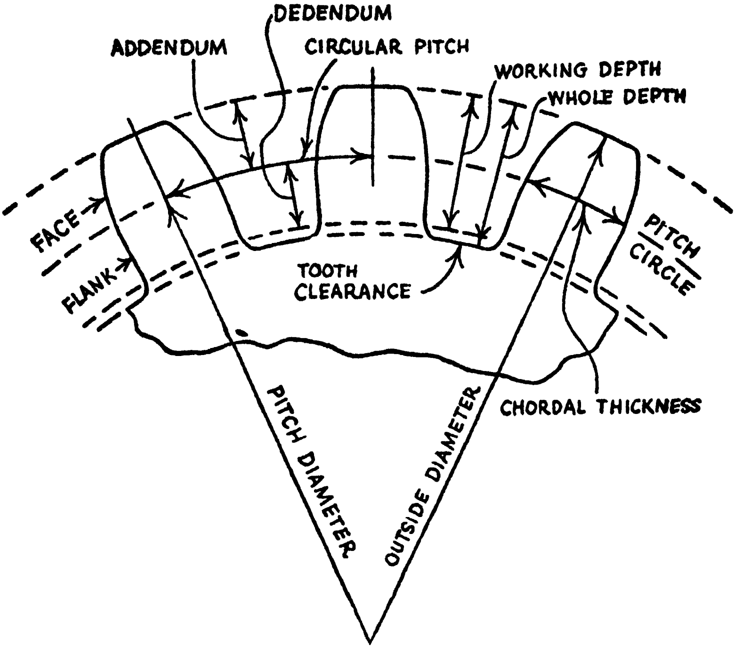


Spiral Gears

Tooth Parts. The terminology of gear tooth parts will be better understood if the idea that all gears are developed from the friction drive is kept in mind. The profile of the tooth has a definite shape which is based on the fact that a point on a tooth must follow a curved path as the point rotates and makes contact with its opposing tooth and then leaves contact. This curve will vary for every diameter of gear, or the theoretical friction roll upon which the gear is based. In gearing the number of teeth is referred to instead of the diameter, since the number of teeth gives an indication of the size of the tooth and gear.

The *pitch circle* of a gear is the line that represents the imaginary surface of the original friction roll. It is of practical value in the drawing of gears and must be considered in gear calculations.

The *pitch diameter* of a gear is the diameter of the pitch circle. The shape and the size of gear teeth depends upon the pitch diameter.



The *addendum* is the radial distance between the pitch circle and the top of the tooth or the outside diameter circle. It is the amount that has been added to the original friction roll.

The *dedendum* is the radial distance between the pitch circle and the bottom of the tooth. Some authorities define the dedendum to include the clearance at the bottom of the tooth also. This is a practical definition, but in theory, at least, the addendum and dedendum are equal since they represent the projection and the groove, respectively, of the original roll. In connection with gear teeth measurements, the addendum must be measured; the dedendum, however, is not measured, and is used only in drawing the gear tooth.

The *working depth* of a tooth is equal to the addendum plus the dedendum.

The *whole depth* is the working depth plus the clearance. As the tooth turns, the top of one tooth would interfere with the bottom of its opposing tooth if it were not for the clearance distance.

The *face* of the tooth is the surface of the tooth from the pitch circle to the top of the tooth.

The *flank* is the surface from the pitch circle to the bottom of the tooth.

The *thickness* of the tooth is measured on the pitch circle. In practice the chordal thickness is used because the measuring instruments measure lineal distance.

The *outside diameter* of the gear is the pitch diameter plus the addendum on each side. It is measured over the tops of the teeth.

Size of Gear Teeth. In reference to screw thread sizes we learned that the relative size of a thread is indicated by the number of threads per inch. This was contrasted to the metric system in which the size of thread was stated directly in terms of pitch or distance from one thread to the next. In the American system it is necessary to divide 1 by the number of threads per inch in order to learn the true size of the thread. A comparable situation exists in reference to gear teeth sizes.

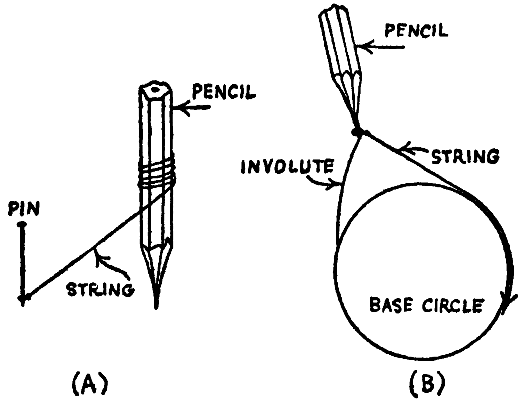
Circular Pitch. Before the development of appropriate mechanisms for accurately dividing a circle in a shop, gears were made from patterns. The patternmaker, working in wood, had to make a form or pattern so that the gear could be cast in the foundry. The gear teeth were cut out of wood and attached to the rim of the gear pattern. The spacing of the teeth was of the utmost importance and had to be accomplished as accurately as possible. The pitch circle was laid out and divided by hand. The distance measured was in reality a chord of the pitch circle. The circular pitch is the distance from a point on one tooth to a corresponding point on the next tooth; it is that point of the pitch circle circumference that would include a tooth and a space. The pitch circumference divided by the number of teeth equals the circular pitch. When gear teeth had to be laid out in the manner described above, gears were sized with pitches that were easy to measure; common fractions such as ½ inch were used to fix the spacing.

Diametral Pitch. The development of dividing mechanism for use in the machine shop has made the circular pitch measure less important. It is still used for large gears which are to be cast, but the smaller gears are now cut from solid metal by the machinist. In the old method of making gears it was advisable to have gear circumference with dimensions that could be easily and evenly divided without awkward fractions. The circumference is not important now in the concept of tooth sizes. It is more natural to think of a circle in terms of its diameter, and the size of gear teeth is now indicated by reference to the pitch diameter instead of the pitch circumference. In gear design today it is more important to keep gear sizes such as distance between gear centers, pitch diameters, depth of teeth, etc. in terms of easily handled fractions of inches.

The *diametral pitch* is the number of teeth on the rim of the gear for each inch of pitch diameter. If a gear of 3 inches pitch diameter has 30 teeth, it is a 10-pitch gear. If on a gear with the same pitch diameter (3 inches) there are 36 teeth, the gear is a 12-pitch gear. A gear with 48 teeth and 3 inches pitch diameter would be a 16-pitch gear. From the foregoing it will be seen that the larger the number of the pitch, the smaller the tooth will be.

Two pitches have been referred to—the circular pitch and the diametral pitch. In practice the word "pitch" is taken to mean diametral pitch, because most of the calculations of sizes, etc. are now based on the diametral pitch system. A *rack* may be considered as a section of a gear with an infinitely large radius. However, for practical purposes, rack teeth are sized to match or fit the teeth of the gear that is to mesh with them. The term circular pitch applied to rack teeth is an inaccurate designation, since it implies measurement on a circle. Linear pitch is a more accurate term to use in referring to rack teeth because it applies to a straight line.

The Involute Curve. Smoothness of operation is essential in the operation of gear teeth. As each tooth comes around and makes contact with its opposing tooth there is a wiping action on the flank and face of the opposing tooth. This action follows an involute curve path. An *involute* curve can be drawn by winding a piece of string around a pencil. If the free end of the string is held on a piece of paper while the pencil is used to draw a curve, unwinding the string as it does so, the result will be an involute.



A second method of drawing an involute is shown in figure (B). From this figure we can see that the involute curve will be affected by the diameter of the disc on which the curve is based. This is equally true as regards the true shape of gear teeth. The profile of the tooth is different for every change in diameter. A 12-pitch gear tooth on a gear that has 24 teeth will have a different profile from a 12-pitch gear tooth on a gear of 64 teeth. The diameters of the base circles are different.

Gear Cutters. It is obviously impractical to have a cutter for every possible profile of tooth in all of the pitches and sizes. It has been necessary to compromise and to standardize gear cutters for practical use. Standard gear cutters are made with eight cutters to each pitch. The cutters will cut all sizes of gears from 12 teeth up to a rack and give satisfactory results. The Brown and Sharpe Manufacturing Co. originated the following system which has been adopted by the other manufacturers of gear cutters.

Involute Gear Cutters

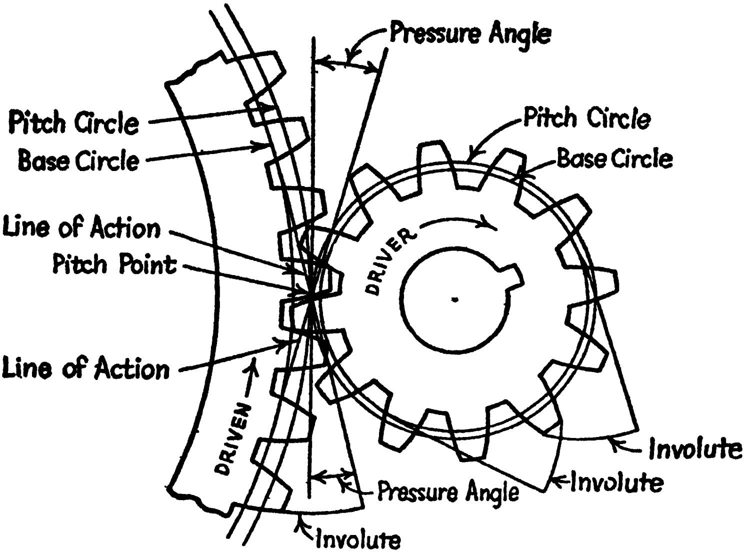
No. 1 will cut gears from 135 teeth to a rack
No. 2 will cut gears from 55 teeth to 134
No. 3 will cut gears from 35 teeth to 54
No. 4 will cut gears from 26 teeth to 34
No. 5 will cut gears from 21 teeth to 25
No. 6 will cut gears from 17 teeth to 20
No. 7 will cut gears from 14 teeth to 16
No. 8 will cut gears from 12 teeth to 13

Each cutter is accurate for the smallest gear in its range; inaccuracy increases as the highest number of teeth in the range is approached. Cutters may be secured in half numbers if greater accuracy of tooth profile is required. Cutters are ordered by pitch and number of the cutter. If a cutter is needed for an 8-pitch gear of 38 teeth, an 8-pitch No. 3 cutter is called for.

Pressure Angles. The involute tooth gears are designed with either a 14½° or a 20° pressure angle. These are standard. To understand the term pressure angle consider a pinion gear that is meshing with a rack. As the gear tooth sweeps around it establishes contact with the top face of the rack tooth; it then exerts a pressure on the rack tooth as it revolves. The rack tooth is pushed along by the gear tooth. The line of action through which the power is being transmitted is at an angle (pressure angle) of 14½° with the axis of the rack. If the gear is of the 20° pressure angle type the line of action is at 20° with the axis.

Gear Cutting Methods. It has already been pointed out that the standard gear cutters are accurate for the smallest gear in their range and that the amount of inaccuracy increases as the upper range of the cutter is approached. The reason for this condition is the impracticality of having cutters for every different profile of tooth in a pitch. As a commercial proposition the revolving formed gear tooth cutter is acceptable. The need for increased accuracy becomes greater at higher speeds. Machinery which is comparatively quiet at low speeds will tend to increase in noisy operation as the speed is increased. Increased accuracy in tooth profile will result in quieter operation at higher speeds and reduced vibration.

Gear teeth are made by milling or generating the gear teeth. The most common process and the oldest is that of milling. This is the method



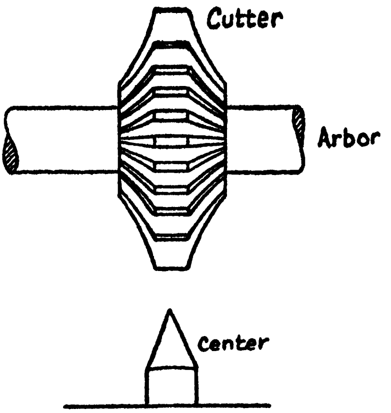
that the machinist is most likely to use, since the generating processes are performed on automatic machines. Milling may also be done on automatic machines, but that is on a production basis and the machines are controlled by operators.

Assuming that the teeth of a gear are to be cut and that the blank has been turned to size, the following method is used.

Milling of Gear Teeth. The pitch of the gear and the number of teeth in the gear are found from the instructions or blueprint that accompany the gear. Knowing the pitch and the number of teeth, the proper gear cutter is ordered by stating the pitch and the cutter number wanted. The cutter is placed on a milling machine arbor so that the center of the gear blank can be reached by the cutter. The cutter should be as close to the column of the machine as possible in order to reduce the possibility of vibration. The cutter should be placed on the arbor so that it will revolve in the right direction. Gears are usually milled so that the force of the cut will be directed against the index head and not the index head tailstock.

When the cutter is assembled and tightened on the arbor it is centered by moving the milling machine table sideways until the center of the head stock is set at the center line of the cutter. Most gear cutters have a permanent line around the periphery of the teeth to show the center of the tooth. This is an approximate method of centering and is satisfactory for most of the gears that have to be cut. If greater accuracy is needed a correction must be applied after the first tooth space has been cut.

The gear blank is forced on to a lathe mandrel and a milling ma-



chine dog is attached. The mandrel is then placed between the index centers and .the tail of the milling dog is clamped in the index head drive plate slot so that there will be no possibility of movement while cutting. When the blank is forced on to the mandrel a small amount of oil should be wiped on the mandrel. This will prevent the blank from "freezing" to the mandrel. The large end of the mandrel is the one to which the dog is attached.

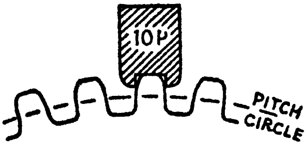
The index head should be set for the desired number of divisions. Indexing is based on the formula, 40 divided by the number of teeth gives indexing required. If the number of teeth to be cut is less than 40, the index crank handle will be moved one full turn plus some fraction of a turn.

Now check the machine for safety. All loose wrenches should be removed. The table feed setting checked, spindle speed checked, etc. The blank is brought up under the revolving cutter very carefully until the cutter grazes the surface of the circumference. If the blank has been coated with blue vitriol or layout fluid on its upper surface it will be a help in securing a proper setting. When the cutter has grazed the surface the graduated dial of the milling machine elevating screw handle should be set to zero. Now move the table longitudinally so that the cutter is between the blank and the tailstock. Raise the table an amount approximately ¾ of the whole depth of the tooth. Using the automatic feed cut the first tooth space.

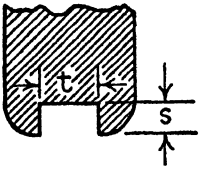
If it is desired to check on the accuracy of the centering of the cutter it may be verified at this time. Turn the mandrel end for end on the centers so that the end which was at the head is now at the tailstock. The blank and mandrel are held securely between the centers, but the dog is to be free. Have the tooth space that has just been cut at the top of the blank. Raise the blank under the revolving cutter carefully until the cutter just grazes the surface of the tooth space. The space should be coated with blue vitriol or layout fluid. If the cutter is on center it will remove the coating evenly from the surface of the tooth space; if it is not exactly on center, it will hit the top of the space on one side and the bottom of the space on the other side. Adjustments are made by carefully moving the cross feed a few thousandths of an inch.

Assuming that the centering is satisfactory and that the first cut has been taken, the blank is indexed and cuts are repeated around the blank. It is good practice to "spot" the blank all around before cutting all around. Spotting is performed by bringing the cutter against the blank until it makes a small spot on the edge of the rim. The table stop should be set so that all spots will be alike. Spotting is a check on indexing. If any error in the indexing should occur the spot will serve as a telltale to show the error.

When the last cut has been made and the cutter is back at the starting point, the table should be raised to the full depth of tooth setting minus about five thousandths. Cut through the space and index. Take the next cut, run the table back and measure the tooth. If it is too large, raise the table a few thousandths and repeat the operations. When the gage shows that the tooth is the right size continue the cuts until the gear is finished.



Gear Tooth Gages. Gear teeth may be measured with a fixed gage or a gear tooth vernier caliper. The fixed gage is the correct size for one pitch. The tooth is correctly sized when the gage is touching at both sides of the tooth and also at the top of the tooth. If the top and one side only are in contact the tooth is too small. If both sides are in contact with the gage but the top is not, the tooth is too large. The gage is made to the measurements of the addendum and the chordal pitch.



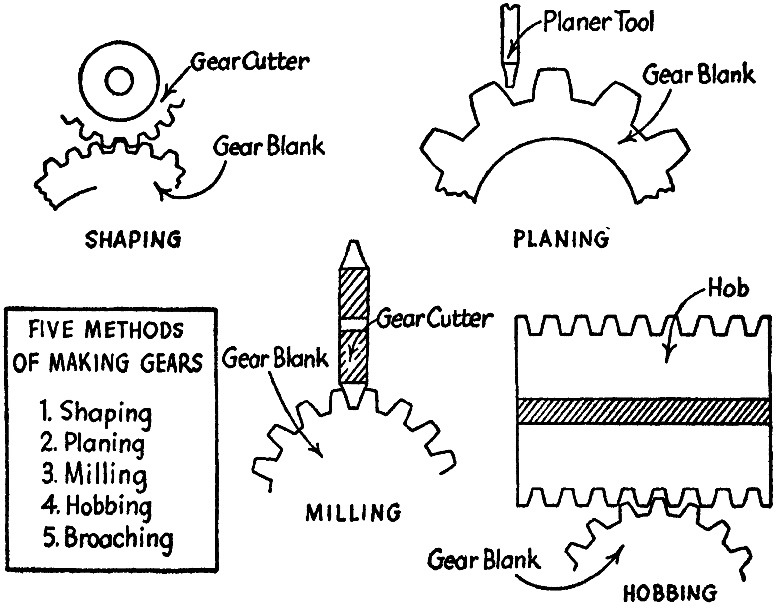
t=thickness
s=addendum

Generating Gear Teeth. If it were practical to make a gear blank of soft material and have a gear made of hard material a gear could be formed by rolling the blank and the gear in contact with each other. The gear would force its way into the blank and the result should be a perfectly fitting pair of gears. The tooth profile would be correct and the contact between the teeth of both gears would be perfect. Of course this is not a practical way to make gears because of material difficulty. If a gear blank were soft enough to be formed in this manner it would be too soft for use.

Gear teeth may be generated by a cutting action. Worm wheel gears have been made by generating since their inception. The process by which worm wheels are generated is called *hobbing.* A cutter or hob is made by first making a worm of tool steel, and then cutting flutes or grooves in the threads to form cutting teeth. In effect the hob works like a tap; as it turns, the edges of the thread cut into opposing metal.

Spur gears, spiral gears or worm wheels can be hobbed. The blank is first turned to the correct shape and size. It is then placed in a milling machine as for ordinary gear cutting. The teeth are roughed or "gashed" into the surface so that there is only a small amount of material left for finishing. The hob, which is exactly the same size as the worm that will engage the worm wheel, is then located in position for cutting. The roughed-out blank is raised until the hob teeth are engaging the tooth spaces. The gear to be cut is left free to turn on centers. As the hob turns it causes the worm wheel blank to revolve and at the same time it cuts its way into the blank. The result is a perfectly formed worm wheel tooth.

Another method of generating gear teeth uses a cutter that is made in the form of a gear. The cutter works with a reciprocating action, and the process is called ***shaping.*** As the gear cutter passes across the surface of the blank it removes stock from the tooth spaces. The blank is caused to turn at an appropriate rate of speed and the gear cutter also turns. The tooth that is formed by this process must have the correct tooth profile because it has been formed under conditions similar to those in which it is expected to work.



Single-edge cutting tools are used in some types of gear generating machines. The action is like that of a shaper or planer. The blank is revolved and metal is removed by the cutting tool as the tooth profile rolls around its axis. Two single-edge cutting tools may be used in the same method. The tools are set so as to cut on both sides of a tooth at once. To understand the action of the gear planer tool, imagine a section of a rack that is made of tool steel, hardened and ground. If the rack section were held in a machine so that it would have a reciprocating motion it could cut the teeth of a gear. If the gear blank revolved while it was being cut the tooth profile would have to be correct.

Very small gears are stamped out in presses. The cutting tools in this case are a punch made in the shape of the gear that is being produced and a die that is shaped like an internal gear. This method of production is limited to very thin gears of small diameter.

Formulas for Spur Gears. *Diametral Pitch.* The size of gear teeth is indicated by the term pitch. When the word pitch is used it is understood to mean the diametral pitch. The diametral pitch may be found by the following formulas.

When the number of teeth and the pitch diameter are known:

$$\text{Diametral pitch} = \frac{\text{Number of teeth}}{\text{Pitch diameter}} = \frac{N}{P.D.}$$

EXAMPLE: Find the diametral pitch of a gear that has 42 teeth and a pitch diameter of 3½ inches.

$$\frac{N}{P.D.} = \frac{42}{3.5} = 12 \text{ diametral pitch}$$

When the outside diameter of the gear and the number of teeth are known the diametral pitch is found by:

$$\text{Diametral pitch} = \frac{N+2}{O.D.}$$

EXAMPLE: Find the diametral pitch of a gear that has an outside diameter of 3 inches and 34 teeth.

$$\frac{N+2}{O.D.} = \frac{34+2}{3} = 12 \text{ diametral pitch}$$

When the circular pitch is known the diametral pitch is found by dividing π (pi) by the circular pitch:

$$\frac{3.1416}{C.P.}$$

Example: If the circular pitch of a gear is 0.314, what is the diametral pitch?

$$\frac{3.1416}{0.314} = 10 \text{ diametral pitch}$$

Exercises

1. A gear has 42 teeth and a pitch diameter of 5¼ inches. What is the diametral pitch?
2. The outside diameter of a gear blank is 5½ inches, and the number of teeth is 64; what is the pitch?

Circular Pitch. Circular pitch is the distance from a point on one tooth to a corresponding point on the next tooth measured on the pitch circle. Having the diametral pitch the circular pitch may be found by:

$$\frac{3.1416}{D.P.}$$

Example: If the diametral pitch of a gear is 8 what is the circular pitch?

$$\frac{3.1416}{8} = .3927''$$

Exercises

1. Find the circular pitch for the following diametral pitch gears: (a) 12 pitch (b) 16 pitch (c) 10 pitch (d) 6 pitch.
2. Find the diametral pitch for the following circular pitch gears: (a) 0.562″ (b) 0.750″ (c) 0.062″ (d) 0.625″.

Outside Diameter. The outside diameter of a gear is the diameter measured over the tops of the teeth. It may be found by the following formulas. Having the number of teeth and the diametral pitch:

$$O.D. = \frac{N+2}{D.P.}$$

Having the pitch diameter and the diametral pitch:

$$O.D. = P.D. + \frac{2}{D.P.}$$

Example 1: Find the outside diameter of a gear that has 38 teeth and a diametral pitch of 8.

$$\frac{N+2}{D.P.} = \frac{38+2}{8} = 5 \text{ inches } O.D.$$

EXAMPLE 2: Find the outside diameter of a 12-pitch gear that has a pitch diameter of 2½ inches.

$$P.D.+\frac{2}{D.P.}=2\tfrac{1}{2}+\frac{2}{12}=2.666''\ O.D.$$

Exercises

1. If a 6-pitch gear has 36 teeth, find its outside diameter.
2. A 20-pitch gear has 48 teeth. Find its outside diameter.

Pitch Diameter. The pitch diameter is the diameter of the pitch circle. It may be found by the formula:

$$P.D.=\frac{N}{D.P.}=\frac{\text{Number of teeth}}{\text{Diametral pitch}}$$

EXAMPLE: If the diametral pitch is 10 and the number of teeth is 56, what is the pitch diameter?

$$P.D.=\frac{N}{D.P.}=\frac{56}{10}=5.600''$$

Number of Teeth on the Gear. If the outside diameter of the blank is known and also the diametral pitch, the number of teeth can be found by: $(O.D.\times D.P.)-2$.

EXAMPLE: A machinist is given a gear blank with an outside diameter of 5″ and is told that the pitch of the gear is 10. How many teeth will he cut in the gear?

$(5\times 10)-2=48$, the number of teeth

Thickness of Tooth. The thickness of the tooth may be found if the diametral pitch is known by dividing 1.57 by the diametral pitch.

EXAMPLE: In the above gear, what will be the thickness of the tooth?

$$\frac{1.57}{10}=.2157'', \text{ thickness}$$

If the circular pitch is known, the thickness of the tooth may be found by dividing the circular pitch by 2.

Depth of Tooth. The whole depth of tooth is found by dividing the constant 2.157 by the diametral pitch.

EXAMPLE: Find the whole depth of cut for the gear mentioned above.

$$\frac{2.157}{10}=.2157'', \text{ whole depth}$$

Exercises

1. A gear has a pitch diameter of 4.300″ and a diametral pitch of 10. What is the outside diameter?
2. What is the thickness of tooth in an 8-pitch gear?
3. What is the whole depth of a 12-pitch tooth?
4. The outside diameter of a gear is 4.625″ and the pitch is 16; how many teeth will be cut on the gear?

Addendum. The addendum is the distance from the pitch circle to the outside circle. Technically it is the amount that has been added to the friction roll in the development of the gear teeth.

$$\text{Addendum}=\frac{1}{\text{Diametral pitch}}$$

EXAMPLE: Find the addendum of a gear of 10 diametral pitch.

$$\frac{1}{P}=\frac{1}{10}=0.100''$$

Clearance at Bottom of Tooth. The working clearance at the bottom of the tooth space is found by dividing 0.157 by the diametral pitch, or

$$\frac{0.157}{D.P.}$$

EXAMPLE: Find the clearance for a 10 diametral pitch gear.

$$\frac{0.157}{10}=0.0157''$$

Working Depth. The working depth of tooth is found by subtracting the clearance from the whole depth.

Center to Center Distance. The center to center distance between two gears is found by the formula: Number of teeth on one gear plus the number of teeth on the other gear divided by 2×diametral pitch, or

$$\frac{N_1+N_2}{2\times D.P.}$$

Exercises

1. Find (a) the outside diameter, (b) the thickness of tooth, and (c) the whole depth for each of the following gears:

28 teeth, 8 pitch
56 teeth, 8 pitch
36 teeth, 12 pitch
84 teeth, 20 pitch
48 teeth, 16 pitch

Note: Remember that when the word "pitch" is used, it means diametral pitch unless otherwise specified.

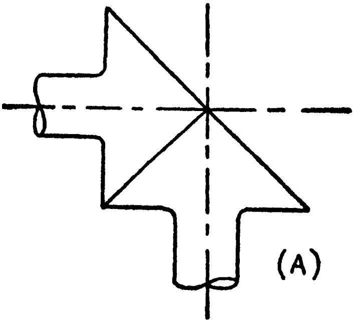
2. Find the addendum, the clearance and the working depth of the following gears:
   8-pitch; 12-pitch; 16-pitch; 20-pitch.
3. What is the center to center distance of the following gears?
   32 teeth, 48 teeth, 10 pitch
   24 teeth, 64 teeth, 12 pitch
   56 teeth, 48 teeth, 12 pitch
4. Find the circular pitch of an 8-pitch gear.
5. Find the number of teeth on a 10-pitch gear with an outside diameter of 4 inches.
6. What is the working depth of a 12-pitch gear?

Exercises

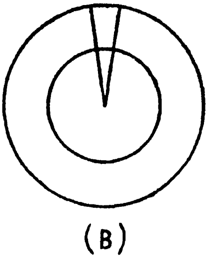
1. What is meant by circular pitch?
2. What is the addendum of a gear?
3. What are spur gears used for?
4. Why are helical tooth gears quieter than spur gears in operation?
5. Name some of the methods by which gear teeth are formed.
6. Why is it impossible to form a perfectly shaped tooth for all diameters in any one pitch?

Bevel Gears. If one has a clear understanding of the development of spur gears from the friction roll cylinders it will be much easier to understand bevel gears and their development. Spur gears transmit motion between shafts that are parallel. If it is necessary to transmit motion between shafts that are at an angle with each other and with centers that would meet, friction cones could be used. The friction cones would be subject to the same limitation of power that exists in the cylindrical friction roll.

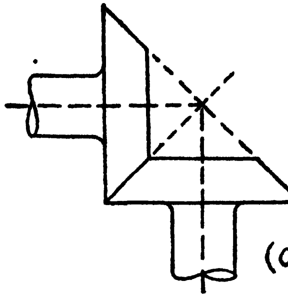
Let figure (A) represent two shafts whose center would meet. On these shafts are placed two cones in contact with each other. If teeth were developed on the cones, as they were in spur gearing, a pair of bevel gears would result. The teeth would start at the large diameter and would become progressively smaller as they reached the cone center. As it is obviously impractical to have the teeth extend to the vanishing point, the cones must therefore be truncated. Figure (B) represents the bevel gear tooth as it


(A)

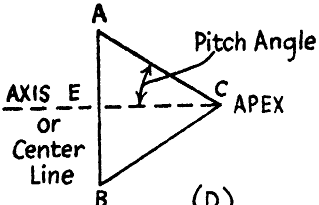
would look if it were cut from the outside to the center and also the effect of truncating the cone. Figure (C) is a side view of the truncation.



(B)



(C)



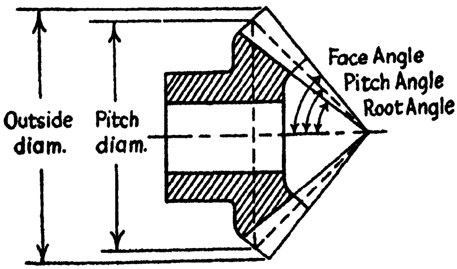
(D)

Figure (D) shows the elements of the pitch cone. The elements apply to the bevel gear just as they apply to the cone. In the figure, the line A-B represents the pitch diameter of the bevel gear that would be built on the cone. The lines A-C and B-C represent the apex distances. The angles ACE and BCE represent the pitch angle.

The pitch angle of a pair of shafts that run at right angles with equal size pitch cones would be 45°. The shafts are at right angles to each other. The cones would therefore have the pitch angle of 45°. If the cones were not equal in diameter, the angles of the cones could not be 45°. If they are equal and at 45°, they would be called *mitre* cones or gears. The pitch angle of bevel gears depends on the pitch diameters of the gear and the pinion.

The pitch diameter of a bevel gear is the diameter of the largest part of the cone or gear; actually the cone has different diameters at every point and the pitch diameter of each point is different. The only pitch diameter that is used in practice is the largest one.

The width of the face of the bevel gear teeth is usually about one third of the apex distance. The length of the tooth is called the *face width*. The bevel gear teeth will have a large end and a small end because of the changing cone diameters.



The pitch angle of the bevel gear corresponds to the pitch circle surface of a spur gear; it is the base upon which is built the tooth addendum, and from which are taken the dedendum and the clearance.

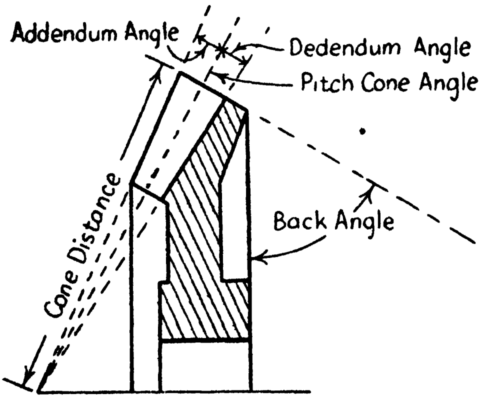
Bevel Gear Cutters. It is impossible to cut an accurate bevel gear tooth with a formed tooth gear cutter because no two points on the tooth are alike. Bevel gears are generated in gear shapers which hold the blank at angles that result in accurate shaping of the teeth. The revolving cutter as used in the milling machine is used where extreme accuracy is not required, or where the gear is wanted in a hurry and no shaper is available.

If a machinist is given the job of making a bevel gear, he must know, in addition to other dimensions, the angles to which he must turn the gear blank and also the angle to which he must set the index head for cutting the teeth. This information is often given on the blueprint or drawing and handbooks supply most of the needed information; however, the mechanic who can check with his own figures or who can supply the missing information is a more valuable man.

The face angle of the blank would be turned by means of the compound rest. The face angle is the pitch angle plus the addendum angle. It could be called the turning angle.

The cutting angle is the pitch angle minus the dedendum angle. The dedendum includes the necessary clearance at the bottom of the tooth space. The cutting angle is also called the *root angle.*

The back angle would also be turned by means of the compound rest. It is equal to the pitch angle.



The addendum angle is found by first finding the tangent of the angle.

$$\tan(\text{Add. angle}) = \frac{\text{Addendum}}{\text{Cone distance}}$$

The addendum of a bevel gear is found in the same way as it is for a spur gear:

$$\frac{1}{D.P.} = \text{Addendum}.$$

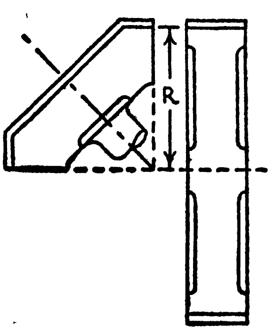
The dedendum angle is found by finding the tangent of the dedendum angle:

$$\tan(\text{Deden. angle}) = \frac{\text{Dedendum}}{\text{Cone distance}}.$$

The dedendum of a bevel gear is found by dividing 1.157 by diametral pitch:

$$\frac{1.157}{D.P.} = \text{Dedendum}.$$

Selecting the Cutter for a Bevel Gear. In selecting a cutter for a spur gear the number of teeth in the gear to be cut determines the number of the cutter that should be used. In selecting a cutter for a bevel gear the number of teeth in the gear to be cut is *not* the basis for selection of the cutter. Cutters for bevel gears are selected as though they were intended for cutting a spur gear that had a pitch radius equal to the back cone radius of the bevel gear to be cut. The reason for this may be seen in the fact that the large end of the bevel gear teeth is conditioned by the back cone radius. The bevel gear teeth are comparable, at the large end, to spur gear teeth which would be cut on a gear that had a pitch diameter equal to the back cone radius multiplied by two. The number of the cutter to be used is found by dividing the number of teeth in the bevel gear to be cut by the cosine of the pitch cone angle. The cutter is then selected from the table which is used for spur gears as well as for bevel gears.



Involute Cutters for Bevel Gears.

No. 1 will cut gears from 135 teeth to a rack
No. 2 will cut gears from 55 teeth to 134
No. 3 will cut gears from 35 teeth to 54
No. 4 will cut gears from 26 teeth to 34
No. 5 will cut gears from 21 teeth to 25
No. 6 will cut gears from 17 teeth to 20
No. 7 will cut gears from 14 teeth to 16
No. 8 will cut gears from 12 teeth to 13

Setting over a Bevel Gear Cutter. The shape of a bevel gear tooth does not change in its outline, but its dimensions become smaller at every point from the large end to the small end. When the tooth is cut with a rotating, formed cutter, it must be thinned so that the change in dimension is approximated. To accomplish the trimming, the head is rotated a slight amount and the milling machine table is moved or set over a slight amount. The combination of set-over and rotation will set the tooth so that the following cuts, one on each side, will approximate the correct size at all points.

The columns are headed by the ratio between the pitch cone radius and the width of the face of the gear. To use the table, find the factor corresponding to the number of the cutter used and the ratio between the face of the gear and the pitch cone radius. Divide the factor by the diametral pitch and subtract the result from one half of the cutter thickness measured at the pitch line.

Table for Finding Set-over Distance for Bevel Gears.

No. of Cutter	3 1	3¼ 1	3½ 1	3¾ 1	4 1
1	.254	.254	.255	.256	.257
2	.266	.268	.271	.272	.273
3	.266	.268	.271	.273	.275
4	.275	.280	.285	.287	.291
5	.280	.285	.290	.293	.295
6	.311	.318	.323	.328	.330
7	.289	.298	.308	.316	.324
8	.275	.286	.296	.309	.319

C_t=Cutter thickness at pitch line.
P=Diametral pitch of gear.
F=Factor from the table.

$$\text{Set-over} = \frac{C_t}{2} \text{ minus } \frac{F}{P}$$

The thickness of cutter may be found by measuring with a gear-tooth vernier caliper. The vertical gage is set to the dedendum distance. The dedendum distance is found by dividing 1.157 by the pitch. This setting of the tooth caliper will enable the width of the cutter at the pitch line to be measured.

Example: Find the set-over distance for a bevel gear of 24 teeth, 6-pitch, with the width of face one third of the pitch cone radius. The pitch cone angle is 30°. The ratio of the pitch cone radius to the face of the gear is 3:1. The gear will require a number 4 cutter. The factor as shown in the table is 0.275. Assume that the cutter-thickness at the pitch line, as found by measuring at this point, is 0.1745.

$$\text{The set-over} = \frac{0.1745}{2} - \frac{0.275}{6} = 0.0387''.$$

Exercises

1. Find the set-over distance for the following gears to be cut on a milling machine; assume that the ratio of pitch cone radius to the width of face is 3 to 1:
   (a) 8-pitch, 32 teeth mitre gear, thickness of cutter=0.129″
   (b) 6-pitch mitre gear with 24 teeth, thickness of cutter=0.260″
2. What is a mitre gear?

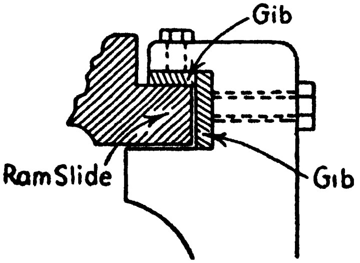
3. What is meant by the expression pitch cone in bevel gearing?
4. What part of a bevel gear tooth is the same as a spur gear tooth of the same pitch?
5. What is meant by thinning the teeth of a bevel gear?
6. What angles of the bevel gear are used by the lathe operator who turns the blank?
7. The outside diameter of a spur gear could be found by adding twice the addendum to the pitch diameter. Can it be done in the same way for a bevel gear? If not, why not?

52. MACHINE BEARINGS

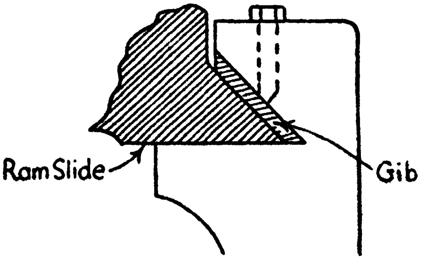
Bearing Surfaces. A machine is a combination of elements which transmit motion from one to the other until the desired force and motion are delivered at the point of productive work. The various elements must be held in their designated places or paths and allowed to move as required by the design. The surfaces which restrict the motion to their predetermined paths are known as *bearings.*

Flat surfaces such as shaper rams are guided by slide bearings. The slide bearing for this type of machine is large because the ram must be supported throughout its length while it travels for the full stroke. The slide bearing must guide the moving member and must restrain it against any motion other than in the plane desired. The ram would have a tendency to lift under a heavy cut so it must be held down. It would have a tendency to move sideways under the pressure of the cut also, so it must be held against this tendency. This type of machine element must be guided in a lateral, horizontal and vertical plane. The bearing surfaces are large and must be well lubricated. They are subject to some wear and provision must be made to compensate for the wear if it is necessary.

The sliding member is either entirely enclosed by the restraining surfaces, or, as in the case of the shaper ram, projections are controlled by the bearings and the main body is controlled by the projections. The usual design for the projections is either the square tee shape or a vee shape.



Tee or Square Sliding Bearing

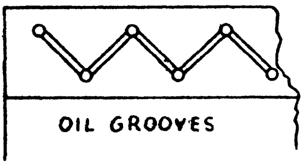


Vee Type Sliding Bearing

The square sliding bearing has two gibs to provide for adjustment against movement in the vertical plane and against movement in the lateral directions. The bolt heads shown in the sketch would probably be headless set screws in actual practice, since they would be more desirable than the projecting head bolt.

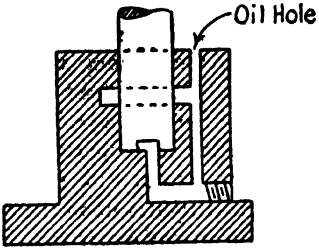
The vee type guide or bearing is called a dovetail because of its shape. It requires one gib only. The proponents of this shape point to this feature of the design as an advantage. It can be seen that the dovetail does restrain any motion sideways or vertically.

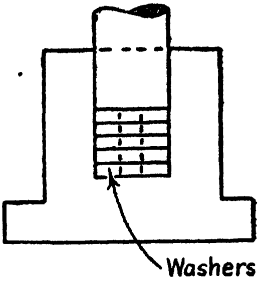
The lubrication of slide bearings presents a different problem from that of other bearings. A much larger surface has to be reached by the lubricant and there is a possibility of losing the lubricant if it is carried out of the bearing by the moving element of the machine. To reduce the loss of oil, wipers made of felt are placed at each end of the bearing. The wipers keep the oil within the bearing surfaces. Oil grooves are cut in the surface of the bearing to carry the oil to all parts of the moving surfaces. Forced lubrication is essential in this type of bearing because of the weight of the moving member and the area that must be covered. Modern shapers are provided with force pumps for this purpose. The pumps deliver oil, not only to the slide bearings, but also to the other moving parts of the machine.



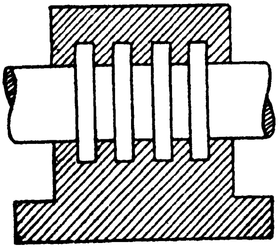
Cylindrical members of a machine are held in radial bearings. The radial bearings are so-called because the weight and pressure of the revolving member are transmitted through its radius. Radial bearings often control machine elements that are rotating at a very high rate of speed. Modern machinery speeds are constantly being increased as new cutting materials are introduced. The use of anti-friction bearings has made the higher speeds possible.

End bearings or *step bearings* are used where the moving member is vertical. In this bearing the shaft rests on its end and the problem of lubricating all of the area includes the necessity of considering centrifugal force. If the shaft revolves at a high rate of speed the oil will be thrown out toward the edges and will be unlikely to work in toward the center. Oil should be introduced at the center of the shaft where it will then be worked out by the centrifugal force. Washers are used in some end bearings. These washers revolve with the shaft but the shaft speed is transmitted at a reduced rate to each washer. The relative speeds between the washers is not as great as it would be in the case of the shaft end revolving directly on a non-moving bearing.





Thrust bearings are used in shafts, such as propellor shafts, where the push of the propellor must be transmitted to the ship and not to the engine. Thrust bearings are used in lathes to absorb the pressure of the cut; they are also found in drill press spindles and in all machinery where the end pressure must be absorbed by the heavier members of the machine and kept from interfering with the rotation of other members. Thrust bearings may be in the form of one collar, or they may be a series of collars, depending on the load to be resisted. The collars, which are on the shaft, rotate with the shaft in grooves which are in the bearing proper. That portion of a shaft which rests or rotates in a bearing is called a *journal.*



Simple Type of Thrust Bearing

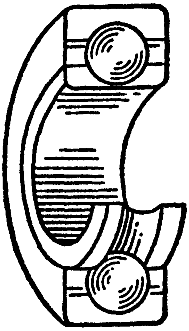
Bearing Metals. If a journal and bearing were made of equally hard material, each member would tend to wear out the other. Friction between them would result in seizing and tearing of both. If the shaft is made of steel and the bearing is made of a softer material such as babbitt, no scratching of the surface of the journal will occur and the bearing will not be scored by the journal. As a rule, like metals do not work well together in bearings; there should be a difference in the surface hardness so that mutual damage will not occur if lubrication fails.

Babbitt bearings are successful because the babbitt metal will not score the journal. If the friction should cause an excess expansion, the babbitt metal will soften and flow under the pressure rather than seize the shaft.

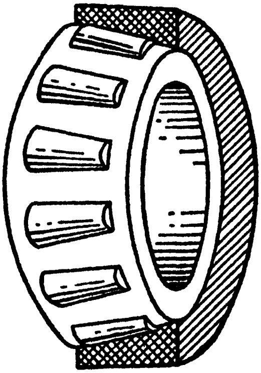
This ability to flow permits oil to establish its film again and so prevents serious damage.

Anti-friction bearings replace the rolling or sliding friction of the solid journal on a solid bearing; here the friction is divided among a number of smaller surfaces. Steel balls or steel rollers carry the load in anti-friction bearings. The balls or rolls are held in position by a *cage*. The cage and its rollers or balls rotate with a *cup*. The shaft is supported by an inner *race* or cone which is tight to the shaft and rotates with the shaft. As the shaft turns, the inner race turns with it and rotates upon the balls or rolls. The balls or rolls also rotate with the shaft and inner race. The total load carried by the shaft is distributed over a number of the rolls and the result is reduced friction at any one point.

Anti-friction bearings are made to handle any type of bearing problem. Some mechanical devices use both ball bearings and roller bearings to carry radial loads and also a thrust load. Anti-friction bearing lathe centers are used in lathe tailstocks to some extent. The danger of a seized lathe center due to improper lubrication is removed when this type of bearing is used. Rollers used in anti-friction bearings may be cylindrical or cone-shaped. The cone-shaped rolls are used for thrust bearings or for radial bearings, or for a combination radial-thrust bearing. Straight rollers are used in radial bearings only.



Ball Bearing Principle



Roller Bearing Principle

Pre-loaded Bearings. In referring to machine tool spindles where anti-friction bearings are used, the term pre-loading is frequently used. Pre-loading means that a specified amount of pressure has been applied to the balls or rolls in assembly. The purpose of pre-loading is to make sure that no vibration occurs when the load of the cut is applied. If the balls or rolls had any slight looseness in the race there would be some vibration under load. The added pressure of a cut does not affect the working of the bearing.

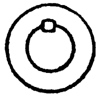
Bearing lubrication must be reliable and adequate. The friction between the two metals in the journal and in the bearing must be replaced by the fluid friction of the lubricating medium. The reliable methods of oiling are: (1) *forced lubrication,* by means of pressure pumps; (2) the *oil bath,* where the revolving parts are partially immersed in oil; (3) the *flood system,* where oil flows from a high point down over the moving parts; and (4) the *splash system,* which is the same as the bath. Oil holes are unreliable because of the possibility of stoppage due to dirt. Wicks, oil pads, etc. frequently fail because of lack of attention by the operator.

Keys and Keyways. Rotating elements of machines such as pulleys, gears, cams, etc. must be fastened to their shafts so that there is no possibility of slip. Many machine parts, on automatic machines especially, must be timed so that their motion occurs at exactly the right time in relation to the other parts of the machine. Keys and keyways are used to prevent rotation of the gear or pulley on the shaft that is carrying them. There are a number of different types of keys, but the most common one is the *square key.* The key is placed in keyways which are cut into the shaft and the hub; when the shaft turns the hub must turn with it.

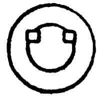
A long keyway on a shaft such as is found on the feed rod of a lathe is called a *spline.* The purpose of the spline is to furnish driving power to the mechanism with which it is connected, even though the mechanism travels from one end of the drive rod to the other. The key that engages the spline is called a *feather.* It is a tight fit in the hub of the gear and a sliding fit in the spline. The carriage can be moved on the ways of the lathe, and the feather moves in the spline at the same time.

The Woodruff key is commonly used in machine work. It is semicircular in shape. The key seat is cut with a special keyway cutter that is known as a Woodruff cutter. The Pratt and Whitney key is rounded at both ends but is rectangular in cross section. The keyway is cut with an end mill.

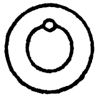
Square Key



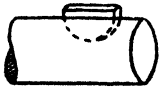
Double Kennedy Key



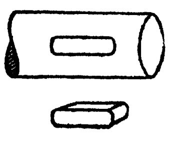
Round Key



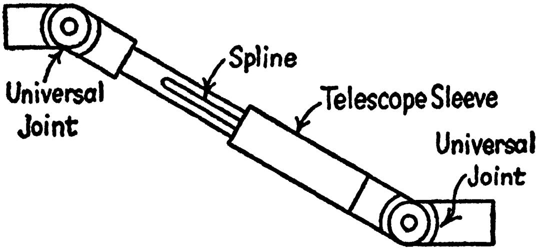
Woodruff Key



Pratt & Whitney Key



In some machines it is necessary to transmit a rotary motion to a gear or shaft that is attached to a movable machine element. The milling machine table is an example of such a requirement. The table must receive power to drive the feed rod. The feed change gear box is located at the rear of the machine and must be transmitted forward and up to the table. The table may be raised or lowered for 15 inches or more, but the rotation of the feed rod must be given at any height. The feed power is transmitted from the gear box to the table through a device called a *universal joint* and a splined rod. The universal joint will rotate a shaft through an angle to follow the elevation of the table. As the table is lowered the spline allows the rod to shorten its length without interfering with rotation.



SELECTED REFERENCES FOR FURTHER STUDY

Banks, C. W. & **Jameson**, J. M. Elementary Practical Mechanics. J. Wiley & Sons.
Berard, S. J. & **Waters**, E. O. Machine Design Problems. D. Van Nostrand & Co.
Felton, R. B. Problems in Machine Drawing. McGraw-Hill Book Co.
Hyland, P. H. & **Kommers**, J. B. Machine Design. McGraw-Hill Book Co.
Jones, F. D. Gear Design Simplified. Industrial Press.
Kimball, D. S. & **Barr**, J. H. Elements of Machine Design. J. Wiley & Sons.
Marshall, W. C. Elementary Machine Drawing. McGraw-Hill Book Co.
Mease, J. A. & **Nordenholt**, G. F. Design of Machine Elements. McGraw-Hill Book Co.
Nachman, H. L. Elements of Machine Design. J. Wiley & Sons.
Smith, W. G. Engineering Kinematics. McGraw-Hill Book Co.
Winston, S. E. Machine Design. American Technical Society.

PART THREE

TRADE SUBJECTS

CHAPTER XI

MACHINE SHOP PRACTICE

Edward N. Wallen

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53. BENCH WORK

What Is a Machine Shop? Machine shop work is that branch of the metal working industries in which metals of various kinds are machined into desired shapes and sizes. The materials worked on may reach the machine shop in the form of castings, forgings or bar stock. Metal is removed as needed from the surfaces and holes are made where needed by means of hand tools and machine tools.

A ***machine tool*** is a machine which may be used for a variety of operations. This is in contrast to the special purpose machines which are used in the manufacturing of great quantities of similar pieces. The following machines are classed as machine tools: engine lathes, milling machines, shapers, planers, universal grinders, surface grinders, and drilling machines. Because of the great adaptability of these machines it may be said that, as a group, they could be used to recreate themselves. Special purpose machines could not be used in this way. Punch presses, screw machines and turret lathes are examples of manufacturing machines. Once these machines are "tooled up," they may be operated by comparatively unskilled workers.

The expression "machine shop" is properly applied only to those shops or rooms in which the machine tools are used and in which a variety of work is done on the machines. Shops which contain special purpose machines are called ***production shops*** and are frequently named for the product which is turned out in them; for example, shell loading room, barrel shop, etc. They are also called by the name of their principal machines; for example, punch press room, drill press room, turret lathe room, etc.

Occupations in the Machine Shop Industries. The workers who operate the special purpose machines after they are "set up," are called ***operators***. They should not be confused with machinists. Many of the special purpose machines are so easy to run that the average person can be taught to run them in a very short time. Because of the length of time that is required to become proficient on all of the machine tools, many workers are specialists on one machine alone. The specialization may also be the result of a shop management system which expects to obtain greater efficiency by routing the work to different machines and workers rather than have each workman go to different machines with his work. When this system is used the workman is known as a "hand"; lathe hand, milling machine hand, shaper hand, etc. A high level of skill on their particular machine is required of these workers or hands. They should be able to read blueprints, use all of the precision measuring tools and layout tools that may be needed in their work, sharpen their own cutting tools, understand the problems of feed and speed, etc. The learning period for this class of workman is, of course, much greater than is needed for the special purpose machine operator.

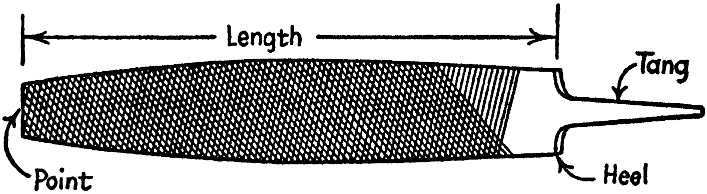
Machinists are skilled in all of the work done on the standard machine tools, they must be able to plan the correct procedure for doing the job, be able to set it up, read the blueprints, make all measurements, sharpen all of the tools, etc. The length of time required to obtain such a wide variety of skills and experiences is, of course, very much more than in the case of a machine hand. The successful machinist who expects to progress in his field must keep informed of new technological developments in metals and machines. His worth depends to a great extent upon the related knowledges which he possesses or acquires. Constant study of the related subjects of his trade will pay dividends to him.

The all-around machinist who progresses as outlined above will probably specialize, eventually, in *tool making*. This class of work may be called graduate machinist work. The tool maker works to close measurements in order to make the tools that are needed to turn out accurate work in the production machines; he also makes punches, dies, special cutters, etc.

Hand Tools. In addition to the ability to set up, operate and adjust the machine tools of his trade, the machinist must also possess a variety of hand tool manipulative skills. Hand filing, hacksawing, chisel and hammer work, tapping, reaming, etc., are some of the hand cutting operations used by the machinist in his bench work.

Files. Files are made of high grade tool steel and are very hard. They are made in a wide assortment of shapes and sizes, literally scores of shapes and sizes. The tool is an old one, that is, it has been found in ancient shops of long ago. They were formerly made by hand. File makers cut the teeth of the file by striking a chisel with a specially shaped hammer, the chisel being guided only by the maker's hand. Great skill was required to space the teeth evenly and in spite of all efforts there were slight differences in the spacing. This was no disadvantage however as the hand cut file performed very well. Machine cut files displaced the hand cut files many years ago, but file makers have developed a method of spacing the cuts so that some of the efficiency, if not all, of the hand cut file has been regained.

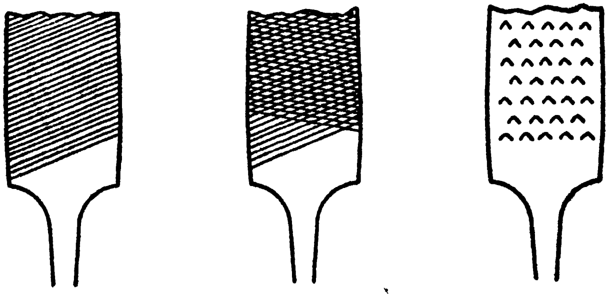
Many files are named for their cross-sectional shape, others have names that are difficult to associate with a shape. There are flat files; round files, straight and tapered, the latter commonly called rat-tail files; three-cornered or triangular files; square files; knife files; etc. A complete listing of the file names and shapes is impractical here. The learner should consult a file maker's catalog to learn more of the names, shapes and uses. An interesting group of files used by toolmakers is the needle files. These are very small files so made that a handle is not needed; they are used for fine, small work.



The Parts of a Hand File

Note: Some files are made with no cutting teeth on one edge or both. These files are known as safe edge files because the files may be used for filing close to a shoulder without cutting into the shoulder.

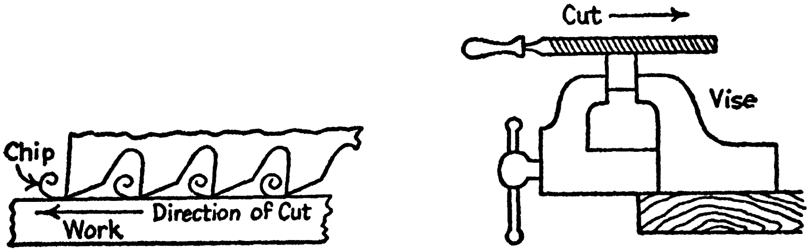
File teeth are cut in a number of ways. There is the single cut which has one line of chisel cuts running from the point to the heel; the double cut which has an overlay of chisel cuts which cross the first or single cut, and the rasp cut in which the teeth are made by a punch rather than a chisel. The many new metals found in industry today have necessitated improvements or changes in the spacing of file teeth and in the angle of the cuts. The softer metals require cuts that will reduce the natural tendency of the metal to clog the teeth.



Single cut Double cut Rasp cut

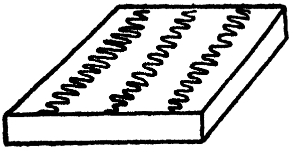
Filing. The art of filing can be acquired only by careful practice. The test of a workman's ability to file is shown in the flatness or trueness of the surfaces which he produces with a file. The good mechanic does not "scrub" his work when filing; he files carefully to accomplish his purpose. The machinist always tries to leave as little material as possible for filing; he machines as much as he can.

Ordinary filing in which the file handle is held in the palm of one hand (usually the right) and the point of the file is held by the fingers of the other is called *cross filing.* The file is pushed across the work without permitting the hands to "rock." The hands must travel in a straight line in order to produce a flat surface. When the file is brought back for the next stroke the pressure must be released or the file picked up entirely to save the teeth. Always remember that cutting teeth are designed to cut in one direction only and are injured if they are improperly used. The cutting edge is backed up by more metal on one side than on the other, the cutting strain is expected to be placed against the other side of the tooth.

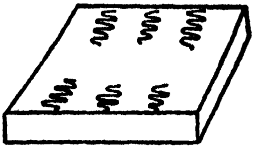


File teeth

An easy way to check the flatness of the surface of work that is being filed is to make a few pencil or chalk marks on the work; then see if the next few strokes will show a high spot by taking off the marks from part of the surface.

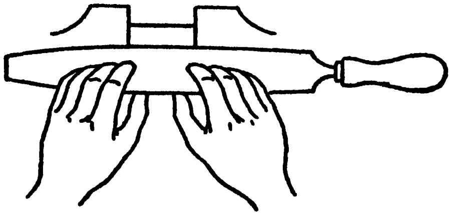


Chalk marks
on work

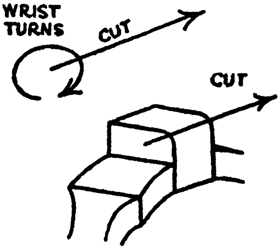


Marks removed from
center shows that
hands are rocking,
surface is not flat

Draw Filing. Draw filing is accomplished by first balancing the file on the work using a slight pressure of the fingers on the file directly above the work. The file should be held by the fingers rather than grasped by the hands, as this gives a more sensitive control and reduces any tendency to rock. Draw filing produces a flatter surface and a better finish on the work than cross filing does. It does not remove stock as quickly. As in cross filing, the file must not be rocked if a flat surface is to be made. In draw filing the file is moved sideways on the work; it is not lifted on the return stroke; it is kept in contact with the work at all times.

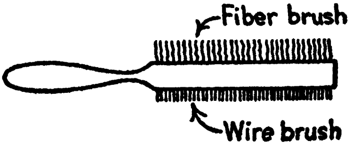


BALANCING THE FILE FOR DRAW FILING



Filing a Curve. Occasionally it may be necessary to file to a curve instead of a flat surface. The best results will be obtained if the file is used as in cross filing and the right wrist is turned while the file travels across the work. This applies to corner curves.

Pinning. When a chip becomes lodged in the teeth of a file it may develop into a pin. After a few strokes the metal particle is built up by more particles and is compressed so that it becomes harder than the work which is being filed; the result is a scratch. Files must be keep free from chips and pins so that scratching will be avoided. A file brush or file card is used to clean out the chips from the file before they form pins. If a pin has formed it must be dislodged by picking it out with a piece of soft metal such as brass; some file cards have a pick attached to them. When filing cast iron, pinning can be reduced by rubbing a little chalk on the teeth of the file; a slight amount of oil on the teeth will do the same when filing steel.



Care of Files. If files are allowed to strike or rub against one another, the result will be damaged teeth. Files should not be thrown into a drawer;

they should be protected when not in use; they should never be struck against anything, they are cutting tools, not striking tools. *Remember*—files should never be used without handles, because the tang of a file, when grasped for use, is held so that it points toward the blood vessels of the wrist. If the file should slip in the act of filing, and there is no handle on it, there is a possibility of serious injury.

Hacksaws. Hacksaw blades are made in different lengths, but the eight- and ten-inch lengths are the ones commonly used. They are classed by thickness of the back of the blade and also by the number of teeth to the inch. In general, blades with 18 or 20 teeth per inch are best for ordinary work, but for thin tubing or sheets, blades with 24 or 32 teeth are best. If coarse tooth blades are used on thin material there is too much strain on each tooth that cuts; the result may be a breaking of the teeth. At least three teeth should be in contact with the material being cut.

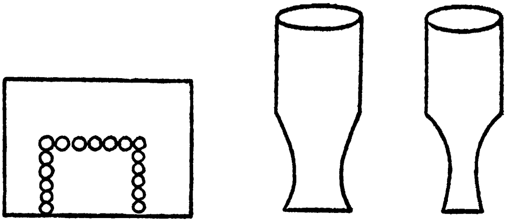
Hacksaw blades are made so that the back of the blade is thinner than the slot that the teeth will cut; in other words, the teeth have "set." They are formed so that they project beyond the side of the blade. When a saw has been in use for some time the set may be worn and the slot will be thinner than when the saw was new. If a blade has to be replaced when a cut is part way through, the new blade must be started very carefully. The new blade is wider than the slot and may jam and be broken if too much pressure is exerted.

The teeth of a blade must point away from the handle of the frame because cutting is done on the push stroke. Pressure must be relieved on the return stroke to save unnecessary wear on the teeth. The best cutting speed is about sixty strokes a minute. A little oil on the back of the blade will reduce binding in a deep slot. Oil is not used when cutting cast iron. The blade must be held tightly in the frame. Most frames are designed so that the blade may be turned 90° for cutting long strips of metal.

Chisels. A machinist should be able to sharpen and use a chisel properly. When cutting thin metal, an eighth of an inch or less, a chisel will work faster and cut just as accurately as a hacksaw, if it is in the hands of a mechanic who knows how to use it. The chisel is a fundamental tool; all other cutting tools, in a sense, are nothing but chisels that are applied by other power than hammer blows. A little study of a lathe tool bit or a milling cutter will show this to be so.

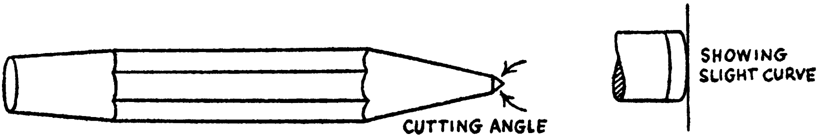
The *flat chisel* is the one that is most frequently used by the machinist. Many people call the flat chisel a cold chisel. All of the chisels used by the machinist are cold chisels; there is no good reason for using the term for the flat chisel alone. The custom is probably derived from the forger who does have a hot cutter or chisel and a cold cutter; the one for hot metal and the other to be used on cold metal.

The chisels of the machine shop are the *flat,* the *round nose,* and the *diamond point* (recognizable by the shape of the point); the *cape* chisel, which is like a flat chisel but is intended for deep, narrow slots; and the *broach* chisel, which is used to part the walls between holes that are drilled close together.



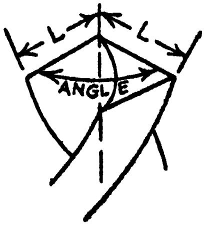
Two views of a broach chisel

The flat chisel should be sharpened so that it has a slight curve on the cutting edge; this makes for easier cutting when parting metal. The angle formed by the sides of the cutting angle should be about 60° for light work, and 90° for heavy work. When using a chisel, goggles should be worn and care taken to reduce danger from flying chips. A chipping guard, usually a screen, is placed to keep chips from flying far. The constant pounding of the head of a chisel will cause a "mushroomed" condition. The metal which forms the mushroom will crack and pieces will fly off with considerable force if the mushroomed metal is not ground away. Grind the mushroom off before it develops to any extent.



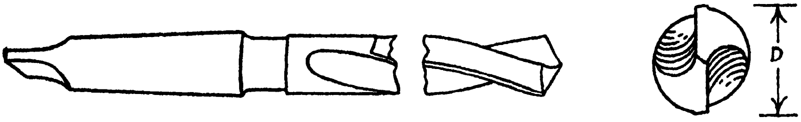
Drills. Twist drills are used for cutting into solid metal in order to make holes of desired sizes. As a rule they have two cutting lips. The lips must be ground to the same angle and length. For general work an included angle of 118° is considered right. Drill grinding gages are available for checking the angle and length of cutting lips. The objection to unequal lips is that the drill will cut a hole that is bigger than expected if one lip is longer than the other. If the angles are not the same, one lip may do all of the cutting, with inaccurate results and possible breakage.

The machinist's work requires a greater number of drill sizes than other trades. Twist drills are grouped in three size classes. The number series runs from No. 1 to No. 60 (.228-.040) and No. 61 to No. 80 (.039-.0135). The 61-80 group is seldom needed in ordinary work and is sold in separate sets. The letter size series runs from A (.234) to Z (.413).

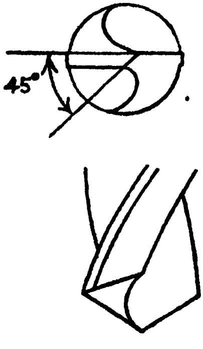


There is a drill for each sixty-fourth of an inch from one-sixteenth up. The largest stock drill is two inches. Holes which are this large are usually bored after drilling with a smaller drill. The large drills would be found more frequently in the heavy machinery shops.

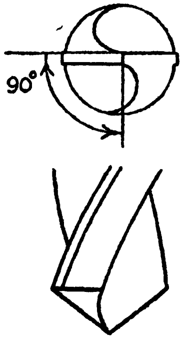
Measuring a Drill. Since the difference in size between twist drills is a matter of a few thousandths of an inch, the scale or the eye cannot be depended upon to detect the difference. Errors that may prove costly will be made if care is not used when selecting a drill for size. Drill sizes are stamped on the shank but frequently become indecipherable because of wear. Drill gages are manufactured for the purpose of saving time when measuring the smaller drills. The smallest hole into which the drill will pass tells the size of the drill. If no drill gage is available, or if the drill is a larger one, it must be measured with a micrometer. When using a micrometer, care must be taken to make the measurement at the widest part of the drill point, across the lands or margins.



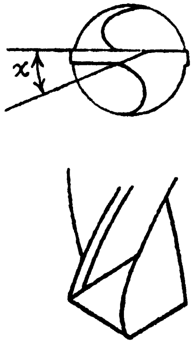
The *flutes* of a drill allow the oil to flow down to the cutting lips and the chips to come up. The *body relief* or *clearance* is designed to reduce the friction between the body of the drill and the walls of the hole that is being made. The *lands* or *margins* of the drill steady the drill in the hole and determine the size or diameter of the drill. Drills are held by the *shank*. Shanks may be straight or tapered. Larger drills are made with tapered shanks so that they can be held in spindles without using a chuck.



CLEARANCE CORRECT

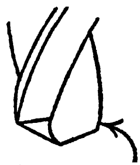


NO CLEARANCE
DRILL WILL NOT CUT



TOO MUCH CLEARANCE
CUTTING EDGE WEAK

Clearance. All cutting edges require clearance so that only the cutting edge is in contact with the surface being cut. Many drills are "burned" or otherwise damaged through the failure of inexperienced or careless workmen to recognize improper clearance angles. The beginner should make a study of correctly sharpened drill points until he can quickly judge the ability of a drill to cut. First look at the point of the drill. If the web is not at an angle of 45 degrees it is not correct; there is either too much or too little clearance at the web. No clearance means that the drill will not cut; too much clearance means that the cutting lips of the drill are weak and will break down quickly. When examining the drill for correct clearance, notice the lands; if they are worn near the point, the drill will heat very quickly because of the wedging effect of the worn lands. The only remedy for worn lands is to grind the point down until the full size lands are reached and then resharpen.


LANDS WORN - WEDGE ACTION RESULTS IN OVERHEATING

Drilling. The type of chip which is coming from the flutes while the drill is cutting is a definite indication of the effectiveness of the cutting edge. If chips are coming from one flute only it is obvious that only one lip is working. Steel chips should appear as tightly rolled spirals. Always remember that excessive pressure and cutting fluid will not take the place of correct sharpening. If a drill is not cutting properly, do not force it; find out what the trouble is and correct it. Sometimes when a drill is withdrawn from the hole or lifted from the bottom, and then drilling is resumed, the drill may not start cutting immediately; this is caused by a chip being caught under the cutting lip and riding around with it. If the drill seems to be slow in resuming a cut, lift it up a little and try again.

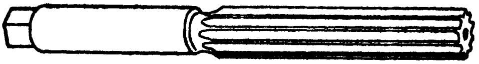
Cutting compounds are used when cutting in order to maintain the proper cutting speeds and feeds without overheating the cutting tool, and also to help make a smoother finish. Cast iron is the only exception to this practice; it is worked dry or with a jet of compressed air as a cooling medium.

The *web* of a drill is made in proportion to the diameter of the drill. A small diameter drill can be started in a centerpunch mark more accurately than a large drill can because of the size of the dead center (web). It is good practice to first drill a pilot or lead hole with a small drill before a large drill is used. Not only will the drill be located better, but it will also cut faster because the dead center will have no work to do. The dead center is the least efficient part of the drill.

As the drill is forced down through the metal, it is overcoming a certain amount of resistance to the push; when it reaches the bottom of the hole and starts to break through, there is a sudden removal of resistance and the drill may "plunge" and "grab." This "grabbing" is the result of the last metal of the hole being caught in the flutes of the drill. When you are nearing the bottom of the hole, you must regulate the pressure so that grabbing will not happen. If the drill does grab, do not try to raise it from the hole. The first reaction on your part must be to *shut off the power.* If the drill should be raised before it is stopped the work may be picked up from the table and then may fly off and cause injury to yourself or someone else.

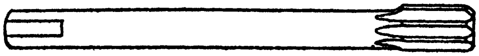
Work that is being drilled should never be held by hand, always clamp it to the table or hold it in a vise that cannot spin. Many cut hands have resulted from failure to observe this precaution. It is good practice to drill a hole in a piece of scrap metal in order to test the drill's cutting qualities and also to check the size of the hole that is being made by it.

Reamers. The holes made by drills are not always satisfactory for use where smoothness and accuracy are desired. If a very accurate hole is needed, it must be drilled with a drill that is under the finish size and then brought to size by *reaming.* The reaming operation is a finishing one; the reamer will not work satisfactorily if too much metal is left for it to remove. The hand reamers are more accurate than the machine reamers and are also more difficult to use.



Hand Reamer

Hand reamers are made with a square shank so that they can be held in a wrench and turned. They should not be used where more than .005 to .008 thousandths of an inch are to be removed. Hand reamers are harder than machine reamers and must never be run by power as they may crack. Special reaming stands are used in some shops, where much hand reaming is done, to help in starting the reamer straight. Starting and keeping the hand reamer straight is of the utmost importance. Straight fluted hand reamers have a decided tendency to chatter; for this reason straight fluted reamers are made with teeth that are spaced so as to reduce the chatter. These reamers are known as "increment cut" reamers. Spiral fluted reamers are also recommended to reduce chattering.



Straight Shank Chucking Reamer

Machine reamers are held in a chuck in the same manner as drills are. There are two types of machine reamers. The rose reamer has cutting edges on the ends only and relatively few flutes. The flutes are wider and serve to steady the reamer cutting edges. The fluted reamer has narrower flutes and more of them; it also cuts on the end. As much as from 1/64 to 1/32 in. may be removed by machine reamers.

Taps and Dies. When threads have to be made in a hole they are usually made by cutting tools known as *taps*. In production work the tapping is done in a tapping machine which resembles a drill press; the taps are made with long shanks so that many nuts can be threaded before the tap is stopped to remove the nuts. These taps are called "tapper" taps because they are intended for use in a tapper machine. Hand taps usually come in sets of three for each size. The first tap, which is intended to start the thread, is called the taper tap, or *starting tap*. It may be recognized by the fact that about six or seven threads at the point are tapered; the first full thread appears after the taper. The second tap is called the *plug tap;* it has three or four threads tapered. If the hole is a through hole (running through the metal), the plug tap will probably be sufficient. The *bottoming tap* is full threaded practically full length and is used to thread to the bottom of a "blind" hole (a blind hole does not go through the piece). Great care must be used in tapping to prevent breakage; tapping is usually one of the last operations on a job and the results of a broken tap may be expensive.

Dies are used for making external threads, threads on the outside of a cylindrical piece. Dies are adjustable for size and are held in die holders. The threads near the beginning of the die are tapered to assist in starting the die. When threading with dies, or taps, they must be turned backwards for a part turn every once in a while in order to break up the chips and prevent clogging. The die is adjustable and the tap is not; therefore the tap may be considered a master tool since the die must work to the threads left by the tap. Oil should be used when cutting threads in order to reduce tearing of the metal.

Tap Drills. A tap drill is an ordinary twist drill that is used for making a hole that is to be threaded by a tap. The tap drill must leave sufficient metal so that threads of sufficient strength may be cut. The expression "½-inch tap drill" does not mean that the drill has a half-inch diameter; it means that the drill is to make a hole into which a half-inch tap will cut threads. The commercial tap drill for this size tap (½) is actually $^{27}/_{64}$ths of an inch. Charts and tables showing tap drill sizes are available in most shops. If the thread is not a standard one, or if the reference material is not at hand the tap drill size may be readily determined by the following formula. Find the double depth of the thread (the depth on one side plus the depth on the other side). The double depth of a standard thread is found by dividing 1.3 by the number of threads per inch. Subtract this (double depth) from the outside diameter of the thread. The result will be the diameter of hole that will leave enough stock for a full thread. Sometimes it is not possible to secure a drill that is the diameter found by the formula. Never attempt to use a drill that is smaller than the diameter, however, as a broken tap may result. Tests have proven that a full thread is not necessary in a nut. If the thread is three quarters as high as the full thread, the loss in strength is negligible, while the breakage of taps is reduced and less power is needed for tapping. The formula .9 divided by the number of threads per inch gives practically two thirds of a thread and may be used as an upper limit to the size of the tap drill; i.e., *O.D.*—.9/*N*. If both the formulas are used there will be a minimum and maximum size drill which can be used as a tap drill.

On looking at tables or charts of tap sizes, or on reading the size stamped on taps, you will probably see numbers like the following: 6-32, 10-24 etc. The beginner is apt to interpret these to mean fractions of an inch and may read them as six thirty-seconds, ten twenty-fourths, etc. This is wrong. The first number means the size, and the other numbers tell the threads per inch. They are referred to as "a six thirty-two" tap, etc. The size number is a gage number. The outside diameter of these taps may be found by multiplying the number by 13 and adding 60 to the result. For example: a 6-32 tap may be figured as follows, 6×13=78, 78 plus 60=138. The outside diameter of the tap is .138 of an inch.

There are two standards of threads in common use in this country, differing from each other only in the number of threads per inch for each diameter. They are the National Coarse series and the National Fine series. Taps are usually marked with the abbreviation for the name, N.C. and N.F. The shape of the thread is the same on both, but a half-inch N.C. tap has 13 threads to an inch and the half-inch N.F. tap has twenty threads to an inch. Be sure that you know which series is called for before you start work. The former names of the series were United States standard, which is now the N.C. series, and the S.A.E. standard (Society of Automotive Engineers), which is now the N.F. series.

Layout Work. "Laying out" is the shop term for the operations involved in marking the location of cuts that are to be made, or the position of holes to be made, on the work. Most of the layout work has to be done on pieces that are to be machined in the shaper, planer, miller or drill press. Layout work may occur before any machining takes place, or at any time during the progress of the work, depending on the requirements of the job. Ability to read and interpret a blueprint is the first requirement for a layout man. In a way, the process of laying out becomes the transfer of dimensions and lines from the print to the piece.

Before any layout work is started on the piece, it should be checked for size. Sometimes it is discovered that the forging or casting is off in some dimension and the layout man must attempt to compensate for the error. The next step is to prepare the surface to receive the lines and markings. A scratch mark on a piece of metal may be difficult to see in some lights, so the surface is prepared in order to give a contrast in color between the scratch and the rest of the surface.

The surface of cast iron may be prepared by rubbing it with chalk or whiting. Blue vitriol, also called bluestone in some shops, is chemically known as copper sulphate and is used to coat steel surfaces. Diemakers used to heat the steel until it became blue as a surface preparation for layout. Layout fluids are now sold which work very well on all surfaces and these are displacing the older methods of surface preparation. If the piece is to be handled very much after layout is complete it is often advisable to make light center punch marks along the lines which might be effaced by handling.

When the surface has been prepared, the layout man must decide where to start the layout. A base line, from which all measurements will be made, is necessary. The base line may be a center line or it may be some other line; the important thing to consider in the selection of the working base is to ensure that all surfaces that must be machined can be machined to the correct sizes. All measurements must be referred back to the base or reference line. Errors have a bad habit of multiplying so all measurements should be made from one base, and not from each other.

Layout Tools. The tools used in layout work are of three classes: *measuring* tools, *marking* tools and *supporting* tools which hold the work while layout is accomplished. The measuring tools are the scales or rules, micrometers, vernier calipers, etc. The marking tools are scribers, surface gages, dividers, hermaphrodite calipers, punches, etc. Angle plates, vee blocks, and small vises are some of the holding tools. A good surface plate is required for good layout work. Surface plates are cast iron plates which have been machined to a flat surface. The top of a bench is not true enough for layout work.

The punches used in layout work are of two kinds. The *center punch* has an angle of sixty degrees at the point, while the *prick punch* has an angle of thirty degrees. It is easier to place a prick punch mark exactly on the intersection of two lines than to place a center punch mark there. The sharper angle makes it easier to see the point and place it more accurately.

In all layout work,the lines must be made sharp and fine. Good work cannot be done with scratched or scribed lines that are blurred or too wide. In very accurate work it is frequently necessary to use a jeweler's eye glass (loop) or pocket magnifying glass to get best results. It must be remembered, however, that judgment must be used in determining the degree of accuracy required. It would be wasting time on rough casting layout, where fractions of an inch are close enough, to use a glass to get to thousandths of an.inch.

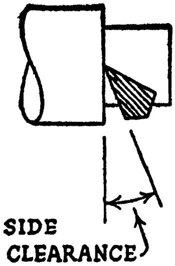
Theory of Cutting Tools. The work of the machinist and metal worker is primarily the removal of metal from the stock so that desired sizes and shapes remain. No matter how much money has been spent for a machine;

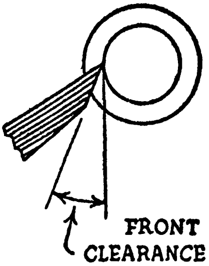
no matter how well it has been designed, its efficient operation is impossible without cutting tools that are correctly made. The machine may cost hundreds or thousands of dollars, the tool may have cost less than twenty-five cents, and still the success of the machine depends on the cutting point. From this it should be easy to accept the idea that understanding and appreciation of the theory of cutting tools is absolutely necessary to the success of the machinist as well as his machine. Many beginners fail to give enough study to this problem; too often they depend on others to sharpen their tools for them, and as a result they lack the ability that they should have. Production workers may get by without being able to sharpen their own tools,—in fact such shops have special tool grinders to do this work—but the skilled man sharpens his own cutting tools.

The first concept to acquire in this study is the principle that *any material properly shaped will cut any other material that is softer than itself.* This seems like a simple statement, and it is; yet a common mistake is to attempt to cut a piece of metal without first finding out if it is soft enough to be cut. A tap may be broken in a hole; someone may try to drill out the tap without annealing it sufficiently, because he lacks understanding of this principle. Annealing means to soften metal.

Hardness is a relative term, but for this problem the following definition works. Hardness is the ability to resist cutting or abrasion. Relative hardness can be determined very quickly and simply by trying to cut the article that is being tested with a properly shaped tool. If it does not cut,—the piece is too hard for the tool.

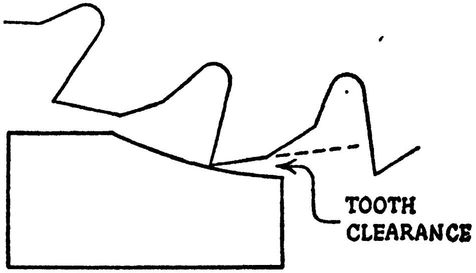
"Properly shaped" is an integral part of the principle of cutting. Make the following experiment: take a sharp pocket knife and lay it flat on a piece of soft wood, with the whole width of blade resting flat on the wood. Try to cut the wood while the tool is in that position. It will not cut, for the reason that the cutting edge cannot reach the wood; the metal back of the blade prevents it from doing its work. Try the same experiment with a wood chisel; you will get the same results. Now lift the back of the blade of the knife or chisel; now the cutting edge can reach the wood and it will cut. The angle which is formed between the flat of the blade and the work is called the *clearance angle.* This is the most important angle on any cutting tool; without clearance a cutting edge cannot reach the material to cut it. The only result of attempting to cut without clearance is the production of heat through friction; some marks may be made on the material, but they are not the marks of cutting, they are marks of tearing. When examining a tool to determine its cutting quality, clearance should be the first thing looked for. The tool may be a drill, a toolbit for lathe or shaper, or a milling cutter or reamer,—but they must all have clearance to cut. Clearance of drills has been discussed on a preceding page.





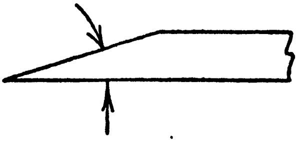
Lathe tool bits need clearance on two sides as shown in the sketches. The side clearance allows the tool to be fed sideways into the work; the front clearance clears the diameter.

Milling cutters have a clearance angle on the top of the tooth to allow the cutting edge to work without having the metal back of the edge drag on the work. The top of the tooth is called the "land."

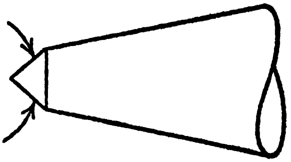


Milling Cutter

The Cutting Angle. Compare the cutting edges of a woodworker's chisel and a machinist's chisel, and notice the angle between the sides that support the cutting edge. The angle formed between the two supporting sides of the cutting edge is called the *cutting angle*. The metal chisel has to have a greater angle because it must cut harder materials than are cut by the wood chisel. The cutting angle of any tool depends on the type of material that it is to cut, the tougher the material the more resistance it will give to cutting and the stronger the cutting angle must be. This rule applies to machine tool cutters as well as to hand tool cutters. There is another consideration in regards to the cutting angle and that is the amount of power that is required to force it to do its work. The lesser cutting angle will cut with less power and may produce a better finish because of cleaner cutting. The cutting angles that are recommended for each metal and type of work represent a compromise between the desire

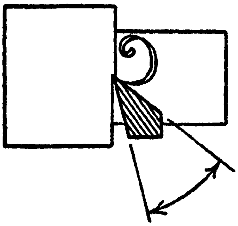


Wood Chisel

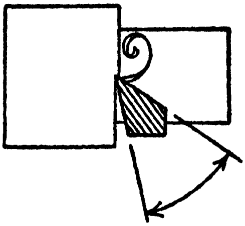


Metal Chisel

to have a stronger cutting angle and the question of clean cutting and less power. When a machine has to be stopped while the tool is being resharpened there is a loss of efficiency, and yet the cuts have to be made with the required finish and without straining the tool. Some authorities consider that a cutting tool that cuts continuously for an hour without resharpening is an efficient tool. Much depends on the material used in the cutting tool as well as on the material that is being cut.



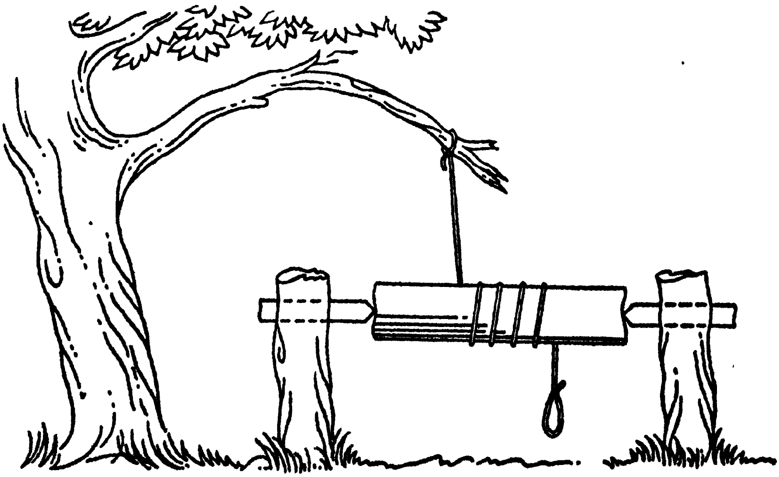
CUTTING ANGLE FOR SOFT STEEL 60°



CUTTING ANGLE FOR CAST IRON 70°

54. THE LATHE

Early Origins. The lathe has been aptly called the backbone of the machine shop. It is perhaps the most important machine in this field. A great variety of work can be done on the lathe by using the numerous attachments that are available. The desire to run a lathe, which most young men have today, is commendable; but the beginner should not be satisfied to know merely which handles must be turned to make the machine move;



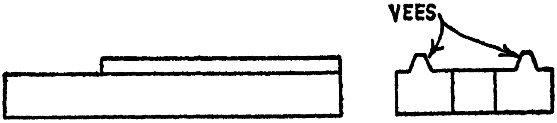
he must study and understand the function of all of the main parts of the lathe and the minor parts as well.

The lathe is the oldest machine, as a type, that we have in the machine shop. It is descended from the ancient potter's wheel which evolved into the so-called tree lathe. The original tree lathe is assumed to have been set up between two posts in the ground. Power for part of the time was probably supplied by the resiliency of a branch above the work. A cord or rope from the branch, wrapped around the work, was pulled causing the work to turn in one direction. When the cord was released the branch pulled the work in the other direction. Cuts were slow and of course inefficient, but that was before the days of speed; since then many centuries have elapsed. The development of the lathe has been greatest during the past half century. In the development of our modern machine, civilization has called for greater improvements than in centuries past.

Types of Lathes. The lathe family is an extensive one. It extends from the little jeweler's lathe which may be seen in many watchmakers' windows to the massive lathes found in the big shipyards, heavy armament plants or railroad shops. It includes lathes which are intended for general work and lathes which are classed as precision lathes because they are accurate to closer measurements than the ordinary lathes.

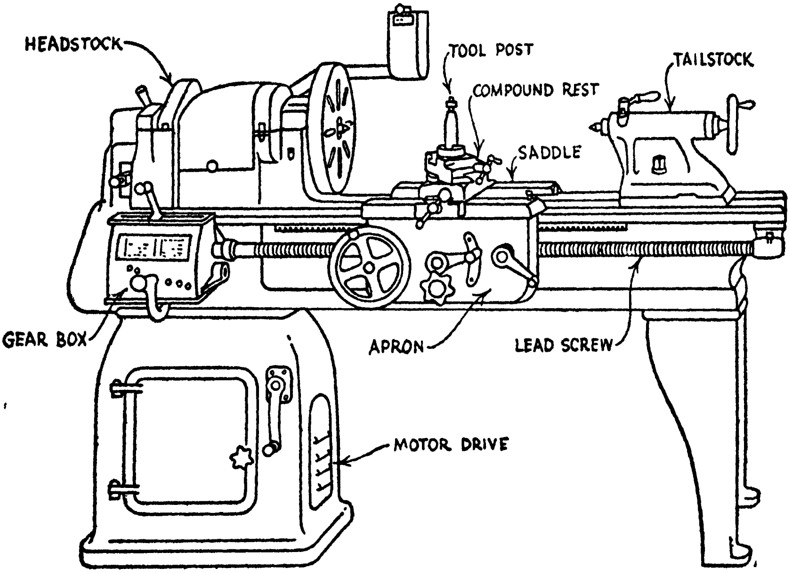
Principal Parts of a Lathe. As you look at the sketch of the crude tree lathe, you will see that the lathe is essentially a device for rotating work past a cutting tool. The machine is intended to turn out cylindrical work. The main parts of the lathe are the *bed* on which is fastened a *headstock*, and usually a *tailstock*. These are the essential parts of the lathe, but of course the modern machine shop lathe has many more details. The plain lathe does not have automatic feeds; it consists only of the bed, the headstock and the tailstock. Lathes of this type are used in machine shops for polishing and filing. Lathes that have automatic feeds are called *engine lathes*.

Lathe beds are made of cast iron or semi-steel. They are first machined nearly to size and then placed in storage so that they may season. (After castings are cut there is a tendency for the casting to warp due to internal strains, hence the seasoning is intended to give the casting a chance to warp before final finishing.) After the seasoning period of several months the beds are taken back to the shop and are machined to size. Scraping is employed where necessary to get better fits. Lathe beds vary from each other in quality of workmanship, weight of metal, and in cross bracing between the sides and material. The better class lathes have heavier beds, fitted with stronger side braces, and are therefore more rigid and can take heavier cuts without vibration. On the top of the bed there are vee-shaped ridges called the *ways* or *vees*. These vees keep the tailstock in line with the headstock, and also guide the carriage in a



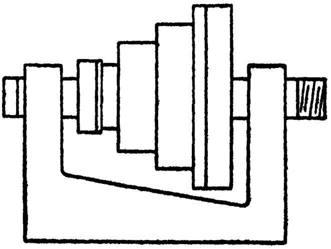
LATHE BED

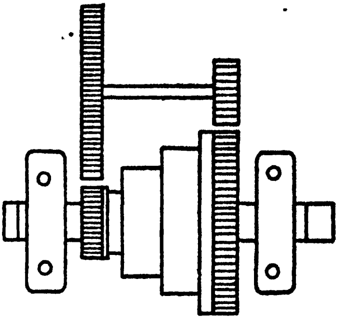
parallel line with the center of the headstock. The accuracy of the cut depends, to a great extent, upon the accuracy of the ways.



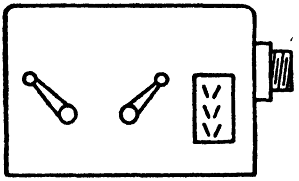
THE PRINCIPAL UNITS OF THE LATHE

The *headstock* of the lathe is fastened to the bed at the left end. There are several types of heads; thus belt driven lathes are usually of the *cone head* type. The steps of the cones are made in different diameters so that speeds can be changed by shifting the belt from one step to another. The step cone is fastened to a spindle which is held in place by the spindle bearings. Back gears are built onto the cone head engine lathes so that additional changes of speed and increased power may be obtained. A hand lever on the side of the back gears moves the gears into mesh and a latch pin disengages the cone from the front spindle gear to permit the power to travel through the gears to the spindle.

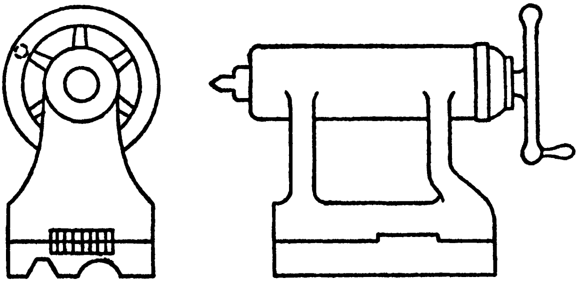




Motor driven lathes are usually of the *geared head* type. Instead of the cone, a series of gears is fastened to the spindle and speeds are changed by shifting the gears with speed-change handles on the outside of the headstock casting. A brass plate showing the positions of the handles for different speeds is fastened to the headstock. Never try to change gears while the lathe is running.

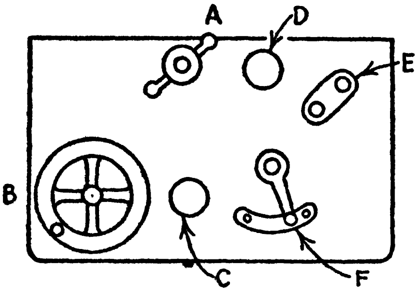


The *spindle* is a hollow shaft which runs through the headstock. The front end has a tapered hole to receive the center sleeve; the outside of the spindle front end is threaded to hold chucks and faceplates. Some modern lathes have what is known as a flanged spindle nose instead of a threaded one. The chucks and faceplates are centered by means of a taper and locked into place with cam locks. The threaded spindle nose is preferred for the lighter lathes because of the greater ease in changing the plates and chucks.



Tailstocks may be moved along the ways of the lathe to accommodate work of different lengths. The base of the tailstock is fitted to the ways and the upper part of the tailstock is designed so that it can be shifted sideways. When a taper is to be cut, the dead center is moved out of line with the live center by moving the entire upper part of the tailstock over. The dead center is held in place by a tapered seat; when the spindle is drawn back by the handle at the rear, the screw that draws back the spindle pushes the dead center out of its seat.

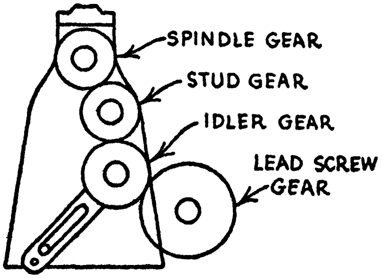
Feeding Mechanism. Engine lathes are equipped with carriages. The carriage is guided by the ways of the bed and is moved by means of the feed mechanism. Hanging down from the carriage, in front, is the ***apron***. The apron carries the feed control handles. Each lathe manufacturer makes his lathe apron a little different from the others. In general, they are equipped with a cross feed handle, a long feed handle, and a split nut handle for thread cutting and automatic feed knobs or levers; some have reverse levers to change the direction of feed. Knowing what to look for, an intelligent person can quickly understand the apron controls of the different lathes.



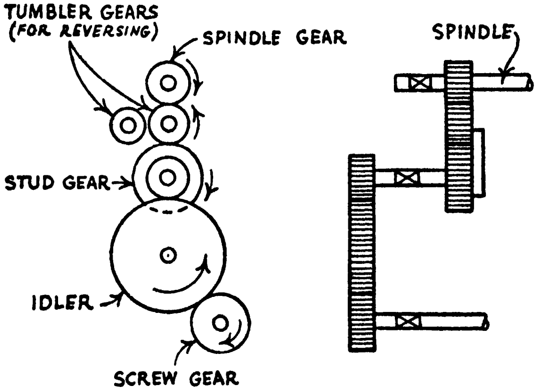
A—cross feed
B—long feed
C—automatic long feed
D—automatic cross feed
E—split nut handle
F—reverse lever.

Under the top overhang of the bed and running through the apron mechanism is found a feed rod or lead screw or both. Some lathes drive the carriage with a splined lead screw, others drive it with a feed rod. A lead screw thread is intended for use only when thread cutting. It is reserved for that purpose because thread cutting is a very exacting job for a lathe and if the lead screw threads are worn by constant use for feeding they may be inaccurate for thread cutting. Where a lead screw is used to drive the carriage for feeding, the drive is not through the threads but instead is by means of a sliding key that moves in the long keyway or spline of the lead screw.

The long feed handle moves the carriage along the ways by means of a small gear (pinion) which meshes with a rack. The rack is fastened to the overhang of the bed. A rack is a straight bar which has gear teeth cut into it.



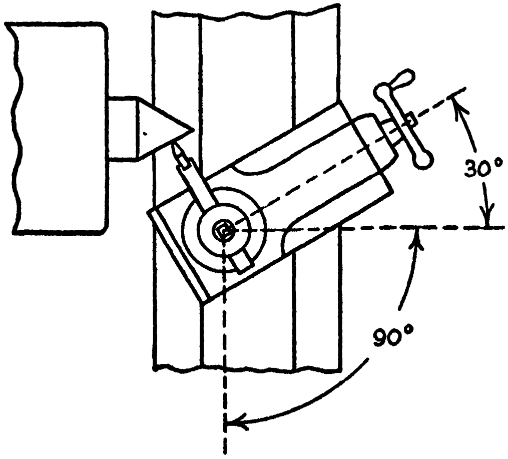
The feed mechanism of the feed rod, lead screw and carriage is connected to the spindle by gears. A spindle gear is fixed to the rear of the spindle, and meshes with another gear which is on a stud that is fastened to the headstock casting. The stud is long enough to receive a second gear which is known as a stud gear. A bracket carries another stud on which are placed gears which mesh with the gear on the lead screw or change gear box shaft. There are two types of engine lathes, the *standard change gear* type and the *quick change* type. In the standard gear type it is necessary to remove the guard from the gears and change gears on the stud, bracket and lead screw when setting up for thread cutting. On the quick change type an index plate shows the position of the levers which are to be moved for different threads. A cone of gears provides a means of changing the ratio.



Gearing from Main Spindle of Lathe to Lead Screw

Where the lathe apron does not carry a reverse lever, a reversing gear mechanism is placed between the spindle gear and the stud gear.

A cross slide is found on the top of the carriage. The cross slide screw moves the tool toward or away from the center line of the lathe. The tool post is usually carried on a compound rest. The compound rest is furnished for the cutting of angles. It may be swiveled to any angle and the tool may be fed by hand to cut an angle according to the setting of the compound rest.



General Care and Oiling of the Lathe. Lathes are carefully made and the parts are accurately adjusted by the manufacturer. When the lathe is installed, it must be carefully leveled to prevent the bed from twisting out of shape and thus throwing the spindle out of alignment. The machine must be oiled at frequent intervals. The spindle bearing should receive special attention as its load is the heaviest. While it is impossible to list here all of the oil holes on the different lathes, it must be remembered that wherever two pieces of metal are in contact and in motion some provision for oiling must be made. You will find the oil holes if you keep this in mind and look carefully. The ways are oiled by first wiping them clean, moving the carriage on the ways, then putting a light coat of oil on the surface of the ways.

Always protect the ways from damage. When removing chucks place a board under the chuck so that it will not touch the ways when it comes off the spindle. A lathe must be kept clean to prolong its useful life. Never try to clean the machine while it is in motion unless you want to lose a finger.

LATHE ATTACHMENTS

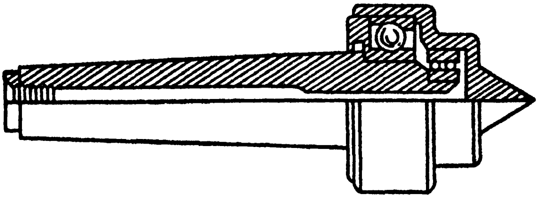
Centers. Many pieces of work are turned between the centers of the lathe. There are two centers; the one in the headstock spindle is called the "live" center because it revolves with the spindle. This center may be unhardened as there is no wear on it and its softness makes it easy to take a light cut over it if necessary to true it up. The center in the tailstock spindle is called the dead center. It should be hardened because the work revolves on it and there is a great amount of pressure exerted by the tool when cutting.

Dead centers should be marked to distinguish them from live centers. This is often done by cutting a light groove around the center above the point. Dead centers should be lubricated with a mixture of white lead and oil. Oil alone would be forced out of the centerhole under the pressure; the mixture will flow satisfactorily and still have body enough to resist the pressure. When it becomes necessary to refinish a dead center it must be ground because of its hardness. Dead centers are probably the most abused attachment of the lathe. Too often, a careless workman will not adjust the piece between centers properly, or he will not remember that metal expands when heated. As turning progresses some heat is generated at the point of the tool, this heat spreads through the piece and the piece expands. A constant watch should be kept on the dead center to make sure that the expansion does not push the piece onto the center too tightly. It is an unpardonable slip to allow a center to become fused into the center hole.

The latest development in dead center construction is the use of anti-friction bearings. These bearings are really live centers; the point revolves with the work and is carried by ball or roller bearings. The new centers are expensive and are not in common use yet because of their cost.

Live Center
(soft)

Dead Center
(hard)



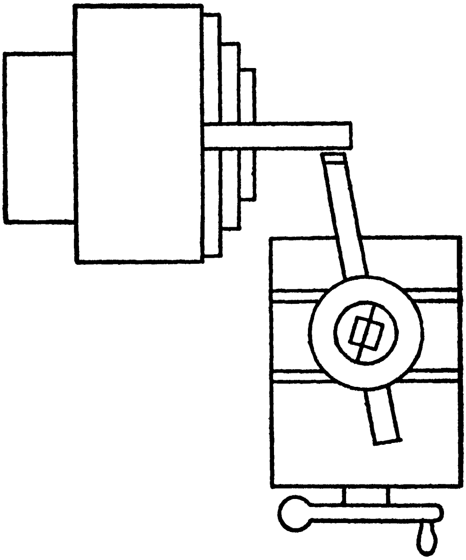
Patented Center with Anti-friction Rollers

Spindle Sleeves. The live center is held in a tapered sleeve which in turn is held by the spindle tapered seat. The use of a sleeve introduces an additional possible source of error. There is always a chance that dirt may get into the sleeve, or that nicks or burrs may be formed by careless workmen. Some shops use a large taper live center instead of the sleeve and regular taper center.

Chucks. Lathe chucks may be three-jawed or four-jawed. Three-jawed chucks are called *universal chucks* because all of the jaws are moved simultaneously. Back of the jaws there is a scroll plate. This is a flat plate which has a thread cut in it. The thread is not like the usual screw thread, as it is made on the face of the plate instead of on a diameter. As the scroll is turned by the key the jaws must move in their slots. Each universal chuck has two sets of jaws. The jaws of this chuck cannot be reversed because of the scroll construction. If it is necessary to hold a large diameter piece the jaws should be changed. It is dangerous to try to hold a piece that is so big that the jaws project beyond the body of the chuck. All three-jawed chucks have been marked with two sets of numbers to identify the jaws that belong to the different chucks. The jaws are also numbered. If it becomes necessary to change the jaws of a chuck, look for the numbers on the chuck and be sure that the jaws you try to insert are numbered in the same way. The slots are numbered in sequence, 1–2–3. The jaws must be inserted in the slots in that order; if they are not, the jaws will not center correctly.

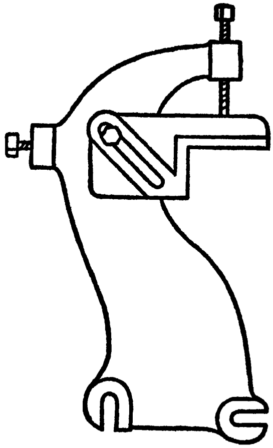
Four-jawed chucks are called *independent chucks* because each jaw moves independently. It is possible to hold a square or round piece of work in the chuck and by adjusting the jaws have it run true. This is not so in the case of three-jawed chucks. Very often three-jawed chucks are strained and as a result the work does not run true. The jaws of a four-jawed chuck are reversed by unscrewing them, turning them around and replacing them in the slot. The four-jawed chuck does not have a scroll plate, each jaw being driven by a threaded screw.

Trueing a piece of work in a chuck seems to bother many beginners. Actually it is simply a matter of finding the high spot of the work and then moving the jaws so as to shift the high spot toward center. If the tool-holder of the lathe is placed so that the back of it is toward the work, and if it is then moved toward the work slowly while the chuck is turned by hand, the high spot is quickly located. If the difference between the high and the low spots of the work seems to be, for example, an eighth of an inch, the opposite jaw is moved back a sixteenth of an inch. Always try to halve the difference.

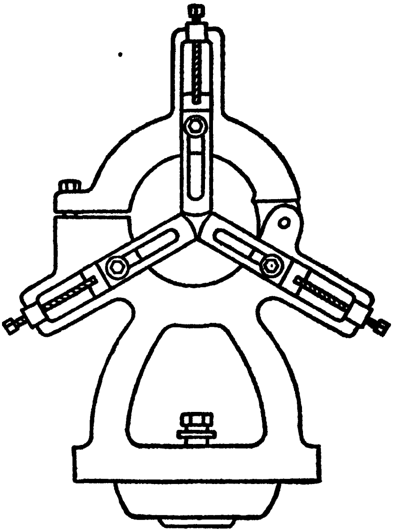


Steady Rests. Steady rests are used to support long pieces of work between centers in order to prevent them from springing away from the tool. They are also used to support a piece that is held at one end only, as in a chuck or on a faceplate. *Follow rests* serve the same purpose as steady rests for between center turning, but the follow rest is not bolted to the bed; it is carried by the cross slide and really follows the cut.

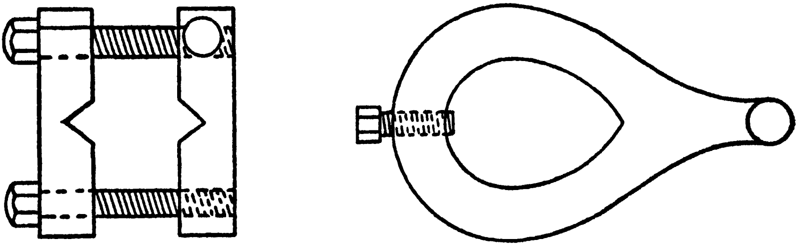
Lathe Dogs. These are clamps which are fastened to the work that is to be turned between centers. The old style dog, sometimes referred to as a driving dog, had a projecting set screw to clamp the work; this, however, is definitely dangerous, as the revolving head may catch a sleeve and cause serious injury. So called safety lathe dogs, which have a hollow head set screw, are much safer. When a finished surface has to be held by a dog, a small piece of brass or copper should be placed between the point of the screw and the work to prevent injury to the surface. Parallel lathe dogs provide a greater range of holding capacity than the solid bent tailed dog, but they must be used with care because they present an even greater hazard than the regular dog.



Follow Rest

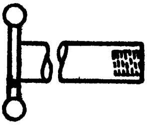


Steady Rest

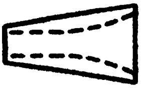


Lathe Dogs

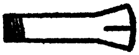
Spring Collets. Spring collets are also known as *spring chucks*. They are made for definite diameters of work and have no adjustment except for a few thousandths of an inch. This adjustment is made by drawing back the collet into an adapter by means of a drawback. The collets are very accurate and work that is held in them runs true. The collets are obtainable in round and in square sizes. The spindle sleeve is removed from the



Drawback



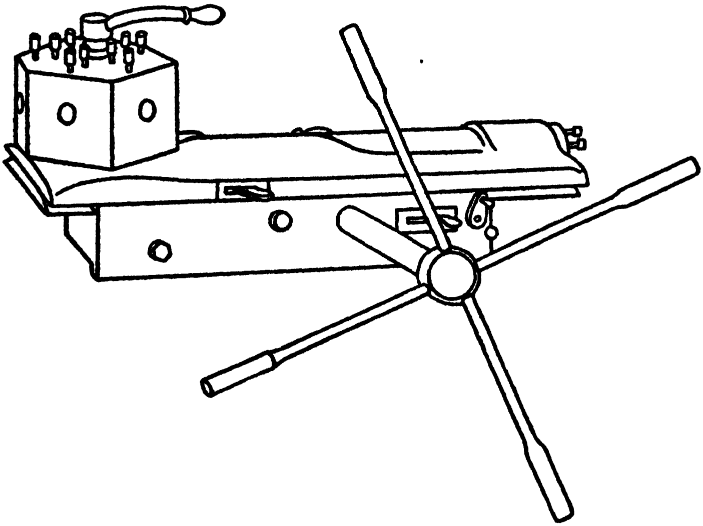
Adapter



Collet

lathe spindle, the adapter is placed in the spindle, the collet is slipped into the adapter, the drawback is passed through the spindle from the back and the collet is pulled back by the threads. As the collet hits the tapered seat of the adapter it is closed on to the work.

Turret Lathes. Interchangeable manufacturing demands that every piece or part of a machine be made to acceptable dimensions. It means that a part ordered as a replacement shall fit into the machine for which it was made without any need for filing or adjusting. Where hundreds of pieces have to be made to meet the close dimensions called for, it would be very expensive to make them on a machine which has to be set for every new cut.



The engine lathe is an extremely versatile machine, and many different operations can be performed on it. Each operation on a piece may call for a different tool or different setting of the same tool. Changing tools or changing the setting of one tool takes time and increases the chances of error. The turret lathe was developed to do lathe operations without the necessity for frequent tool changes.

The turret of a turret lathe replaces the tailstock of the engine lathe. While greater efficiency is obtained from a lathe that is designed and built as a turret lathe, an engine lathe can be converted into a turret lathe by removing the tailstock and replacing it with a turret. The turret is, in effect, a multiple tool holder. Each tool that may be called for is held in a station of the turret. The tools may be drills, reamers, taps, dies, turning tools, boring tools, etc. As the turret is advanced toward the work the tool does its job, then the turret is moved back away from the work and is turned so that the next tool is in place for its cut. The usual number of stations on a turret is six. However, this does not mean that the total number of operations is limited to six. An additional turret is often used on the cross slide where it takes the place of the conventional tool post. In addition to the turret tool post or carriage turret, there may be one placed in back of the work. It can be seen from the foregoing that a completely tooled up turret lathe with all of the available attachments can perform a great many operations without having to be reset for each one.

The tools used have to be designed so that they can be held in the turret. This is the work of the tool designer or tool engineer. The tools are made by a toolmaker. Having the required tools available, the big job is then to set each tool in its turret station; adjust the turret travel so that the proper length of cut will be made; and then adjust the cutting edges so that the correct diameters will be produced. When this is accomplished, the machine can be operated by a comparatively unskilled workman. The work must be checked for size and finish constantly, to guard against tool wear or shifting.

There are two classes of work for the turret lathe: *chuck work* and *bar work*. In the first, the work is held in chucks and each piece must be loaded into the machine by hand. In the second, the stock is fed through the spindle until it reaches a work stop. The stop determines the length of material that extends out of the chuck, and a collet type chuck holds the stock firmly while it is being worked on. The bar stock is supported at the rear of the lathe to prevent whipping as it revolves.

Automatic turret lathes are usually the smaller sized lathes and are commonly known as *automatic screw machines*. The feeding of the cutting tools and the feeding of the stock for the next cut are automatically controlled so that once the machine is set up and started, it repeats the operations until the stock is exhausted. Semi-automatic turret lathes are the most common. These machines have power feeds for all of the tool operations, but the stock must be placed in position for the cut by hand. Hand turret lathes are those in which the stock is placed in position for the cut and the tools are fed to the work by the hands of the operator. There are no power feeds.

Turret Lathe Tools. The chucks used in turret lathes are of practically the same type as those used in the engine lathe. There are, however, chucks which operate automatically for use in the full automatic machines.

All of the tools used in the turret must be fastened to or held by the turret. Since the operations all start from the end of the piece instead of from the diameters, the tools have little or no resemblance to engine lathe tools except in the principles of cutting, which are the same for all tools.

Diameters are turned by box tools or hollow mills. The *box tool* is a device for holding cutting tool bits in place and allowing the finished portion of the work to feed into the hollow space back of the cutting edges. The work is supported by a back rest to keep it from springing away from the cutting tool. Some box tools have several cutting edges so that several diameters may be turned at the same time.

Hollow mills are used for turning small diameters. As the work is reduced in size it extends into the space of the hollow mill back of the cutting teeth. The hollow mills are intended to turn or cut one diameter and are not adjustable.

Threads are made, when the length of the work will permit, by taps and dies. While solid taps can be used there is a loss of time, if the machine has to be stopped and the tap backed out of the threaded hole. Collapsing taps are used because it is not necessary to reverse the machine to draw out the tap. Instead, the tap collapses after it has finished cutting, and it is then drawn straight back. Self-opening die heads are used for the same reason. When the die has finished its work, it opens up and may be drawn back from the thread without loss of time.

Where the thread is too long for a tap or die, the thread is cut in the same manner as in an engine lathe. A thread cutting tool is placed in the cross slide tool holder and the thread is "chased" by feeding the carriage with the lead screw.

Cross-slide tools are held in the tool post of the lathe. The tool post may have a number of tool stations, and there may be a back tool post as well as the usual front post. Tools which cut from the back of the work have to be placed in position with their cutting edges down instead of up. All of the cutting tools of engine lathe work are found in use in the turret lathe and, in addition, there is a tool known as a forming tool.

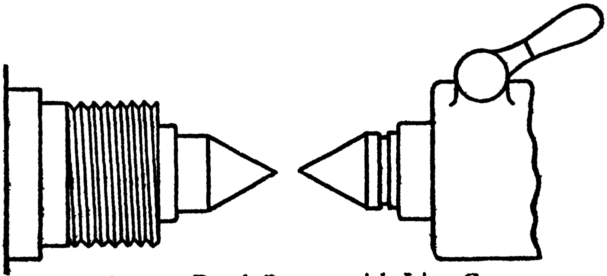
The *forming tool* is designed to produce a groove or other diameter that requires a straight in feed. The forming tools must be made and set in the lathe very accurately in order to produce correct diameters. This is a real problem when the tool must make several diameters at one time. Forming tools must be sharpened by grinding on the face or upper part of the tool so as to preserve the correct shape. They are never sharpened by grinding on the sides.

Tooling up the turret lathe is a very important job. It requires a wide knowledge of the tools that are available or that can be adapted. It requires an ability to plan the sequence of the operations so that one operation will not interfere with the completion of a following operation. The planning may be done in the engineering department of the shop or it may be left to the turret lathe foreman or set-up man. To be successful in this work requires a lot of study and experience. The turret lathe manufacturers have given much attention to the developing of men for this work. Some have publications which act as clearing houses for ideas on difficult problems in turret lathe work. For the beginner, a thorough knowledge of engine lathe work is indispensable. The knowledge gained in connection with engine lathe work will help tremendously in understanding the problems of the turret lathe set-up man.

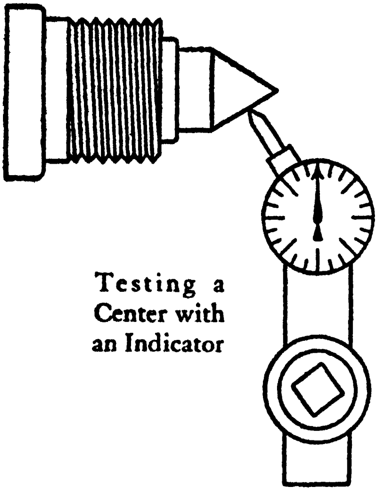
55. USING THE LATHE

Lathe Turning. This is the commonest operation on an engine lathe. Lathe hands in some European countries are called "turners" rather than lathe hands, as in America, because of the frequency of this operation. *Turning* may be defined as a lathe operation which reduces the diameter of the piece of work. Turning may be done with the work held between centers, or in a chuck, or with the work bolted to a faceplate. The operation may be *straight turning* to produce a straight cylinder; or *taper turning* to produce a cone-shaped cylinder; or *form turning* to produce a combination form as for a machine handle; or *radius turning* to produce a ball shape or concave shape.

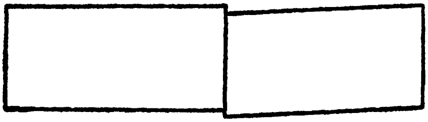
Straight turning between centers requires that the tailstock center be in line with the headstock center. Accuracy in the lathe starts at the headstock; hence the first requisite is to have a true running live center. The spindle sleeve must be clean and free from chips or dents, etc., or it cannot run true. In cleaning the spindle sleeve seat of the lathe, a cloth may be pushed into the hole and twisted around or it may be pushed through the spindle. The spindle must be cleaned while the power is off. Never attempt to clean out a revolving spindle as you may lose a finger in the attempt. When you are sure that the sleeve is clean and that there are no dents to interfere with its running true, it may be placed in the spindle seat. The live center should be inspected and cleaned as carefully as the sleeve. Sighting across the lathe at the live center while it is running should show whether the center is true or not. Hold a piece of paper back of the center so that the background will be shut off and better light obtained. Hold a pencil in your fingers and brace your hand against some stationary part, then carefully advance the pencil point against the revolving center: if it is true you will feel no high spot. For very accurate work it may be necessary to use an indicator.



Lining up Dead Center with Live Center



Testing a Center with an Indicator



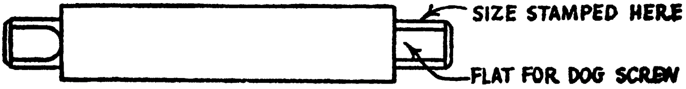
Piece Turned with Inaccurate Live Center

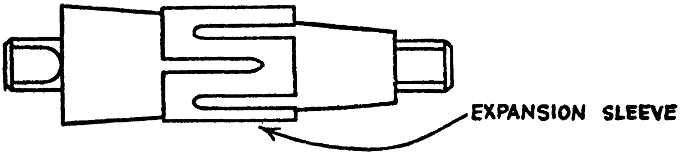
If the live center does not run true the work turned on it will be eccentric. The figure shows the results of turning on an untrue center; the cuts which were taken from both ends and which should have met in the center are not concentric.

Lining up the dead center with the live center may be done in several ways, depending on the degree of accuracy required. There are graduations at the rear of the tailstock which may serve as a rough check. The graduations should not be depended on for any but rough work. The tailstock may be moved up to the headstock and both centers may be in close proximity to each other. This will allow a comparison of the points to be made. Look down from the top at both centers; if they are not in line, the tailstock should be adjusted. One of the easiest ways of finding whether or not the centers are in line is to take a light cut and then with a micrometer, measure the piece at both ends of the cut. If there is any variation in the diameters, it is proof that the dead center is out of line.

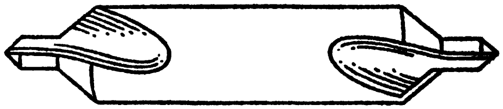
Proving bars are sometimes used by toolmakers to check alignment of centers before a cut is taken. The proving bar is a piece of metal that has been made to correct diameters; that is, the diameters at both ends are the same. The proving bar is placed between centers, an indicator is held in the tool post and the point of the indicator is brought up against the bar. When the carriage of the lathe is moved, the indicator will show whether the centers are in line, or how much they are out of line. If the centers are in line, a straight cylinder will be turned; if they are out of line the piece will be tapered.

Mandrels are used to hold work that cannot be turned on self-contained center holes. A mandrel is a bar of steel, usually tool steel hardened and ground, having a slight taper so that pieces may be held tightly. It is forced into the hole of the piece that is to be turned by means of an arbor press. A lathe dog is fastened to one end and the faceplate turns it on the centers. Mandrels are made in standard sizes. The size is stamped on the large end of the mandrel so that the user will know which end is to be forced into the hole. A little oil on the mandrel will prevent trouble when forcing the mandrel out of the hole. Expansion mandrels are provided for use where the solid mandrel may not be able to grip the work tightly enough. This condition may arise where the hole is slightly oversize.

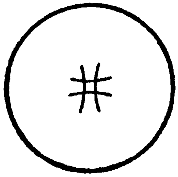


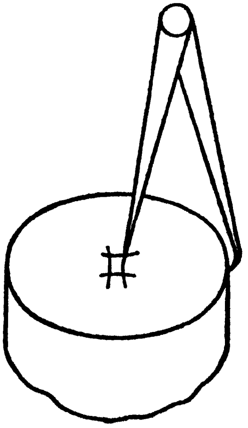


Long pieces of solid material are turned on their own centers. A combination drill and center reamer or countersink is used to drill the center hole after it has been located. The center of a piece of material may be found in several ways. If it is square or rectangular, diagonal lines may be scribed from corner to corner, the intersection of the lines may be considered the middle of the end. A center punch mark should be made at the intersection. The point of the center drill will be started in the center punch mark.

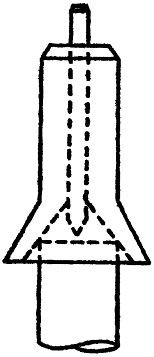


Combination Drill and Countersink

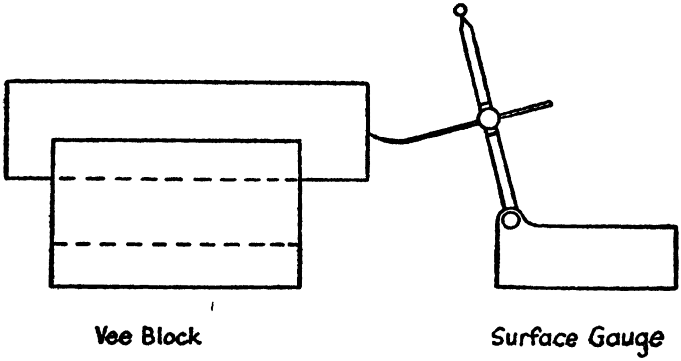




The center of a round piece may be found by using a pair of hermaphrodite calipers or a pair of dividers. The end of the work should be chalked to show the lines more plainly. The calipers or dividers should be set slightly larger than one half of the diameter. Four lines should be made, one from each end of two diameter spacings. On small diameter work a bell center punch is frequently used. The end of the stock should be free from burrs and should be square for accurate results. The bell punch must be held straight with the piece, otherwise the centerpunch will hit off center.

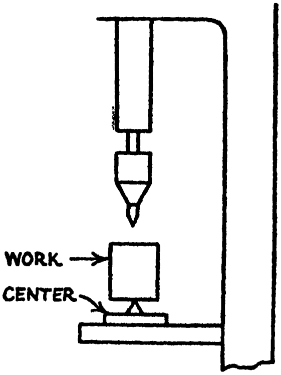


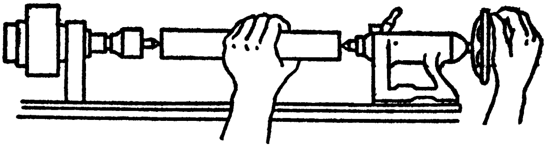
A vee-block and surface gage are used to find centers by resting the work on the vee-block, setting the surface gauge scriber point close to the center of the piece and scribing lines. If close accuracy is not required, i.e., if the material is large enough to "clean up" easily when it is turned, the work may be held in a three-jawed chuck and the center may be drilled with a combination drill held in a drill chuck by the tailstock spindle. The accuracy in this method depends upon the accuracy of the three-jawed chuck.



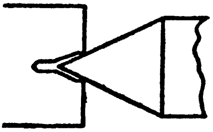
When the centers are located and center punched the drilling of the center holes may be done in a drill press or a lathe. Drill presses may be used if a pointed center is available. The point center fits into a hole in the drill press table and should be in alignment with the drill press spindle.

The speed lathe or the engine lathe may be used for the same purpose. The center punch mark of one end is placed against the point of the center, the combination drill is held in a drill chuck in the spindle and is fed into the work. Care must be taken when starting the drill to prevent breakage.

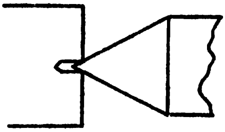




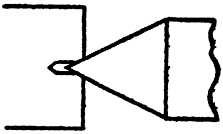
The size of the center holes is a matter of judgment; it depends on the use for which the center holes are intended, the diameter of the stock, and the amount of material to be removed from the ends. Engineering handbooks give tables for use in making centerholes, but as a rule good judgment is all that is required. If the piece must run on centers, the hole is made large enough to stand the wear; if the piece is to be turned on centers and then will probably not run on centers again, a small hole may be used. Small diameter pieces would look bad if the center hole were too large, and if much stock has to be removed from the end the center holes should be deep enough so that recentering will be avoided.



TOO DEEP



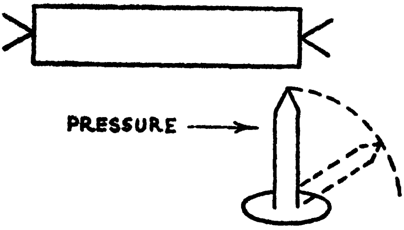
NOT DEEP ENOUGH



CORRECT

When the piece has been centered, it must be held between the centers of the lathe with the right pressure. If too tight, the centers will be injured by excessive friction and resulting heat. If too loose, the centers will wear quickly because the piece will be pressed away from the lathe centers and inadequately supported. Be sure the centers are free from dirt and chips and that there is a drop of lubricant in the dead center when you put the work on the centers. Do not lubricate the live center. Bring up the dead center into the centerhole carefully—do not come up with a bang. As soon as contact is felt between the center and the piece, stop and tighten the tailstock spindle locking device. Try the dog in the faceplate slot, and if it feels tight loosen the dead center. Practice this setting and have some expert test your setting until you have developed the sense of touch that is needed to determine correct pressure.

The turning tools must be placed in the tool post so that the point of the tool is not below center. The center should be checked by bringing the tool point up to the dead center before the work is in place. The tool may be slightly above center but never below. If it is below center there is a tendency for the work to climb up onto the tool. This may cause an accident, especially on thin work. If the tool is set too high, it will rub against the work without cutting.



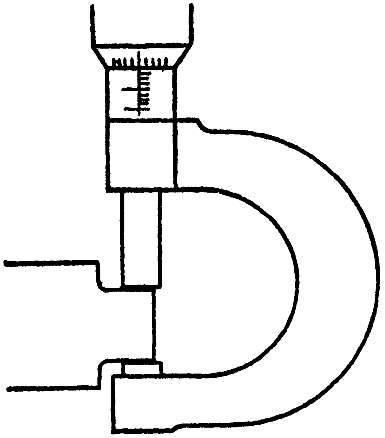
The turning tool should extend from the tool post at right angles to the work. If this setting is not made the tool may loosen up under the pressure of a heavy cut and swing into the work. If the tool is straight a heavy cut will swing the tool away from the work.

The toolholder should always be held as short as possible. It should not project out from the tool post any more than is necessary to reach the work. The toolbit should also be held close. It should be remembered that the work which the tool is expected to do puts a very heavy strain both upon the point and upon the toolholder. In fact, the toolholder will spring a little under a heavy cut. If the tool has been placed much above center, the spring caused by a heavy cut will make the tool cut deeper than is expected.

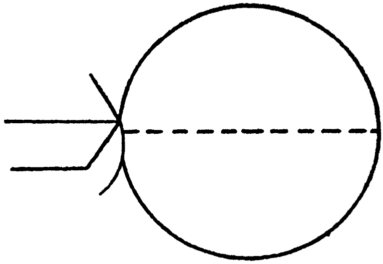
Depth of Cut. The machinist is expected to work to thousandths of an inch; the diameters which he turns to are usually given in thousandths. Each lathe cross feed handle has a graduated collar which tells how far in the tool is carried at each cut. It must be remembered that as the work revolves the cut is continuous; therefore, if it is desired to remove .020 in., the tool is advanced .010 in.; the depth of cut is .010 in., the reduction in diameter is .020 in.

When working to a finish diameter, the tool should be fed only far enough to use a micrometer. An eighth of an inch is far enough; the micrometer will make contact for the measurement on this width. The desired diameter is subtracted from the reading and the tool is advanced one half of the difference. Some mechanics prefer to advance the tool a little less than one half to play safe. This is good practice if you are not sure of the accuracy of your cross feed screw. Remember the possibility of spring in the tool; it will spring more when a heavy cut is taken. If you have been working to a diameter, the last measurement should have been made after the tool has taken the full depth of cut for a short distance (⅛ in.). If you keep working down with light cuts and then feed the tool across to take the full depth of all your adjustments, it may cut deeper than you expect when the full strain is put upon the point.

Straight Turning. Straight turning in a chuck is a little easier than turning a piece between centers, as the problem of overheated centers does not exist. One difficulty of turning in a chuck is the possibility that the work may spring under the pressure of the cut. This is a positive danger if the piece is projecting from the chuck to any extent. The work should be held as short in the chuck as is possible, just as the toolholder and



Measuring a Cut When Working to a Diameter

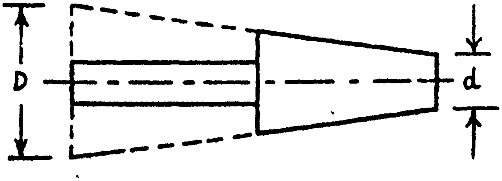


Showing How the Tool If above Center will be Sprung down into the Work under a Heavy Cut

toolbit must be held close. If the work is at all weak it should be supported at the end by the dead center. A center hole must be made and lubricant applied as in center turning. Very often it is not possible to reach the dimensions with the toolholder at right angles to the cut. Offset toolholders are available for this possibility and should be used. If an offset toolholder is not at hand and the toolholder has to be held so that it points toward the chuck, it must be tightened well and watched carefully to guard against an unexpected undercut.

Taper Turning. This has all of the problems of straight turning plus the problem of setting the tailstock over or setting the taper attachment so that the desired taper will be produced. Taper turning in a chuck requires the use of either the compound rest or the taper attachment; the tailstock cannot be used.

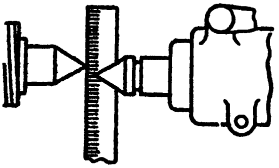
Every piece that is to be tapered must be considered as though it were tapered for the full length even though the taper actually is to be made for a portion of the length only. This is necessary because the tailstock must be offset one-half of the total taper for the piece. If the piece shown in this sketch were tapered the full length, the difference between the large diameter and the small diameter would be the total amount of taper. The taper per inch of the



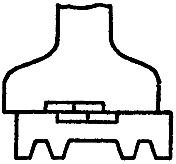
piece multiplied by the length of the piece will give the total taper; this divided by two will give the amount that the tailstock must be set over. If the diameters at both ends of the taper are given and the taper runs the full length of the piece, simply subtract the small from the large diameter and divide by two.

If the taper does not run the full length, the taper per inch must be found. This is done by subtracting the small diameter from the large diameter, dividing the result by the length of the tapered part to get the taper per inch, and multiplying the T.P.I. by the length of the piece. This will give the total taper, which is then divided by two.

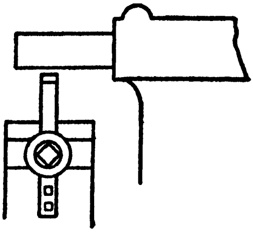
Measuring the amount of set-over may be done in several ways.



(A)



(B)



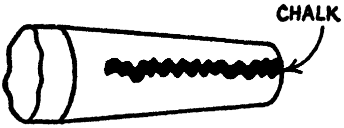
(C)

In sketch (A) above, a scale is being used to measure the amount of offset. In sketch (B) the offset is measured with the aid of the graduations on the back of the tailstock. In sketch (C) the back of a toolholder has been brought up against the tailstock spindle while the tailstock was in line with the live center; the tool is then drawn back a distance equal to one-half of the total taper (the offset), and the tailstock is then moved over until it touches the toolholder again. In using this last method (C) care must be taken to compensate for backlash. Lost motion in machinery is called *backlash*. It exists to some degree in all machines and is due to wear between parts.

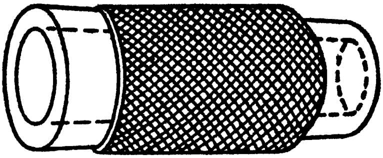
It must be remembered that all tapers must be "proven" despite the care that is taken in offsetting accurately. The center holes in a piece of stock are seldom considered when calculating the offset and yet they do affect the distance between the live and the dead centers.

Tapers are proven by trying them in a gage. Three chalk marks are placed on the taper. The piece is then placed into the gage and is given a half-turn. If the chalk has been rubbed off evenly the tapers agree; if not, it is necessary to reset the tailstock. If the chalk is rubbed off the small end of the taper it shows that the tailstock must be moved nearer to the operator's side of the lathe. If the chalk is rubbed off of the large part of the taper the tailstock is set over too far and must be moved back.

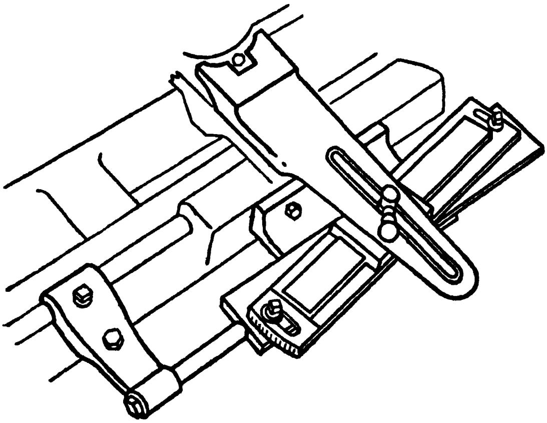
When tapers are cut on offset centers, the center holes of the piece are



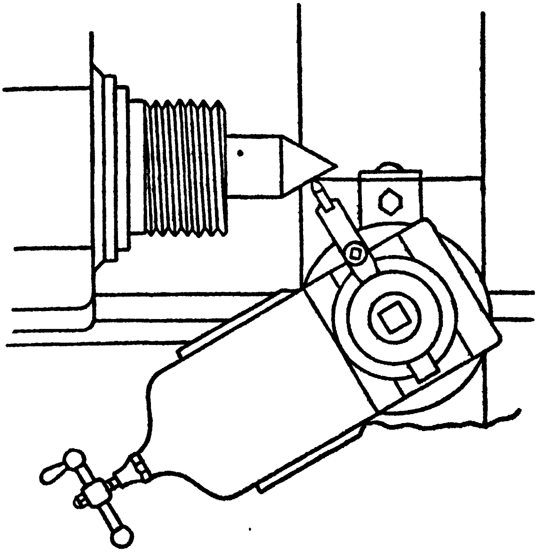
PIECE TO BE PROVEN



TAPER GAUGE



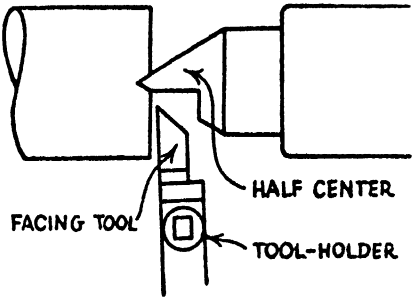
worn out of their true cone shape. Taper attachments make it possible to turn a taper while the lathe centers are in line with each other and thus prevent the wear on the center holes. The taper attachment consists of a slide which is held on a bracket at the back of the carriage in such a way that when the tool is fed along the ways the slide will cause the tool to follow a path that gives the taper desired.

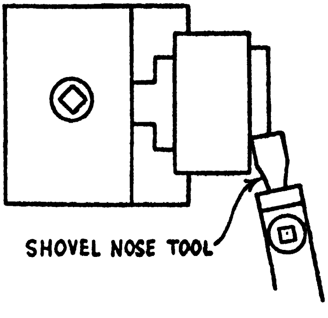


Compound Rest. Sharp tapers and angles may be turned with the compound rest. The compound rest may be swiveled to any desired number of degrees. There is no automatic feed on the compound rest and it must therefore be fed by hand. While the compound rest feed screw is being turned to feed the cutting tool into the work it is good practice to lock the carriage so that it will not move. This is done by means of the carriage clamping bolt which may be found either in the front or the back of the carriage, on top.

Facing a piece in a lathe is sometimes called *squaring*. It may be defined as a cut that reduces the length of the piece. The operation of facing should precede the operation of turning as a rule. If the piece is to be turned on centers, it is important that the ends be faced or squared first in order to avoid a lop-sided center hole. If the area of the center hole is not the same all around, the side that has less contact with the center will wear faster than the other side, and the result will be an oval-shaped center hole.

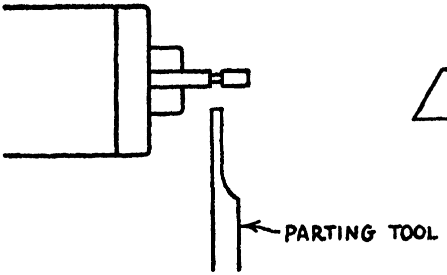
When facing a piece that is held between centers a half center is a great aid. This is a center that has had almost half of the angle ground away on one side. The facing tool bit is ground so that it can reach into the work near the center. The cut is a shearing one as the tool is drawn out by means of the cross feed handle. Facing cuts are made from the center out to the diameter as a rule. On large diameter facing cuts, where a sizable amount of stock is to be removed, a different tool is used and the cut is made from the outside in toward the center. Large diameter facing is called *radial facing*. Large diameter work that is to be faced in this manner is held in a chuck.

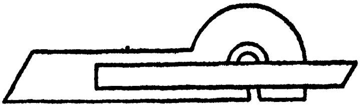




Shoulder turning is the name applied to a turning cut that goes up to a shoulder. It includes a turning cut and a facing cut. Never feed up to a shoulder by the automatic feed, always stop the feed when the tool is about an eighth of an inch away from the shoulder and finish the cut by hand.

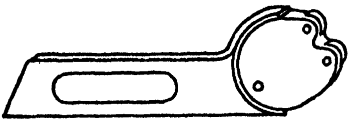
Parting or *cutting off* requires a long bladed cutting tool which is fed straight in to the work. A groove is cut until the piece is parted from the main body of the stock. Successful parting requires a lathe that has a good front bearing, a strong chuck, and no play in the cross slide. Always stand to one side of a parting tool. If you are cutting steel use oil on the tool. The tool must be set on center. A spring tool is best for parting. Parting should not be attempted on a piece that is held between centers; it is a chuck job.



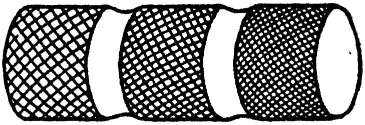


SPRING PARTING TOOL

Knurling is an embossing process. Indentations are formed on the surface of the work for the sake of appearance or to give a gripping surface for a handle. The diamond knurl is usually used and the knurls are made in three sizes of diamonds: fine, medium and coarse. Although some chips are made with a knurling tool, they are very fine and the operation cannot be classed as a cut, although it is customary to speak of cutting a knurl. The operation is a rolling one in which metal is displaced so that it is partly pushed up above the surface and partly compressed down into the indentations. Plenty of oil must be used when knurling, and particular attention must be given to the centers. More pressure is exerted against the work by the knurling tool than by a cutting tool. The knurling tool is brought up against the work and forced in about a sixty-fourth of an inch. The tool is fed by power to the other end of the work and the feed is reversed before the tool leaves the work. The knurling tool should not be withdrawn from the work until the knurl is finished.



Knurling Tool for Lathe

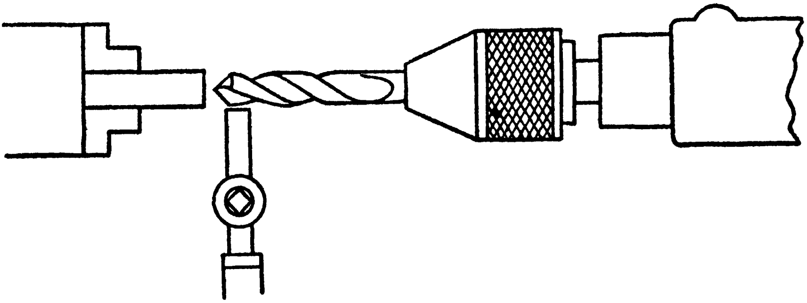


Sample of Knurling

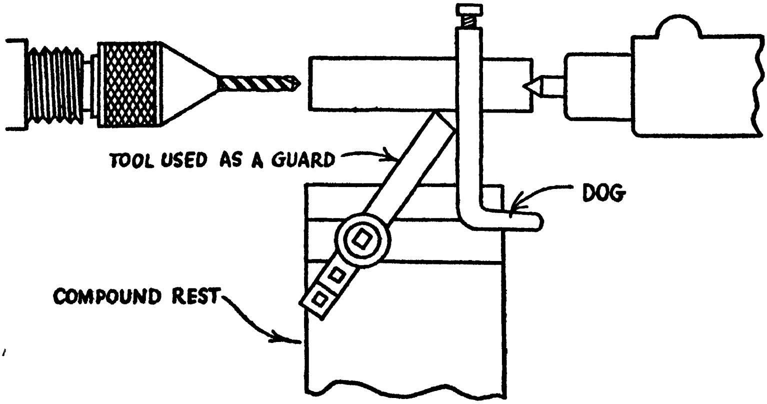
Boring is the process of enlarging a hole with a single point tool that is held in the tool post. This operation will produce a more accurate hole than a drill will. The boring tool does not follow the hole and therefore, if the hole is eccentric or oval-shaped, the boring tool will make it round. The boring tool is a weak tool; it extends further from its support than a turning tool does and as a result the cuts must be light. If a heavy cut is taken, the hole may become "bell-mouthed" as a result of the spring of the tool.

The hole may be bored to diameter but satisfactory results are obtained by boring to within five thousandths of the finish size and then reaming the hole with a hand reamer. Never use power with a hand reamer.

Drilling in a Lathe. If the work to be drilled is held in the chuck, the drill is held in a drill chuck in the spindle of the tailstock. The piece must be faced smooth so that no tip remains in the center of the piece. A combination drill is a good tool to start the hole with because it is shorter than the regular drill and will not wobble as quickly. When the spot has been made for the drill, a lathe toolholder should be reversed in the tool post and brought up to the drill to support it. Support does not mean push against. Too many beginners try to force the drill over, unintentionally, by pressing it with the toolholder. The drill must be started carefully, and if it does wobble it must be checked before the full diameter of the drill is in the hole. Once a drill has reached its full diameter in a hole there is no way to correct eccentricity.



Sometimes it is necessary to drill a hole through a fairly long piece or to drill the hole after it has been turned on centers. This may be done by resting the piece against the dead center, holding it with a dog that

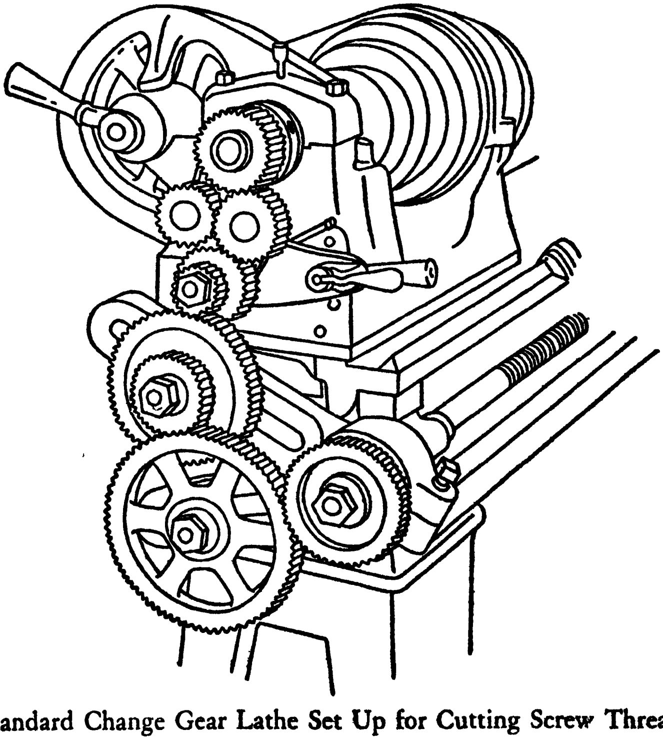


rests on the compound rest, and drilling with the drill chuck held in the spindle. The hole is drilled about half way through from each end.

The operation of drilling against the dead center is a risky one. If the work should come off the dead center while the drill is turning, it will be caught and twisted. The drill may break or the operator's hand may be injured. The reversed toolholder must be used in this type of drilling to keep the work from leaving the dead center. No undue pressure should be exerted by the toolholder, however, for otherwise the drill will not drill the hole in the center.

NOTE: The operator must grasp the dog with his hand and hold it back against the dead center while starting the drill. When the drill has reached its full diameter the toolholder guard is placed in position. The carriage is fed along slowly with one hand while the tailstock spindle is fed with the other. The dog is not expected to push the toolholder and carriage, as this would place a twisting force on the drill.

Thread Cutting. Cutting a thread in a lathe is often called "chasing a thread." A tool properly shaped and properly set so as to form the correct



Standard Change Gear Lathe Set Up for Cutting Screw Threads angle is moved along the axis of the work by the lathe screw. The number of threads per inch cut on the work depends on the number of times the piece of work turns while the tool travels one inch.

The relation between the turning of the spindle, which turns the piece, and the travel of the tool is determined by the gearing which is placed between the spindle gear and the lead screw gear. While it is true that many lathes are of the quick change type, and it is a simple matter to set the gears on these lathes, the beginner should understand how to set up a standard change gear lathe for thread cutting.

Let us suppose that all the gears shown in the figure have the same number of teeth. Then, if the spindle turns once, the lead screw will turn once. If the lead screw has four threads to an inch, the tool will travel one inch when the lead screw has turned four times. It would, under these conditions of gearing, cut four threads per inch. Now let us put a gear on the lead screw which is twice as large as the one on the spindle; the spindle must now turn twice in order to make the lead screw turn once. The tool will still travel an inch while the lead screw turns four times, but the spindle and the work will have turned eight times. The result will be eight threads per inch on the work. The same principle holds for all threads. The number of threads on the lead screw is to the number of threads on the screw to be cut as the number of teeth in the driving gears is to the number of teeth in the driven gears.

The stud gear is a driver, the lead screw gear is driven. If two gears are used on the intermediate gear stud (sometimes referred to as idler gears) the first one is a driven gear, and the second (the last one on) is a driver. In working out the ratios, remember that all figures which are obtained from the lead screw number will be teeth of driver gears, while all figures from the thread to be cut will be driven gears.

In working out a problem in gearing, it is first necessary to find out what gears are supplied with the lathe. If the lead screw has four threads per inch, the gears will probably start at twenty-four teeth and progress by four teeth, i.e., 24–28–32, etc. Write the number of the lead screw over the number of threads per inch on the thread to be cut. Factor the numerator and the denominator of the fraction thus obtained, or multiply both by the same number until a fraction is obtained which has numbers corresponding to the number of teeth on gears that are available.

Example 1: To gear a lathe to cut 12 threads per inch when the lead screw has four threads per inch proceed as follows: $\frac{4}{12}=\frac{24}{72}$ (both sides have been multiplied by the same number, 6). A 24-tooth gear on the stud and a 72-tooth gear on the lead screw will give the desired ratio.

EXAMPLE 2: Gear a lathe to cut 24 threads per inch, using a lathe that has a four-thread-per-inch lead screw. Consider the fraction $\frac{4}{24}$; this ratio is too "great" to be practical for simple gearing, so compound gearing must be used. The fraction will have to be split in the following manner: $\frac{4}{24}=\frac{2\times 2}{6\times 4}$. The first pair of numbers is to be multiplied by 12, the second pair is to be multiplied by 12 also. The results will be: $\frac{24}{72}\times\frac{24}{48}$. The gears above the line have been derived from the lead screw number and are drivers; the gears below the line have been derived from the number of the thread to be cut and are therefore the driven gears.

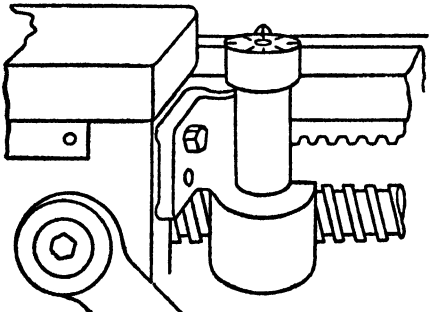
Lathes are usually supplied with two of the smallest number gears; that is why two 24-tooth gears are used in the preceding example. It is not necessary to use the same number to multiply both fractions; one could have been multiplied by 12 and the second pair by 14. As long as both parts of the fraction are multiplied by the same number, the two fractions can be treated independently.

Setting the Tool. The life of the screw thread and its strength depend on the correct fit between the sides of the thread in the nut and on the screw. The threading tool must be ground to a sixty-degree angle. The center gage is used for testing the angle of the tool bit. A true running and medium or fine grain grinding wheel should be used for grinding threading tool bits. The angle of the tool bit may be correct, but, if it is not set in the toolpost so that the angles are also correct in relation to the axis of the work, the thread will be as inaccurate as though the tool had been ground incorrectly.

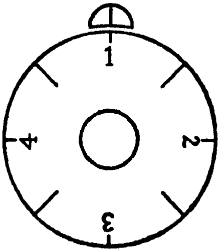
Feeding the Threading Tool. If the compound rest is set straight, and the tool fed in the same manner, there is a severe strain on the point of the thread tool. This is due to the cut being taken on both sides of the tool at the same time. To avoid the strain from the double chip, most mechanics prefer to set the compound rest at an angle and feed so that the leading edge of the tool bit does the cutting. The compound rest may be set at an angle of 30 degrees or, as some mechanics prefer, it may be set at 29 degrees. The final cuts when finishing the thread are taken with a straight in-feed from the cross slide screw. This will smooth out the side down which the tool was fed.

Cutting the Thread. A number of cuts must be taken when cutting the thread. The starting cuts may be ten or fifteen thousandths deep but, as the tool moves in, the cut will have to be lighter because of the heavier strain. Plenty of oil should be used when chasing; the oil should be applied with a small brush. A small tin pan or cover of a soap can should be placed under the threaded piece on top of the cross slide to catch the oil which will run off the piece. The lead screw is engaged by the split nut handle on the cutting passage of the tool. The tool is withdrawn at the end of the cut, the split nut is disengaged, and the carriage is run back by hand. The chasing dial, mounted on the end of the carriage, is an indicator to tell when the split nut should be engaged. The center portion of the dial moves inside of the outer ring. The center portion carries lines which are numbered. When the right line approaches the index line on the outer ring the handle is moved to engage the split nut. If the lead screw has an even number of threads and the thread that is being cut has an even number of threads, the split nut may be engaged at any line. If the thread being cut is odd, the split nut is engaged at every other line.

The chasing dial is intended to save time. If the lathe had to be reversed and the tool fed back by the lead screw, much time would be lost. On short threads the loss of time might not be important, but on long threads it becomes expensive.



(a) Chasing Dial



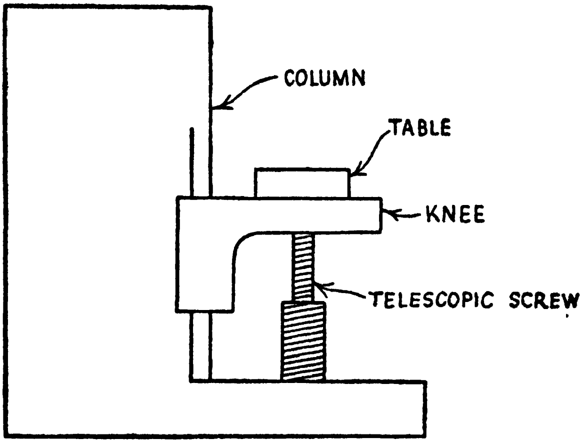
(b) Face of Chasing Dial

56. MILLING AND GRINDING

Milling Machines. *Milling* is a machine operation in which excess metal is removed from the work by feeding it past a revolving cutter. The milling machine was developed specifically for the purpose of producing work in quantity. Eli Whitney, the inventor of the cotton gin, invented the milling machine in order to turn out musket parts for a government contract. He may have taken the idea from the circular saw used for wood cutting. His machine consisted of a table on which the work was held, a screw to move the table, and an adjustable arbor to carry the cutter.

Types of Millers. Milling machines may be divided roughly into two types, the production millers and tool room millers. The *production type miller* adjusts the depth of cut by changing the height of the milling cutter. The table is moved automatically. This type of miller is known as the Lincoln type. There are many variations of production milling machines which have been designed for different industries. The mechanic's concern is with the tool room miller and not with the production type.

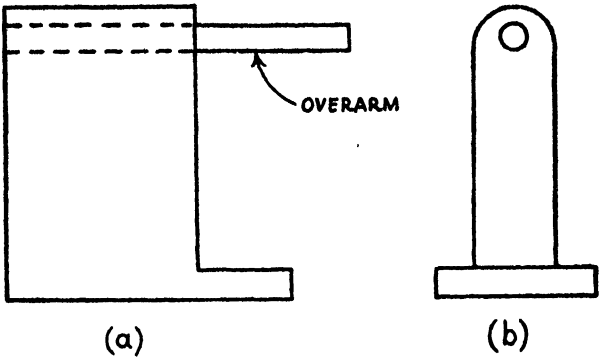
Tool room millers are usually of the knee and column type. The table is held on a knee which is raised along a column by a telescopic elevating screw which pushes it up from the bottom. The column is an integral part of the large box casting which contains the gears and spindle for driving the cutters. The base of the milling machine is hollow and acts as a catch basin for the cutting fluids that are used on the cutters. This fluid runs back into a settling tank where it is strained and pumped back to the cutter.



The spindle of the machine is a hollow shaft of tough steel. It must be perfectly square with the column in order to make accurate milling possible. The knee is clamped to the column by bolts which must be loosened for elevating or lowering and tightened before a cut is made.

Through the top of the box casting there runs an overarm, the function of which is to support the arbors which carry the milling cutters. In milling, each tooth of the cutter produces a separate strain as it cuts; the successive strains would cause chattering if the cutter were not strongly braced. Some milling machines have one large round bar for the overarm, others have square sectional arms, some are dovetail in shape, and some have two round arms side by side. This last arrangement is very rigid.

Some milling machines are driven by means of a cone head pulley, similar to some types of lathes, while others are driven through gears. The geared milling machines have two or more handles on the side of the body, which are shifted to give varying speeds to the spindle. An index



(a) (b)

plate shows the location of the handles for each speed. At the rear of the machine there will be found another group of levers and an index plate which give changes of feed.

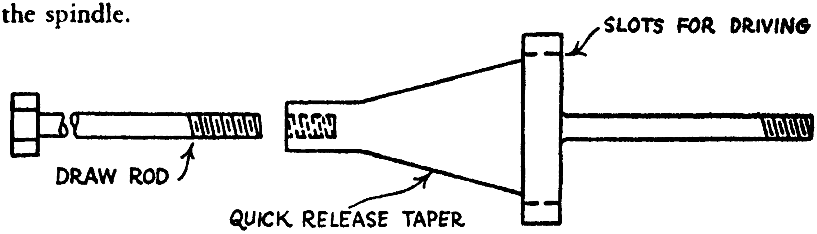
The term *speed* in the milling machine refers to the peripheral travel of the milling cutter. The term *feed* relates to the table travel under the cutter..The older type machines had index tables that showed the number of thousandths of an inch travel of the table for each revolution of the spindle. The later type index tables show the distance traveled by the table in one minute.

Knee and column machines are of two types. If the table can be swiveled on a base, i.e., set at an angle to the column, it is known as a ***universal milling machine;*** if the table cannot be swiveled it is called a ***plain milling machine.*** The universal milling machine was developed for the purpose of milling the spiral flutes of twist. Several years ago a milling machine maker brought out a milling machine in which the table could be swiveled and also tilted. This machine is called the ***omni-versal milling machine.***

Feeds on a Milling Machine. Most milling machines of the tool room type have automatic feeds for three planes. The table can be fed lengthwise, crosswise and up and down. Each feed works independently of the others and only one can be used at a time. The feeding takes place through a universal joint feed rod that extends from the feed gear box to the underside of the table. The rod is telescopic so that the table can be raised or lowered without affecting the feed gears. A shear pin is provided so that the gears will be saved from breakage in case of a sudden overload. The shear pin is located in the feed rod. If a vertical feed is being used the locking bolts of the knee must be loosened.

When lowering the table by hand, care must be taken to loosen the clamping bolts before trying to lower the table. If an attempt is made to lower the table with the bolts tight the backlash of the screw will cause the table to drop a few thousandths of an inch when the bolts are loosened. This will result in a shock to the elevating screw.

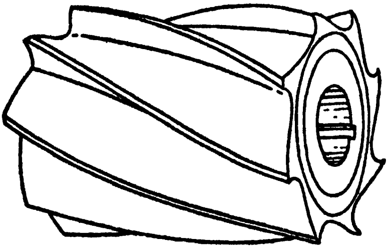
The spindle of the machine used to contain a No. 10 "Brown and Sharpe" taper seat. This is a self-holding taper, that is, the slant sides of the taper would grip the sides of the seat and hold firmly. When the taper remained in the seat for long periods of time it often froze in the seat and became a problem to knock out. Many machines still are used with the slow taper. To prevent freezing of the taper shank a little oil should be wiped on to the shank before it is placed in the spindle. The newer machines are being made with a self-releasing taper spindle. The slant on the new spindle taper is steep enough to prevent freezing. It is tapered 3½ inches per foot. The arbor is turned by keys which are attached to the spindle collar and which fit into slots in the collar of the arbor. In this type of arbor the taper is used for centering, not for holding or for driving. The arbor is held in the spindle by a drawback rod which is threaded on one end and which screws into the end of the arbor pulling it back into the spindle.



Milling machine arbors are provided with a number of collars of varying lengths so that the arbor mills can be placed where most convenient for use. The assembled cutter and collars are tightened by a nut on the threaded end. When loosening or tightening this nut the end of the arbor must be supported by the overarm support to avoid bending the arbor. The overarm support is bolted to the overarm and extends down to the end of the arbor.

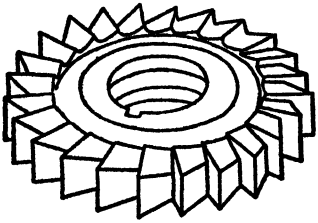
Milling Cutters. Milling machine cutters may be held on the milling machine arbor; or they may be held by means of a shank, either tapered or straight; or they may be fastened to the spindle of the machine. There are so many different sizes and shapes of milling cutters that it is quite impossible to list them all. The variations in any one style of cutter will be a difference in size and in the type of tooth. The learner should acquire a knowledge of the type names of milling cutters and also an understanding of the differences in the shape of teeth, including the reason for the difference. Milling cutters should be referred to by their type name and their size; also, in some cases, the "hand." Some cutters are right handed, others are left. This does not apply to plain arbor mills, but is important in calling for angular, radius or shank cutters.

Plain milling cutters are intended to mill flat surfaces. These cutters are held on arbors. The teeth may be cut in straight lines or in helical lines (Note; Some "old-timers" still use the expression "spiral mill" when they mean helical mill). Plain milling cutters have no teeth on the sides; they do have a keyway for use when taking heavy cuts. For ordinary tool room milling the keyway is seldom used, but if there is much metal to be taken off the keyway prevents slipping of the mill on the arbor.

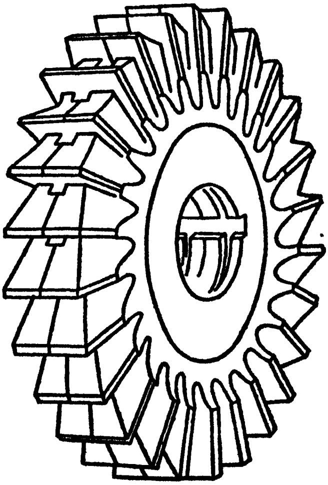


Spiral Tooth Arbor Mill

Plain milling cutters are designated by type of tooth (i.e., spiral, straight, nicked, inserted), by the diameter of the mill, and also by the width of the cutter. For example, a workman may need a spiral toothed arbor mill three inches in diameter and five inches long. The name slabbing cutter is also used to classify this type of cutter. The operation of milling a flat surface is called *slabbing*.



Side Mill



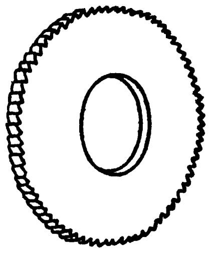
Interlocking Cutter

Side milling cutters are held on arbors. They are also called *straddle mills.* Their purpose is to cut not only on the upper surface of a piece of work but also on the sides. They are often used in pairs for milling both sides of a piece simultaneously. The teeth of straddle mills extend down the sides for a short distance only.

Interlocking cutters are used for milling slots of a definite width. The width of the cut is kept constant by placing thin washers between the two cutters. This becomes necessary after the cutters have been sharpened to compensate for the removal of metal by grinding.

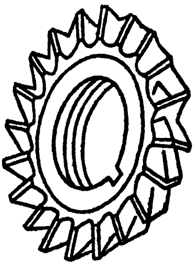
Slitting saws, usually called saws, range from about a sixteenth of an inch in width to 3/16 in., depending on requirements. Slitting saws for special purposes will vary from the very thin saws used in jewelry work to the huge saws used in armor plate work. The average machine shop slitting saws run from 2½ in. diameter to about 6 in. diameter. Keys should be used with the larger diameter saws because of the difficulty of holding the saw by the arbor nut pressure against a heavy cut.

Screw slotters are used for making the slots in the heads of screws. They are made in specified widths and are not expected to make deep cuts. They are hardened but are not ground on the sides. They may be recognized as screw slotters by the black finish which results from the hardening process.



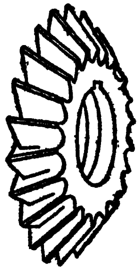
Screw Slotter

Angular cutters, or angle mills, are either of the angle type or the double angle. The single angle cutter has one straight side and is designated by the number of degrees formed by the angle of the teeth on the face. The teeth on the side are for the purpose of obtaining a smoother finish when cutting grooves. Double angle cutters may be symmetrical (that is, they may have teeth that form the same angle on both sides of the center), or they may have different angles on both sides. Angle mills used for milling the flutes of reamers, taps, etc., are usually made with a 12° angle on one side, and 40, 48, or 53° on the other. Angle cutters must be called for by "hand." A left hand cutter must turn counter-clockwise when cutting. The hand of a cutter may be determined by observing in which direction the cutter must turn in order to have the cutting edges of the teeth hit the work first.

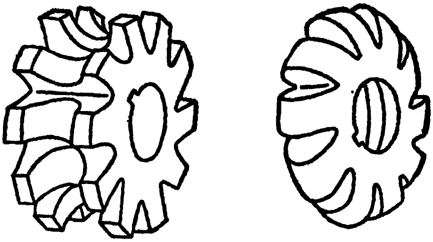


Single Angle Mill

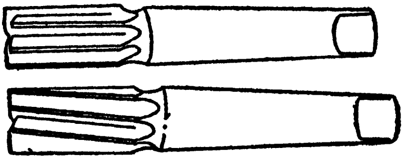
Radius mills are also called corner-rounding cutters. They may be right hand, left hand or double, depending on the side or sides from which they mill the corner. Convex cutters and concave cutters are used for milling full or part of half circles.



Double Angle and Reamer Cutter


Concave Convex

Formed cutters are specials. They are made to produce surfaces which may be a combination of flats, curves or angles. The formed cutters are intended for production work. The expense of making a special formed cutter is justified when the number of pieces to be made to the special shape is enough to show a real saving on each piece. Very frequently the formed cutters have to be made in sections and interlocked so as to produce the desired shapes. All cutters used for making gears are of the formed cutter type. These cutters must be sharpened by grinding on the face of the cutter in order to preserve the outline.

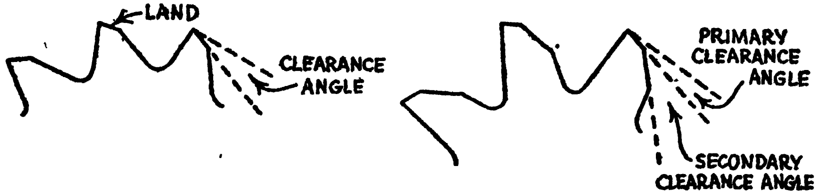

End Mills

Shank cutters include the end mills, tee-slot cutters, keyway cutters, etc. End mills are made with a tapered shank which is placed in a tapered sleeve. End mills are made in both spiral and straight tooth styles; they are also made right hand or left hand. They cannot be used to cut straight into a piece of metal unless they are of the two-lipped type which is really a form of drill. The end mill will cut in only a short distance because of the center space which has no teeth. When using a spiral end mill, the hand of the spiral and the hand of rotation are important. The hand of spiral should oppose the hand of rotation, otherwise the mill will loosen and be pulled out of the sleeve.

Shell end mills are used in order to save the expense of replacing the shank when the cutter is worn out or broken. They are larger than the usual run of end mills. End mills range in size from about a half-inch diameter to about an inch and a quarter. Beyond those sizes the shell end mill design becomes uneconomical.

Spindle mills are held on the spindle directly. They are large in diameter and are intended for heavy facing jobs where the work is held so that a vertical cut can be taken.

Types of Teeth. In practically all of the cutters used in milling machines there are considerations of strength and free cutting of the teeth involved. Before discussing the differences in design, the terminology of the tooth parts should be understood.



Saw Tooth Type Coarse Tooth Type

(Showing Lands Greatly Exaggerated).

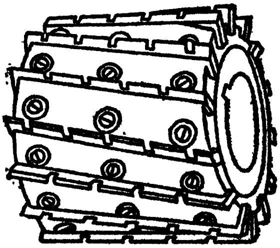
The top of the milling cutter tooth is called the *land*. It has a clearance angle so that it does not drag on the work. Back of the land is the supporting side of the tooth, which gives chip clearance for the following tooth. The curve at the bottom of the tooth is called the *gullet*. The coarse tooth milling cutters have fewer teeth and each tooth is made stronger than the saw tooth type. The land, the gullet and the first clearance angles are the same as in the saw tooth, but there is a secondary angle at a point about half-way down the chip clearance slope which is called the *secondary clearance*.

If the face of the tooth is radial, that is, if it would intersect the center of the cutter if extended, it has no rake. If the face is "hooked" the line will fall back of the center of the cutter; the tooth is said to have rake.

Spiral tooth cutters reduce vibration, because a tooth exerts a shearing or slicing action on the work. They engage the metal gradually instead of in a series of shocks as would be the case in a straight toothed cutter. The following tooth starts its cut before the working tooth has left the surface completely.

A mill cutter that has a width of several inches would normally make a chip of the same length. This would require a large amount of power because of the width of the chip. Nicked teeth cutters are designed to break up the chip into sections in order to reduce the power needed.

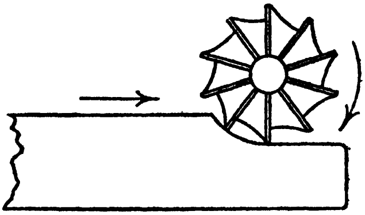
The body of large mills is sometimes made of cheaper metal to save expense. The teeth are made of the higher priced metals and are inserted into the body; they are locked into place and may be removed if replacement is necessary.



Inserted Tooth Mill

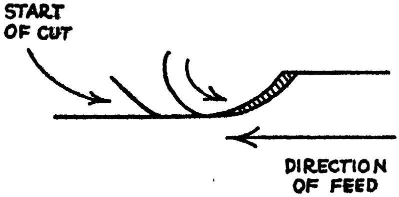
When a milling cutter is working there is a very great strain on the arbor or shank. The overarm and the overarm arbor support is used to resist the upward strain of the cut which tends to spring the arbor. In spite of the bracing given to the arbor there will still be a little spring that cannot be entirely avoided. If a mill is allowed to rotate on the surface of a work (which it has been cutting) with the table feed stopped, the mill will make an undercut. The strain of the cut has been removed and the arbor straightens itself; the result is a slightly deeper cut. When taking a finish mill cut, do not stop the feed until the cut is finished.

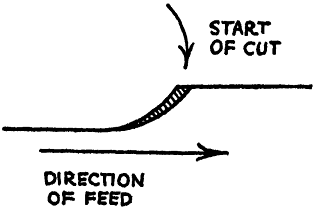
Direction of Cut. A milling cutter must rotate so that the cutting edge of each tooth strikes the work first. This seems like an obvious requirement that needs no mention, and yet, all too frequently, cutters are allowed to run backwards and an attempt is made to mill the work with the cutter in that condition. The work should advance to the cutter so that the cutter cuts from the bottom of the work up. This is called feeding against the cutter.



Feeding with the cutter is the term applied when the work is fed in the same direction in which the bottom of the cutter is turning. With the usual type of milling machine, it is not good practice to feed with the cutter because there is danger of the work being dragged into the cutter. There will also be a tendency for the work table to vibrate more when climb milling is attempted than is true with regular milling. This is due to the backlash which exists in most feed screws.

Despite the objections mentioned above, there is an advantage in the shape of chip which the climb milling procedure makes as compared with the chip of ordinary milling. In ordinary milling, where the work is fed against the cutter, the tooth engages the work gradually at the start of the chip; as the tooth rotates the chip becomes heavier due to the feed of the table. The result is a wedge-shaped chip, with the heaviest section formed when the tooth is leaving the work; the heat generated is greatest at the end of the cut as a result. In climb milling the tooth engages solid metal at the start of the cut and works towards the thin end of the wedge at the finish. This lessens the heat generation.

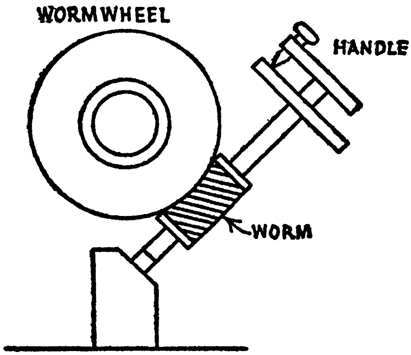




Holding the Work When Milling. Work may be held in a vise on the table, or it may be bolted to the table, or held in a fixture, or held by an index head. The greater part of milling machine work is held in the table vise. Vises are of several types. There is the plain vise, the swivel vise and the toolmaker's vise. The swivel vise can be rotated on its base so that the jaws may be placed parallel to the column or at 90° to it. Any angular setting in a horizontal plane may be made. The toolmaker's vise, also called a universal vise, can be tilted in the vertical plane and rotated in a horizontal plane. This vise is needed when compound angles are to be milled. When work is held in a vise, it is usually placed on parallels. The function of the parallels is to raise the work so that the cut can be made without danger of cutting into the vise jaws. In general, work should be held at a height that will keep the milling cutter at least an eighth of an inch from the jaws. The work must be down on the parallels, otherwise there is no certainty that the work is level. A soft hammer of lead, copper, etc., is used to tap the work down. Do not hammer the work down, for light taps will do the job much better. Heavy hammer blows will raise the work. Do not tighten the vise any more after tapping the work, for the added pressure may spring the jaws enough to raise the work.

Index Heads. *Indexing* is the process of moving a piece of work on its axis for a desired definite amount. Gears, milling cutters, etc. are indexed to get correct spacing of teeth. Index heads are also known as dividing heads. The operation of indexing is also a dividing operation.

The *index head* consists of a hollow casting called the body which is mounted on a base and supported so that it can be tilted. A spindle extends through the head. The spindle is threaded at one end to receive chucks; it is also fitted with a tapered seat for holding the index center. A wormwheel is attached to the spindle inside of the head; the wheel has forty teeth. A worm engages the wormwheel and is carried on a shaft that extends out of the side of the head. When the worm is turned by a crank handle, it in turn causes the wormwheel to turn. This turns the spindle and the work. Since the worm is a single thread helix, and the wormwheel has forty teeth, the handle must be turned forty times to make the wormwheel and spindle turn once. From this it can be seen that the problem of indexing resolves itself into determining how far the handle must be moved to secure the desired amount of rotation or movement of the piece on its axis.



The formula for simple indexing is 40 divided by the number of teeth or cuts required equals the distance to move the handle.

$$\frac{40}{N} = \text{Indexing.}$$

Example: Find the indexing for 80 teeth.

$$\frac{40}{80} = \frac{1}{2}, \text{ or the handle must be turned for half a revolution.}$$

Index heads are equipped with a number of plates which have circles containing different numbers of holes. The purpose of the index plate is to make it possible to turn the index crank for a part of a turn accurately. The plates furnished by different companies differ from one another in the number of plates supplied and in the number of holes in the circles. One manufacturer supplies three plates with holes as follows:

first 15–16–17–18–19–20
second 21–23–27–29–31–33
third 37–39–41–43–47–49

The sector arms make it possible to move the crank without having to count the number of holes for each move. In setting the sector arms, the number of spaces between the holes should be counted, not the holes themselves. Some heads have a plate fastened to the spindle outside of the head. A number of holes are in the plate near its circumference. The number is usually 24. A pin extends from the body of the head and may engage the holes if desired. To save time where a small number of divisions such as 2–4–6–8–12– have to be made, this rapid index plate is used. The worm is dropped from the wormwheel by means of a cam, the spindle is turned by hand, and the rapid index pin determines the distance that the spindle is moved.

Although a great many different divisions can be obtained by using the plates furnished with the index heads, there are some numbers which cannot be indexed in the ordinary manner. In differential indexing a train of gears is attached to the head so that the index plate will be moved while the crank is turning the spindle. This obviously puts the index hole that the crank pin is being moved to, for each cut, in a different relative position.,The pin has to move a greater or shorter amount to reach the designated hole. The motion of the spindle is a differential between the motion of the crank and the movement of the plate.

Compound indexing serves the same purpose as differential indexing. It is accomplished without gears. The crank is moved in one direction for the designated number of holes and then the plate and crank are given another movement by hand which adds to, or subtracts from, the first movement of the crank. Charts are furnished with the milling machine index heads to show how to set up for the type of indexing that the head is designed to do.

Grinding. This operation was formerly used only for the removal of hard metal in order to finish tools. It has become one of the principal manufacturing techniques. This change of purpose has been brought about through the higher standards of accuracy which now prevail in industry; through improvements in the forging processes, which make it commercially practical to forge to close limits; and through improvements in the grinding machines and in grinding materials. The automobile and airplane industries have been the leaders in adapting the grinding processes to the production of higher classes of surface finish and of accuracy.

One of the comparatively new grinding machines used is in the thread grinding field. Taps are made, by some companies, by being first turned, fluted and then hardened. The hardened blank is then placed in a thread grinding machine and the threads are ground into it. This process has the advantage of more uniformity of thread and hardness. If the blank warped during the hardening, the grinding of the threads would remedy the condition.

Smooth running high speed gears require a very accurate tooth shape and fit. If the shape of the teeth is not perfect a noisy gear will result at high speeds. Hardened gears will of course resist wear and will have a longer life than soft gears. Manufacturers recognized the desirability of producing hardened gears, but also knew that hardening would probably cause some warping or change in the tooth spacing and outline. The gear grinding machines were developed so that gears could be hardened and then ground to a perfect form.

Centerless grinding has been developed for the rapid grinding of work on a production basis. No centers are required in the piece of work. The major elements of the centerless grinding machine are the grinding wheel, the work rest and the regulating wheel. The regulating wheel is a grinding wheel usually made with an elastic bond. Its function is to feed the work against the grinding wheel and if necessary through the space between the wheels. The work rest is a metal blade which keeps the work at the correct height in relation to the wheel centers.

Straight pieces of work of one diameter are ground by passing them between the wheels of the grinding machine. This may be an automatic operation with the pieces fed in from a hopper on one side to a catch basket on the other. The speed of the travel is determined by the setting of the regulating wheel, also by its diameter and surface speed. Straight in-feeds are used for grinding pieces that cannot be traversed by the wheels. In this method the work rest moves the work toward the grinding wheel until the desired diameter is reached. Irregularly shaped pieces, such as machine handles, may be ground by using a wheel that is trued to produce the shape; the feed is straight in. Tapered work is ground by feeding the work from one side up to a stop; the wheels are trued to an angle that will give the taper called for.

The machines thus far mentioned fall into the special purpose and production type of machine; they are not, as a rule, found in machine shops. The grinding machines of the machine shop are the surface grinders, for plane surfaces; cylindrical grinders for internal and external cylindrical work; cutter grinders for sharpening milling cutters, etc.

The *surface grinder* has a flat table which reciprocates under the grinding wheel. The work is held on the table by means of a vise or as is more common, by a magnetic chuck. The table is fed so that the surface of the work will be ground while the table is reciprocating. The hand wheel which lowers the wheel is graduated so that a fraction of a thousandth of an inch may be easily controlled.

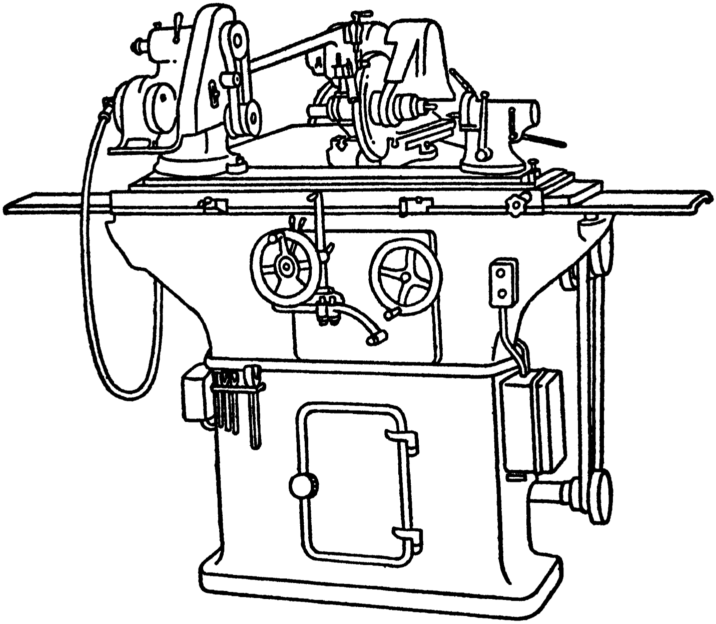
If direct current is not obtainable for the magnetic chucks a generator must be installed. Magnetic chucks will not work on alternating current. Permanent magnetic chucks are now available which do away with the necessity of using the electromagnet type. When work has been held on a magnetic chuck it may retain some magnetism when it is removed from the chuck. Shops that are equipped with such chucks should have a demagnetizer in order to remove any residual magnetism from the work.

Some surface grinders are equipped with attachments that make it possible to flood the work to keep it cool. This is not usual in ordinary machine shop work, however. The average depth of cut for a surface grinding machine is seldom more than a thousandth of an inch. Care must be taken to prevent overheating the work. Hard grinding wheels are more likely to burn the work if not used with a coolant.

In the *cylindrical grinding machine* the table is traversed on vees past the wheel. The work and wheel both revolve. The finish of the work is affected by the speed of its rotation and travel. The work is held on two dead centers, which reduces the possibility of eccentricity that might result from a live center that might not be true.

The headstock and the tailstock which carry the work are mounted on a swiveling table which in turn is carried by the sliding table. To grind tapers, the swivel table is placed at an angle and locked into position. The setting of the sliding table is not affected.

Internal grinding is accomplished by special attachments. The grinding wheel head is replaced by an internal grinding head. This carries a small diameter wheel on a slender shaft so that the inside of a hole can be reached and ground. The work is carried in a chuck. Internal grinding must be a slower operation than external grinding because of the weakness of the shaft and the small diameter of the wheel. Cylindrical grinding machines are fitted with pumps and water guards so that the work may be kept cool while the grinding is carried on.



Universal Grinding Machine
Motor Driven

Grinding machine spindles turn at a very high rate of speed. The wheel must be carefully mounted on the arbor and must run true to prevent vibration which would result from an unbalanced condition. The wheel has a lead bushing center which should be a good fit on the arbor; it is supported by large flanges which are held firmly by the arbor nut. In tightening the nut, excessive pressure must be avoided or a cracked wheel may result. Soft washers are placed between the flanges and the wheel to lessen the chances of breaking and also to even up the pressure. Wheels must be inspected before they are placed on the arbor. A wheel that is cracked would be a dangerous implement. A light tap with a screwdriver handle should tell whether or not the wheel is cracked; a dead sound is given off by a defective wheel.

Wheels must be trued after they are mounted. Trueing a grinding wheel is an operation in which a diamond is used to remove some of the surface of the wheel. The diamonds used are commercial diamonds, not those used for jewelry. The diamond is mounted in a holder which is held by a fixture on the table of the grinding machine. The wheel is passed across the diamond until the surface is running true. If the wheel must be shaped for a special form or angle, the trueing or forming is also obtained with the diamond.

Dressing a wheel is the term applied to an operation which is similar to trueing. The diamond is used as in trueing, but the purpose of dressing is to clean the surface of the wheel. This becomes necessary if the wheel is glazed or loaded. A wheel is glazed when the grinding particles have become dulled without being forced out of the bond. An ideal bond is one that retains the grinding particle until it has become dull and then, due to the extra pressure which a dull particle would exert, permits sharper grains to come in contact with the work. A wheel is loaded when the surface is carrying particles of the material which is being ground. This happens when grinding soft metals if the wheel speed is not right.

Grinding Wheels. The subject of grinding wheels is too lengthy to go into in much detail here, but there are certain terms which must be understood by the machinist. These terms will now be explained.

A *grinding wheel* is composed of small particles of abrasive which are held together to form the shape of the wheel. An abrasive is a very hard substance which forms sharp edges and corners. The materials used as abrasive are so hard that a fracture of the grain will always result in sharp edges.

Natural abrasives are mined in different countries. Turkey and Tripoli furnish emery; corundum is mined in Canada; Indiana has good deposits of sandstone which is made into wheels for the softer tools such as woodworkers use. *Artificial abrasives* are manufactured in great quantities and have supplanted the natural abrasives to a great extent. They are produced with the aid of electric furnaces. The particles of abrasive are graded for size through sieves. The size of the particles is referred to as the grain. The grain is numbered according to the number of spaces per inch in the sieve which was used to select it.

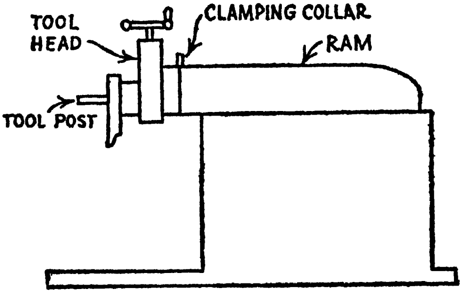
The *bond* of a wheel is the material used to hold the grains. The bond may be clay which is vitrified into a kind of porcelain, or rubber, or shellac, or resin. The wheels that are bonded with rubber or shellac are called elastic wheels. They are used in thin wheels and for work where a more brittle bond would cause breakage. The structure of a wheel is the density of the abrasive grains and their arrangement in the bond. If a wheel is formed with greater pressure while it is in the wheel mold it will have a denser structure than if formed with less pressure.

The new cutting material, cemented carbides, are very much harder than the other metals, and ordinary abrasives have not been successful in grinding the new material. Diamond wheels have been developed for the purpose of grinding the cemented carbides. The diamonds used are again the commercial diamonds which are not desirable for use as jewelry. The diamonds are crushed and bonded on to a special composition wheel. The layer of diamonds will be from a sixteenth to an eighth of an inch in thickness, depending on the cost.

57. SPECIAL MACHINE TOOLS

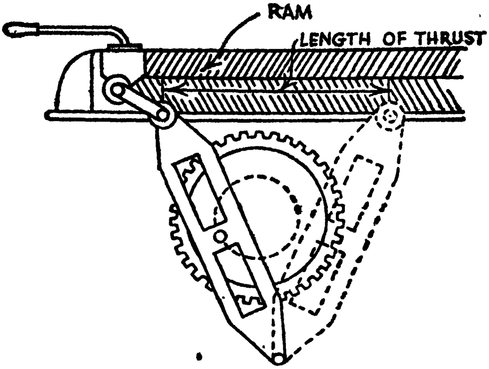
The Shaper. The function of the *shaper* is the production of small flat surfaces. Milling machines and planers are also used for this purpose, but the shaper has certain advantages of easier set-up and handling that make it superior to them for machine shop use. The shaper is one of the machine tools; it is not intended for manufacturing in quantity. The shaper cuts flat surfaces by means of a single point tool, very similar to a lathe tool, which is carried back and forth over the work taking a chip on the forward stroke. The work is held in a vise, usually, and is fed by the automatic feed mechanism so that the tool will make successive chips.

The tool is held in a tool post at the forward end of the *ram,* which is the principal moving part of the shaper. It is a heavy, strong iron or semisteel member which is reciprocated by mean of gears and levers. The leading end of the ram carries a tool head which in turn carries the clapper box and tool post. The tool is raised or lowered for its cut by means of the hand screw of the tool head. The tool head may be swiveled so that angular cuts can be taken. The head is clamped in position by a screw which is usually on the collar back of the tool head. The edge of the collar is graduated into degrees for setting to angles.



The ram is moved by an arm, called a vibrating arm, which is connected to a block carried by a large gear known as a bull gear. The bull gear and arm are inside of the body casting of the machine. Since all jobs that the shaper must do are not of the same length, it is necessary to provide a means of changing the length of stroke. The length of a shaper stroke should be at least three-quarters of an inch longer than the length of the work. The ram is set so as to carry the tool past the work, on the cutting stroke, by an eighth of an inch. To change the length of the stroke the block is moved in its relation to the center of the gear.

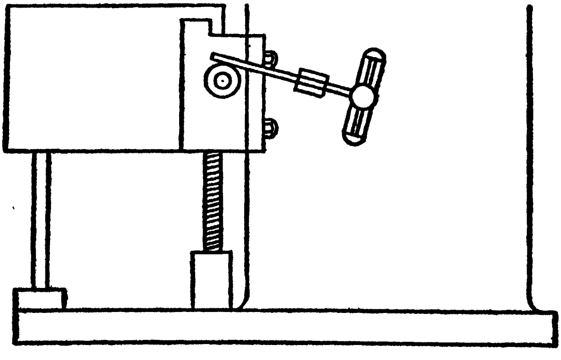
The ram is fastened to the internal mechanism of driving by a clamp block. This clamp block may be loosened by a lever that is found on the top of the ram toward the rear. The purpose of the clamp block lever is to



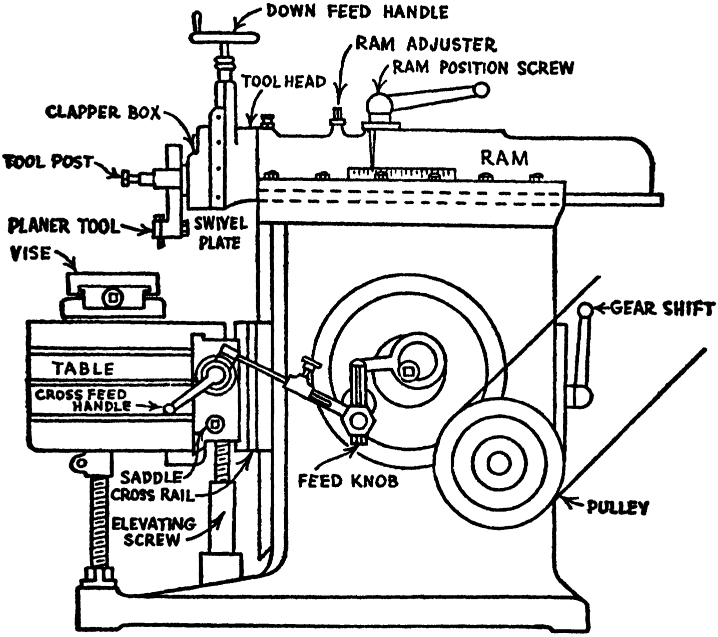
Showing Drive of Crank Shaper

allow a change of position of stroke to be made. Frequently when the ram is set for length of stroke it will be found that the work will not be covered by the tool travel. When this occurs, the clamping block is loosened and the position of the block in relation to the ram is changed by turning a handle which is at or near the forward end of the slot in the ram.

Much of the work done in a shaper is held in the shaper vise. The vise is clamped to the table which is in turn carried by the cross rail. The cross rail is clamped at whatever height is required to the column of the machine. The outer end of the table is supported by a brace which must be loosened when the table is to be lowered. Some shaper tables may be swiveled for angular cuts. These are called universal tables.



Shaper Tools. The tools used in a shaper resemble lathe tools in their general shape. There is no rocker in the tool post of the shaper and therefore the front clearance angle of the tool is not changed as it is in a lathe. The work of the shaper is flat and there is no reason for a great amount of clearance. The lathe tool should have from 12° to 15° of front clearance; the shaper tool needs only from 3° to 5°. The shaper tool must be stronger than a lathe tool because the cut it makes is intermittent rather than continuous. There is a distinct shock on the tool at the start of every cut.

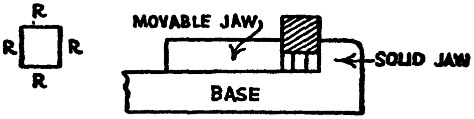


The Shaper

Holding the Work. Shaper vises are made very strong and the jaws open to a great capacity. The vise jaws are deep and the work is usually held on parallels so that the tool can cut where needed without danger of hitting the jaws. If castings are to be held in a vise, the jaws of the vise should be protected by strips of cardboard or brass placed between the work and the vise. Although the work may be bolted directly to the table, this is seldom necessary because of the capacity of the vise. Angle plates or knees are sometimes bolted to the table and the work clamped to the angle plates. Cylindrical work may be shaped with the aid of *shaper centers,* attachments which are fastened to the table in place of the vise. The work is held on centers and the cutting tool is lined up with the center axis of the work; as the cutting progresses, the work is rotated on the centers.

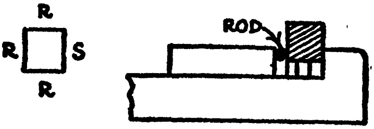
The test of a shaper-hand's skill rests on his ability to shape square objects. By square objects we do not mean merely square to the eye, but true squares, that is, 90° when measured with an accurate machinist's solid square. It does not mean simply taking four cuts and having a finished piece that looks square. In order to appreciate the difficulty of making a piece of work exactly 90° in a shaper, certain facts must be considered. The cut is made by the tool which is carried by the ram. The ram is practically independent of the table setting. If there is any sag in the table the path of the tool will not be parallel to it. If a chip, a piece of dirt, or a small burr should be under the vise, the effect will be the same; the cut will not be parallel to the table. If the jaws of the vise have been sprung so that they do not stand exactly parallel to the column, the work will not be square with the ram. If the piece is not down on the parallels it will not be square.

Method of Squaring Stock in a Shaper. The following principles should be studied and understood so that the problem of squaring stock may be solved. The size of the piece does not matter providing it falls within the capacity of the vise. Assume that the piece has four rough sides, such as a casting or forging or bar stock would have. The first step is to take a cut on one surface. Place the piece on parallels so that the top surface may be worked on by the tool. Tap the piece lightly with a lead or other soft hammer. At least one of the parallels should show that the piece is touching it. Sometimes it is impossible to get the piece down on both parallels when it is too rough.

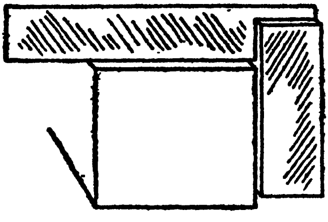


Stock with Four Rough Sides

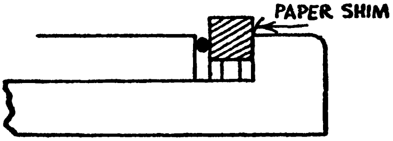
After the top surface has been planed the piece will have three rough sides and one smooth side. Place the smooth side against the solid jaw of the vise and put a solid round rod of convenient size (three-eighths of an inch is usually good) between the movable jaw and the piece of stock. The round rod acts as a pivot and transmits the pressure of the movable jaw to the work so that the work is pressed firmly against the solid jaw. This is necessary because the movable jaw may have a tendency to spring up from the base and if it did so the piece would not be pressed squarely against the solid jaw.

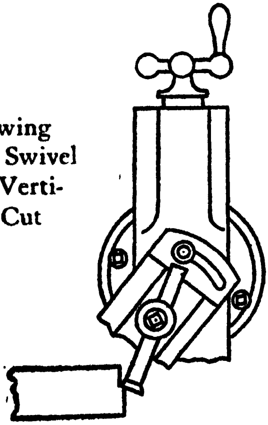


When the second side is planed the piece may be tested for squareness. First be sure that the square is true; it is good practice to check squares frequently by comparing them with one another. When using a square to test a piece the correct method is to press the heavy member of the square against the piece, then carefully slide the square down on to the piece until the blade of the square touches. This should be done while the piece and square are held toward the light. If the line of light between the blade and the piece is not even the piece is obviously not square.


Solid Steel Square

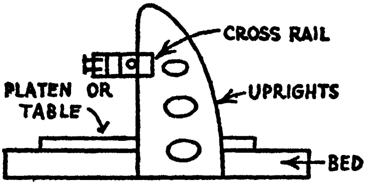
Some of the possible causes of error in squaring a piece have already been mentioned. The trueness of the solid jaw is usually the cause of error. If the piece is found to be out of square, a correction will have to be made with the solid jaw of the vise. Since the error is seldom greater than a thousandth or two, paper may be used. A thin strip of paper (cigarette paper is preferred by most mechanics) is placed so that it rests between the solid jaw and the upper part of the work. The effect of the paper is to compensate for the probable spring in the solid jaw. When the amount of error has been determined, as well as the thickness of paper required to correct the error, the same thickness can be used for all subsequent cuts on the piece.



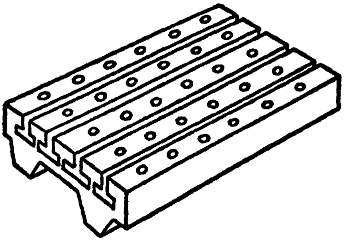

Showing Apron Swivel for a Vertical Cut

End Cuts. When the end of a piece of stock must be planed it is usually better to do it with a down-cut of the tool. The piece is placed in the vise so that one end projects from the side of the vise. The tool is set in the tool post so that it will reach the work. The clapper box must be swung away from the end that is being cut; if not, the box will cause the tool to drag on the work during the return cut.

The Planer. The planer is the largest machine in a machine shop. It is intended to be used for finishing large machine elements such as machine bases, lathe beds and other pieces that are too large for milling machines or shapers. Planer work resembles shaper work in the fact that single pointed tools are used to remove stock. The tools are the same as those used in shapers except for size. Planer tools are much heavier. Unlike the shaper, the planer tool does not reciprocate: it removes stock while the work passes under it, the feed being accomplished by moving the tool at the beginning of each stroke.



The principal parts of the planer are the platen or table, the uprights, the cross rail, the bed, the tool heads, and the driving mechanism. The platen is a heavy cast iron piece that is made with a number of tee-shaped slots for holding tee head bolts, and a considerable number of straight holes into which stops are placed. The side pieces or uprights are heavy, well braced sections of iron which stand on both sides of the bed and which carry the cross rail. The cross rail extends from one side of the planer to the other. It carries the tool heads for horizontal planing and is equipped with a feed screw for automatic and hand feed. The tool heads are massive as compared to the tool head of the shaper. They hold the cutting tool by bolts and straps (metal pieces) instead of with a tool post as does the shaper. The bed is made with vee-shaped guides in which the table slides and is thus held in alignment.


Platen Showing Tee Slots and Poppet Holes

Planers are classified by the maximum cross section size of the work that can be held on the table and passed under the tool head. A 30-inch planer can take a piece 30 inches wide and high. Planers range from the 24-inch size up. The maximum size depends only on the requirements of the buyer of the machine. Large planers are made to special order where needed. The length of the bed is also variable. Lathe manufacturers use planers with beds of twenty, thirty or even more feet in length. The extra length is used so that a number of lathe beds may be fastened to the platen at one time and finished together. Openside planers are equipped with only one upright. They are useful in the smaller sizes as the limitation of the distance between the two uprights of the standard planer is removed. Pieces may be fastened to the platen and may extend on one side. Planer drives are of several types. The older design used a rack under the platen and a pinion gear meshing with the rack. Another type drives with a wormwheel and worm. Hydraulic drives are the latest design.

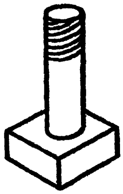
Planer Work. The operation of a planer, once the work is fastened to the platen, appears to be simple and it is. The travel of the table is determined by the setting of trip dogs on the side of the table. These dogs strike trip levers which cause the travel of the table to be reversed at the end of each stroke. The tool setting is a matter of good judgment and simple mathematics. The automatic feed is set by a ratchet wheel and feed pawl.

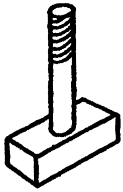
The skill of the planer operator is shown by his ability to determine correct methods of fastening the work to the table. He must be aware of the nature and properties of metals, especially as regards the strains that may exist within the metal, or those which may be introduced by improper clamping methods. He must also be able to plan the handling of heavy pieces. Many pieces which are worked on a planer must be lifted by overhead cranes; this calls for good judgment in methods of attaching chains or cables to the piece to lift it.

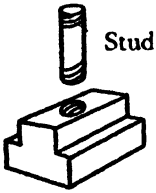
All castings cool from the outside toward the center, since metal contracts as it cools. The outside of a casting shrinks before the inside has assumed its final position. The result is the setting up of *internal strains.* When the outer surface of a casting is removed, the piece has a definite tendency to change shape and size. This is the reason for the seasoning of machine beds and other metals, since warping would be detrimental. If a casting has to be machined all around, it should be planed on all sides to give an opportunity for the internal stresses to exert themselves in all directions before the final finishing. Seasoning is of course also desirable. It would be useless, for example, to plane one side, season the piece, and then plane another side. The same condition exists with cold rolled steel. Cold rolled or drawn steel is finished cold in the steel mill. The final passes through the rolling mills are made while the piece is cold. The result is a clean surface, but the piece has internal strains which have been introduced by the compressing of the metal in the rolls. The outer skin of cold rolled steel must be removed all around before satisfactory straight surfaces can be made and kept.

External strains are introduced when clamping the piece if the piece is not properly supported. Clamps and bolts are often required to fasten the piece to the platen. If the piece is not "shimmed" or "blocked," the clamps will bend the casting. When the casting has been planed and the clamping pressure removed the surface will return to its normal plane and the finished surface will not be straight.

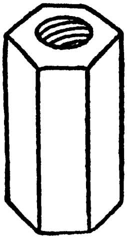
Furniture. Planer work demands a large assortment of bolts, metal straps, stops, shims, etc. These pieces are called *furniture* by the planer-hand. Here is a list of some of the more commonly used pieces of furniture. *Bolts* for the planer are made with a square head; they are called square head bolts. The head is slid into the tee-slot of the platen and put into position for clamping. The height of the bolts will vary as a matter of course.


Square Head Bolt

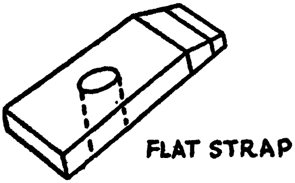

Tee Head Bolt

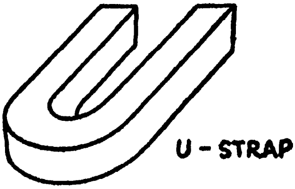


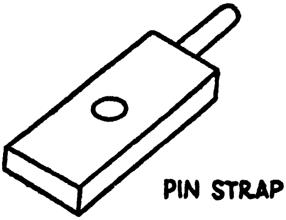
Tee Head

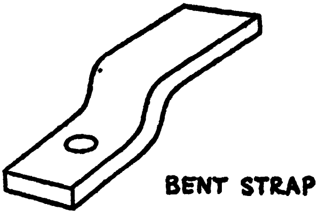

Long Nut
for Joining Studs

Sometimes it is impossible or inconvenient to slide a square head bolt in from the end of the platen slot. In this case a tee head bolt is used. This type of bolt may be put into the slot at any point, given a half turn, and fastened. Studs are threaded at both ends and may be screwed into a tee head. They may be built up to obtain greater height by using long nuts into which two studs are threaded. Clamps or *straps* are flat pieces of steel supported at one end by blocks and tightened down so that the other end fastens the work to the table. The height of the block in relation to the height of the work is very important. The block should not be lower than the work and it should not be much higher. If the block is too low the strap will exert a side push against the work and the work may be pushed out from under the strap. If the strap is too high there will not be enough bearing on the work and the strap will probably bend. The blocks should be slightly higher than the work to compensate for the natural bending of the strap when the nut is tightened. *Shimming* is the term applied to thin pieces of metal placed under a piece of work to level it up. *Blocking* is the expression used when the piece cannot be termed "thin." Blocking is also called packing. It is done by using pieces of metal

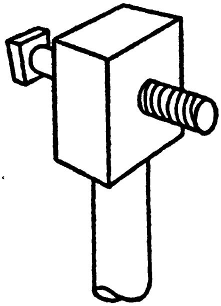


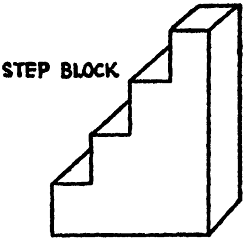






or wood that may be available. Jacks are very useful if the height of the work permits their use. Step blocks are also very helpful and much neater.





Poppets are placed in convenient holes in the platen and are used to force "hold-downs" against the work or to bear directly against the work itself. *Planer vises* are sometimes used for small work. The vise is very heavy as compared to other vises. The movable jaw is forced against the work by two large set screws.

Broaching Machines. In the machines that have been mentioned thus far, the cutting tool has been of the single point type such as in the lathe, planer and shaper; or the multiple cutting edge type that is used in the milling machine and drill. The multiple cutting edge tool has been a revolving one. The cutting edge of the milling cutter takes its chip and returns for another chip on its revolution. Broaching started as a hand operation in which a cutting tool was forced through a hole in order to enlarge or shape it as desired. The broach was pushed through by means of an arbor press or was forced through by blows.

Essentially, a *broach* is a multiple cutting tooth tool. The teeth are arranged in a straight line, each tooth removes a small amount of material as the broach is forced through the piece. The amount of metal removed by each tooth is only a few thousandths of an inch and the tool is passed through the work once. Broaching machines were inevitably designed to apply the principles of the method more efficiently. The machines are designed either to pull the broach through or to push it through. The pull-type machine will handle a longer broach than the push-type will and consequently more stock can be removed. A broach that is pulled through does not have the tendency to bend that the push broach has. Broaching machines proved so satisfactory that the method is now being applied to external broaching. Pieces that are to be surface-broached are held while the broach passes over their surface, removing the waste material. In a sense the operation is similar to that of passing a large file over the surface. External broaching is proving to be a valuable method of finishing surfaces and will probably find greater use in the future.

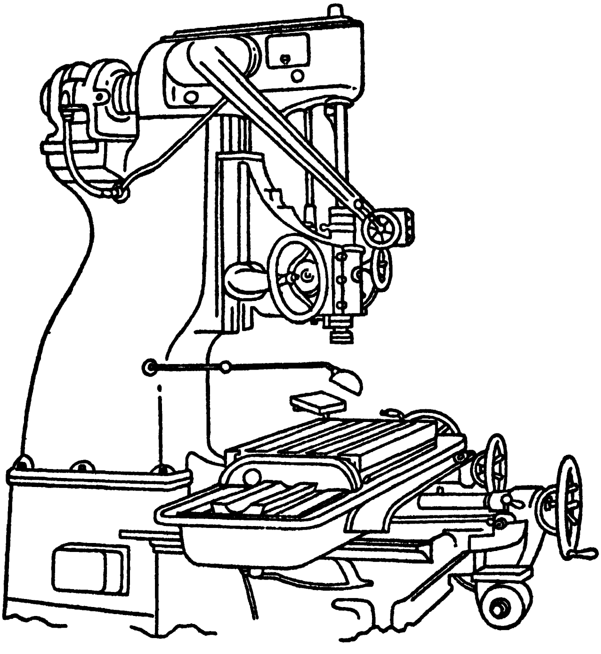
Metal Cutting Band Saws. The metal cutting band saw is changing the technique of the die maker and is saving a tremendous amount of time. Formerly when a die was to be made, the outline of the die was laid out on the stock, then a second outline was laid out inside of the full-sized outline. A series of center punch marks was made along the inside layout. These punch marks had to be spaced carefully so that the edges of the drilled holes would just touch each other. When the holes had been drilled the waste metal was knocked out and the die maker filed the die to the desired outline. This was clearly a slow process.

When a metal cutting band saw is available the die maker now makes his full-sized outline and then drills one hole inside of the layout. The band saw is then broken, passed through the hole and joined together again by electric welding. The welding unit is built into the machine itself. After the saw has been welded it is ground on the sides to reduce any thickness at the weld. This grinding unit is also a part of the machine. The band saw cuts the waste metal from the rest of the piece in much less time than was possible with drilling. The cut can be made close to the outline so as to reduce filing; in addition, there is a saving of material in many cases, since the part cut out may be useable. The machine will also do filing. Special band files, which are made by securing a series of file sections to a band, are inserted in the same manner, and very accurate, quick filing to the line is thus made possible.

Jig Borers. Jig- and fixture-making is a specialized branch of the machine shop trade. The workmen who specialize in this form of work are toolmakers. A *jig* is a device which is designed to hold the work and guide the tool. A *fixture* is a device to hold the work only. Both mechanisms are intended for use in production work. When the jig or the fixture is made it is used by production workers who repeat the same operations over and over again. The jig will always hold the work in the correct position and the drill or other cutting tool will always be guided into the exact location.

Interchangeable manufacturing demands close tolerances in dimensions. The holes in the jigs which are to guide the drill must be exactly located. There may be a number of holes in the jig plate all of which must be held to center distances which are expressed in thousandths of an inch.

The toolmaker's method of locating the hole spacing is called the *button method.* In this method the toolmaker first locates the position of the centers of the holes as accurately as possible; he then drills and taps a small hole. The "button" is a hardened and ground cylinder which has a hole that is larger than the screw that passes through it and is threaded into the tapped hole. The buttons are shifted until their center distances are correct. Measurements are taken with a micrometer or vernier and the button is tightened so that it will not shift after it has been located. The piece of work is then fastened to a lathe faceplate, and with the aid of an indicator, the toolmaker trues up a button by shifting the piece of work. The button is removed when the location is complete, whereupon the toolmaker proceeds to drill and bore. The other buttons are trued in the same manner. It is readily seen that this method is quite slow and subject to the chance of error.



Giant Jig Borer

Jig borers are machines which are designed to drill and bore holes accurately. The table of the machine can be moved so as to place the work under the drill so that center distances of holes are correct to thousandths of an inch. Some jig borers are equipped with vernier scales on the table so that longitudinal and transverse travel can be determined by vernier readings. There is no dependence on the accuracy of the table screws which may be worn. Other jig borers use size blocks to check the placement of the table. The size blocks are accurate in dimension to a few millionths of an inch.

SELECTED REFERENCES FOR FURTHER STUDY

Atlas Press Company. Manual of Lathe Operations and Machinists' Tables. Atlas Press Co.

Barrett, J. W. Machine Shop Operations. American Technical Society.

Burghardt, H. D. Machine Tool Operation. (2 vol.) Part I: The Lathe, Benchwork, and Work at the Forge. Part II: Drilling, Planing, Milling, Grinding. McGraw-Hill Book Co.

Colvin, F. H. & Stanley, F. A. Drilling and Surfacing Practice. McGraw-Hill Book Co.

Colvin, F. H. & Stanley, F. A. Grinding Practice. McGraw-Hill Book Co.

Colvin, F. H. & Stanley, F. A. Turning and Boring Practice. McGraw-Hill Book Co.

Halsey, F. A. Methods of Machine Shop Work. McGraw-Hill Book Co.

Jones, Harry. Progressive Lessons in Machine Shop Practice. Bruce Publishing Co.

Leonard, W. S. Machine Shop Tools and Methods. J. Wiley & Sons.

Pratt, J. A. Elementary Machine Shop Practice. D. Van Nostrand & Co.

Smith, R. H. Advanced Machine Work. Industrial Education Press (Boston).

South Bend Lathe Co. How to Run a Lathe. South Bend Lathe Co.

Turner, F. W., Perrigo, C. O. E. & Bertrand, A. Machine Shop Work. American Technical Society.

Turner, W. P. Machine Tool Work: Fundamental Principles. McGraw-Hill Book Co.

Warner-Swasey Co. Turret Lathe Operator's Manual. Warner-Swasey Co.

CHAPTER XII

WOODWORKING AND PATTERNMAKING

William H. Ruten

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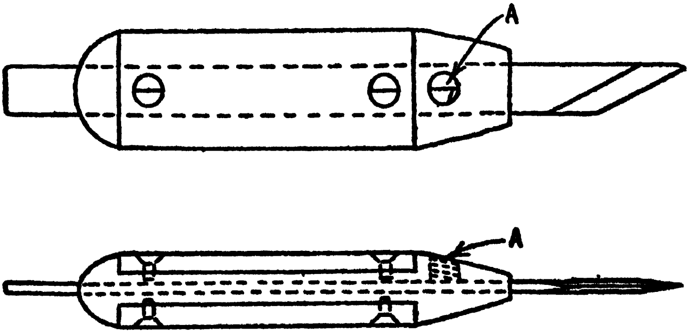
58. HAND CUTTING TOOLS

The Bench Knife. The *bench knife* is the woodworker's most important bench tool. It is indispensable for whittling and shaping small parts. Among the better workmen, it is preferred to the lead pencil as an instrument for marking lines on the surface of wood when work is being measured to size for cutting. This is especially true when the work must be measured, or "laid out," very accurately, as in pattern making. A line scored in the wood by means of a knife is preferred to a line drawn by means of a lead pencil, because the knife line is cut cleanly into the wood at least one thirty-second of an inch in depth, while the pencil line is merely a surface mark which might become rubbed off or otherwise blurred during the subsequent handling of the work. However, where accuracy is not so important, or where the surface would be defaced by knife lines cut into it, a pencil line would be used.

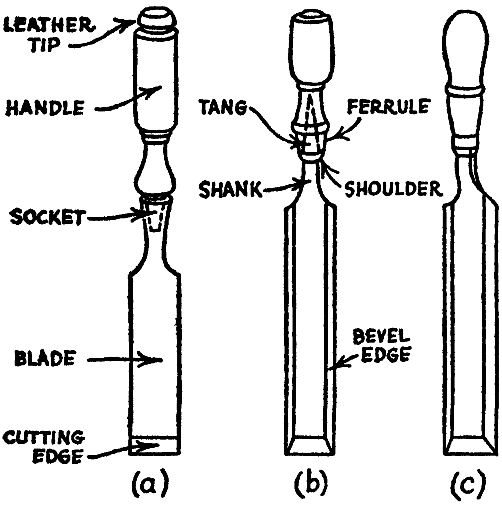
A popular type of woodworker's bench knife is shown below. The blade is nine inches long, one-half of an inch wide, and one-sixteenth of an inch thick. It fits in a slot which extends all the way through the handle. The handle is approximately five inches long, and is made of brass with a



Woodworker's Bench Knife

hardwood face inserted on each side. The blade is held in place in the handle by means of a headless screw A, which may be tightened or loosened with a small screw driver. As the cutting edge becomes worn, the screw A may be loosened, the blade advanced in the handle a desirable distance, and then the screw retightened. The handle may be used indefinitely. New blades may be purchased and inserted in the handle to replace worn blades whenever necessary.

The Chisel. The woodworker's bench chisel consists of a thin, flat blade of steel about six inches long, with a cutting edge at one end and a handle at the other end. The blade is about one-quarter of an inch in thickness at the handle and tapers slightly toward the cutting edge. Chisels range in width from one-eighth of an inch to two inches. The size of a chisel is indicated by the width of the cutting edge. Chisels are made in two different styles, known as *framing* and *paring*, as shown in (a) and (b). The blade of the framing chisel is thicker and stouter than the blade of the paring chisel, and is attached differently to the handle. The framing chisel is intended for heavier work, where pressure to force the cutting edge into the wood is frequently applied to the leather-tipped handle by blows from a mallet. The handle is fitted into a socket in the metal of the blade, which tends to keep the handle from splitting. The blade of the paring chisel is longer, thinner, and lighter in weight than the blade of the framing chisel. In its better form, the two edges of the blade are bevelled, which enables the cutting edge to cut into corners more cleanly. The handle-end of the blade consists of a shank, a shoulder and a four-sided tapered tang, which is driven into the wood handle to hold the handle to the blade. To keep the handle from splitting, it is bound at its lower end by a metal band, known as a ferrule. Pressure, to force the



Framing Chisel

Paring Chisel

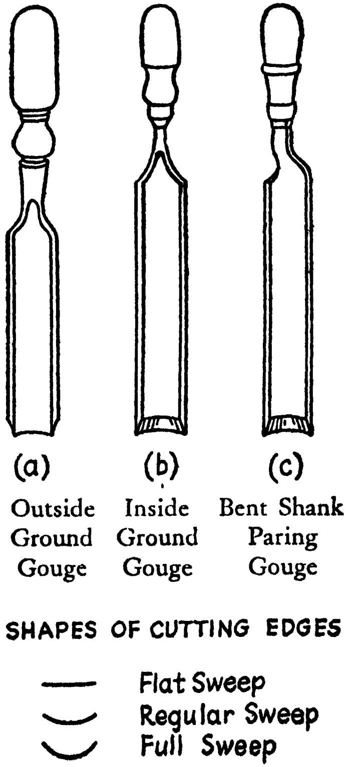
Bent Shank Paring Chisel

Woodworker's Chisels

cutting edge into the wood, is applied to the handle of the paring chisel solely by a steady push with the hand. The cutting edge at the end of the blade is developed by grinding the face at an angle of approximately 20° with the back. The back of the chisel is kept perfectly flat.

The paring chisel is further adapted to the lighter work required in pattern making by a special style shown in (c), known as a *bent shank paring chisel.* The shank and handle have something of the appearance of a bricklayer's trowel. This style is preferred by many patternmakers because the off-set handle enables the chisel to be used flat on surfaces that are longer and wider than the length of the blade without marring the work or injuring the knuckles of the hand of the operator.

The Gouge. The chief difference between a chisel and a gouge is that the blade of a chisel is flat throughout its length, while the blade of a gouge is concave throughout its length. A *gouge* may be thought of as a chisel with a curved cutting edge. The blades of gouges are made in three different shapes of cutting edge, known as *flat sweep, regular sweep,* and *full sweep.* The cutting edge is shaped to form an arc of a circle. Gouges may be had in socket handle, tang handle, and bent-shank styles, the same as chisels. They are made in widths varying from one-eighth of an inch to two inches. The size of the gouge is indicated by the length of a straight line measured from one end of the cutting edge to the other. The bevel forming the cutting edge of the gouge may be either on the convex side or on the concave side. When the bevel is on the convex side, the gouge is known as an outside ground gouge, or simply as a gouge. When the bevel is on the concave side, the gouge is known as an inside ground gouge, or is sometimes called a paring gouge. The outside ground gouge is more useful as a general purpose tool, and is therefore preferred by the carpenter and the cabinetmaker, while the inside ground gouge is especially adapted to shaping curved inside cavities of some length, and is there-


(a) Outside Ground Gouge

(b) Inside Ground Gouge

(c) Bent Shank Paring Gouge

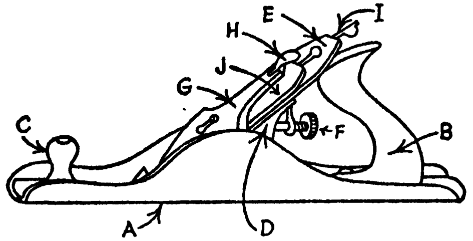
SHAPES OF CUTTING EDGES

Woodworker's Gouges

fore preferred by the pattern maker. The bent shank style handle is associated with the inside ground cutting edge. When so associated the gouge becomes known as a bent shank paring gouge. A gouge will make a very smooth cut if the cutting edge is given a slight circular movement as it is pushed forward into the wood.

The Plane. When it is necessary to remove surplus material from a board, or to produce a smooth surface on a board, the woodworker uses a tool known as a *plane.* Its cutting action is made possible by a thin, chisel-like cutter, called a plane iron, which projects very slightly through a slot in the bottom of the plane. When the plane is being used, it is pushed over the surface of the board by hand, the bottom of the plane being in contact with the surface of the board. The plane iron, projecting through the slot in the bottom of the plane, engages the surface of the board and shaves off a thin piece of wood, known as a shaving. The thickness of the shaving may be controlled by the distance the cutting edge of the plane iron projects below the bottom of the plane. This distance may be easily regulated by means of a brass adjusting nut found on the underside of the frog which supports the plane iron in place, as shown at (F) in the accompanying figure.

A. Bottom
B. Handle
C. Knob
D. Frog
E. Plane iron
F. Plane iron adjusting nut



G. Lever cap
H. Cam
I. Plane iron lateral adjusting lever
J. Chip breaker

Woodworker's Hand Plane

When the brass adjusting nut is moved toward the frog of the plane, the plane iron is drawn in; when the nut is moved toward the handle, the plane iron is pushed out in relation to the bottom of the plane. In the figure, A represents the bottom of the plane, which is in contact with the surface of the board to be planed; B is the handle, which is grasped in the right hand to push the plane over the surface; C is the knob, which is grasped in the left hand to aid in pushing the plane against, as well as over, the surface to be planed; D is the wedge-shaped frog for supporting the plane iron; E is the plane iron, the lower end of which is bevelled and sharpened like a chisel cutting edge; F is the brass plane iron adjusting nut; G is the lever cap which holds the plane iron against the frog; H is the quick acting cam which may be raised when it becomes necessary to remove the lever cap so that the plane iron may be taken out for sharpening; I is the plane iron lateral adjusting lever, which aids in regulating the evenness of the shaving being cut; J is the chip breaker, which is fastened to the plane iron to aid in curling the shaving as it comes from the cutting edge.

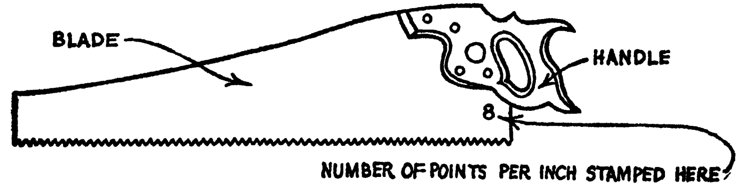
Planes may be had in many sizes and styles. Only those most generally used by the woodworker will be described. The *smooth* plane, which is about nine inches in length, is used for smoothing surfaces without regard for straightness. The *jack* plane is about fifteen inches in length. It is used to true up surfaces and edges quickly from rough stock to prepare them for the jointer. The *jointer* plane is about twenty-two inches in length. It is a finishing plane and is used wherever straightness is required. The jointer will produce a straight surface as well as a smooth surface. The *fore* plane is a short jointer intended for use on smaller work; it is about eighteen inches in length. The *block* plane, which is about six inches in length, is intended for planing across the end of a board. The block plane differs from the planes just described in that the cutter is inverted, and is supported at a greater incline, which makes it more adaptable for end-grain planing.

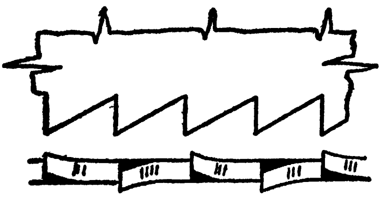
The Hand Saw. A *saw* consists of a very thin blade of steel, with a handle at one end, and a series of pointed cutting teeth along one edge. The operation of cutting through a board, to separate pieces from it, with a saw, is known as sawing. Saw cuts are made along the side of lines drawn on the wood, to the outside of the line, on the waste stock, so that the wood will not be undersize after it has been sawed. The saw is made to cut by grasping the handle of the saw in the right hand, with the index finger of the right hand extended along the side of the handle to assist in guiding the saw. The thumb of the left hand is placed aside the line drawn on the wood at the point where the cut is to be made. The toothed edge of the saw blade is placed against the left thumb nail at an angle of about sixty degrees with reference to the surface of the board, and the saw drawn lightly toward the operator. Drawing the saw toward the operator is known as the backward stroke, because the saw cuts when it is being pushed away from the operator. The first backward stroke enables the operator to balance the saw in preparation for the first cutting stroke. Short, light, easy, back and forth strokes should be taken at first until the saw starts forming a slot, known as a *kerf*. The length and rapidity of the strokes may be increased until the full length of the saw is used.

According to use, a saw may be considered as being one of two kinds. The saw used for cutting across the grain of the wood is known as a *cross-cut saw*. The saw used for cutting in the same direction as the grain of the wood is known as a *rip saw*. The chief difference between them is the shape of the teeth formed on the cutting edge. The teeth of the cross-cut saw are like small knife points, arranged to give *two rows of cutting edges*, one on each side of the saw. Each alternate cutting tooth has its cutting edge on the opposite side of the saw blade. This enables the saw to cut the wood at two places, along the same line, at the same time. The one set of teeth with cutting edges on the right side of the saw, cut into the wood at the right side of the saw blade; at the same time, the alternate teeth that have their cutting edges on the left side of the saw cut into the wood on the same line on the left side of the saw blade. This cuts the wood fibers at two places, close together, at the same time. The portion between the two saw cuts is pushed out of the kerf as *sawdust.*

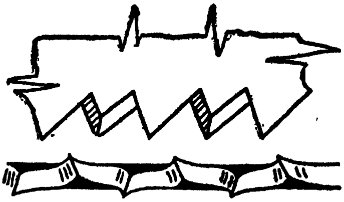
The cutting action of the rip saw is not at all the same as the cross-cut saw, since it is not necessary to sever completely the wood fibers when cutting with the grain. It is more a matter of *separating* the fibers. The teeth on the cutting edge of the rip saw are like a series of small chisels, one behind the other. A small piece of wood is cut off and pushed out of the kerf in front of each tooth. Each successive tooth, as it follows in line, cuts off and pushes out another small piece.

The coarseness of the saw is determined by the number of teeth to an inch of length, and is indicated by a number stamped on the saw blade near the handle. The size of the saw is the length of the cutting edge in inches. The length of the blade for common work is usually twenty-eight inches. Each alternate tooth of a saw is bent outward in the opposite direction; the bending of the teeth in this manner is said to give the saw "set." The set of the teeth keeps the saw from binding as it cuts into the wood, thus enabling it to make a kerf wider than the thickness of the saw. The accompanying figure shows a hand saw and enlarged views of the shape of the teeth, the set, the cutting action, and the saw kerf of a cross-cut and a rip saw, respectively. For very fine work, a saw known as a *back saw* is used, because of the metal reinforcing strip across the back

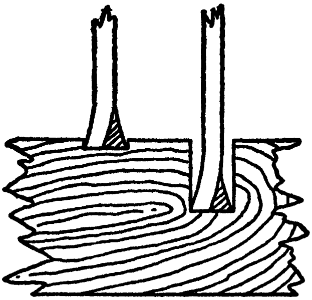




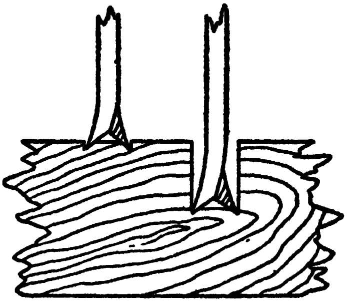
Enlarged View of Rip
Saw Teeth



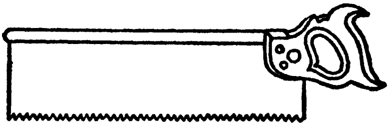
Enlarged View of Cross-cut
Saw Teeth



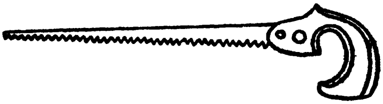
Enlarged End View of Cutting Action of Rip Saw, Showing Set of Saw and Saw Kerf



Enlarged End View of Cutting Action of Cross-cut Saw, Showing Set of Saw and Saw Kerf



Back Saw

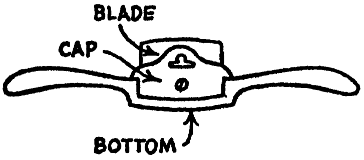


Compass Saw

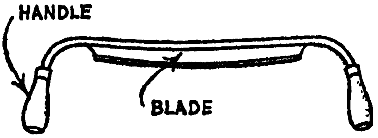
of the blade to keep it from bending while in use. The teeth are shaped somewhat like the teeth on a cross-cut saw, only they are smaller, averaging about fourteen points to an inch of length. This saw is intended for use on small, light work at the bench. It is made with blades of various lengths from ten to fourteen inches.

The compass saw has a blade that tapers its entire length, coming almost to a point at the end. In its smaller sizes it is known as a *key-hole saw;* in its larger sizes it is called a *compass saw.* As its name indicates, it is used for cutting key holes and other small irregular holes. A hole large enough to insert the point of the saw must first be bored through the wood with an auger bit before the saw can be started. The compass saw is a heavier tool, well adapted to sawing the average run of curved work. The shape of its teeth is a cross between a cross-cut and a rip saw. It has a very wide set to keep the blade from binding. The blade of the keyhole saw is usually about six inches in length, while the blade of the compass saw is usually about fourteen inches in length.

The Spokeshave. The *spokeshave* is intended to be used to smooth the surface of irregularly shaped and curved work, in much the same way that the plane smoothes the surface of flat work. The blade and cutting action of the spokeshave are similar to the blade and cutting action of the plane. The chief difference between the spokeshave and the plane lies in the length of the bottom that is in contact with the work, and in the position of the handles. The bottom of the spokeshave is short, and the handles are at the sides of the cutter; these features make it more readily suited to narrow work of irregular outline. The shortness of the bottom permits it to conform to the shape of the curve. The handles at the sides of the cutter permit the hands of the operator to clear the surface of the work. The spokeshave is usually pushed when in use, but it may be pulled if desired. The frame and handles are usually made of one piece and may be either of wood or metal. The cutter is usually about two inches wide.



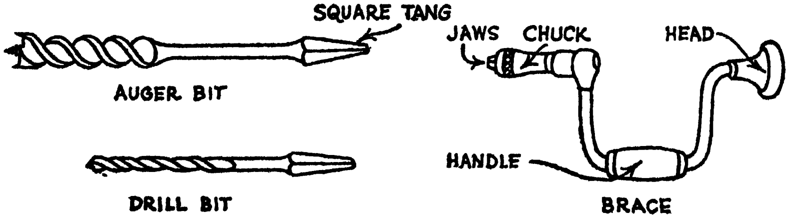
Spokeshave



Draw Knife

The Draw Knife. The *draw knife* is really a wide chisel with a short blade, which is pulled into the work by means of the handles, instead of being pushed into it as in the case of the chisel. It is used as a roughing tool to remove waste stock when there is not enough to saw off but too much to plane away. It is used on both straight and curved surfaces. In use it is drawn toward the operator. The length of the blade is usually about twelve inches. Because its long exposed blade makes it a dangerous tool to use, many woodworkers prefer to rough out the general shape of their work with chisels, gouges, and spokeshaves in preference to using the draw knife.

Boring Tools. The tools used for making round holes in wood are called *boring tools.* They consist of cutting tools, called *auger bits* and *drill bits,* and a crank-like holder, called a *brace,* which is used to give the bits rotating motion. Bits have a square tang at one end which fits into the chuck of the brace. To insert a bit in a brace, the chuck is grasped firmly in the left hand, the head is rested against the body of the operator, and the handle is cranked counterwise until the jaws have opened sufficiently to



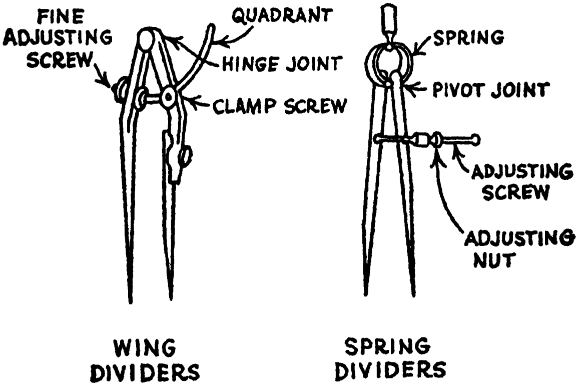
Boring Tools

receive the square tang. The bit is then placed between the jaws of the chuck and the cranking motion is reversed until the jaws close firmly on the bit. The brace and bit are now ready for use. Holes to be bored are usually located by indicating their centers by two intersecting lines called *center lines*. The point of the bit is started in the wood at the point where the center lines intersect.

Auger bits may be had in sizes, varying by sixteenths of an inch, from one-quarter to one inch. Drill bits may be had in sizes, varying by thirty-seconds of an inch, from one-eighth to one-half of an inch. The size is indicated by a number stamped on the tang. A number four on an auger bit means four-*sixteenths,* or one-quarter of an inch. A number four on a drill bit means four *thirty-seconds,* or one-eighth of an inch.

59. LAYOUT TOOLS

The Dividers. *Dividers* consist of two slender, tapered legs pivoted together at one end in such a way that with the aid of adjusting screws the distance between their pointed ends may be set to different dimensions. They are used for scribing circles and arcs, for dividing circles and lines into a series of duplicate dimensions, and for transferring measurements from the rule to the work and from the work to the rule. Dividers may be had in two different styles, *wing dividers* and *spring dividers.* The wing dividers are the heavier of the two styles, and are in great favor for general use; but where greater accuracy is required, the more lightly-constructed spring dividers are to be preferred.



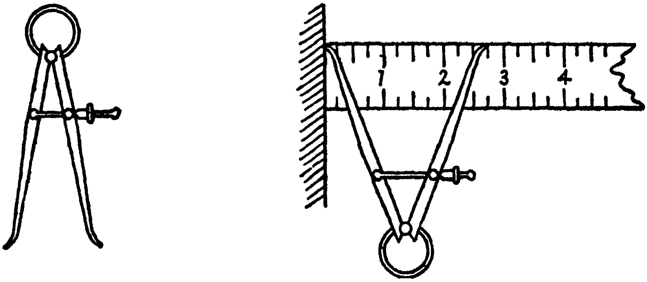
Dividers

The distance between the points of the wing dividers may be regulated by means of the clamp screw which is in contact with the quadrant. The distance may be further regulated by means of the fine adjusting screw.

One of the points is removable, and is of such size that a lead pencil may be inserted in its place. The distance between the points of the spring dividers may be regulated simply by turning the adjusting nut on the adjusting screw. The legs are constantly under pressure to hold them apart, and against the adjusting nut, by the action of the spring. Some tool manufacturers incorporate the quick adjustment features of the wing dividers into the spring dividers by providing a patented split nut, which, when relieved of the pressure of the legs by pressing the legs together, may be slid quickly back and forth over the adjusting screw to any desired position. As soon as the pressure of the leg is placed against it, it loses its quick-action features and may again be moved back and forth along the adjusting screw by simple turning, just as with a solid nut.

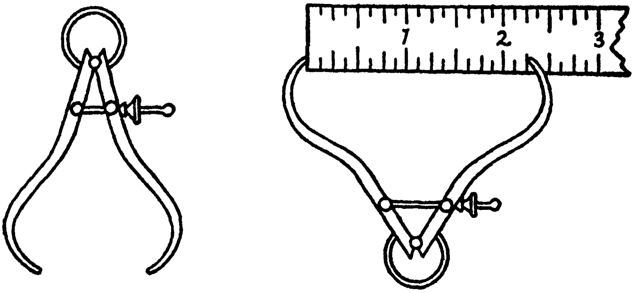
The distance between the points is often set to some definite measurement with the aid of the rule. In making the adjustment between the points, the one-inch line is preferred to the end of the rule as a starting point for the measurements, because the rule cannot support a divider point on its end very accurately. The one divider point is placed on the one-inch line and the measurement computed from there, and the workman mentally adds one inch to the dimension desired. If the dimension is to be four inches, the one point is placed on the one-inch line, the other moved to the five-inch line; if the dimension is to be two and one-half inches, one point is placed on the one-inch line and the other on the three and one-half inch line. Any inch-line on the rule may be used as a starting point provided the necessary computations are made to permit the points to be adjusted to the desired dimension. If a two-foot rule is available, some workmen prefer to use the ten-inch line as a starting point because it is less likely to lead to error due to the necessity of adding one inch to the desired dimension. By dropping the first digit from all subsequent measurements on the rule, the ten-inch line may become the zero line, the eleven-inch line the one-inch line, the twelve-inch line the two-inch line, etc. A dimension of four inches would be transferred to the dividers by placing one point on the ten-inch line (the zero line) and the other on the fourteen-inch line (the four-inch line). A dimension of two and one-half inches would be made by placing one point on the ten-inch line and the other on the twelve and one-half inch line, the ten-inch line becoming the zero line and the twelve and one-half inch line becoming the two and one-half inch line.

The Calipers. Calipers are similar to dividers in construction, except that the legs are not straight, and they do not come to a point. They are curved either in or out. Calipers are used for measuring distances, either between or over surfaces, or for comparing distances and sizes with the dimensions on a rule. The distance between the end of the legs may be adjusted in much the same way as with dividers. When adjusting a calipers to a definite dimension, the ends of the legs are placed on the graduations of a rule and are then either adjusted in or out until the desired dimension between them is obtained. In adjusting an outside calipers, the end of one leg may rest against the end of the rule, while the other leg is adjusted to the desired dimension on the rule. In adjusting an inside calipers, the end of the rule is placed against a flat surface in such a way that the end of one of the caliper legs may be placed against the same surface as a starting point for regulating the distance between the legs. The accompanying figures show inside and outside calipers, and the methods for setting the legs to definite dimensions.



Inside Calipers

Setting an Inside Calipers to 2¾″ on the Rule



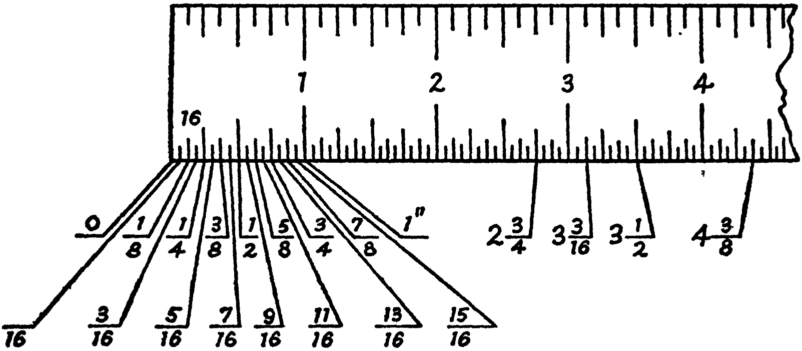
Outside Calipers

Setting an Outside Calipers to 2¼″ on the Rule

Calipers, Inside and Outside, and Methods for Setting Legs to Definite Dimensions

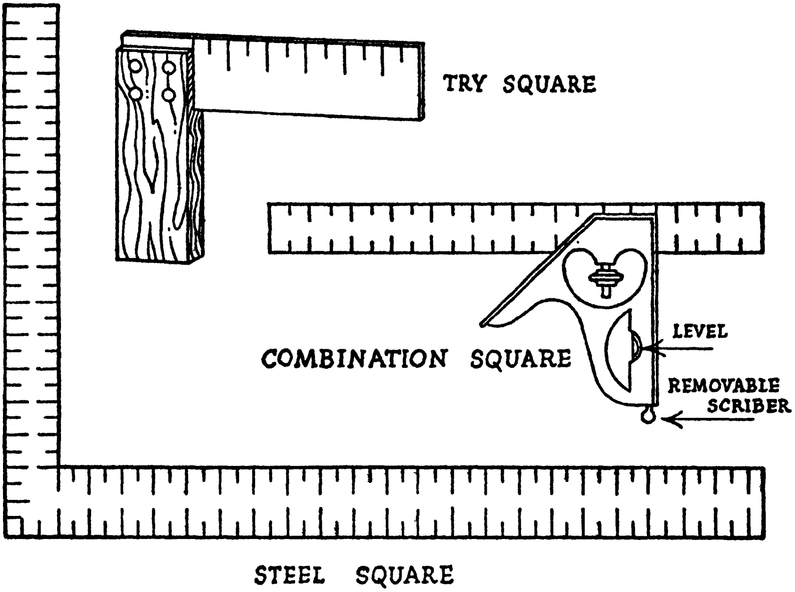
The Scale. A straight, thin, flat piece of metal graduated in inches and fractional parts of an inch, used to make linear measurements, is frequently called a *scale* to distinguish it from a straight, thin, flat piece of wood graduated in inches and fractional parts of an inch, which is called a *rule*. In reality, both are rules, one being a rule made of metal, the other a rule made of wood. Draftsmen and designers frequently use a scale when called upon to draw an object at a reduced size. When the plans of a building or a machine are drawn on a sheet of paper much smaller than the actual size of the object, the drawing is said to be drawn to scale. Representative measurements are used instead of actual full-size measurements. A measuring instrument for determining representative measurements is called a *scale.* A measuring instrument for determining actual measurements is called a *rule.*

Rules may be had in many different lengths, widths, thicknesses, styles, and types of graduations. The size of a rule is expressed by its length in inches or feet, as, a six-inch rule or a two-foot rule. In steel rules, the six-inch, the twelve-inch, and the twenty-four inch rules are the more popular sizes. The type of graduations which divides the inch into eighths and sixteenths on one side of the rule and thirty-seconds and sixty-fourths on the other side is the most common. Rules made of wood are usually two feet or more in length. For convenience in carrying, they are made of a number of pieces hinged together so that they may be folded into shorter lengths. A two-foot rule may be folded so that it does not occupy a space over six inches long. In learning to read a rule, it is necessary to become familiar with the number of divisions into which each inch is divided, then to count these divisions from left to right.



Showing the Fractional Value of the Division Points on a Rule Graduated in Sixteenths of an Inch

The Square. The woodworker usually has three different styles of squares in his tool kit: the steel square, the try square, and the combination square. The *steel square* consists of two thin blades of steel, one twenty-four inches, the other approximately sixteen inches in length, joined together permanently at right angles to each other. The *try square* consists of a shorter blade of steel set squarely into a head which is thicker than the blade. The head may be made either of wood or of metal. The *combination square* consists of a blade of steel, along which the head may be slid and fastened at any desired position. One face of the head makes a right angle with the blade, while the back side of the head makes an angle of forty-five degrees (called a miter) with the blade.

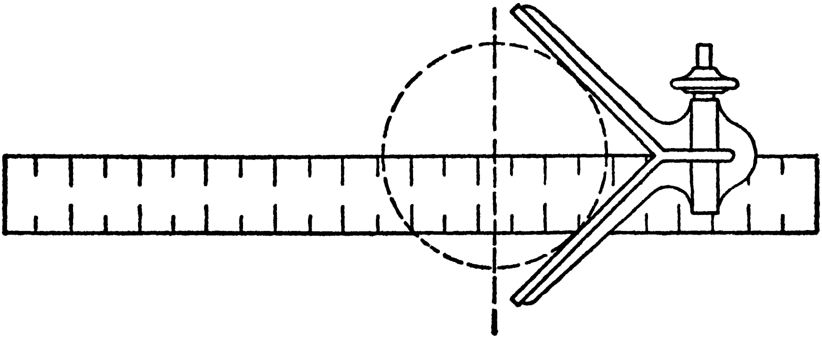


Showing Steel Square, Try Square, and Combination Square

Squares are used for marking lines at right angles to other lines, or for marking lines at right angles to the edge of a board, and also for testing angles that are intended to be right angles. But the steel square has many other uses. It is used by the carpenter to solve house framing problems. With it he can figure the length and angles of roof rafters, the length and angles of braces, the laying out of stairways, and many other construction details. The long arm of the square is called the *body,* the short arm the *tongue,* and the outer corner where the body and tongue meet is called the *heel.*

The try square is much smaller than the steel square. A common size has a blade six inches in length. The head is thicker than the blade so that it may be used against the edge of a board while the blade lies flat on the top surface of the board when used to scribe a line at right angles to the edge. Besides being used to scribe lines at right angles to the edges of boards, the try square, as its name indicates, is used to test angles by trying the square against the work. When used for this purpose the head is always held rigidly against a true surface.

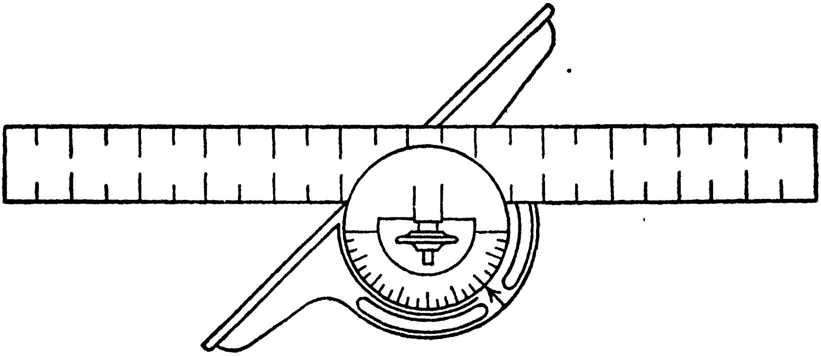
The combination square is intended for use where greater accuracy is required. The fact that the head is adjustable on the blade gives the square a wide range of uses. The end of the blade may be used inside of hollow areas to test their depth or to test the squareness of their sides with the surface of the work. The head may be removed entirely from the blade and the blade used as a rule. It is called a combination square because it may be used to mark and test an angle of forty-five degrees as well as an angle of ninety degrees. It may be used as a depth gage, a height gage, a level, a scriber, and as a rule. A spirit level and a handy scriber-point are contained in the head. The level is a permanent part of the head. While the square is being used to test squareness, it may at the same time be used to tell whether a surface is level. The scriber is held frictionally in the head by a small brass bushing. The handle of the scriber projects from the bottom of the head of the square in a convenient place to enable it to be taken out quickly.



Showing the use of a center head in scribing center lines on the end of cylindrically-shaped material

The Center Head. The *center head* is an attachment for the combination square. The head of the square may be unclamped and entirely removed from the blade and the center head substituted in its place. The center head is used to locate the center on the end of shafts, bar stock, and other cylindrical material. The cylindrical bar on which the center is to be located is fitted against the two faces of the center head simultaneously so that the blade will lie flat on the end of the bar, as shown above. With a scriber-point, a line is drawn on the end of the bar, using the edge of the blade as a guide. The bar is now rotated approximately one-fourth of a revolution under the tool. Another line is scribed on the end of the bar. The point of intersection of the two lines indicates the center of the bar.

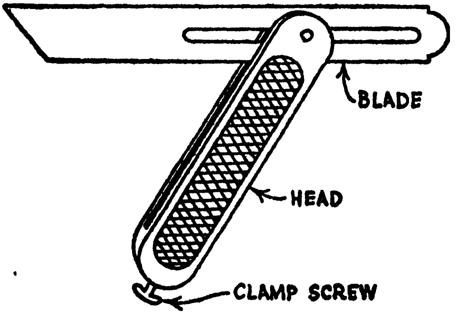
The Bevel Protractor. The *bevel protractor* is another attachment for the combination square. When the protractor head is substituted on the blade of the combination square in place of the square head, the tool may be used to mark and test angles of any degree from 0° to 180°.



Bevel Protractor

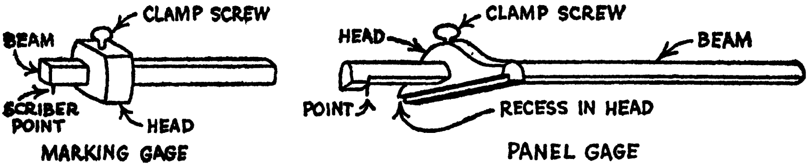
The blade is fitted into a slot in a graduated disc which rotates in a complete circle and may be secured at any angle in the head. The blade is held in place by means of an adjusting nut in much the same manner in which it is held in the square head and center head. The working face of the head extends on both sides of the blade; this permits an angle or its supplement to be checked with the same setting. When the blade is set at an angle of ninety degrees with the head, the protractor makes a small convenient T-square.

The Bevel Square. The *bevel square* consists of a head and a blade similar to a try square. The difference between a bevel square and a try square is that the blade of the try square is fastened permanently to the head at an angle of ninety degrees, while the blade of the bevel square may be adjusted and secured at any angle with the head. To adjust the blade of the bevel square, the clamp screw is loosened and the blade moved in the head until the desired angle is reached, when the clamp screw is tightened. It is used for marking and testing angles other than ninety degrees.



Bevel Square

The Marking and Panel Gage. A *marking gage* is used to scribe lines on the surface of a board parallel with an edge. It consists of a head sliding on a beam to which it may be fixed at any desired point by means of a clamp screw. At one end of the beam is a scriber-point which is usually filed to a flat chisel edge, parallel to the head, so that the scribing action will be similar to cutting rather than scratching. In setting the gage, the clamp screw is loosened, the head slid along the beam until the desired distance between the point and the head is reached (being measured with a rule) and the clamp screw is then tightened. In use, the head is pressed firmly against the edge of the board, the scriber-point is brought in contact with the surface of the board, and the gage slid along, using the edge of the board as a guide. The scriber-point will cut a thin, fine line in the surface of the board, parallel to the edge.

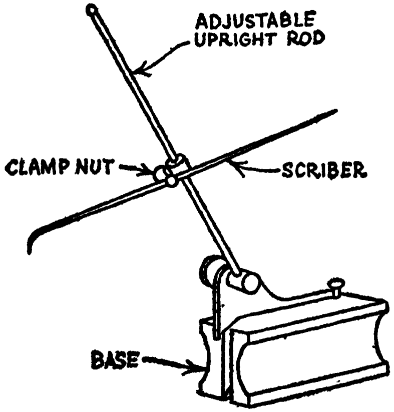


MARKING GAGE

PANEL GAGE

Showing Marking Gage and Panel Gage

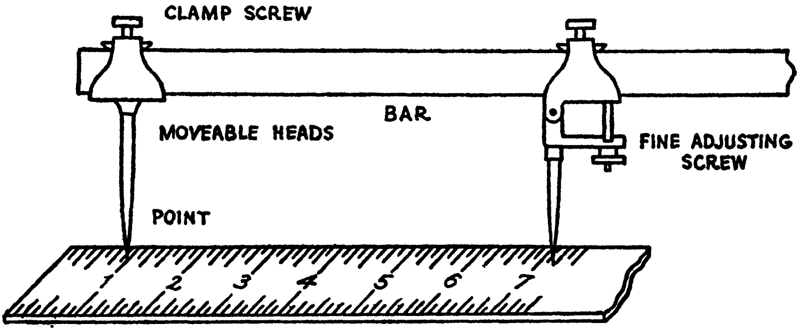
A *panel gage* is used for scribing lines at a considerable distance from the edge of the board. Its action and construction are somewhat similar to that of the marking gage. The beam and head are longer and larger. The length of the head is increased sufficiently to warrant ample support along the edge of the board while scribing a line. In addition, a recess is cut in the face of the head along its full length to enable the head to rest on top as well as against the edge of the board. This insures greater rigidity at the scriber-end of the beam when the scriber is brought in contact with the board.



Surface Gage

The Surface Gage. The *surface gage* consists of a heavy base, an upright rod to which is fastened a scriber, held by a clamp nut. The scriber is used for scribing lines at definite heights from a base surface and for transferring measurements from one piece of work to another. By resting the base of the surface gage and the work to be scribed on a true surface, such as a cast iron surface plate, it is possible to adjust the scriber to any desired height with the aid of a rule, and then to scribe horizontal lines on the vertical surfaces of the work, parallel with the base or surface plate. The base has a V-shaped groove cut through the bottom so that it may be rested on a cylindrical surface as well as a flat surface. The use of the surface gage is not limited to scribing lines on vertical surfaces; it is also frequently used as a height gage.

Trammel Points. *Trammel points* are used on work which is too large to handle with ordinary dividers. This tool consists of a long bar and two sliding heads designed to slide along the bar and to clamp at any desired distance. Each head supports a medium-length steel point similar to the pointed leg of a divider. One head is usually equipped with a fine adjusting screw which makes it possible to refine the measurement after the heads have been clamped to the bar. When setting the trammel points, the one head is securely clamped at one end of the bar, the clamp screw of the other head is loosened, the head moved along the bar until the point reaches the desired distance from the first point, and the clamp screw is then tightened. If the measurement is not exactly as desired, the point may be moved back and forth slightly by means of the fine adjustment screw, as shown above. This setting is done with the aid of a rule in much the same manner as the distance between divider points is adjusted.



Showing How Trammel Points Are Set to a Definite Dimension with the Aid of a Rule

60. PRINCIPLES OF PATTERN CONSTRUCTION

SOLID PATTERNS

Provision for Shrinkage of Metal. When metal is heated from the solid state to the liquid state, the absorption of heat by the metal causes it to increase in volume, or expand. Likewise, when metal is cooled from a liquid state to a solid state, the loss of heat by the metal causes it to decrease in volume, or contract. Since a casting is made by cooling a mass of molten metal in a mold of the desired shape, some provision for the decrease in volume of the metal must be made on the pattern that is to be used in shaping the mold. This provision is called an *allowance for shrinkage.*

The amount that is to be allowed for the shrinkage of the metal which takes place in molding the casting depends upon four conditions:

1. The kind of metal used in the casting; some metals shrink more than others.
2. The size of the casting; thick castings shrink less than thin castings because they solidify more slowly, and thus permit the adding of more metal to make up for that which is lost in contraction.
3. The shape of the casting; a casting that is long in proportion to its width will shrink differently than one which is more compact, even though both castings are of the same weight.
4. The temperature of the metal when poured into the mold; the hotter the metal, the greater the shrinkage in cooling.

The shrinkage allowances commonly made by pattern makers for different kinds of metals and different sizes of castings are as follows:

METAL AND SIZE OF CASTINGS	SHRINKAGE ALLOWANCE IN INCHES PER FOOT
CAST IRON:	
small or thin castings	1/8
medium size	1/10
large, thick	1/16
MALLEABLE IRON:	
small or thin	1/8
medium	1/10
CAST STEEL:	
small or thin	1/4
medium	3/16
large	5/32
BRASS:	
small or thin	3/16
medium	1/8
BRONZE:	
small or thin	3/16
medium	1/8
ALUMINUM:	
small or thin	3/16
medium	5/32
large	1/8
MAGNESIUM:	
small	5/32

A casting is considered small if its greatest length is under twenty-four inches. It is considered thin if its thickness is under three-eighths of an inch. It is considered as being of medium size if its greatest dimension is under forty-eight inches. It is large if it measures over forty-eight inches.

To provide for the shrinkage of the metal in the mold while the casting is being made, it is necessary to make the pattern oversize by the amount that the metal will shrink so that the hardened casting will be of the desired size. If a standard rule were used in making a pattern, it would be necessary when making a measurement to calculate and allow for the shrinkage. This is eliminated by the use of a special rule, called a *shrinkage rule*. A shrinkage rule is an oversized rule graduated in such a way that it maintains an exact shrinkage relation between all graduations. In appearance it looks like a standard rule, but if the two were compared, the shrinkage rule would be found to be longer. To avoid mistakes, shrinkage rules are plainly marked for the shrinkage allowance they are intended to make. A rule marked "SHRINKAGE ⅛ INCH PER FOOT" will be ⅛ inch longer per foot than a standard rule. The 12⅛ inches are taken as one foot. This lengthened foot is subdivided into twelve equal parts representing inches, and each of these parts is subdivided into eighths or sixteenths. By the use of this rule the proper shrinkage allowances are made on a pattern for a casting whose shrinkage is on the basis of ⅛ of an inch per foot. Shrinkage rules are made either of wood or steel in twelve or twenty-four inch lengths. Care should be taken to use the rule whose shrinkage allowance corresponds to the shrinkage of the metal from which the casting is to be made. Shrinkage rules may be procured for all standard metal shrinkages.

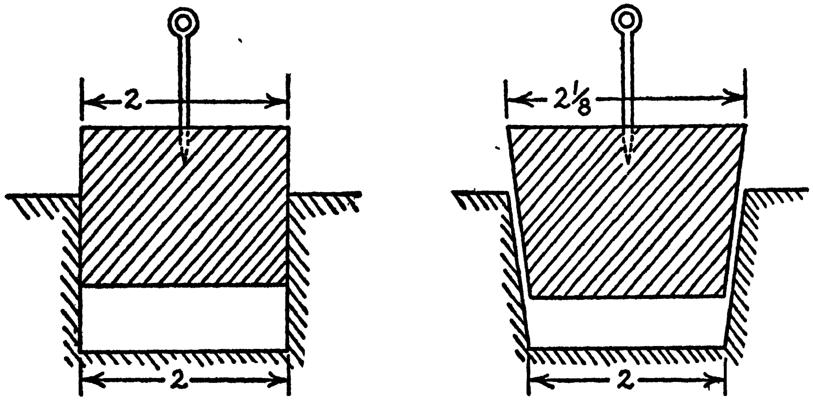
Provision for Draft. The allowance known as *draft* may be defined as the taper or slant put on the vertical surfaces of a pattern to enable the molder to withdraw the pattern from the sand mold. A pattern must be made so that the large end comes out of the sand first. If the sand is disturbed when the pattern is removed, it makes patching of the mold necessary and sometimes leaves the sand so insecure that it washes into the mold when the metal is poured. The amount of draft to be allowed varies greatly and depends upon the following conditions:

1. The use to which the pattern is to be put; patterns for parts which are to fit together without machining, such as cast gears, should have very little draft.
2. The height of the drafted surface. A vertical surface twenty-four inches high requires less draft per inch than one twelve inches in height.
3. The location of the surface, whether it is an external surface or an internal surface. On the inside surface of a hollow pattern such as a ring, there must be extra draft so that the sand forming the inside will not break away from the mold when the pattern is withdrawn.

4. The position of the surface in the mold, whether it is in the cope or in the drag. A surface from which the cope half of the mold is lifted must have additional draft.

In general, "give the molder all the draft you can," keeping the amount consistent with the use of the casting. Nothing increases the difficulties of the molder more than too little draft on the pattern.

The allowance for draft is always added to the dimensions given on the drawing, never taken from them. The pattern is the largest on the surface that is uppermost when the pattern is lifted from the mold, as shown below. No hard and fast rule as to the amount of draft to allow can be given. The minimum amount that might be allowed for draft is one-eighth of an inch per foot of surface. However, for smaller surfaces, one-sixty-fourth of an inch per inch of surface is a common allowance for outside surfaces, and one-thirty-second of an inch per inch of surface on inside surfaces.



This Pattern Is Difficult to Lift from the Mold because of the Parallel Vertical Sides.

This Pattern Is Easy to Lift from the Mold because Draft Has Been Added to the Vertical Sides.

Showing Pattern Without Draft and the Same Pattern After Draft Has Been Added.

Allowance for Finished Surface. In addition to the allowances for shrinkage and draft, there is still another allowance to be made. This is an allowance for machining the surfaces of the casting. In building machinery, it is often necessary to fit two or more castings together to make a complete machine. Whenever two castings are intended to fit together, true surfaces must be developed on each casting. This is done in the machine shop by the machinist, and is a process which removes the rough metal from the surface until a smooth surface is produced. When the words "machining" or "finish" are used in connection with pattern making, they always refer to work to be done on a casting in the machine shop by the machinist. Since patterns are generally made from machine shop drawings, which give only the finished sizes of the casting and not the pattern sizes, the pattern maker must watch for finished surfaces so that he can add additional stock on those surfaces of the pattern.

The following symbols are used on drawings to indicate that the machinist is to do some work on the casting and that the pattern maker must make some provision on the pattern so that this work can be done.

SYMBOLS USED TO DESIGNATE FINISH

f=FINISH
F.A.O.=FINISH ALL OVER
FINISH
TURN
BORE
TAP
ff=FILE FINISH

COUNTERBORE
C'BORE
DRILL
COUNTERSINK
REAM
BROACH
SPOT FACE

When a surface is to be threaded, the style of the thread and the number of threads to the inch are indicated by a note on the drawing.

The amount of finish to be added on the surfaces of the pattern depends on four factors:

1. The *kind of metal* of which the casting is to be made.
2. The *size and shape* of the casting.
3. The *method used for machining* of the casting.
4. The *grade of finish* required on the casting.

Kinds of Metal Used for Castings.

A. *Cast Iron.* Cast iron usually has a hard scale on the surface. Sufficient stock must be added to permit the machinist to put the cutting tool under the scale for first roughing cut. A light cut over the scale would in many cases ruin the cutting edge of the tool.

B. *Malleable Iron.* Malleable iron has a tendency to become somewhat twisted and distorted in the process of annealing. Because of this, the same full allowance is made as in cast iron, even though it does not have as hard a scale.

C. *Brass.* Brass is usually cast with greater care than most metals and therefore requires less stock allowed for finish.

D. *Aluminum.* Aluminum does have a hard surface. It machines easily and therefore requires less finish allowance than most metals.

E. *Steel.* Steel requires a greater allowance for finish than any other metal. It casts rather roughly, shrinks excessively, and has a tendency to become twisted and distorted during the annealing process.

Size and Shape of Casting. Since the amount of shrinkage, warping and distortion which will take place in a large casting is rather uncertain, ample allowance for finish must be made "to be on the safe side." The pattern maker reasons that it is better on a large casting to allow a little more finish than is actually required by the machinist, even if it takes more work in the machine shop, than to allow an insufficient amount and run the risk of having the casting rejected on account of being undersize. The larger and thicker the casting, the greater is the amount to be allowed for finish.

Method Used for Machining of Casting. Methods of finishing or machining the casting also affect the amount of stock to be allowed. A file finish, or a disc-grind finish, for instance, requires a smaller amount of stock to be removed than a finish produced by a cutting tool. Castings to be finished from the rough on disc grinders require only sufficient finish to take care of draft and possible warpage. On small and medium-sized work that is to be finished by means of cutting tools in the lathe, shaper, planer, or milling machine, usually one-eighth of an inch added stock will be sufficient if the casting is of iron. Usually not more than one-sixty-fourth of an inch is allowed for a file finish.

Grade of Finish on the Casting. When a particularly smooth, clean surface is required, an added allowance is made to insure the removal of any imperfections which are liable to exist near the surface of the rough casting. All four of these factors must be considered in deciding on the amount of stock to be allowed on a surface for finish. The drawing that is furnished to the pattern shop gives the finished dimensions of the casting and indicates the kind of finish desired. The pattern maker must add whatever finish is called for to those surfaces indicated by the finish symbols. It is very necessary that he know the method the machinist will use to remove the metal in order that he may add the required amount of finish when it is called for by such signs as f, ff, bore, disc grind, etc. In general, it may be said that the amount should be increased in proportion to the size of the casting, and should be sufficient so that the machinist, when taking the first cut on the surface, may get the point of the cutting tool beneath the sandy scale that is present on the surface of a casting and still have enough stock for a second cut followed by a finishing cut. An extra allowance is usually made on the cope side of large castings to permit machining to the sound metal that lies beneath the impurities that always float to the top of a mold while the metal is in a liquid state.

The following allowances for finish are commonly made by pattern makers:

METAL AND SIZE OF CASTING	FINISH ALLOWANCE IN FRACTIONS OF AN INCH
CAST IRON:	
Small	1/8 to 3/16
Medium	3/16 to 1/4
Large	1/4 to 3/4
MALLEABLE IRON:	
Small	1/16 to 1/8
Medium	1/8 to 3/16
CAST STEEL:	
Small	3/16 to 1/4
Medium	1/4 to 3/8
Large	3/8 to 3/4
BRASS, BRONZE, ALUMINUM, MAGNESIUM:	
Small	1/16 to 3/32
Medium	3/32 to 5/32
Large	5/32 to 3/16

Finishing Pattern for Preservation. To prevent wood patterns from absorbing moisture from the damp molding sand in which they are used, it is necessary to coat them with a wood preservative. If wood patterns are permitted to absorb moisture, they may change their shape, due to swelling or warping, or the glued joints may come apart. The wood preservative most commonly used is called *shellac.* It may be likened to an easily applied, quick-drying varnish. Shellac fills the pores of the wood and provides a hard, smooth surface which enables the pattern to be withdrawn easily from the molding sand.

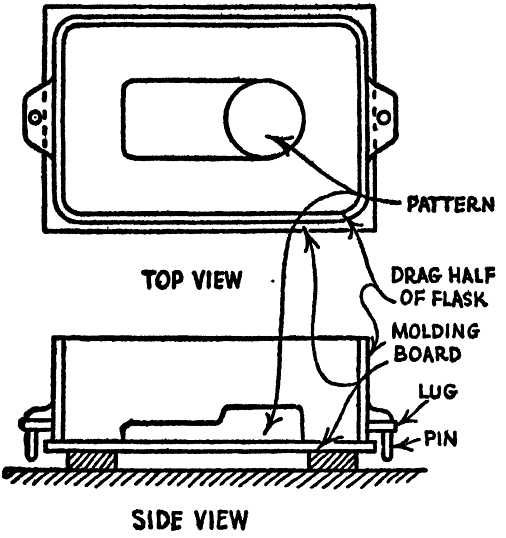
Shellac comes into the shop in the form of orange colored flakes known as *gum shellac.* It is prepared for use by dissolving the flakes either in wood alcohol or denatured alcohol, both of which are poisonous if taken internally. The flakes are put in a wide-mouthed bottle or earthen jar which should be closed tightly. Alcohol is poured over the dry flakes in the proportion of one and one-half pounds of flakes to one-half a gallon of alcohol. This mixture is then allowed to stand for several hours, being stirred occasionally. It will dissolve into a thin, orange-colored, varnish-like liquid. Shellac is best kept in a glass or copper container, as it will readily discolor through contact with metal other than copper or tin. The muddy color it acquires when exposed to the atmosphere in an open container can be cleared up by stirring in a little powdered oxalic acid. If shellac becomes too thick for use at any time, it may be thinned by adding alcohol. Shellac seems to work best when slightly thinner than machine lubricating oil.

SPLIT PATTERNS

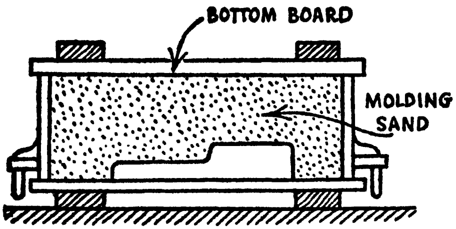
Reason for Using Two Pieces. Castings are made by pouring hot, fluid metal into molds made in damp sand. Molds are made by pressing sand around a wood form of the same size and shape of the desired metal casting. The wood form is called a *pattern.* The sand is held together while it is being pressed around the pattern by a box-like frame, called a *flask.* A flask is composed of two sections, an upper and a lower half. The top half is called the *cope,* the bottom, the *drag.* Pins and lugs are attached to opposite sides of the flask to permit the two halves to be taken apart and placed together again in the same alignment. When the sand is firmly pressed around the pattern, the two halves of the flask are taken apart and the pattern lifted from the sand, leaving a cavity or mold. The sides of the pattern are made slightly tapered, with the biggest side at the top so that the pattern can be lifted out of the sand easily without disturbing or breaking the sides or the edges of the mold.

It is well to know the operations involved in the making of a mold so that the making of the pattern can be properly planned. The making of a mold requires the following materials and equipment: properly prepared molding sand; a two-part flask; two flat molding boards of the same length and width as the flask; a screen for sifting sand; a shovel for shovelling sand; a rammer for ramming sand into the flask around the pattern; a strike-off bar for scraping the excess molding sand from the top of the flask; a draw spike for driving into the pattern so that it may be lifted from the mold; a sprue pin for making a round vertical hole through the cope into which the melted metal is poured; a vent wire for punching ventilation holes in the sand mold; a gate cutter for forming a channel in the sand to permit the melted metal to flow from the bottom of the sprue hole to the mold cavity; and a molder's trowel for patching the mold and smoothing the surface of the sand.

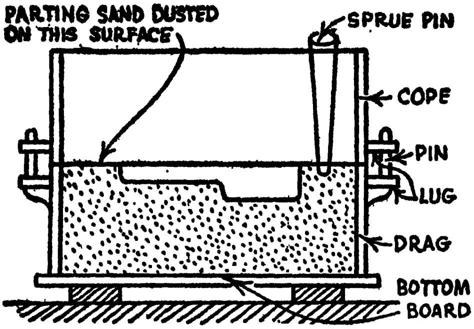
The use of this material and equipment will be discussed further during the following description of the making of a mold. One of the flat molding boards is placed on the work bench. The pattern and the lower section of the flask are placed on the molding board, as shown in (a) below. The pattern is covered with sifted molding sand to a depth of approximately one to two inches. The drag flask is then heaped full of unsifted molding sand. The sand is rammed well around the inside of the flask and over the pattern. The surplus sand is scraped off even with the top of the flask with the strike-off bar. Numerous holes are punched in the drag to within one-half inch of the pattern with the vent wire; this is to make the sand porous so that the steam will penetrate readily through the sand. The bottom board is now inverted and placed on top of the drag, as shown in (b). The bottom board is clamped to the molding board, and the drag is then rolled over on the bench. The molding board is now on top and the bottom board is on the bottom. The molding board is removed and laid aside. The surface of the sand is smoothed with the molder's trowel. The surface is dusted with a very fine powdered sand, called *parting sand*. The parting sand forms a coating over the surface of the drag which prevents the sand that is going to be put in the cope from sticking to the sand already in the drag. It is necessary to separate the cope from the drag in order to lift out the pattern; the use of parting sand makes this possible. The cope half of the flask is placed on top of the drag, as shown in (c). A sprue pin is



(a) Showing Pattern and Drag Half of Flask Placed on Molding Board Ready to Receive Molding Sand.



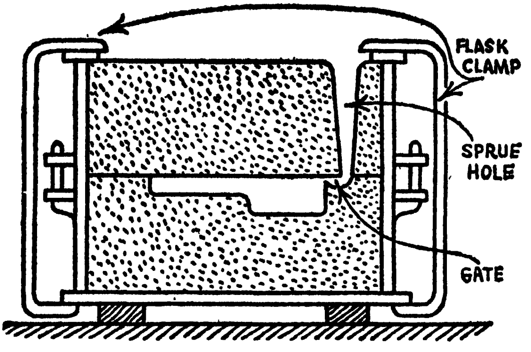
(b) Showing Bottom Board on Top of Drag Filled with Molding Sand Ready to Roll Over.



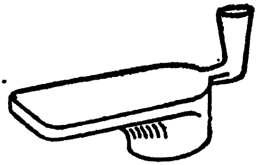
(c) Showing Drag Half of Flask Rolled Over, Resting on Bottom Board; Molding Board Removed from Top of Drag; Cope Half of Flask Positioned on Top of Drag by Means of Lugs and Pins; Sprue Pin in Place; Cope Ready to Receive Molding Sand.

placed to form a hole through the cope for the purpose of allowing the metal to enter the mold. Molding sand is sifted over the pattern to the extent of from one to two inches. Unsifted molding sand is added until the cope is heaping full. The molding sand is rammed in the cope in much the same manner that it was rammed in the drag. The excess sand is struck off the top of the cope. The cope is vented with the vent wire. The sprue pin is removed. The pouring hole is made funnel-shaped at the top by pressing the sand back with the fingers. The cope is lifted off and stood on edge at one side. The draw spike is driven into the pattern. The pattern is rapped, and then lifted from the sand. A channel, called a *gate,* is cut in the top of the drag with the gate cutter, extending from the bottom of the sprue hole to the mold cavity. The mold is inspected for imperfections. The cope is picked up and replaced on top of the drag. Before the metal is poured, the cope and drag are either clamped together or else weighted by placing a heavy weight on top of the cope to keep the cope from shifting by the pressure of the hot metal rushing into the mold. After the metal is poured and has been allowed to cool, the clamps or weights are removed, the flask is taken apart, and the casting is examined. The sprue and gate are broken from the casting. The excess sand is removed from the casting by brushing with a wire brush. Fig. (d) shows the mold completed, ready to receive the molten metal. Fig. (e) shows the rough casting before the sprue and gate have been removed. Fig. (f) shows the finished casting, ready for delivery to the machine shop.

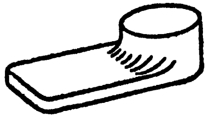
Patterns are designed and made for but one purpose, and that is to form a mold in sand of a desired shape. A pattern must be made so that it may be easily lifted from the sand. It is by removing a pattern from the sand that the mold is formed. The shape that a pattern is to take does not always permit it to be made of but one solid piece of wood. Frequently a pattern is made of two pieces, split through the middle for convenience in lifting it from the mold. A cylinder represents the simplest



(d) Showing Pattern Removed, Gate Cut, Mold Clamped, Ready to Receive Metal.

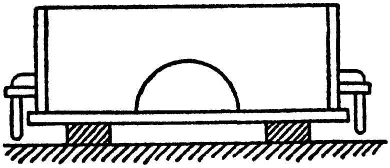


(e) Rough Casting

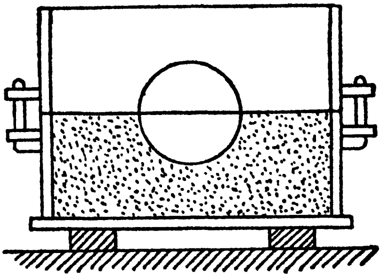


(f) Finished Casting

example of a two-piece pattern. If a cylindrical pattern were made of one piece, difficulty would be experienced when removing it from the sand mold. By splitting the pattern through the middle, a flat surface is produced along the joint which makes it possible to place half the pattern flat on the molding board when ramming the sand in the drag, as shown in (g). After the drag is rammed, it is rolled over in the usual manner. The other half of the pattern is placed on top of the half that is bedded in the drag. The cope half of the flask is placed on top of the drag half. Molding sand is rammed in the cope; Figs. (g) and (h) show these operations. When the cope is separated from the drag, the top half of the pattern is lifted off with the cope. Half of the pattern is bedded in each half of the flask. Each half of the pattern can be easily lifted from its



(g) Showing One Half of Two-piece Cylindrical-shaped Pattern Placed on Molding Board Ready for Receiving the Molding Sand in the Drag.



(h) Showing the Drag Half of Flask Rammed and Rolled Over, with Second Half of Two-piece Pattern Placed on Top of First Half; Cope Half of Flask Is Ready to Receive Molding Sand.

respective half of the mold. The mold may then be closed for pouring. The molding advantages presented by splitting the pattern make this practice desirable wherever possible. The two pieces of a two-piece pattern are held in proper alignment with each other by placing two wood pins in one half, which fit into two corresponding holes in the other half.

Use of Color to Distinguish Patterns from Core Prints. The United States Bureau of Standards, Department of Commerce, working with a joint committee of the American Foundrymen's Association, in 1930 adopted a standard color-marking for producers and users of wood foundry patterns and core boxes. The purpose of the color scheme is to indicate to the molder what parts of the casting are to be cored, what parts are to be machined, and what parts are to be solid. The adopted standard color markings for patterns and core boxes are as follows:

1. Surfaces to be left unmachined are to be colored *black.*
2. Surfaces to be machined are to be colored *red.*
3. Seats of and for loose pieces are to be marked by *red stripes on an orange background.*
4. Core prints and seats for loose core prints are to be colored *orange.*
5. Stop-offs are to be indicated by *diagonal black lines on an orange base.*

Some variations of the colors are permissible within reasonable limitations. The colors may be obtained by mixing suitable inexpensive pigments with orange shellac to produce the color desired. Lamp black added to orange shellac will produce black shellac. Pattern red added to orange shellac will produce red shellac. Orange shellac is the natural color of shellac. White shellac is orange shellac that has been bleached by treatment with caustic potash and chlorine. White shellac is not generally used for pattern work, but it is sometimes used in the process of finishing furniture.

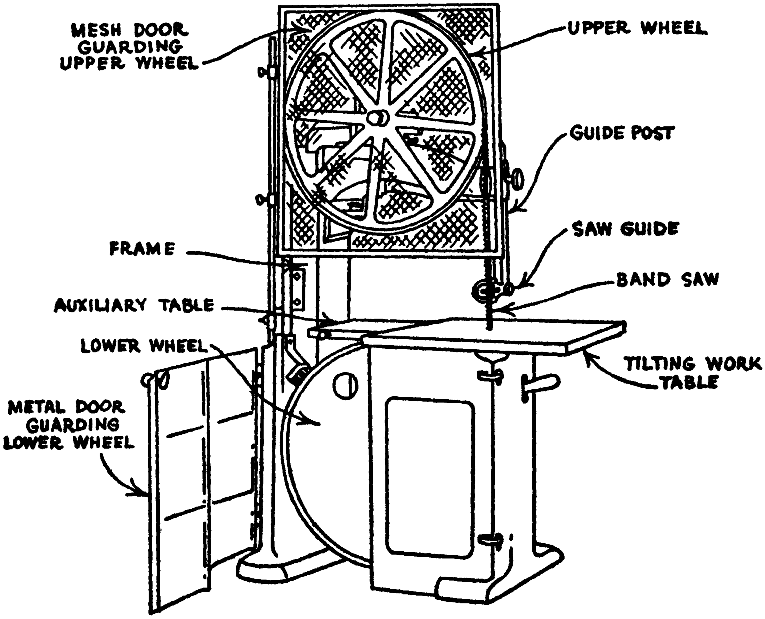
Methods of Marking Patterns. When a pattern is completed, and before it goes into service in the foundry, it is good practice to give it an identification number. This number usually appears on the surface of a pattern as raised lettering. Sometimes it is preferred that the lettering be cut into the surface. The main object of giving patterns a symbol number is to furnish a means of identification of the particular part when it becomes a casting in an assembled machine. Machine parts become worn or broken and need to be replaced. The symbol number on the casting identifies the part when ordering replacements.

The letters most commonly used are made of soft white metal so that they can be easily bent to irregular contours when placing them on the surface of a pattern. The back side of the letters is coated with shellac before they are placed in the desired position on the surface of the pattern. As far as possible the letters are placed so as not to interfere with the lifting of the pattern from the mold. By placing the letters on the drag side of the pattern a clearer outline will be obtained on the casting than when they appear on the cope side. The sizes of letters obtainable vary from ⅛ of an inch to several inches; usually lètters from ⅜ to ½ inch in height are used. Frequently the identification number to be placed on a pattern is the same number that is found on the working drawing of the part. This system of numbering has a decided advantage when broken castings are to be replaced. The number on the casting will tell what working drawing to look for, and when the drawing has been located, the exact original size of the part can be determined.

61. MACHINE TOOLS

The Band Saw. The band saw is a power-driven, fast-cutting machine tool used for sawing wood to desired shapes. It consists of a continuous toòthed band, or belt, of flat steel; a heavy cast iron frame; two wheels, one directly above the other, with wheel guards; saw guides; a work table; and a driving mechanism.

The saw blade is a long, narrow, flexible, endless ribbon of steel with saw teeth filed on one edge. It runs over the face of two narrow rimmed



Band Saw

wheels, much as a belt runs over the face of two pulleys. The two wheels rotate on separate shafts, which are mounted in the large, heavy cast iron frame. The frame also supports the work table through which the saw passes in a downward direction. The wood that is to be sawed is placed on top of the table and pushed into the saw by hand.

The two wheels may be made either of cast iron with spokes, or sheet metal with a disc web instead of spokes. The rims of the wheels are about one and one-half inches in width, and are covered with a band of thick rubber which serves as the tread of the wheels on which the saw tracks. The lower wheel, under the work table, is the power wheel. It is mounted on one end of a shaft which is held in positive, non-changing alignment with the frame of the machine. It can be rotated, but its shaft is rigidly supported in one horizontal position at all times in the base of the machine. The opposite end of the shaft either contains the driving pulleys, or is coupled directly with an electric motor for transmitting power to the machine. If the machine is belt-driven, the shaft contains two pulleys, one rotating loosely on the shaft, the other keyed permanently to the shaft. When power is turned into the machine the belt is shifted from the loose pulley to the tight pulley by means of a belt-shifting mechanism. If the machine is directly connected to an electric motor, the power is turned into the machine by throwing a switch. In either case, when the power is turned into the machine, the lower wheel starts to rotate. The lower wheel drives the saw over the upper wheel. The upper wheel acts as an idler, and permits the saw to pass over it in the same way that a belt passes over a pulley.

Operation. The saw blade is under tension while it is running. Ordinarily the tension is tested by noting the stiffness of the blade near the table. The upper wheel can be adjusted vertically to provide the tension and to accommodate saws of different lengths. Also, the upper wheel can be tilted forward and backward for the purpose of "tracking" the saw. Tracking the saw means making the saw run at a certain place on the face of the rim to keep it from being forced off the rim entirely. If the top wheel is not in exact vertical alignment with the lower wheel, the saw will not run smoothly on the rims of the wheels and may be forced off. The adjustments on the upper wheel are made at the time a new saw is placed in the machine. Once these adjustments are made they seldom need attention during the life of the particular saw blade.

The saw, while running, passes over the rims of the upper and lower wheels, and through a slot in the middle of the work table. In placing a new saw in the machine, it must pass through this slot from the front edge of the table to the middle before it can be placed on the wheels. This slot weakens the table somewhat and care must be exercised not to use the table for supporting work for which it was not intended. Over-loading the table may spring it and throw it out of true surface alignment. The table is made in two parts. The larger part through which the saw runs is adjustable to different angles in relation to the saw. It may be tilted to the right and the left of the saw. The smaller part of the table, which is called the auxiliary table, is permanently fastened to the frame of the machine.

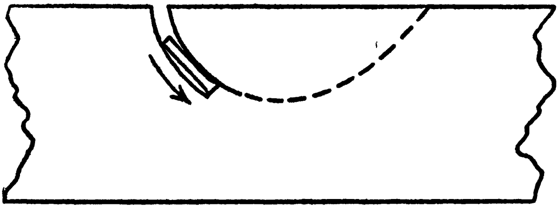
The work table may be adjusted to any angle between 90° and 45° on the right side of the saw, and any angle between ninety and eighty on the left side of the saw. In setting the saw table to an angle, a lever arm is released on the underside of the table to the back of the base. The table is tilted to the desired angle by pushing down or lifting up on the right hand edge of the table. A graduated segment underneath the table to the front of the base indicates the angle of the table in relation to the saw. When the proper angle is reached, the lever arm is tightened. It is important to be able to adjust the table to any angle when sawing out stock for a pattern; in this way it is possible to apply the correct amount of draft to the sides of the pattern while cutting out the parts.

The saw blade travels upward on the left-hand side of the wheels, and downward on the right-hand side. It is the downward moving portion of the saw, on the right-hand side between the two wheels, that does the actual cutting. This section of the saw is supported by two guides, one above the table, and one below the table. The guides keep the saw from being pushed from the rim of the wheels while the work is being pushed against the teeth of the saw. The guide above the table is adjustable upward and downward to accommodate different thicknesses of work. It is fastened to a square guide post which slides in a vertical bearing in the cast iron frame of the machine. The proper height of the upper saw guide is between one-quarter and one-half inch above the top surface of the work. The upper wheel of the band saw is usually guarded by a door made of heavy wire mesh or sheet metal; the lower wheel by a door of wood or cast iron. The lower wheel is usually driven at approximately 600 revolutions per minute, which in a wheel three feet in diameter would develop a surface speed of about 5600 feet per minute. This means that the saw is passing through the wood at a speed faster than a mile a minute. When the high speed of the saw is considered, a better appreciation of the fine quality of the machine and of the saw itself is obtained.

The size of the band saw is designated by the diameter of the wheels on which the saw tracks. A sixteen-inch band saw is one with wheels sixteen inches in diameter. Band saws may be had in sizes from twelve to thirty-six inches. Band saw blades are obtainable in widths varying from one-eighth of an inch to several inches.

Range of Work. The band saw is the most useful of all types of power-driven saws for the pattern maker. It was originally designed to saw curved work, but by using saw blades of different widths, it may be used for almost any sawing operations, whether they are straight-line sawing or curved-line sawing. The width of the blade is selected to suit the work to be done. If the saw is to be used for cutting curves of small radii, the saw blade must be narrower than if it is to be used for larger curves or for straight line sawing. Because a band saw blade will cut circles or other curved lines in any direction it is especially useful for sawing intricate parts of patterns. However, it can only be used when the curve starts at the outside edge of the stock. For sawing interior curves, it is necessary to use a reciprocating jig saw, where the end of the saw can be passed through the interior of the design through a previously bored hole.

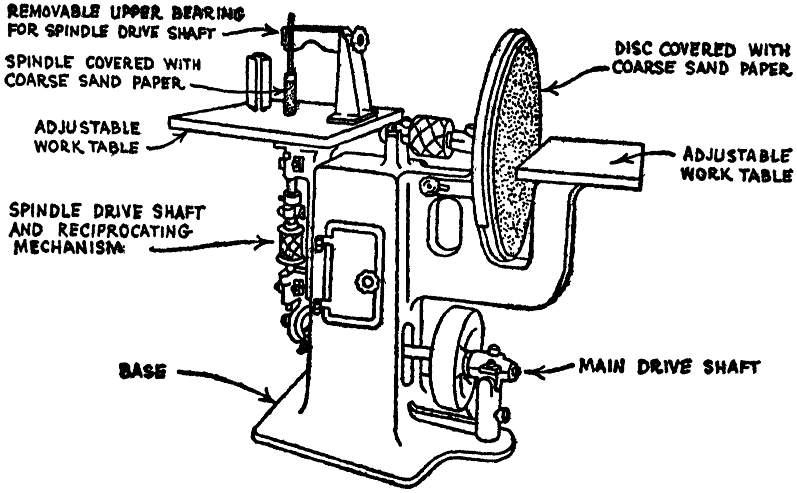
The accompanying figure shows an enlarged view of a band saw blade following a curved line in a piece of wood. It can be seen that the side of the blade is tangent to the curve not at the cutting edge but at the middle of the blade. The width and the thickness of the blade limit the side and the clearance. Some band saw blades are made as wide as twelve inches. These blades are used to cut boards from logs in a saw mill. Frequently teeth are filed on both edges of wide band saws so that boards may be sawed from logs while they are being pushed into the saw and while they are being brought back to their original position.



A Band Saw Following a Curved Line

Most band saws have removable ripping gages that may be bolted to the work table when ripping operations are desired. More often the saw is operated without gages for holding and guiding the work. The common procedure is to draw or scribe a line on the surface of the wood where the cut is to be made. The work is rested on the table and then fed into the saw by pushing and guiding with the aid of both hands.

Disc and Spindle Sander. A combination disc and spindle sander is shown below. It consists of a base; a revolving disc; a revolving spindle; two work tables, one each for the disc and the spindle; and the necessary driving mechanism. The base is made of cast iron, in one-piece construction. It supports all the various parts of the machine. The disc may be had in diameters ranging from sixteen to thirty inches. The size of the machine is determined by the diameter of the disc. A disc of larger diameter requires a base of heavier construction. The disc itself is generally made of steel in the larger sizes, and of aluminum in the smaller diameters. It is


REMOVABLE UPPER BEARING FOR SPINDLE DRIVE SHAFT
SPINDLE COVERED WITH COARSE SAND PAPER
ADJUSTABLE WORK TABLE
SPINDLE DRIVE SHAFT AND RECIPROCATING MECHANISM
BASE
DISC COVERED WITH COARSE SAND PAPER
ADJUSTABLE WORK TABLE
MAIN DRIVE SHAFT

Combination Disc and Spindle Sander

accurately machined to a true surface and precisely balanced so that it may revolve at high speeds without vibration. The outside face of the disc is covered with coarse sand paper. When the sand paper becomes worn it is necessary to remove the disc from the machine and replace the worn sand paper with new. The sand paper may be purchased in round sheets of various diameters to fit different disc sizes. The sand paper sheets are secured to the face of the disc by using special sand-paper-disc cement, shellac, or a patented clamping ring. Some manufacturers provide a guard which covers the entire back and edge of the disc; the guard is not only a safety device, but may also be incorporated with a suction system to draw sand paper dust away from the disc while in operation.

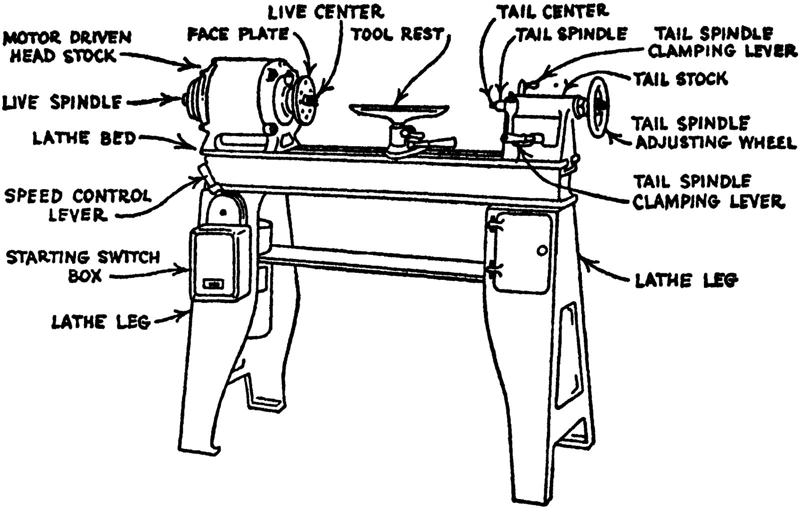
The drive shaft to which the disc is secured is usually of ball bearing construction. It is intended to operate at a running speed of approximately eight hundred revolutions a minute, depending on the diameter of the disc. Discs of larger diameters revolve at slower speeds than smaller discs.

The spindle construction is somewhat more complicated. When in operation it not only revolves, but it is made to reciprocate up and down at the same time. This up and down motion brings new sections of sand paper continually into cutting position. This gives the paper longer life and better cutting qualities. The spindle itself, the part that does the cutting, is really a separate sand-paper-covered sleeve, approximately eight inches in length and from one and one-half inches to six inches in diameter, which fits on the spindle drive shaft of the machine. The spindle drive shaft passes through the middle of the sleeve. The sleeve is held in place by means of collars and bolts. The upper end of the spindle drive shaft is supported by means of a removable, overhung bearing, attached to a bracket. Several removable, flat, iron rings of various inside diameters are provided for use at the center of the table to accommodate sleeves of different outside diameters. The work table for both the spindle and the disc may be tilted to various angles. All moving parts are driven from a main drive shaft in the base of the machine. The machine may be either belt- or motor-driven.

Range of Work. The combination disc and spindle sander is adaptable to straight, curved, and irregularly shaped work. The procedure usually followed in working wood to a desired shape is, first, to draw, or lay out an outline of the shape on the surface of a board, then to saw the board roughly to shape on the band saw, then to finish the rough saw cut to a finished shape by using chisels, gouges, spoke-shaves and sand paper. With the aid of the sander, the work can be taken directly from the saw to the sander and much of the tedious time-consuming hand tool work eliminated. By holding the work against the fast revolving disc or the fast revolving spindle, the excess wood on the outside of the layout lines may be ground away rapidly, accurately and smoothly. The disc is especially adapted to square and angular surfaces and outside curves; the spindle, to inside curves. The work is placed on the table and brought in contact with the fast moving, sand-paper-covered surfaces of either the disc or the spindle by hand. By moving the work back and forth slightly across the cutting surface of the disc or spindle while in operation, a very smooth surface is readily obtained.

Wood-turning Lathe. A wood-turning lathe, as shown below, consists of a bed supported on two legs, a headstock, a tool rest, and a tailstock mounted on the bed. The bed is the main body of the lathe. The top of the bed is accurately machined to provide a true surface on which to support the headstock, tailstock, and tool rest. The headstock is bolted permanently in position on the left end of the bed, while the tailstock and the tool rest are made adjustable so that they may be clamped at any position on the bed.

The headstock of the lathe provides a support for the live spindle. It is the live spindle that rotates the work in the lathe; it is hollow its entire length. The right-hand end of the live spindle, toward the inside of the machine, is threaded to receive the face plate. The hole through the center of the spindle is tapered at this same end to receive the tapered shank of the live center. In a motor-driven headstock, the main shaft of the motor also serves as the live spindle of the lathe. In a belt-driven lathe, a cone pulley is keyed to the live spindle between two end-bearings.



Motor Driven Wood-turning Lathe

By shifting the belt on different size steps of the cone pulley, varying spindle speeds are obtained. In a motor-driven headstock different spindle speeds are obtained by a variable-speed control switch.

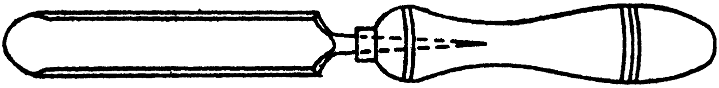
The tailstock furnishes a support for the tail, or dead spindle, which in turn supports, at its inner end, the cup, or dead center. It is called the dead spindle because it does not revolve when the power is on, as the live spindle does. By turning the wheel on the outside end of the tail spindle, the latter may be adjusted forward or backward in the tailstock. The tail spindle may be locked at any desired position in the tailstock by the spindle clamp on the top of the tailstock. The tailstock may likewise be clamped in any desired position on the lathe bed by means of the tailstock clamp. The tool rest is provided with a clamping lever for fastening it to the lathe bed in any desired position. Wood-turning lathes are usually provided with three different spindle speeds: 1200, 1800, and 2400 revolutions per minute.

Type of Work. Wood turning is done by mounting and rotating a piece of wood in the lathe, while a cutting tool is brought in contact with its revolving surface. As the wood rotates past the cutting edge, the tool cuts it away. By appropriate manipulation of the cutting tool, the work is given the desired shape.

There are two distinct methods by which rotating wood may be cut to a desired shape. One is known as the *cutting method,* because the cutting tool actually cuts the wood away. The other is known as the *scraping method,* because the cutting tool does not cut, but rather scrapes the wood away. There are two distinct kinds of wood turning to which each of these two cutting methods applies. One kind of wood turning is known as *face plate turning,* because the wood is fastened to a face plate and rotates when the face plate is rotated. The scraping method applies to face plate turning. The other kind of wood turning is known as *between center turning,* because the work is mounted in the lathe between the live and dead centers. The live center has several spur points which are driven into the work. When the live center rotates, the work mounted between the lathe centers rotates. The cutting method applies to between-center turning. The two different methods become necessary because of the two different directions of the grain of the wood in the work being turned. Face plate work is known as "cross grain"; between-center work is known as "with the grain."

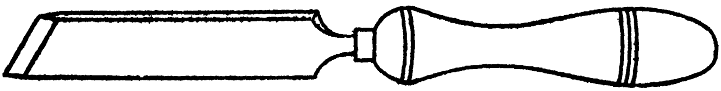
Turning Tools. The cutting tools used in wood turning are somewhat similar to the woodworker's chisels and gouges. They are, however, longer and heavier and are called turner's chisels and turner's gouges. Sometimes they are referred to as turning tools.

The wood turner's *gouge* is a heavy tang-handled gouge whose cutting edge is ground in such a way that when seen in a plain view it is approximately a parabola. This tool is used as a roughing tool in cutting a rough, square block to a cylindrical form, between centers. Smaller sized gouges are used as finishing tools on concave curves. The gouge is a cutting tool, not a scraping tool, and is therefore never used on face plate turning.



Wood Turner's Gouge

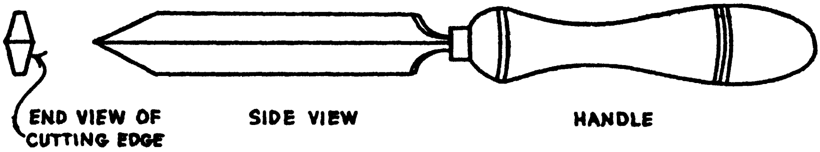
The wood turner's *skew chisel* is a thin, flat, tang-handled chisel with the cutting edge at an angle to the side edges of the tool. The cutting edge is beveled on both sides, as shown. The point of the cutting edge at the acute angle is called the "point," and the point at the obtuse angle is called the "heel" of the tool. The skew chisel is used as a finishing tool in cutting plain cylindrical surfaces, in cutting end grain, and in turning convex curves. The skew chisel is also a cutting tool, and not a



Wood Turner's Skew Chisel

scraping tool; like the gouge, therefore, it is used for between center turning only.

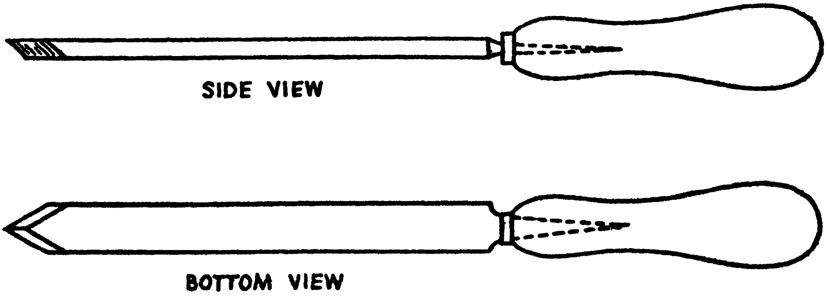
The *sizing tool* has two distinct uses, and two names. It is referred to interchangeably as a sizing tool and as a parting tool. When used as a sizing tool, it is made to cut a flat, bottom groove squarely into the work, to accommodate the points of the outside calipers in testing the diameter of the work for size. When used as a parting tool, it is thrust squarely into the work, cutting a deep groove, and separating the finished work from the rough ends that are used to hold it in the lathe. The cutting edge is the thickest part of the tool, and is at the middle of the sides; this shape gives the cutting edge strength as well as clearance. The sizing tool is used for between center work only.



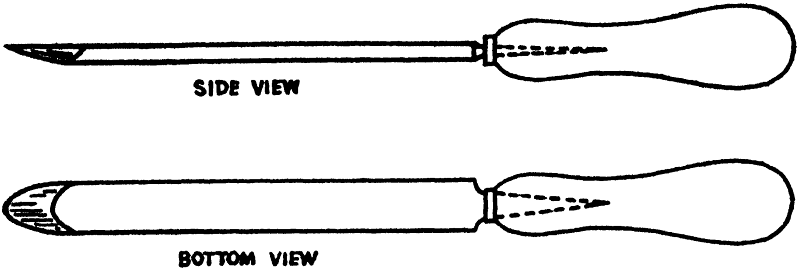
Wood Turner's Sizing Tool

The ***diamond point,*** the ***round nose,*** and the ***straight wood-turner's chisel*** differ from each other only in the shape of their respective cutting edges. All three are flat, tang-handled, scraping chisels that may be used alternately for both face plate and between center turning. The cutting edge of the diamond point scraping chisel is shaped to a diamond point, as shown. On face plate work, it is used as a roughing tool. On between center work, it is used as both a finishing tool on cutting end grain, and as a finishing tool on small convex curved surfaces.

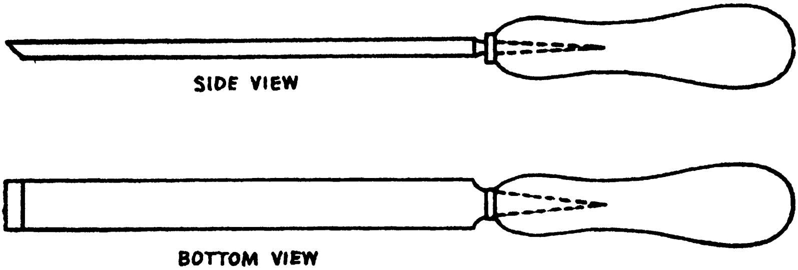
The cutting edge of the round nose scraping tool is shaped either in a semi-circular or a parabolic form. On face plate work, it follows the



(a) DIAMOND POINT SCRAPING TOOL



(b) ROUND NOSE SCRAPING TOOL



(c) STRAIGHT NOSE SCRAPING TOOL

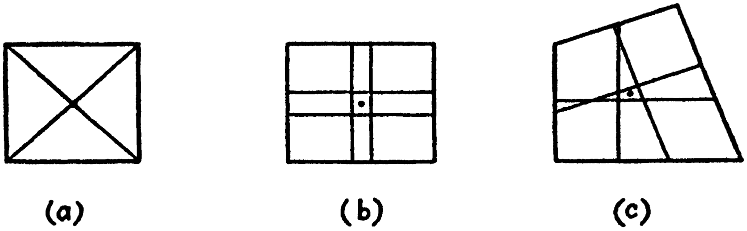
Showing the Side and Bottom Views of the Diamond Point, Round Nose, and Straight Nose Wood Turner's Scraping Chisels

diamond point chisel in furthering the rough shaping operations. In some face plate turning the round nose tool is used for the rough shaping as well as the final finishing operations. On other types of face plate work it is used for the rough shaping only. On between center work the round nose chisel is used as a finishing tool on concave curve surfaces where the gouge will not meet the needs due to the particular shape of the work.

The straight nose chisel is used as a finishing tool on both face plate and between center turning. Face plate work is usually started with the diamond point, followed by the round nose, and finished with the straight nose. The tool sequence varies with the shape of the work. Sometimes the work is of such shape that it may be started by the diamond point, then rough-shaped by the round nose, and finished by the round nose. Work between centers is usually rough shaped by the gouge and finished with the skew chisel. If extreme accuracy is required of between center work, the final finishing is done by scraping with the straight nose chisel.

All wood turner's tools have long handles so that their cutting action may be controlled more easily.

General Procedure: Mounting and Centering. In preparing a piece of wood to fit in between the centers of a lathe, it is necessary to saw the two ends square with the sides. On each of the two ends the axial center is located. This may be done by employing either of the two methods shown.


(a) (b) (c)

Methods of Locating the Axial Center on the End of a Piece of Wood.

If the block is square or rectangular in section, the center may be quickly located by drawing lines diagonally, from corner to corner, with a rule and a pencil; the intersection of these lines is the center of the piece. Another method, applicable to a section of any form, consists of setting the marking gage to a dimension approximately equal to the distance from the edge to the center, and then scribing four lines on the end of the piece, each parallel to one of the four edges. The shape of the section is thus reproduced in miniature on the end of the piece and the position of the center may be located by inspection.

After locating the position of the center on both ends, the piece is ready to be placed in the lathe.

1. Release the tail stock clamping lever, and move the tail stock out of the way to the right.

2. Remove the live center from the live spindle by pushing it out with a bar that may be inserted in the live spindle at left-hand end.

3. Stand the piece of wood on end on the bench. If the size of the piece does not permit its standing on end, clamp it in the bench vise with one end up.

4. Place the point of the live center against the point located at the center of the piece of wood. With a mallet drive the live center into the end of the piece of wood. The spurs of the live center should cut securely into the wood.

5. Replace the live center in the live spindle, keeping it attached to the piece of wood that is to be turned. Bring the tail stock forward into position. The point of the dead center should be about an inch away from the end of the wood. Clamp the tail stock to the bed of the lathe by means of the tail stock clamping lever. Turn the tail spindle adjusting wheel so that the dead center cuts into the end of the wood, the point

of the dead center and the center on the end of the piece coinciding. Next move back the dead center and oil it, and again move it into position, being careful that it is not so tight as to prevent the free rotation of the lathe. Finally, clamp the tail spindle firmly in place with the tail spindle clamping lever. The tail spindle should not project from the tail stock by more than an inch.

6. Adjust the tool rest so that it is about one quarter of an inch away from the corners of the stock when the wood is revolved in the lathe. The height of the tool rest should be such that when the tool is in cutting position, the hand grasping the tool handle and the fore arm are horizontal, with the body in an erect, easy position. The arm should be close to the side of the body. For most work the rest will be lower when using the gouge and the parting tool than when using the skew chisel. The tool rest should always be as close to the work as possible.

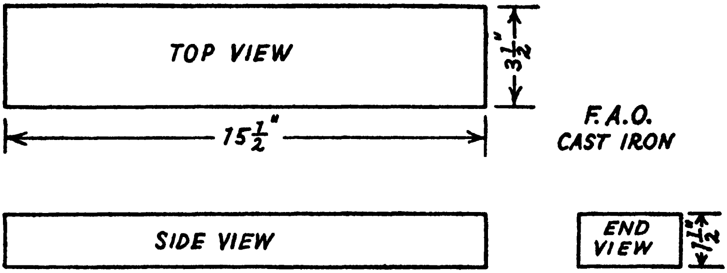
62. PATTERN CONSTRUCTION

Making a Solid One-Piece Pattern. Before beginning work on a pattern, it is well to have in mind a clear picture of what is to be done and how it is to be done. To aid in developing a clear understanding of the work, the drawing or blue-print of the casting to be made is very carefully studied. The drawing gives the size and shape of the casting, the material from which it is to be made, and indicates whether there is to be any subsequent machining or working of the part by the machinist before the casting is ready for service. While studying the drawing, a full size, sectional view of the casting is made on a flat board, called a layout board. The material of which the casting is to be made determines the proper shrink rule to be used in marking off and drawing the dimensions on the layout board. When the full size drawing is completed on the layout board, it is called the pattern layout, or simply referred to as the layout of the job. From the layout, we determine how the pattern is to be lifted from the molding sand to form the mold; what surfaces of the pattern are to be drafted, or tapered, to aid in the removal of the pattern from the sand; and what allowances, if any, are to be made for coring or machining.

When these preliminary matters have been determined, the next step is to consider how different pieces of wood might be joined and fastened together to form the desired shape of the pattern. Small, simple shapes can be cut from one solid piece of wood, whereas complicated shapes require the joining and fastening together of many different pieces of wood.

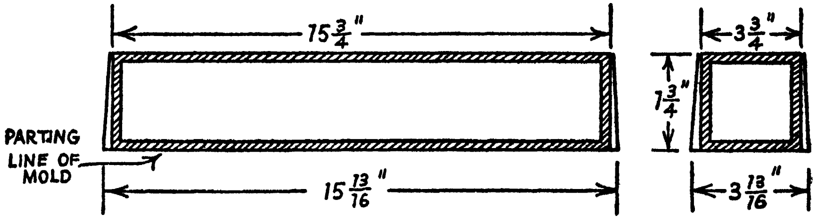
When a pattern can be made from one solid piece of wood it becomes known as a one-piece pattern, to distinguish it from a pattern whose construction requires more complicated methods. A solid one-piece pattern is made by cutting out a piece of wood a little oversize from the size required for the finished pattern. On this piece of wood, lines are drawn outlining the exact size and shape of the pattern. After the lines are drawn, all the surplus wood on the outside of the outline is removed by using whatever cutting tools are required in order to shape the piece to the outline.

Below is a mechanical drawing of a casting. It tells the pattern maker that the desired casting is rectangular in shape; that it is 1½ inches in thickness, 3½ inches in width, and 15½ inches in length; that it is to be machined on all surfaces; and that it is to be made of cast iron.



Mechanical Drawing of Cast Iron Part, 1½"×3½"×15½", Machine-Finished on All Surfaces.

The next figure shows a pattern maker's layout of the casting. Draft and finish are added. The parting line is indicated. The dimensions are made with a shrinkage rule, allowing ⅛" per foot shrinkage, which is the shrinkage allowance for cast iron. From this layout the pattern maker determines the size of the wood needed from which to make the pattern. A list of the size of the pieces of wood needed is called a bill of material. The bill of material for this pattern would appear as follows:



Pattern Maker's Layout, Showing Parting Line and Allowances for Finish and Draft.

1 piece, white pine, 2"×4"×16"

A piece of this size is obtained from the lumber rack by sawing it from a board of a larger size. This piece then becomes known as the rough stock. The rough stock is now ready to be reduced to final dimensions. Working a piece of wood to dimensions is a reducing process. The amount of waste material to be removed depends upon the size and shape of the finished work. Usually one quarter of an inch oversize in thickness, width and length is sufficient.

Detailed Procedure. The following is a description of the procedure to be used in making this one-piece pattern.

1. Select the better of the two wider sides, and plane it just enough to clean it off and make it a true surface. If there is any warp on the surface of the board, the high edges or corners are to be noted and planed down first. If the two edges are high, the first plane cuts should be along the edges. If two diagonal corners are high, the strokes of the plane should be from one of the high corners diagonally across the center of the stock to the other high corner. When the high edges or corners are removed, proceed to plane the entire surface lightly. Take as few strokes with the plane as necessary. Do not make the mistake of trying to take a heavy cut. The shavings should be very thin if satisfactory results are to be attained. Should the grain of the wood tear out, and the surface become rough, turn the board around so as to begin the stroke from the opposite end of the board.

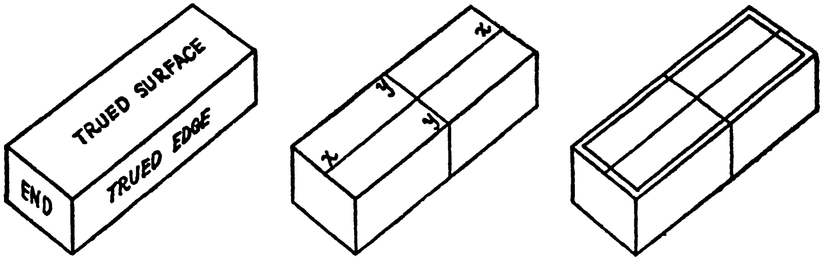
When starting the stroke, the forward end of the plane should be pressed down firmly by pressure exerted on the knob. When the plane is well onto the board, the pressure on the knob and the handle should be the same. As the plane passes off the board at the end of the stroke, the pressure should be taken from the knob and additional pressure exerted downward on the handle. A true surface cannot be produced unless the plane rests flat upon the board throughout the entire length of the cut. As the planing proceeds, the trueness of the surface should be tested from time to time with the long corner edge of the plane, testing lengthwise, crosswise, and diagonally at several points. A true surface is straight as to its length and width and has its four corners in the same plane. This may be easily determined by placing the planed surface against a surface that is known to be true, such as a surface plate, a machine work table or any handy, true metal surface.

2. Select the better of the two edges and plane it until its surface is true and square with the face just planed. The edge is more easily planed if the stock is held in the bench vise. The accuracy of the edge in relation to the surface is tested with the try square.

3. Set the marking gage to one and three-quarter inches. With the marking gage lay off the thickness from the trued surface, scribing the

line completely around the stock. With the plane remove the surplus stock to the scribed line. Plane and test this surface for trueness the same as the first surface. The stock is now planed to the required thickness, and one edge is true. It is now ready to have the outline of the pattern laid out on it.

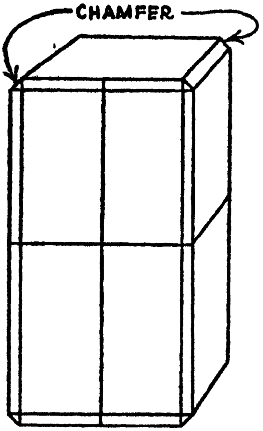
4. Scribe center lines on the two trued surfaces, as shown in (b). Lines in the same direction as the grain are scribed with the marking gage. Lines across the grain are cut with a bench knife used against the blade of a try square. Center line x-x, Fig. (b), divides the width of the top and bottom surfaces approximately in half. Center line y-y, Fig. (b), divides the length of the top and bottom surfaces approximately in half. With the center lines as a guide, the outline of the pattern is laid out on both the top and bottom surfaces. The pattern outline on the top surface will measure 15¾ inches in length, and 3¾ inches in width, the measurements being made with a shrink rule allowing ⅛ of an inch per foot shrinkage. The pattern outline on the bottom surface will measure $15\frac{13}{16}$ inches in length by $3\frac{13}{16}$ inches in width. With the pattern completely laid out, the next operation is to remove the surplus wood on the outside of the outline. This is done by removing the stock on the ends first, and then the edges.



(a) (b) (c)

Showing the Stock Ready to Be Laid Out at (a); Top and Bottom Surfaces Are True, One Edge Is True. Center Lines Are Laid Out on Top and Bottom Surfaces, at (b). Outline of Pattern Laid Out from Center Lines on Top and Bottom Surfaces, at (c). Center Lines and Layout Lines on Top Surface Only Are Visible.

5. Select the better of the two ends, and plane it until the wood has been removed to the layout line on the top and bottom surfaces. While planing the end, the stock is held in the bench vise. The plane is held at an angle, but the direction of the stroke is parallel to the long edge corners of the surface. This enables the wood fibres to be severed with a shearing cut. The corners at the edges are chamfered to prevent the splintering out of the grain of the wood while planing the end surface, as here shown.



Showing Chamfered Corners to Facilitate End Grain Planing.

6. Chamfer the corners at the other end. Plane the wood away to the layout line, in the same way as the first end.

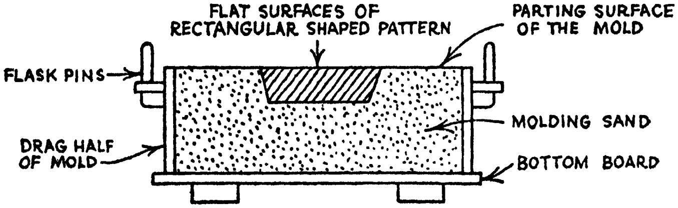
7. Plane one edge until the wood is removed to the layout line. Watch the line on the top surface as well as on the bottom surface. Remove the wood *to the line,* but do not remove the line.

8. Plane the opposite edge until the wood is removed to the layout line, the same as for the first edge. The pattern is now shaped to size, and is ready to be finished.

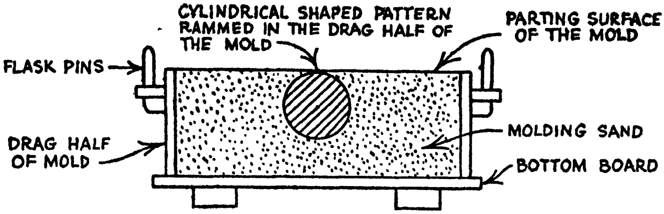
9. With a piece of very fine sand paper (#2/0), sand paper the surfaces lightly, rounding the corners neatly and carefully.

10. Since the casting is to be machined on all surfaces, the pattern is colored red on all surfaces. Shellac the surfaces of the pattern with red shellac. First apply one coat, allowing it to dry for approximately half an hour. Sand paper the pattern carefully and apply a second coat of red shellac. The shellac should not have a consistency greater than that of ordinary milk. If the shellac is too heavy, it will not dry in the time allowed, nor will it give a smooth finish. After the second coat of shellac has dried, sand paper the surfaces again carefully, and apply a third coat. After the third coat dries, the pattern is completed.

Making a Split or Two-Piece Pattern. Generally speaking, castings of a rectangular shape can usually be cast from a solid pattern, made from one piece of wood. Castings of a cylindrical shape, however, present a very different foundry problem. Rectangular shapes have flat surfaces. One of the flat surfaces can usually be made to lie in the same plane with the parting surface of the mold, the surface between the cope and the drag, as shown in the accompanying figure. Cylindrical shapes, however, do not

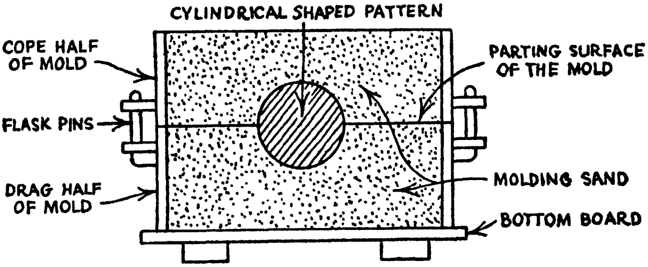

FLAT SURFACES OF RECTANGULAR SHAPED PATTERN
PARTING SURFACE OF THE MOLD
FLASK PINS
MOLDING SAND
DRAG HALF OF MOLD
BOTTOM BOARD

Showing a Flat Surface of a Rectangular-shaped Pattern Lying in the Same Plane with the Parting Surface of the Mold

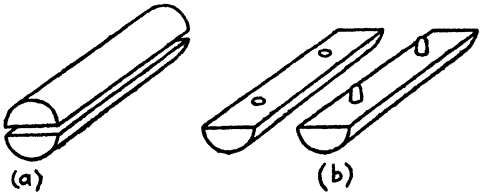

CYLINDRICAL SHAPED PATTERN RAMMED IN THE DRAG HALF OF THE MOLD
PARTING SURFACE OF THE MOLD
FLASK PINS
MOLDING SAND
DRAG HALF OF MOLD
BOTTOM BOARD

Showing that a Solid Cylindrical-shaped Pattern Presents a Problem when Trying to Lift it from the Mold

present flat surfaces which can be made to lie in the same plane with the parting surface of the mold. If a cylindrical-shaped pattern were made solid, in one piece, as in the case of a rectangularly-shaped pattern, it could not be lifted from the mold, as is apparent from the sketch shown. In order to lift the pattern from the mold, it would be necessary to have the parting of the mold pass through the center of the cylindrical-shaped pattern. In order to do this easily, the pattern for a cylindrical-shaped casting is not made of one solid piece of wood, but is made instead of two separate pieces, exactly alike, each piece representing one-half of the completed pattern. The two pieces are held in proper alignment with each other by two dowel pins; each pin is of a different diameter so that the two halves will fit together in only one way.



Showing a Cylindrical-shaped Pattern Placed in a Mold in Such a Way that it may be Lifted from the Sand Easily

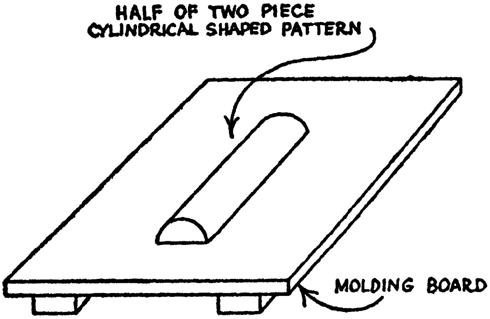


A CYLINDRICAL SHAPED PATTERN MADE OF TWO SEPARATE PIECES BEING ASSEMBLED

THE TWO PIECES OF A CYLINDRICAL SHAPED PATTERN SHOWN SEPARATELY BEFORE BEING ASSEMBLED

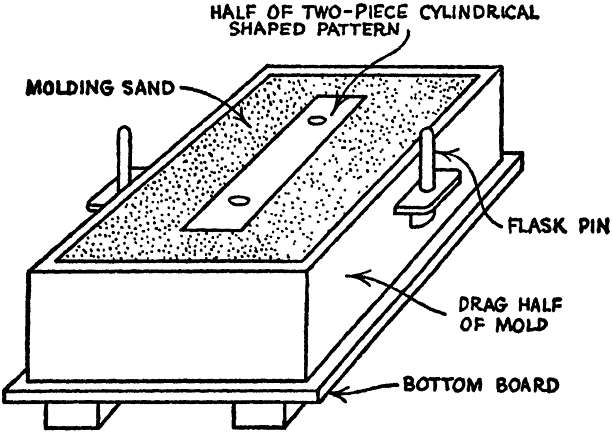
Showing a Two-piece Cylindrical-shaped Pattern

In fitting the two halves together, holes are bored in each piece the same distance from each other. Dowel pins are glued in the holes of the one piece. The top ends of the dowel pins are rounded off so that they will fit easily into the corresponding holes of the other piece.

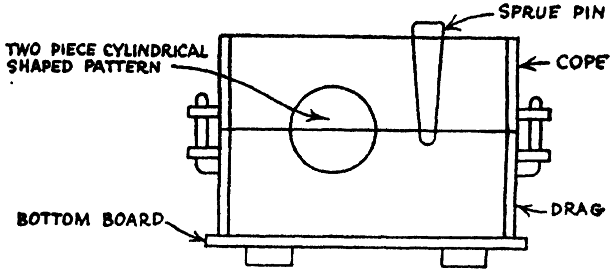


Showing the Half of the Two-piece Cylindrical-shaped Pattern which does not contain the Projecting Dowel Pins, placed on a Molding Board, Ready to be Rammed in the Drag Half of the Flask

In making the mold, the half of the pattern that does not contain the projecting dowel pins is laid flat on the molding board, as shown herewith. The drag half of the flask is inverted on the molding board, filled with sand, rammed, and rolled over in the usual manner. The other half of the pattern, the one with the projecting dowel pins, is now placed in position on top of the half already rammed. The cope half of the flask is placed in position on top of the drag half. The sprue pin is placed in position. The parting surface, between the cope and the drag, is sprinkled with parting sand. The cope half of the flask is filled with molding sand, rammed, and struck off, as shown. The sprue pin is withdrawn, and the cope half of the flask is



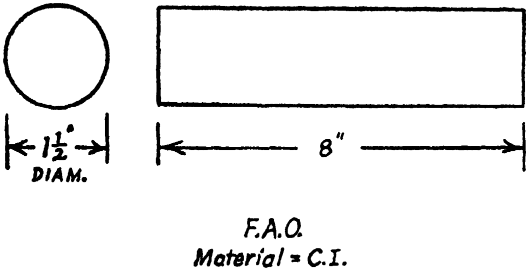
Showing One Half of a Two-piece Cylindrical-shaped Pattern Rammed in Drag lifted off. The pattern is removed from the cope and the drag. The gate is cut. The cope is replaced. The mold is ready for pouring the metal. By splitting the pattern in half, and making it from two pieces of wood, it becomes possible to remove the pattern easily from the mold. Such a pattern is known as a ***split pattern,*** or is sometimes called a ***two-piece pattern.***



Showing Two-piece Cylindrical-shaped Pattern, Completely Rammed in Mold. Mold Ready to be Separated to Remove Pattern

Procedure for Making a Two-piece Pattern.

1. Study the following drawing carefully. It shows a cylindrical-shaped casting, one and one-half inches in diameter by eight inches in length, to be made of cast iron and machined on all surfaces. Allowing for machining, the casting in the rough should measure one and three-quarters inches in diameter by eight and one-quarter inches in length. To aid in drawing the pattern from the mold, the pattern should be made of two pieces of wood, each representing one-half of the completed pattern. The one piece should be fitted with projecting dowel pins that fit into corresponding holes in the other piece. The two pieces should be fitted together with dowel pins while the wood is in the semi-rough state, being prepared for turning in the lathe. Allowances must be made on the ends of the wood for holding the two pieces together with wood screws while turning them to shape in the lathe. Approximately one and one-half inches added to each end will be sufficient for holding the two pieces together and for mounting in the lathe.

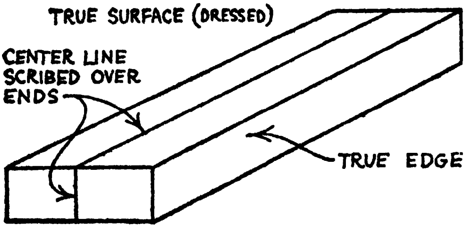


Cylindrical-shaped Casting

2. Secure two pieces of wood, each piece to measure one inch thick, two inches wide, eleven inches long. The two pieces must be the same length, and the ends must be square.

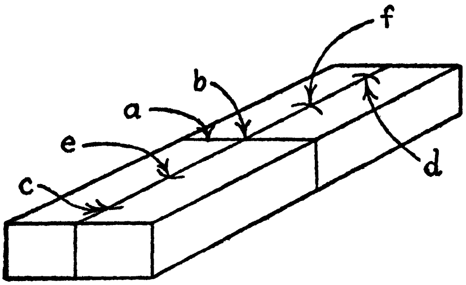
3. Dress one surface and one edge of each piece.

4. Scribe center lines on the dressed surface of each piece with the marking gage, extending the center lines over the ends, as shown herewith.



Center line scribed on surface and over ends

5. Measure the length of the stock along the center line; divide the measurement by two. Starting at one end of each piece, measure off on the center line the distance equal to one half the length. At this point square a line across both pieces, as shown at (a).



Center Line Squared across the Stock at (a), halfway between the Ends. Length of Completed Pattern Laid-off at (c) and (d). Center Lines for Dowel Pin Holes Laid-off at (e) and (f)

6. Open the dividers to four and one-eighth inches. Place the end of one leg of the dividers at point (b); scribe two arcs, cutting the center line four and one-eighth inches on each side of point (b), as shown at points (c) and (d). The distance between (c) and (d) represents the length of the completed pattern. These are reference points. The dowel pin holes must be laid out inside these two points.

7. Open the dividers to two and three-quarters inches; place the end of one leg at point (b), and scribe two arcs cutting the center line at points (e) and (f). These points are the centers of the dowel pin holes. They are to be laid off on both pieces.

8. At point (e) on both pieces, bore a hole with an auger bit three-eighths of an inch in diameter, and one-half an inch deep. At point (f) on both pieces, bore a hole one-quarter of an inch in diameter, and one-half an inch deep.

9. From a piece of three-eighths of an inch diameter dowel stock, saw

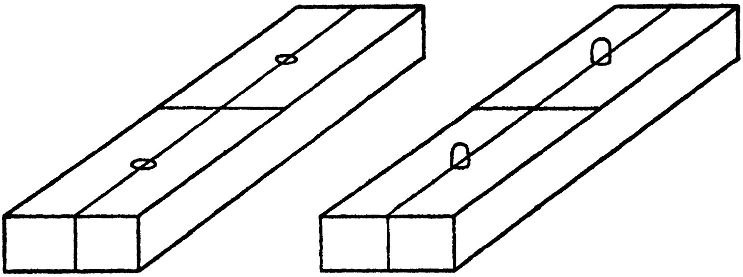
off a piece three-quarters of an inch in length. From a piece of one-quarter of an inch diameter dowel stock, saw off a piece three-quarters of an inch in length.

10. Round off one end of each piece of dowel with a bench knife.

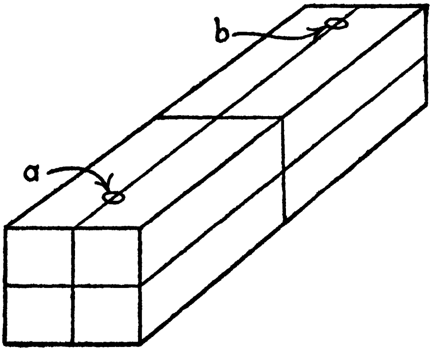
11. Take up one of the pieces of pattern stock and drop a little glue in each of the two dowel pin holes.

12. Place the sawed end of the two dowels in the two dowel pin holes respectively. The round ends of the dowels are now projecting from the pattern stock approximately one-quarter of an inch.

13. Take up the other piece of pattern stock and fit it to the first piece by means of the dowel pins and the dowel pin holes. The two pieces appear as here shown. When fitted together, they appear as shown at the left.



Showing Both Pieces of Pattern, Ready to be Fitted together by means of Projecting Dowel Pins and Dowel Pin Holes



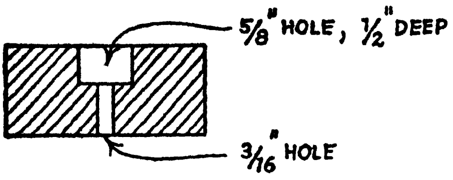
Showing Pencil Line drawn over top, connecting the Center Lines on the Ends. Centers Laid-out for Screw Holes for Holding the Two Pieces together while Turning to Shape in the Lathe

14. Connect the center lines on the ends with a pencil line over the top surface of the stock as indicated.

15. Scribe lines three-quarters of an inch from each end, locating points (a) and (b).

16. With a #10 auger bit, 5/8″ diameter, bore two holes each 1/2″ deep, one at point (a), the other at point (b). These holes are to be used for wood screws to hold the two pieces together while turning to shape in the lathe.

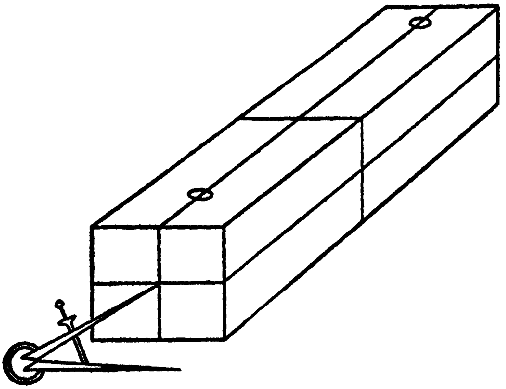
17. At the center of each ⅝″ hole, drill a 3/16″ diameter hole through the top piece, as shown.



Cross-sectional View through Center of Screw Hole, showing ⅝″ Hole and 3/16″ Hole

18. Insert a one and one-half inch, number ten, wood screw (1½″–#10) into each hole and tighten down with a screw driver, drawing the two pieces together solidly.

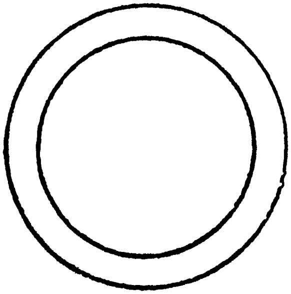
19. Insert the point on the end of one of the legs of a dividers into each end of the stock at the point where the scribed center lines cross the joint between the two pieces. This enlarges and identifies the point on each end where the lathe centers will be inserted into the stock, when mounting between centers in the lathe.

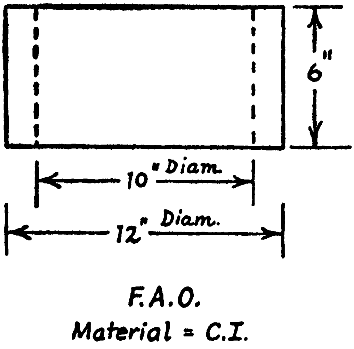


Enlarging the Point where the Center Line Crosses the Joint between the Two Pieces. The Lathe Centers are Inserted in the Ends of the Stock at these Points

20. Mount the stock between centers in the lathe and turn to size i.e., 1¾″ in diameter by 8¼″ in length. Refer to section entitled "Wood-Turning Lathe —General Procedure," if further instructions are necessary.

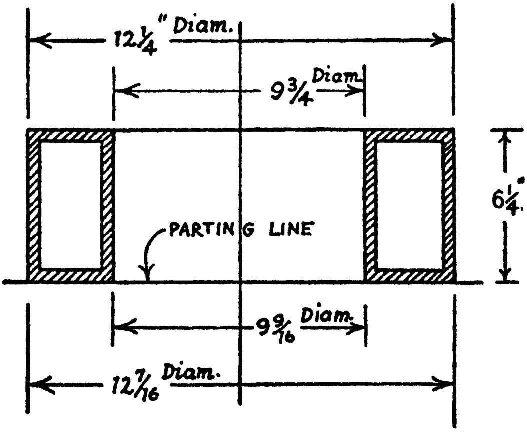
Making a Segment Pattern. Large cylindrical-shaped patterns, in the shape of rings in which the diameter is greater than the height or length, are made by gluing up several layers of segments. Segmental work comes under the class of built-up patterns. Solid patterns and two-piece patterns are usually made from lumber sawed directly from boards. Built-up patterns refer to patterns that are made of many different pieces, either for the purpose of adding strength to the



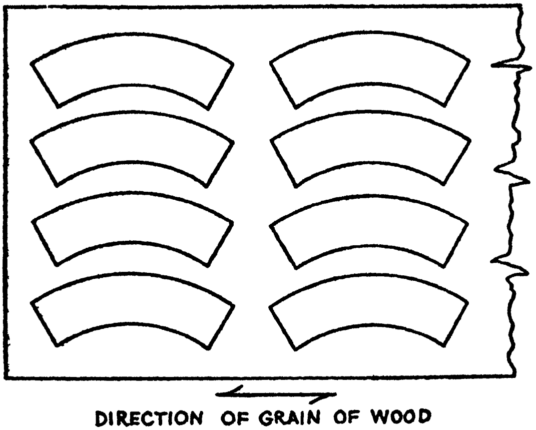


Mechanical Drawing of Ring-shaped Casting, 12" in Diameter on the Outside, 10" in Diameter on the Inside, 6" High; made of Cast Iron, and Machine-finished on all Surfaces

pattern, or to keep the pattern from warping. Segments are built up in much the same manner that bricks are built up in a wall, each one overlapping the joint of the two below it. This construction is not confined to any one type of pattern, but lends itself well to rings and wheels, whether they are made solid or split. When patterns for ring-shaped castings are made of one piece of wood, they cannot be depended upon to retain their shape. But by gluing up several layers of segments, the warping of each piece is counteracted by the warping of adjacent pieces, and great strength is obtained. It is common practice in work of this kind to use six segments in each layer. This is done because it is easy to divide each layer into six parts. The radius of the circle is equal to the length of the segment, as shown below. The grain of the wood lies in the same direction as the length of the segment.



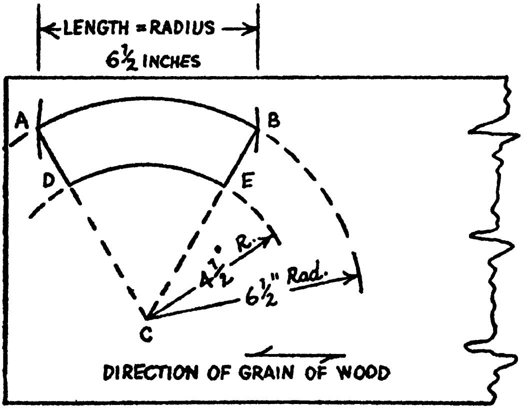
Pattern Lay-out of Casting Shown above; Parting Line determined; Finish and Draft added



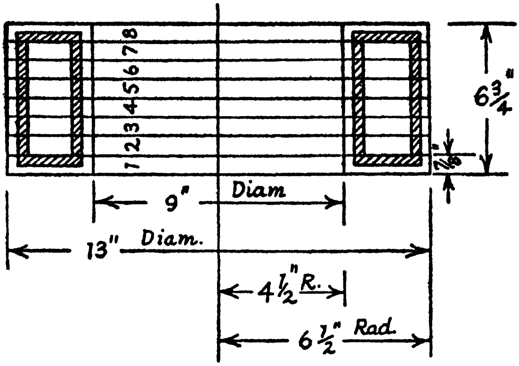
Showing a Method of Laying-out Remaining Segments on Surface of a Board, by drawing a Pencil Outline around the Template

When constructing a segmental pattern, a template of one of the segments is laid out and sawed to shape on the band saw first. With the aid of the template, other segments are laid out on the surface of a board, as here indicated.

It is the laying out and shaping of the first segment that is very important. After the first segment is laid out and sawed to shape, it becomes known as a *template.* The template is then used for laying out the other segments by simply placing it on the surface of a board of the correct thickness, and marking a pencil line around it. Six separate segments are required to make a layer. If the board from which the segments are cut is between thirteen-sixteenths and seven-eighths of an inch in thickness, eight layers will be needed to build up the pattern to the desired height. This means that forty-eight segments are needed for the construction of the pattern.

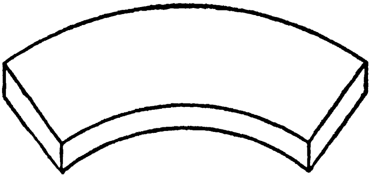


Showing Outline of First Segment Laid-out on Surface of Board



Segmental Construction Details Superimposed on Top of Pattern Layout

The size of the first segment, or template, is obtained from the pattern layout. A pair of dividers, or trammel points, is opened and set to six and one-half inches. With the dividers thus set, one of the legs is placed at a point on the surface of a board, e.g., point C. An arc, such as AB, is scribed on the surface of the board with the other leg of the dividers or trammels. On this arc, with the setting of the dividers unchanged, the distance AB is spaced off. The distances between the points A and B, between the points A and C, and between the points B and C, are equal. This distance is the radius of the outside circle; it is also the chord, or length of the segment, and is equal to six and one-half inches. By drawing a line through points A and C and through points B and C, the ends of the segment are determined. The dividers is now set to four and one-half inches, the inside radius. One leg is placed at point C, and the arc DE, forming the inside of the segment, is scribed. These various operations are shown completed above; the figure at the left shows the same segment after it has been sawed from the board. It is now known as the "template." How the template may be used to mark off the outline of other segments has already been indicated above.



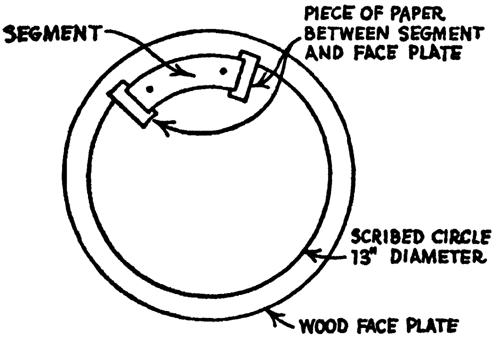
Showing First Segment sawed from Board. Used as a Template in Laying-out Remaining Segments

Procedure for Making a Segment Pattern.

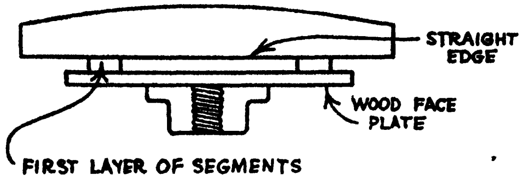
1. Study the drawing on p. 785. It shows a ring-shaped casting, 12″ in diameter on the outside, 10″ in diameter on the inside, 6″ in height, made of cast iron, and machined on all surfaces.
2. Obtain a board, approximately 1″×8″×18″, to be used for a layout board.
3. Dress one surface and one edge of the board.
4. Make the pattern layout, using a ⅛″ per foot shrink rule.

- (a) Reproduce the lines full size shown in dimensioned view.
- (b) Add ⅛″ finish on all surfaces, indicated by the cross-hatched area.
- (c) Add the draft to correspond with the dimensions shown.
- (d) Determine what thickness the lumber must be from which the segments are to be cut, in order to divide the height of the pattern into an even number of layers. In this particular case, lumber approximately ⅞″ in thickness would require eight layers of segments.

5. Obtain a board approximately 1″×8″×12″. Dress to a thickness of ⅞″.
6. In the upper left hand corner, lay out the first segment.
7. Saw out this segment carefully; it becomes the template to be used for laying out the other segments.
8. Lay out the rest of the segments on the surface of the board.
9. Saw the segments on the band saw.
10. Trim the ends of the first segment, the template, to its scribed outline, lines AD and BE.
11. Using this segment again as a template, trim the ends of all the segments to correspond. With the exception of fitting the last segment in each layer, the segments are now ready to be glued together. This is usually done on a wood face plate.
12. Obtain and true up a wood face plate approximately 14″ in diameter.
13. Scribe a circle approximately 13″ in diameter on the surface of the face plate.
14. Fasten one of the segments to the face plate with two nails, placing a piece of paper under each end, between the segments and the face plate. The purpose of the pieces of paper at the joints is to prevent the segments from sticking to the face plate.
15. Glue a segment to each end of the first segment, nailing the segments to the face plate with a piece of paper under each end.
16. Glue and nail the remaining segments of the layer in this same manner, one at a time, with a piece of paper under each joint. Try the sixth segment in place before gluing it to see if it needs further fitting.


SEGMENT
PIECE OF PAPER BETWEEN SEGMENT AND FACE PLATE
SCRIBED CIRCLE 13″ DIAMETER
WOOD FACE PLATE

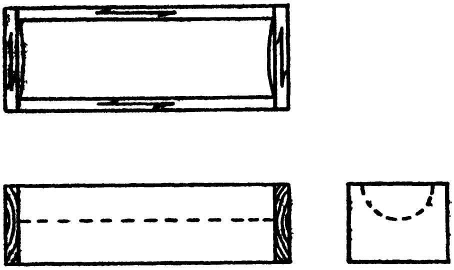
Showing one segment nailed to face plate with a piece of paper under each end



Showing a Straight Edge being Fitted against the Top Surface of the First Layer of Segments to make a True Surface for Adding the Next Layer

17. After the sixth segment has been glued and nailed into place, drive the heads of all the nails below the top surface with a nail set.
18. Place the face plate in the lathe, and true off top surface of layer of segments with the aid of a straight edge. The straight edge must fit against the top surface of the layer of segments.
19. Remove the face plate from the lathe. Glue the next layer of segments on top of the first layer in the same manner that the first layer was applied to the face plate, eliminating, however, the pieces of paper, and applying glue between the layers as well as the ends of all the segments. Care must be taken to "break the joints," that is, to have each segment lap across the joint of two segments in the first layer, very much as bricks are laid up in a wall.
20. After the second layer is glued and nailed, the face plate is returned to the lathe and the top surface made true.
21. Continue laying up the layers of segments in this manner until the entire eight layers are glued and nailed.
22. Allow the glued up segments to stand overnight (for the glue to set) before turning the pattern to size. If it is felt that the nails in the first layer will not hold the completed structure securely to the face plate, several wood screws may be driven through the face plate from the back side into the pattern.
23. When the pattern is turned to size, the screws through the back side of the face plate, if any, are removed. The pattern is pried from the face plate with a wide chisel. The protruding nails are withdrawn, and other nails are driven into the same holes to secure the first layer to the rest of the pattern.
24. The pattern is now ready for finishing.

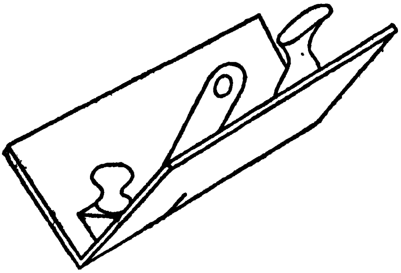
Making a Straight Round Core Box. A straight round core is usually made in what is called a half-round core box, as shown below. This gives the green core a flat surface on which to rest while it is being baked. Two cores are made from the half round core box. After they are baked, the two cores are pasted together on their flat surfaces to make one complete cylindrical core. This method prevents flattening of round sides, which would be the case if round cores were made of one piece and rolled out on flat plates to bake.



Showing Three Views of a Half-round Core Box

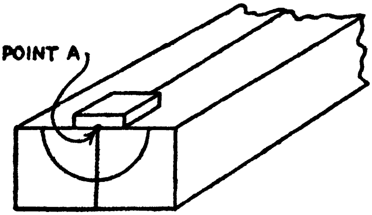
Half-round core boxes are made by first laying out the outline of the core on a piece of wood of the correct length, width, and thickness. The wood inside the outline is removed roughly with the aid of gouges. The finished dimensions are obtained by planing with a core box plane as shown.

Procedure for Making a Half-round Core Box.



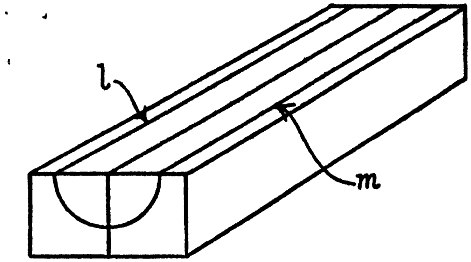
Core Box Plane

1. Assume that a half-round core box is needed to produce a round core 10″ long by 2½″ in diameter.
2. Obtain a piece of wood 2″×4½″×10½″ (2″ thick, by 4½″ wide, by 10½″ long).
3. True one face, one edge, and both ends, making the piece 10″ long.
4. Scribe center line along dressed face and over both ends, working from dressed edge with marking gage.
5. Place a small block of wood on the dressed face, even with one end, as shown.

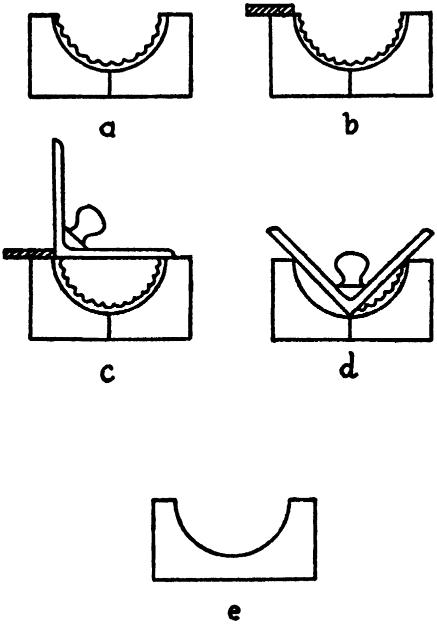


Block of Wood Placed on Dressed Surface, even with One End, to Accommodate Point of Dividers when Scribing Circle on End of Stock

6. Set a dividers to 1¼″. Place one leg at point A. Scribe half circle on end of piece, as shown.
7. Scribe half circle on other end in the same manner.
8. With marking gage, working from dressed edge, scribe the lines marked l and m, representing the outside diameter of the core, across the dressed face, as shown.
9. Clamp stock in bench vise. Rough out outline of box with gouge, as shown in (a) below.



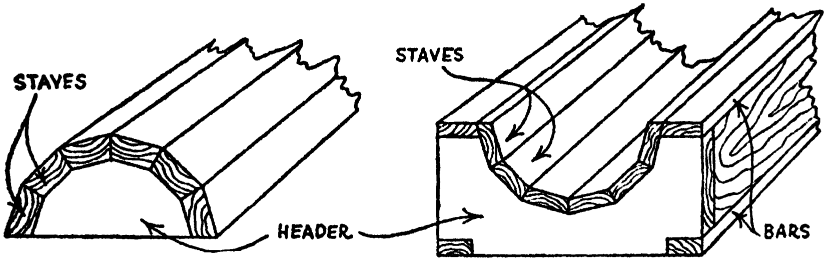
Outline of Core Laid-out on Stock



Showing Various Steps in Making a Half-round Core Box

10. Obtain a thin strip, approximately 10″ long, with a straight edge. Tack it temporarily on the dressed surface, with the straight edge coinciding with one of the scribed lines, as shown in (b).
11. With the core box plane resting on the dressed surface and against the strip, as shown at (c), plane away the wood to a depth of approximately ⅛″.
12. Turn the piece end for end in the vise, and repeat these operations along the other gaged line.
13. Remove the strip entirely, and continue planing until half of the box is planed, as shown in (d).
14. Turn the piece end for end in the vise, and repeat these operations on the remaining half of the box, planing until the place is reached where the planing left off while planing the first half. The piece now appears as in (e).
15. Draft the ends slightly, cutting under from the dressed surface.
16. Obtain two pieces of wood, ¾″×2″×4½″, to be used for ends on the box.
17. Nail the ends to the box; plane the ends evenly with the outside of the box.
18. The core box is ready for finishing.

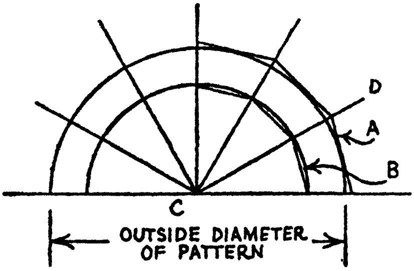
Making a Stave Pattern or Core Box. Large cylindrical patterns and core boxes are made by gluing staves to headers, something like the construction used in making barrels. The headers provide stock on the ends for fastening the staves. These patterns usually follow the two-piece, or split pattern, type of construction. Two halves are required to make a complete pattern, and one half-round core box for making the cores. The pattern and core box are just the reverse of each other in construction, as shown below. In the pattern, the inside of the stave is fastened against the header, and the outside is shaped to size. In the core box, the outside of the staves is fastened to the header, and the inside is shaped to size. The number of staves to be used, and their thickness, depends upon the size of the pattern. The greater the number of staves, the nearer to finished size the work can be built. The correct shape of the headers, and the width and angle of the staves, can easily be obtained from a full-sized layout. The outside diameter of the rough size of the pattern is scribed on the layout board. This semicircle is divided into an equal number of parts by spacing with a dividers. If six divisions are to be made, the dividers would be set to half of the radius. Lines such as CD are drawn through the spaced off points to the center of the circle; these lines establish the width and side-angles of the staves. A chord A is drawn tangent to the circle; line B is drawn parallel to line A, representing the thickness of the stave, and establishing the size of the header.



HALF OF TWO-PIECE PATTERN

CORE BOX

Showing Construction of Stave Pattern and Core Box



Full-size Layout of Stave Pattern Construction

The method of laying out stave core boxes is the same as laying out patterns, except that the core diameter is used as the basic line instead of the pattern diameter. The subsequent spacing is figured outwardly instead of inwardly in establishing the header size. Bars, serving as braces, are used to connect the headers together, as shown above.

SELECTED REFERENCES FOR FURTHER STUDY

CAMPBELL, H. L. Metal Castings. J. Wiley & Sons.

LEWIS, M. S. & DILLON, J. H. Instruction Sheets for General Shop: Foundry. McGraw-Hill Book Co.

MCCASLIN, H. J. Wood Pattern-Making. McGraw-Hill Book Co.

RICHARDS, W. H. Principles of Pattern and Foundry Practice. McGraw-Hill Book Co.

RICHEY, J., MONROE, W. W., BEESE, C. M. & HALL, P. R. Pattern Making. American Technical Society.

WENDT, R. E. Foundry Work. McGraw-Hill Book Co.

CHAPTER XIII

METAL TRADES

Edward N. Wallen

63. PRESS WORK

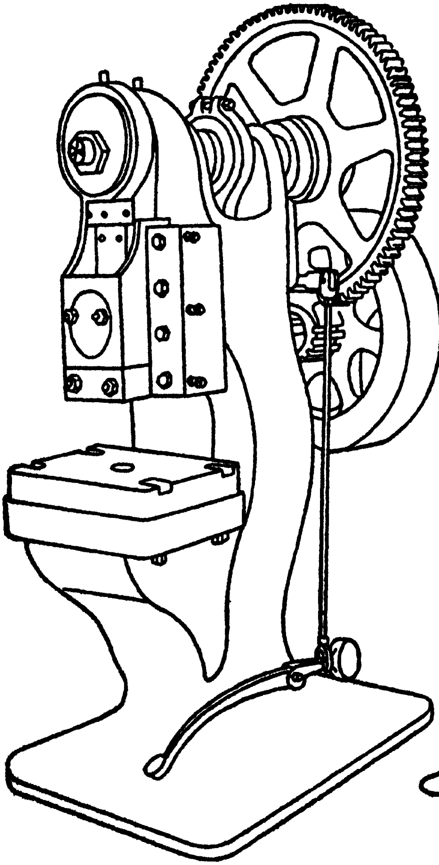
The Punch Press. The press has made possible the production on a commercial basis of a great many articles whose cost would be prohibitive if other manufacturing methods were used. The punches and dies used in press work are often very costly, but the speed with which the required articles are produced and the simplicity of operation after the punch and die are set up makes the unit cost of production very small. Punches and dies are made by a specialized tradesman who must be first of all a good machinist. Die-making is a branch of toolmaking and is probably the highest paid. There are many different types of dies and progress in their application is being made constantly.

Types of Presses. Presses range in size from small bench presses used in jewelry and other light work to the gigantic presses used in the making of automobile bodies. It is difficult to classify presses because of the many different types. They may be classified by use, by construction, or by the design of their frames. The suggested classifications are frequently found in combination in the same press. Manufacturers usually list their presses according to the intended use of the press. In this classification there are presses for punching or blanking out metal; for drawing metal into elongated shapes such as cartridge cases; for embossing metals; for forging hot metals; for trimming, perforating, etc. The presses may be of the single action, double action, multiple crank, toggle, or cam type of construction; this refers to the mechanism by which the slide is forced down to do its work. The frames of a press may also be used as a means of classification; some frames are inclinable, others are straight-sided, or they may be gap, arch, adjustable bed, etc. The feed may be by hand or automatic. Some presses are operated by means of a foot treadle or pendulum, although the majority are motivated by power.

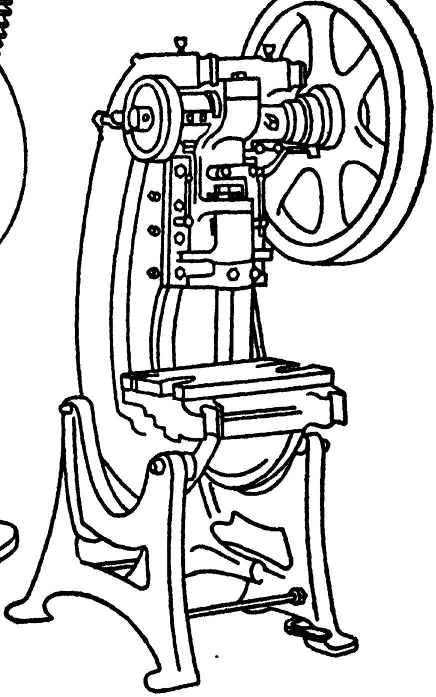
Inclinable presses may be used with the frame in a vertical position, or it may be inclined from the vertical. The purpose of inclining a press is to make it easy to clear the die of finished work. Some dies deliver the finished part at the top of the die and the part then slides off of the die by gravity. If the same die were used in a vertical frame press it would be necessary to remove the part by hand or construct some mechanism to remove it. Inclined presses are built with the frames held at the required angle. They should not be confused with inclinable presses which are adjustable for angles.

A *straight-sided* press is built with the press sides or frames vertical. These presses are intended to handle heavy press jobs without distortion of the frame. They are extensively used for heavy stamping, punching, and trimming operations. *Arch* presses are practically the same as the straight-sided press in all but the frame. The frame of the arch press is in the form of an arch.

Punching presses have a slide at the front of the frame instead of be-



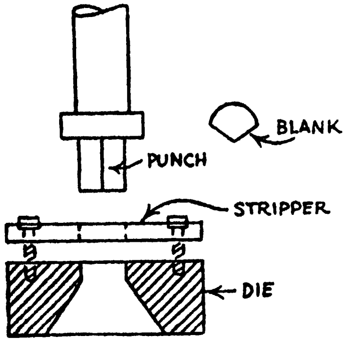
Punch Press



Inclinable Press

tween the sides of the frame. A shaft extends through the frame to the rear of the machine. A pulley or gear is attached to the shaft. The slide is moved by a crankpin on the front of the shaft. The vertical position of the slide may be varied by an adjustment between the crankpin and block.

Punch Press Dies. The most commonly used is the *blanking die.* In this die a punch, which is held in the punch carrier of the press slide, is forced down against the sheet metal that is being blanked. The punch shears a piece of the sheet metal through the die. The waste metal of the strip is forced off of the punch, on the return stroke, by means of a stripper plate. The alignment of the punch with the die is assured by guide pins. The sketch of a simple blanking die shows the clearance that is given in the die to permit the blanks to fall through without binding; this clearance is usually from about 3° to 5°.

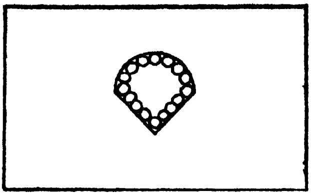


Simple Blanking Die

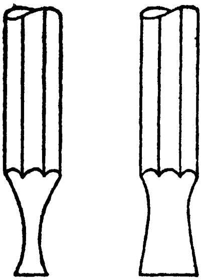
Dies and punches are made of tool steel. The accuracy of the punch when made is determined by the probable number of pieces that the die is expected to make. If the die is to be used for the production of a great many parts it will have to be made more accurately than it would if only a small number were to be made. This greater initial accuracy is required to compensate for the unavoidable wear that will occur during use.

The diemaker would shape his die blank to the thickness and outline called for. He would then lay out the outline of the piece that is to be blanked. If the shop is equipped with a metal cutting band saw, the diemaker would then drill a hole inside of the outline, break the band saw, pass the saw through the hole, weld the saw ends together and proceed to saw out the waste material. The metal cutting band saw is equipped to weld the saw ends and grind down the thickness of the blade at the weld. After the waste material was removed the diemaker would file to the outline of the blank using a template or gage to check his work.

If no band saw is available the diemaker must remove the waste material by the following method. Inside of the outline of the blank he must scribe another outline. On this he makes a series of center punch marks spaced so that the edge of one drilled hole will just touch the edge of the adjoining hole. He then drills holes all around the layout, and then cuts the thin wall which is left between the holes with a broach chisel. The die is then filed to fit the template. This method is slower and more laborious than the metal band saw method and is seldom used in up-to-date shops.



Die Blank Drilled



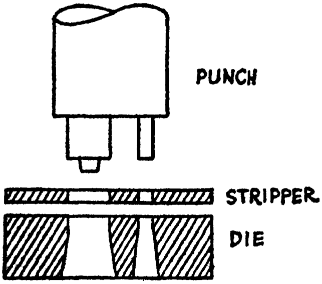
Broach Chisel

When the die is finished it must be hardened. Special, non-warping steels are used for the making of dies, for if the die warps to any extent during the hardening process it may be ruined. Slight warping is corrected by *stoning* or grinding the die until it again fits the template.

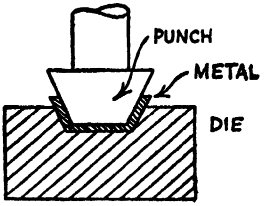
The punch is roughed out as closely as possible to the shape of the die and is then held in a punch press and carefully forced into the die. This is called "shearing in" the punch. After the punch is hardened it must be checked for fit because of the possibility that it may have warped.

Multiple blanking dies are made with a number of punches and die openings. The position of the punches in a multiple blanking die is very important. Aligning one punch with a hole is a simple matter, but where there are a number of punches to be aligned real skill is required.

Follow dies are made for work where a piece must be cut from the strip and also be perforated or punched. These dies are made so that one part of the die cuts the blank and another part perforates it. The stock is fed along through the die so that a completed piece results from each stroke of the press. The perforation is usually made first; the stock is then moved along until the pierced section is under the blanking punch. *Gang* or *multiple* dies differ from the follow dies in that they are made to produce a number of duplicate pieces at each stroke; each punch of a gang die cuts its own blank. There is no following punch to perform a second operation. In a *compound die* the upper and lower members of the die assembly carry a punch and a die with a stripper plate. Compound dies produce more accurate work than the regular blanking and follow dies do. The stock is supported between the two opposing stripper plates and die faces while the piercing and blanking is carried out.

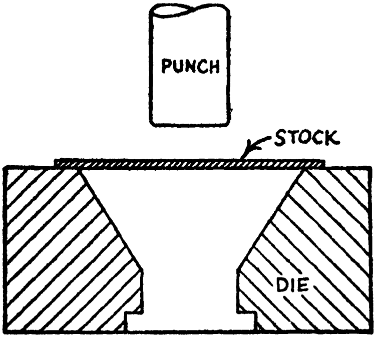


Bending dies are made for the purpose of bending wire or sheet metal parts into required shapes. The bending is accomplished by forcing the material into a depression by a punch which corresponds in shape to the depression, or by slides which force the metal into the shape wanted.

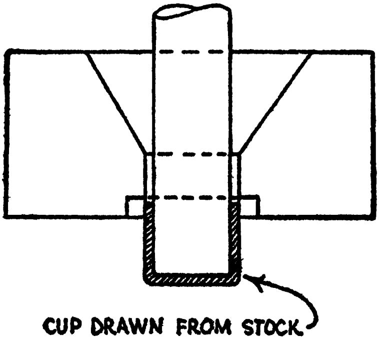


Simple Bending Die

Drawing dies are used to draw parts from flat metal into cylindrical or other shapes. The blank is first cut from the strip metal and is placed in the drawing die. The punch descends and forces the blank into the die, thus forming it into a shell or cup. Drawing dies may be made so that blanking and forming are combined. They are called combination drawing dies. Some drawing dies are made with a knock-out to force the finished cup out of the depression; others are of the push-through type where the finished cup is pushed down through the die opening by the next cup that is made.

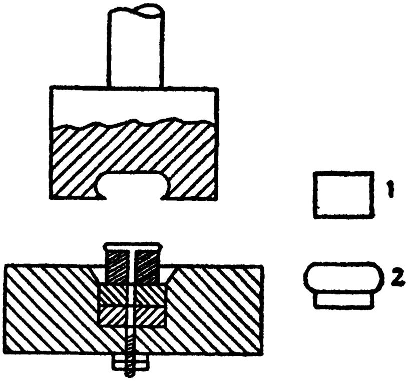


SIMPLE DRAWING DIE



Bulging dies are used to form shells that have a larger diameter than the opening. The cup or shell is drawn on a drawing die and is then placed over a mushroom plunger. The punch has a cavity in its face which determines the outline of the finished bulge. Rubber discs are under the metal top of the bulging die. When the punch descends on to the cup it compresses the rubber which forces the metal out into the space of the punch face; when the punch ascends, the rubber returns to normal and the cup can be lifted off of the die.

Embossing dies are used to form raised designs or letters, etc. upon the surface of the work. Embossing differs from forming in a relative sense. Designs made by embossing dies are shallow, whereas forming dies shape the whole piece. The raised circles on tin cans are examples of the work of embossing dies.



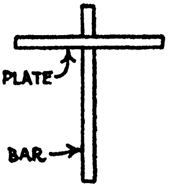
Coining dies are used to form designs on metals, jewelry, silverware, etc. They operate in very powerful presses. In coining work both elements, upper and lower, are dies. The stock is pressed between the two dies and the metal is compressed to form the design.

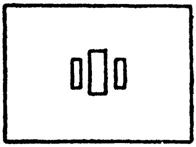
Curling dies are used for forming curled edges around drawn cups, etc. Curling dies that are used to form a bead or curl around a wire are usually called wiring dies.

Perforating dies are used to make large numbers of holes in sheet metal. Strainers, sifting devices and other objects in which a large number of holes must be made are products of the perforating dies. These dies are also called piercing dies.

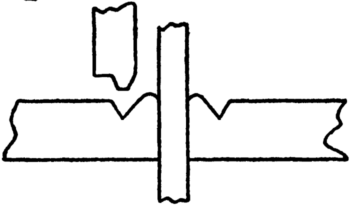
Swaging dies are used to displace metal to fill up space or to produce rounded edges or ends. They are sometimes used in assembling mechanical devices. An example of this assembly use of swaging dies is found in telephone apparatus where thin brass plates are fastened to flat galvanized bars; the bar fits through the plate and the plate is then swaged so that the displaced metal tightens the plate against the bar.

The dies mentioned in the preceding pages are for the most part simple examples. In production work the dies are often made in combined forms so that a die may do a number of the operations which have been described here. The diemaker's art and the design of dies is responsible for the rapid development of economical manufacturing of many products whose cost would be prohibitive to the consumer if they were made in any other way. The initial cost of dies is often very great. A die may cost thousands of dollars to make, but when the cost of the die is spread over the number of finished pieces made by the die, the unit cost of production is reasonable.





TOP VIEW SWAGED



ENLARGED VIEW SHOWING SWAGED EFFECT

Hand-fed presses have been responsible for many injuries to workmen. All presses should be equipped with safety guards that will make it impossible for the operator's hands or any part of the operator to be in a hazardous position when the press slide operates. Some presses are equipped with "electric eyes" (photoelectric cells) to prevent the slide from moving while any part of the operator's body is in a dangerous position. Another form of safety device is the pendulum guard in which a swinging pendulum sweeps across the front of the machine as the slide is coming down thus knocking away anything that would be too close. Another type is used in which the operator's hands fit through bracelets which are attached to the press so that the hands are forcibly pulled away as the slide descends, if they are not already removed.

Die Casting. Die casting has become an important factor in the rapid production of interchangeable parts for many mechanical devices. Radiator grills of some cars, typewriter parts, gears and many other products are now die cast. A die casting requires little or no finishing after it is made beyond the removal of the *flash* which may develop if the die is worn at the edges. Die casting as such is not new; it was used to manufacture lead bullets before the American revolution. The term die casting, however, should be applied only to the casting of molten metal under pressure, since the ordinary pouring of lead into a mold is not generally included in the modern concept of the process known as die casting. A well-known example of die casting is the production of printer's type. The Linotype machine is a die casting machine in which lines of type are set up and cast in rapid succession.

Essentially the die cast machine includes a melting pot for the cast metal, the die which determines the shape of the finished product, a plunger activated by hand or compressed air, and a valve to control the passage of the molten metal to the die.

Die Casting Alloys. The metals used in die casting have a comparatively low melting point. They are generally alloys of two or more of the following metals: lead, tin, zinc, aluminum, copper, antimony, and magnesium. Usually, one of the constituents predominates, and is referred to as the *base;* thus a lead-base type might consist of 85% lead alloyed with tin and antimony; etc.

A typical tin-base die casting alloy is the following high grade Babbitt metal (S.A.E. specifications, ♯10 Babbitt):

Copper	4.5%
Antimony	4.5%
Tin	91.0%

Such an alloy would be used for shaft and connecting-rod bearings in automotive and aircraft production. Another example of a typical die casting alloy is the following zinc-base type, used for automotive parts and many household appliances:

Copper	10.0%
Aluminum	4.0%
Magnesium	5.5%
Zinc	80.5%

Pressure die castings made with this alloy are easily machined, have high ductility and tensile strength, and are resistant to corrosion.

The dies used in casting are very expensive to make, and the method of die casting is applied only to parts which must be produced in quantities that make it possible to regain the cost of the die.

Operation. In common with most machine types, die casting machines are also made to operate automatically. The operator of an automatic machine has only to fill the secondary melting pot with metal and carry away the finished product. The secondary melting pot is used to melt the pigs of metal so that the main melting pot temperature will not be lowered through the introduction of cold metal. After the casting has been made and removed from the die it must be trimmed to remove the sprue and flash. The *sprue* is the excess metal that is left at the pouring point; the *flash* is a light fin that appears around the edges of the die if there is any lack of tightness between the two halves of the die. All die castings are water cooled to ensure quick cooling. If the rate of cooling is slow the casting will be weak because the crystals of the metal will be large.

Casting dies correspond to the cope and drag of a sand mold but are so different in all other respects that a further comparison is difficult. The dies are usually made of machine steel because hardness is of little or no importance and the heat of the molten metal would soon draw the temper of a hardened die. The dies have to be made with an air vent so that the air within the die can escape as the molten metal is forced in.

64. FORGING AND WELDING

Hand Forging. Metals reach the machine and manufacturing shops in the form of rods, bars, castings or forgings. Forging may be defined as the plastic working of steel. Steel, when heated to a sufficiently high temperature, becomes plastic to some degree; it can be caused to flow into different shapes by the application of pressure such as that received from hammer blows. When a piece can be forged to its approximate finished shape and size there is a decided saving of material and machining time. Forged articles also possess qualities of strength and durability that are superior to the same qualities in articles that are cut from the solid stock. Most of the forged products are made with the use of power hammers and forging dies; however, the mechanic occasionally has a job of hand forging to do and he should have a knowledge of the tools, materials and methods used in hand forging.

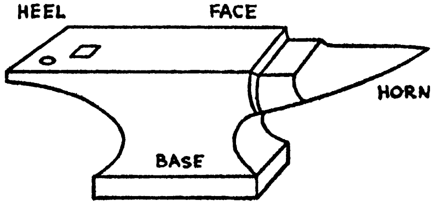
Forges are of three types; they are fired by gas, oil or coal. The coal fired forge is used a great deal in field work because it is readily portable and may be set up quickly without the necessity of having gas connections. It is the traditional forge of the blacksmith. Essentially it consists of a fire pot, grate, blast pipe, blower, coal box, water tank and hood. Strictly speaking, the coal forge burns coke, that is, the bituminous coal which is used for fuel is wetted down by the smith and is thus formed into coke. For good results the coal must have a low sulphur content, since sulphur is injurious to the qualities of steel. Coking the fuel drives off most of the gases which are contained in the coal and gives a clean fire with little smoke.

Gas fired furnaces are in common use in machine shops. They are cleaner to handle and easier to operate than coal forges. The gas fired furnace is equipped with a blower and an automatic shut-off valve to cut off the gas supply in the event that the air supply fails. In lighting a gas furnace the air must be turned on before the gas is. If the air blast should be cut off the result would be an explosion of the gas in the furnace chamber. When shutting down a gas furnace the gas is turned off first and then the air may be turned off.

The mixture of gas and air should be proportioned properly for best results. If too much air is admitted the flame will be what is known as an oxidizing flame; the excess oxygen in the flame will attack the surface of the steel and will form a heavy black scale. The surface of the metal will probably show surface pits when the scale is removed due to the unevenness of the scale formation. If too little air is used the flame is called a reducing flame. The result of a reducing flame is surface softness. The outer skin of the metal loses some of its carbon content as a result of a reducing flame and the character of the steel at the surface is definitely changed. A neutral flame has just the right proportion of air and gas; it may be recognized by watching the interior of the furnace—slight wisps of flame will be seen near the top of the fire chamber.

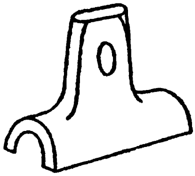
The following experiment shows the appearance of the different types of flames. Place a small piece of wood in the fire chamber of the furnace. It is assumed that the furnace is lit. Slowly shut the gas valve and watch the piece of wood. (Note: Safety first—always look into a furnace from the side or from a safe distance in front.) Now open the gas valve to allow more gas to enter. Repeating this experiment a few times you will notice that you can cause the wood to blaze or to go out while still in the furnace. An excess of air will cause a definite flame to appear on the surface of the wood and the wood will burn; a deficiency of air will put out the fire on the surface of the wood and it will be baked, not burned. The neutral mixture of gas and air will cause the wood to smolder without breaking into flame.

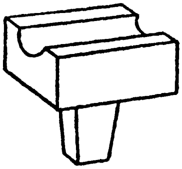
Tools Used in Hand Forging. The blacksmith's anvil is his workbench. The anvil is interesting in that its design has not changed for centuries. Its shape and proportions are admirably suited to all types of light forging and there seems to be no need for improvement. When we consider the changes in design that have occurred in practically all other forms of tools it would seem that the ancients who developed and made the forge did an excellent job. Anvils were formerly made of wrought iron with a plate of tool steel welded on to form a face. They are now frequently made of cast steel. Anvils are sold by weight, i.e., the price is quoted at so much per pound. The principal parts of the anvil are the horn, the face, the heel, and the base. In the *heel* of the anvil there are two holes; one is square and the other round. The square hole is called the hardie hole and is used for holding the square shank tools used in forge work. The round hole is the pritchel hole; it is used for punching holes through the metal and for bending small round stock.

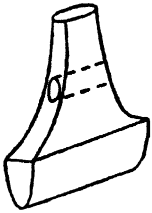


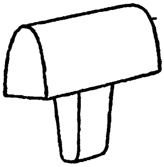
Swage blocks are heavy rectangles of steel or wrought iron. The edges of the block are notched with a variety of semi-circular grooves. The body of the block has a number of differently sized and shaped holes through it. The swage block is used for drawing round stock, for piercing metal, and for light bending.

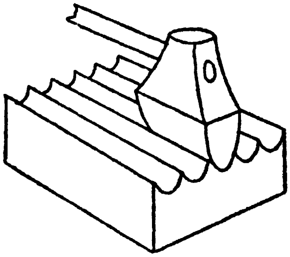
Anvil Tools. The anvil tools of the blacksmith have a square shank to fit into the hardie hole of the anvil. Most of the tools are made in pairs; that is, one tool is placed in the hardie hole, and the same type of tool, its counterpart in a tool of similar shape but fitted with a handle, is held on top of the work. The anvil tools that are used on the bottom are called *bottom tools* and the handled-tools are called *top tools.* The swage is used for shaping, sizing and smoothing round forgings. They are designated by the diameter of the round groove which shapes the forging. The eye of the top swage is made so that the forger's helper can stand at the side of the anvil. *Fullers* are used for necking and grooving forgings, and also for drawing down a forging to a smaller size. Fullers are also made so that they can be used close to a corner or to start a square corner. Fullering is the name given to the operations which are accomplished with the fuller tool. The sketches show two examples of a fuller in use. The bars of steel are presumed to have been heated to a forging heat so that they are semi-plastic; the fuller is being used to force the metal into new positions so as to increase the length and reduce the cross-sectional area.

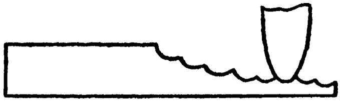

TOP SWAGE


BOTTOM SWAGE

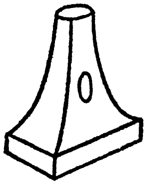

TOP FULLER


BOTTOM FULLER

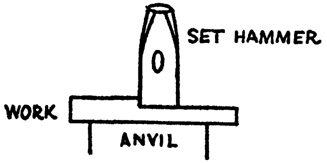

Fullering a Bar to Widen It.

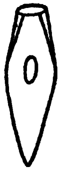

FULLER USED TO DRAW OUT A BAR

Hand Forge Tools. After a piece is drawn out by means of the fuller it must be flattened to remove the ridges that the fuller has made. The tool which the forger uses for this purpose is called a *flatter;* it resembles somewhat a sledge-hammer head, but is not used for striking. It is held on the hot metal by means of its handle and is struck with a sledge. The set hammer is used in much the same manner but its purpose is to produce sharp shoulders on the work.

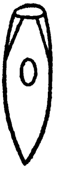


FLATTER



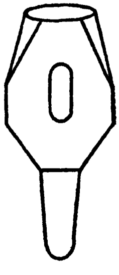


HOT CHISEL

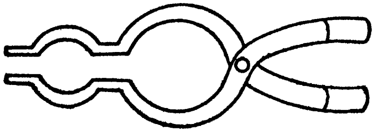


COLD CHISEL

The blacksmith's chisels are held by a handle so that they can be struck with the sledge. There are two types of chisels; one is the *cold chisel* which is used for nicking cold metal, the other is the *hot chisel* for cutting hot metal. In cutting cold metal the smith does not attempt to cut through the piece: he nicks it on both sides and breaks it off. The hot chisel cuts through the piece as a rule because hot metal cannot easily be broken; it will only bend. Punches are used for making holes in heated metal. They are held by a handle as are the other top tools; their working end has been drawn out to form a round or square punch.



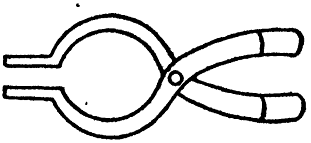
Punch.



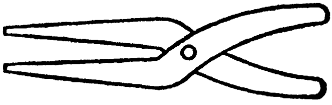
Double Pick-up Tongs.

When a piece of heated metal is being worked on by the smith it must be held securely by tongs. The fit of the tongs is important, and there are a number of different shapes, each for its special purpose. *Pick up tongs* are rarely used for forging; they are intended for picking up pieces of metal that may be around the forge. It should be remembered that it is not good practice to attempt to pick up metal that is lying around near a forge—it might still be hot enough to burn severely. Single pick-up tongs are used for picking up heavier pieces; the jaws are stronger and heavier. *Straight lip tongs* are used for holding thin flat work. The stock in the jaws is heavy, and the jaws may be adjusted by the smith to handle individual needs. *Box tongs* are used to hold rectangular work; since work may be jarred loose from the grip of flat tongs, box tongs are best because they grip the work on four sides. There are a number of other tongs which are used by the forgeman, such as rivet tongs or bolt tongs, side tongs, link tongs, and eye tongs; each has a distinctive shape which is easily recognizable as to its purpose and name. The hammers used in forge work include the *ball peen hammer* of the machinist, but very much heavier in weight. The smith's ball peen is from 2 to 2¼ pounds in weight. He also uses *cross peen* hammers of about the same weight. The *sledge* hammer is used in several weights depending on the class of forging which is being done; it will run from 10 to about 18 pounds.

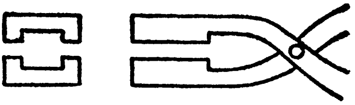
The tools used in forging are made of several kinds of steel. The cutting tools, such as chisels and hardies, and also the hammers, are made of tool steel. These tools are heat treated to combine hardness with some toughness. The fullers, swages and other tools that must stand considerable battering are made of medium carbon steel; they are also heat treated to increase their toughness. Tongs are made of low carbon steel; they are not heat treated as they need possess no hardness, but must instead be tough.



Single Pick-up Tongs.



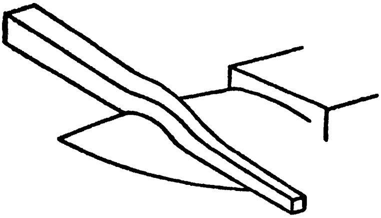
Flat Tongs.



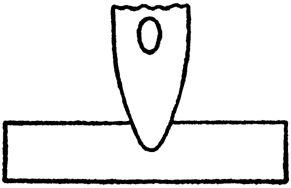
Box Tongs.

Forging by Hand. In general, hand forging operations are those which are intended to alter the shape of the metal, to bend it, to make holes through it and to weld it. The technical names of the forge operations are *piercing, drawing, upsetting, offsetting, shouldering, welding,* and *bending*. For any of these operations the temperature at which the metal is worked is important. The heat must be high enough to give the required amount of plasticity without overheating. A knowledge of the nature of steels and irons is essential to successful forging; this topic will be discussed later. A forging heat is one which is adequate for the shaping of metal under pressure; a welding heat is higher. Wrought iron can be safely heated to a white color, about 2700° F. Steels vary in the correct forging heat, depending on their composition; steel of about 0.50% carbon should not be heated beyond a yellow color, or about 2200° F.

Drawing. In this operation the length or width of the metal is increased. When a hot piece of metal is struck by the hammer the particles of metal in the vicinity of the blow or pressure are displaced and do not recover their original position. If the piece were held on the face or top of the anvil while struck, the blow would tend to flatten it in all directions. The horn of the anvil is used for drawing, or the rounded edge of the fuller is used, depending on the size of the piece which is being drawn. The metal is caused to flow lengthwise rather than sideways when it is forged on the rounded surfaces.

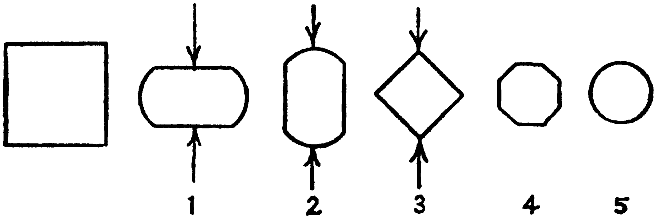


Drawing out a Piece on the Horn of the Anvil.

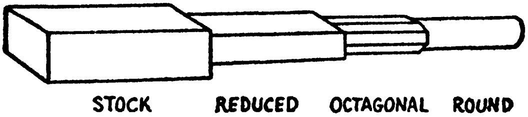


Using a Fuller to Start a "Draw" on a Piece.

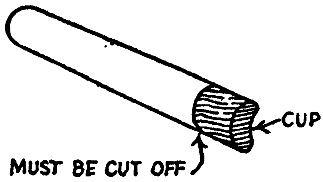
If a round or square piece is to be drawn into a smaller cross-sectional area it must be worked progressively; that is, the forging must be done on all sides in approximately equal amounts to avoid producing longitudinal fractures and seams in the finished piece. A round bar that is to be reduced to a smaller diameter should first be forged to a square of about the same cross-sectional area as the finished round bar is to have. It is then forged to an octagonal shape and finally brought to the round shape by using the right size swage for finishing.



Steps in Drawing to a Reduced Round Cross Section.



In any type of forging the stock must be heated slowly to the forging temperature to ensure even heating throughout the piece; the forging should stop when the piece has fallen to below a dull red color. If the heating is not uniform the flow of the metal will not be uniform, and the ends of the forged piece will be defective. When the end of a forged piece is uneven it must be cut off or "cropped," involving a loss of metal.

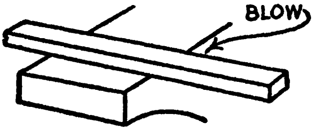


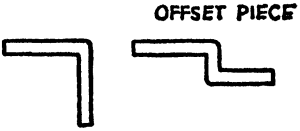
The length of time required for heating depends on the shape and composition of the metal.

The pressure exerted on the piece being forged is also of great importance. If the blows or pressure are light, only the surface metal will flow, and the center of the piece will not be affected. This will result in the "cupping" effect shown in the sketch. Judgment must be used to determine the force of the blow as well as the heat.

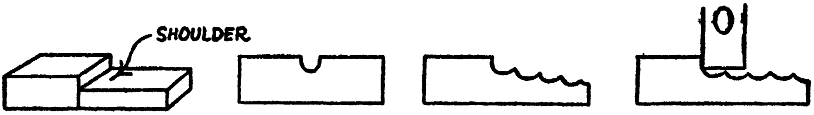
Upsetting. This is a forging process in which the cross-sectional area is increased. It is the reverse of the drawing operation. Bolt heads are formed by the upsetting process. Force is applied to the end of the heated stock and the result is a thickening of the bar. The method of upsetting is determined by the size and shape of the piece; it may be accomplished by hammer blows, or by ramming the piece against an unyielding surface, such as the anvil face or some block of metal. When a short heavy piece is to be upset so as to increase its diameter throughout its entire length, even heating is necessary, as it is on all forging operations. If the piece is heated more at the ends than in the center, the result will be an hourglass shape; if the piece is heated more in the center than at the ends, the piece will be bulged in the middle. The effect of the sledge or hammer blow must be felt throughout the piece in order to produce even results. If a piece is to be upset at one end only, as in the case of a bolt head, it should be heated at the end that is to be upset. If it is impossible to heat locally the piece should be cooled through the part that is not to be upset by immersion in a cooling bath.

Offsetting. This means the production of a piece with an offset in its length. For example it may be necessary to produce a piece that will have two right-angle bends in it to give two parallel lines. The bends may be made by striking the piece to form it over the edge of the anvil, or it may be necessary to use square pieces of stock as a forming die.





Shouldering. This refers to the production of a shoulder at some point on a bar of stock. The shoulder may be made on one or more sides. The fuller, set hammer and flatter may be used for the production of a shoulder. The fuller is used to start the reduced section, and to draw it out if necessary; the set hammer is used to make the square corner; and the flatter, to smooth the results of the fuller drawing.



FINISHED SHOULDER (a) START (b) DRAWN (c) SET HAMMER SQUARING SHOULDER

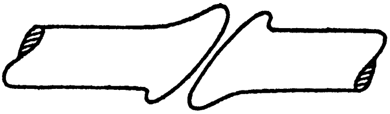
Welding. This term is applied to the joining by cohesion of two pieces of metal. Cohesion must not be confused with adhesion in this connection. In a properly made weld the surfaces of the two pieces will have completely blended and become fused together so as to make a solid junction at the weld. The surfaces which are to be welded must be clean, and no oxidation or foreign matter must be present to prevent a perfect intermingling of the surface particles. When a weld is properly made it should be very difficult to locate the line of junction between the two original pieces. The piece should be as strong at the weld as it is at any other section of equal cross-sectional area.

A welding heat may be determined visually by the appearance of the heated piece. The color is white and the surface of the metal will have a greasy appearance due to the readiness of the metal to flow. When it has begun to melt and is ready to weld, it is placed in contact with another piece at the same heat, and the two surfaces will flow into each other; they must be assisted, of course, by pressure from the blows of the sledge or hammer. A *flux* is also needed to keep the oxygen of the air from combining with the white hot metal during the short time that it is exposed to the air while being placed in position for the blows and during welding. The purpose of the flux is to prevent oxidation. Sand and borax are both used as fluxes in welding. Sand has a high fusion point, but in combination with ferric oxide it becomes fusible at a fairly low temperature and is therefore useable as a flux.

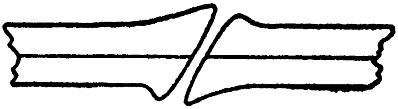
The greatest obstacle to a successful weld, aside from the proper temperature which must be reached and maintained until the weld is completed, is the formation of a film of oxide on the surface of the hot metal. Oxides form slowly at low temperatures and very quickly at high temperatures. The type of flame in the furnace has a distinct effect on the formation of the oxide; an excess of air in the flame will therefore increase the tendency of the iron to combine with the excess oxygen. The iron in the steel (Fe) combines with the oxygen (O_2) to form Fe_2O_3. Another difficulty that arises from the formation of Fe_2O_3 is the fact that the remaining carbon in the zone that is oxidized will pass off as a gas CO or CO_2; this leaves the zone adjacent to the weld with a lower carbon content than the rest of the metal, and the weld will therefore have a tendency to weakness. It must be remembered that a flux has no other function than the protection of the surface to be welded against oxidation. If an excess of flux is used and if it is not forced out from between the welded surfaces it, too, will have a deleterious effect; it will make a dirty weld, and the foreign matter will prevent a proper cohesion of the metals.

Preparing Metal for Welds. There are a number of standard welded joints which must be shaped before the weld is attempted. In all of these joints the preparatory shape is intended to ensure good contact at the center of the weld and the forcing out from between the weld of any foreign matter such as excess flux. The preparation of the joint is called *scarfing*. Scarfing includes the upsetting of the metal at the point to be welded to allow for the expected loss through oxidation and hammering.

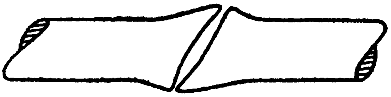
The *lap weld* is the most commonly used weld and is so called because the edges of the pieces lap over each other.



Round Stock Scarfed
for a Lap Weld.

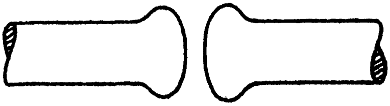


Square Stock
Scarfed for Lap Weld.



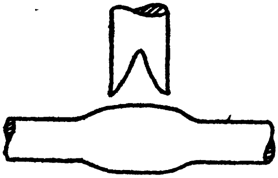
Poor Scarfing—Poor Welding.

Butt welds are seldom made on small pieces but are made on large shafts, etc. The weld is made by ramming the prepared ends together and is reliable if properly made. It is common practice to start the weld with the pieces in the fire and finish the weld, after the pieces have been joined at their centers, by removing them from the fire and using the anvil and hammer blows. The scarf for a butt weld is easily made by heating the end, upsetting it, and shaping it so that it is convex.

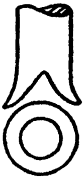


Scarf for Butt Weld

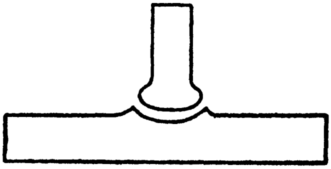
Jump welds are made by joining the end of one piece of metal to the other at a right angle. It is a difficult weld to make because there is not as much opportunity for hammering as there is in other welds. The scarf must be very carefully made and fitted. When two flat pieces are to be jump welded, the upright member is given a convex scarf which is fitted to a concave scarf of the horizontal member. When round pieces are jump welded the upright member is cleft and the horizontal member is upset. The cleft will fit over the bulge of the upset section and will weld satisfactorily if properly fluxed and heated.



Upset and Cleft for Round Jump Weld.

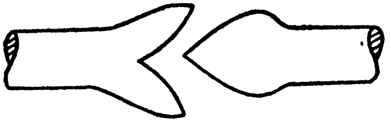


End View.

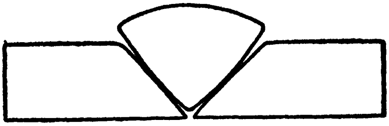


Flat Pieces Prepared for Jump Weld.

Cleft welds are used to join high carbon steel to low carbon steel, and for joining pieces that have a large cross section. When high carbon steel is to be welded to low carbon steel by a cleft weld, the high carbon steel is made the insert or wedge of the cleft to protect it from loss of carbon content.



Low Carbon High Carbon
Cleft Weld



Vee Weld.

Sometimes a weld must be made on pieces that are so shaped or so large that they cannot be upset and scarfed in the manner used for smaller pieces; the *Vee weld* is used in such cases. An insert is prepared to fit into the notch which is formed by the two ends which are placed in position to receive the insert. It is obvious that this type of weld can be employed only on pieces that are so heavy that they will retain their position during the welding process.

The use of pressure welds such as have been described and which are also known as blacksmith's welds is limited to situations where no hazard to personnel is involved. There are so many factors involved in the making of a blacksmith's weld and so much depends on the personal skill of the smith that such welds should be restricted. The human element enters into the making of a pressure weld to such an extent that the area of the welded section should be materially increased in order to ensure safe working strength with safety. Some of the factors involved in the making of a blacksmith's weld are: (1) the weldability of the material; (2) the furnace; (3) the atmospheric conditions of the flame; (4) the fuel; (5) the formation of scale, dirt or oxides; (6) the flux; (7) the temperatures of the two pieces; (8) the possible difference in the metal of the two pieces; (9) overheating; (10) decarburizing of the metal which is being heated and welded; (11) the possible difference in shape and cross-section of the two pieces; (12) the ability of the workman; and (13) the change in grain structure of the welded parts.

Grain growth or crystallization of metals is a phenomenon that is associated with high temperatures. The strength of a metal is correlated with the size of the grains that compose the metal; the greater strength as a rule is found in fine-grained metals. When a steel is heated to a very high temperature the grains show an increase in size. If the heat has not been too high the metals may be heat-treated to restore the grain size to a stronger one, but if the steel has been "burned" this is practically impossible. All welded parts should be annealed in the vicinity of the weld to refine the grain structure. This is done by heating the metal to a bright red color and allowing it to cool slowly. Mechanical working of metal will reduce the grain size to some extent and forge welds should be given a thorough working at the weld and in the vicinity of the weld. Generous upsetting for the scarf of the weld point should be the rule also so that the excess material will make it necessary to work the welded section back to size and thus will compress the grains.

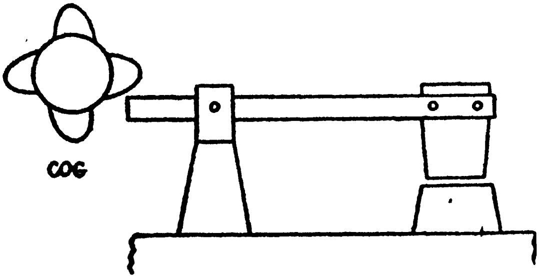
The foregoing discussion of forge work and forge welding has been necessarily brief. The maxim that there is no learning without doing is very pertinent to this work and to all welding processes. Practice and work with metals plus a diligent study of the materials used is the only way to become competent in the field.

Machine Forging; Power Hammers. The size of work that can be handled by man power is obviously limited by the strength of a man, while the size of work than can be handled by machine power is limited only by the capacity of the machines, which can be built as large as needed. Both forging and welding require quick heavy blows to produce the necessary metal movement before the metal loses its heat. The blow must be heavy enough to be felt deep in the metal. The complete forge shop is equipped with the necessary power hammers, which are of several types.

Drop Hammers utilize the force of gravity. The hammer is attached to a board or arm which is raised vertically above the work; when it is released, the force of gravity pulls it down and it strikes a blow with a force depending on its weight and upon the distance dropped. Drop hammers are used to a great extent when forging with a die; the heated metal is placed over the die and is forced into the shape of the die by the blow of the drop hammer. The weight of the hammer used will depend on the maximum size of the stock to be forged; stock of 2 inches diameter will take a hammer of 200 pounds, whereas a hammer of 20,000 pounds is used for a piece 36 inches thick.

Helve hammers have the hammer attached to a beam which is lifted and dropped by a cam, or, in the primitive form, by a cogged wheel. Helve hammers are suited to light work and can deliver a great number of blows quickly. The helve is made of wood, but the fall of the hammer is not due to gravity alone. Buffers of rubber or springs are compressed as the hammer head is raised, and these buffers push the hammer down when the helve is released by the rotation of the cam or eccentric.

Trip hammers resemble a punch press in operation. The length of the stroke can be adjusted and the hammer is operated by a foot lever. The trip hammer is belt driven and the foot lever operates a clutch for starting and stopping. The hammer moves vertically in guides so that the die faces are always parallel. The number of blows per minute that can be struck by the helve hammer or the trip hammer ranges from 15 per minute to 450, depending upon the size of the hammer.



Principle of Helve Hammer

Steam hammers are actuated, as the name implies, by steam pressure. The hammer is raised and controlled by slide valves in the same manner as a steam engine piston. Expert steam hammer men can control their hammers to deliver light or heavy blows, and to stop the hammer in its path wherever they want it to stop. It has been claimed that a good steam hammer man could crack the shell of an egg without spilling the contents. Steam hammers are classified by their falling weight, which is the combined weight of the hammer, piston rod, rings and die. The largest hammer ever built is about 80,000 pounds. This hammer was built for forging shell casings.

General Welding. Welding as performed at the forge has its limitation. It is not possible to bring all shapes or sizes of metal to the forge, nor is it possible to weld in place. Another limitation to the pressure type of weld is the fact that only the ferrous metals respond satisfactorily to this method. The other methods of welding, electric arc, acetylene torch, thermit, are more adaptable. They can be used for a variety of metals other than the ferrous as well as on the ferrous metals. It is possible to take the heat to the part to be welded and to do welding in place.

The principles of welding are the same whether the weld is made by the forge method or by other methods. The forge method may be termed a *plastic weld* in contrast to the *fluid weld* of the newer methods. In gas and thermit welding the welding metal is in a more fluid state than is the metal when welded at the forge. In either case the heat must be great enough to cause the metal to become at least semi-liquid. In gas welding the welding rod is liquefied. There must be a homogeneous union of the metals, and there must be no foreign matter or oxide between the surfaces of the metals which are to be fused as such dirt would prevent a proper fusion.

Acetylene welding uses the two gases, acetylene and oxygen, mixed in a blowpipe or torch to form a flame of very high temperature. The gases are supplied in containers (cylinders) which are connected by hose lines to the torch. Valves are provided for regulating the mixture of the gases and the flame. The torch is designed so that tips with different sizes of orifice may be used. The various sized tips are intended to produce a larger or smaller flame according to the thickness of the material which is being welded.

Cutting torches are designed, as their name implies, for the purpose of cutting metal. The action of cutting is accomplished by first heating the metal with a mixture of acetylene and oxygen and then cutting off the acetylene, keeping a stream of pure oxygen directed on the metal. The burning with the oxygen in contact with the hot metal increases the temperature of the metal, and in a sense the metal burns itself up. Iron and steel are easily cut with the cutting torch, but other metals are more difficult to cut because they form oxides that fuse at higher temperatures than that of the metal.

Oxygen is stored in steel cylinders which are strong enough to resist the high pressure of 2000 pounds per square inch. Each standard cylinder will hold about 224 cubic feet of oxygen. Precautions against fire must be taken. Where a number of cylinders are to be stored they should be kept in specially built rooms equipped with adequate fire protection devices; such rooms must be clean as oily or greasy materials will cause serious fires if there is any leakage.

Acetylene is also stored in cylinders. The acetylene cylinders are filled with a porous material, the purpose of which is to divide the space within the cylinder into innumerable small spaces to prevent the acetylene from breaking up under pressure. When acetylene is to be charged into the cylinder, the cylinder is partly filled with acetone. Acetone dissolves many times its own volume of acetylene. The acetylene-bearing acetone permeates the porous filler of the cylinder and the resulting small globules are safe for transportation and use. Acetylene will explode at a pressure of 15 pounds per square inch even if no air is present. The cylinders contain about 300 cubic feet of gas at a pressure of 225 pounds per square inch.

The actual welding is done by melting a filler (or, as it is commonly called, a welding rod) into the space between the metals which are to be joined. The metal adjacent to the space must of course be heated to a plastic state so that a perfect union can be made. Welding rods are made of a number of materials because the welding metal must be similar to the metals which are being joined. Fluxes must be used with practically all metals except low carbon steel. A flux is seldom needed when welding low carbon steel because the oxide that is formed during welding remains liquid and is readily displaced by the molten metal; it does not interfere with the union of the metals.

In the discussion of forge work the several types of flames were mentioned. It was pointed out that a neutral flame, one that did not contain an excess of air or gas, was preferred. The same is true of the welding torch flame. A neutral flame is best for most work. The adjustment of the regulating valves controls the type of flame that is produced. Some operators judge the flame by its appearance, while others observe the sound of the flame as the gases emerge through the orifice. The beginner would do well to secure the opinion of an experienced welder and to practice regulating the flame until the sound and appearance of a neutral and reducing flame are clearly understood.

Brazing is a process in which parts are joined by means of a spelter solder. *Spelter solder* is a mixture of copper and zinc. The operation is somewhat similar to that of welding in that the spelter is melted into the preheated space between the metals which are to be joined. A flux is needed when brazing. The blowpipe is used as it is for regular gas welding. The temperatures needed for brazing are not as high as in welding.

Use of the Torch for Hardening. A relatively new process in machine work is local hardening of gear teeth and machine parts by means of the welding torch. The part to be hardened is heated to a hardening temperature and a stream of water is directed on the spot or area. This technique has increased the resistance to wear without the possible distortion that would result from heating and quenching large areas.

Thermit Welding. Thermit is a mixture of finely divided aluminum and iron oxide. The aluminum in the mixture has a high chemical affinity for oxygen, and when the mixture is ignited by the application of sufficient heat the chemical reaction between the aluminum and oxygen proceeds so rapidly that the iron in the mixture melts. The iron flows into the welding space and unites with the parts to be welded. Thermit is safe to handle as it requires a high temperature to become ignited. The ignition temperature is about 2800° F., but the resulting temperature of the chemical reaction is about 4500°. Thermit may be ignited with a match, but the safer practice is to stick the match in the mixture and light it with a long hot rod to avoid the spattering of the thermit when it catches fire. A mould must be built around the parts which are to be welded. The thermit powder is placed in a crucible where the ignition takes place. The mould must be made with vents, pouring gates, etc., just as in the case of a regular mould in foundry work. The parts are preheated and then the thermit is ignited. The molten steel which results from the ignition flows from the bottom of the crucible into the mould where it fills the space between the parts and forms a homogeneous mass.

Electric Welding. The passage of an electric current through an area of increased resistance results in an increase in heat at and adjacent to the area of high resistance. This fact is utilized in electric welding. An electric current is caused to flow so that the high temperature results in the melting of metal at the parts to be welded. The molten metal joins the parts as in gas welding or other forms of welding. The electric current is brought into proximity with the metal to be welded by means of an electrode. As the current flows through the electrode an arc is "struck." The operator must have the ability to hold the electrode so that a steady arc is produced. The arc depends on the distance between the electrode and the mass of metal that is being welded. It can be seen that this calls for a steady hand and good control.

Arc welding may be done with alternating current or direct current. The alternating current used in ordinary electric wiring is usually of 60-cycle frequency. Normal frequency welders use a 60-cycle current, but the effect of the alternation is a series of intervals during which no current flows to the arc. This makes it difficult to produce a stabilized arc. Arc welding machines are built in both the 60-cycle frequency and also in higher frequencies so as to reduce the intervals during the cycle. Direct current arc welding makes it possible to control the polarity of the arc. The part that is being welded is used as the positive electrode and the rod is used as the negative electrode. The purpose of this is to concentrate the heat where it will be most effective. It is estimated that two-thirds of the heat of the arc is generated at the positive electrode and only one-third at the negative electrode. In alternating-current welding the heat is the same at both electrodes since the polarity shifts with each cycle.

Spot Welding. This refers to electric welding in which the current flows through a copper conductor until it reaches the pieces which are to be welded. The increased resistance to the passage of the current results in a fusion between the parts. Spot welding is used for light sheet work. It may be done by hand or by machine. The hand-operated spot welders have the advantage of portability and can be used in places that are difficult to reach. The machine spot welders are well suited to the rapid quantity production of light, sheet metal products.

65. HEAT TREATMENT OF STEEL

The Nature of Steel. The characteristics and properties of the ferrous alloys can be altered by proper heat treatment. The machined piece that is produced in a machine shop often has to be heat treated before it is acceptable for use. Heat treatment may be performed for the purpose of making a piece of metal soft so that it can be machined easily; or the piece may be toughened to resist strains, hardened to resist abrasion or cutting, or normalized to remove internal strains. The softening process is called annealing; it will be described later.

An understanding of the nature of the ferrous metals is necessary in order to understand properly the various processes of heat treatment. The development of new steels, each with different characteristics and each calling for a slightly different heat treating technique, has complicated matters somewhat. Metallurgists have brought out many new steels and other metals in the last forty years. A complete discussion of the subject is impossible here because of space limitation and therefore only a general treatment will be given.

The basis of steel is iron. When carbon is mixed with the iron in certain proportions the resulting metal is known as steel. Steel can be defined as a chemical combination of iron and carbon. The percentage of carbon that is contained in the steel determines the hardenability of the steel. It should be noted here that iron as cast also contains carbon, but in a different manner than does steel. The dark color of iron is due to its carbon content; it is the result of excess carbon which is not combined chemically with the iron.

An elementary comparison for the purpose of better understanding may be made by considering salt and water. As every one knows, salt will dissolve in water up to a point which is known as the saturation point. From that point on the salt will remain undissolved in the liquid. Iron, likewise, will absorb a certain percentage of carbon but will fail to do so after its "saturation point" is reached and the carbon will remain in the metal mass uncombined or undissolved. In making steel the carbon is removed from the molten iron and a definite percentage is then replaced or added; this process changes the "cast iron" into steel.

The percentage of carbon in the steel is referred to as being so many "points." One per cent of carbon by weight would be 100 points of carbon. An 85-point steel would contain 85/100 of one per cent carbon. The number of "points" of carbon that can be combined with iron before the "saturation" point is reached is about 170; after that point is reached the metal is in the iron class. In practice, steels are seldom made with more than about 150 points of carbon, as they would then be too hard and brittle.

In shop language steels are referred to as low carbon or soft steels, medium carbon steels, and high carbon or tool steels. The low carbon or soft steels contain from a trace of carbon to about 25 or 30 points. Low carbon steels do not contain enough carbon to permit hardening. Medium carbon steels contain up to about 60 points of carbon and respond slightly to heat treatment. They can be toughened to some extent, but do not make fine cutting tools. The tool steels range between 65 and 120 points of carbon. Some steels for cutting hard materials are made with up to 140 points of carbon.

When a grain of iron unites chemically with carbon it forms a grain which is known as *cementite.* This is sometimes referred to as hardening carbon. The percentage of cementite in the steel (in other words, the percentage of carbon, since that is contained in the cementite grain) determines the type of heat treatment that must be used in order to harden the steel. If the percentage is low it is impossible to harden the steel without chemicals.

Tool steels (high carbon) are hardened by heating to a temperature depending on the carbon content. As the steel is heated it passes through several transformations of structure. The steel men know these structures as *pearlite, sorbite, troosite, martensite,* and *austenite.* The first, pearlite, is so called because its appearance under the microscope suggests the layers of a pearl. The other structures are named after metallurgists who discovered them. With each change of temperature there is a change in the characteristics of the metal. When the steel has been heated to slightly above the martensitic range it possesses its maximum hardening power. If heated much beyond that stage there is a possibility of an increase in the size of the grains. This phenomenon is called grain growth; the grains are coarse and the steel will be brittle.

If, when the steel is heated to its best hardening range, it is quenched rapidly, the characteristics of hardness will be fixed into the steel. The quenching must be done at a fast rate to prevent the steel from dropping back to the softer structures of sorbite and pearlite.

When heating steel that is to be hardened several considerations must be kept in mind. The steel must be heated evenly, for if there are thin sections as well as heavy sections, the thin sections could be overheated before the thicker sections were ready for quenching. The piece must be heated evenly throughout its mass.

When the steel has reached the temperature at which it may be hardened it is said to have reached its critical point. The critical point is the point of transformation from one structure to another. A piece of steel may have several distinct transformation points occurring at different temperatures, but the transformation point which marks the change into martensite is the critical one for hardening. There are several ways in which the critical point can be verified. If the steel is watched it will be noticed that the color of the metal changes slightly, and a darkening or flush appears to pass over the metal in spite of the fact that heat is being added steadily from the fire. Another phenomenon that is connected with the critical point is the loss of power to attract a magnet. If a magnet is suspended from a wire or rod and brought near the steel it will react to the steel until the critical point is reached; after that the magnet is no longer attracted to the metal. Of course the magnet cannot be allowed to become heated or it will lose its magnetism. If a pyrometer were applied to the steel the indicator would show a steady rise in temperature until the critical point is reached. At that point the indicator would pause, and, if sensitive enough, would drop back slightly. This shows that a momentary loss of heat occurs at the critical point, which accounts for the darkening of color or flush which has been mentioned as occurring at this point.

The quenching medium or bath varies for different steels; some are quenched in water, some in oil, and some are air-cooled. The commonest steel is the water-quenching steel. Oil-hardened steels are recommended for die work and for tools in which distortion or warping must be kept down to a minimum. The capacity of the quenching tank must be large enough to extract heat from the steel so fast that the transformations referred to will not take place. When the steel is plunged into the tank it should be moved around in the water until it turns black. If this is not done, steam bubbles may form on the surface of the metal and the result will be a soft spot. The water will be kept from the surface where the steam bubble is, and the temperature at that point will not be reduced fast enough to preserve the martensitic structure which is desired.

After a rapid quench, the steel will be hard; however, the mechanic should always test the steel with a good file to prove its hardness. The file teeth should slip over the metal and not bite if the steel has been properly hardened. A characteristic of steel that has been hardened is its increased brittleness. If a tool were put into service directly after hardening, the chances are that it would quickly break.

Tempering of Steel. Tempering is a heat treating process which follows hardening. Its purpose is to increase the toughness of the steel without materially reducing the hardness. It is accomplished by reheating the hardened metal to a temperature that will give the desired amount of toughness. Some shops are equipped with tempering plates or ovens which may be adjusted to give the temperature needed, but most machine shops are not so equipped. The machinist polishes the piece that has been hardened and then carefully exposes the polished piece to the heat of the furnace. As the piece warms up a coating of oxide forms on the surface. The color of the oxide is an indication of the temperature of the metal. When the desired color is reached the piece is cooled in the quenching bath to prevent any increase beyond the desired temperature. If the tempering is carried too far the hardness will be lowered too much and the piece will have to be rehardened.

TEMPERING COLORS

Pale yellow . . .	430	degrees	Fahrenheit
Light straw .	450	"	"
Dark straw .	470	"	"
Brown	490	"	"
Brown turning to purple	510	"	"
Purple	530	"	"
Bright blue .	560	"	"
Dark blue	600	"	"

Tools are tempered to colors and temperatures according to their use. Cutting tools such as chisels are made tougher than the cutting tools used in machines, drills, etc.; the chisel has to stand more shock than other cutters. Wrenches and screw drivers are tempered higher than cutting tools; their hardness is required to resist wear, not to cut other materials.

In general, the following colors are used for tool tempering:

Pale yellow	430	degrees. . . .	Cutting tools for lathes, etc.
Light straw	450	"	Milling cutters and reamers.
Dark straw	470	"	Taps and dies.
Purple	530	" . .	Screw drivers, wrenches, etc.

The process of tempering as described is a double operation; the hardening completed, the piece is cooled and polished, and then tempered. Some tools are shaped so that the process of tempering can be speeded up. A chisel, for example, may be heated to a hardening heat, immersed in the quenching bath for about half of its length and moved around in the water until the end is black. The chisel, still held in the tongs, is then polished for a portion of the length by means of a piece of emery cloth wrapped around a stick or file. No attempt is made to produce a good polish, the object is simply to remove the black oxide so that clean metal can be seen. The heat that is still in the black part of the chisel will creep down until it reaches the clean part and the tempering colors will appear. This method of tempering saves time and also leaves a soft end on the chisel. The butt of a chisel should be soft as the hammer blows would crack a hardened butt.

Annealing is a process of softening steel by heating it to above its critical point and then allowing it to cool as slowly as possible. Boxes of sand or ashes are sometimes used for annealing purposes. The hot piece is placed in the box and covered with sand or ashes to retard the rate of cooling. It can be seen that annealing is the opposite of hardening in purpose and method of cooling. A so-called rapid anneal is sometimes used where time must be saved. It is not as effective as a full anneal. In a rapid anneal the piece is heated and then allowed to cool naturally in air until it is black; this process may be repeated.

The low carbon steels cannot be hardened in the same manner as the tool steels because they lack sufficient hardening carbon. Machine parts that are made of low carbon steel may be given a hard case by adding carbon to their surface. This changes the surface metal into high carbon steel at that point. The process is known as case hardening or pack hardening, depending on the technique used.

Case hardening is accomplished by heating the steel to a bright red and then placing it in contact with a carbon-bearing chemical. Cyanide of potassium is very effective for this purpose, but is a deadly poison. There are several patented compounds on the market which are used for case hardening in place of the old method of cyaniding. The piece absorbs enough carbon to change the nature of the surface metal and it is then reheated and quenched just as a piece of solid, high carbon steel would be. The depth of the case depends upon the temperature at which the metal was placed in contact with the carburizing material and also upon the number of times the process was repeated before the final quenching. It seldom exceeds 5 or 6 thousandths of an inch, however. No tempering is necessary after case hardening because the piece has a soft core which has retained its toughness. No brittleness has developed.

Pack hardening is another technique of case hardening. The case secured by pack hardening is much thicker than can be produced with the cyanide or other compounds. It may be 20 to 30 thousandths thick. Pack hardening is a much slower process than ordinary case hardening. The pieces which are to be pack hardened are placed in iron boxes and are surrounded by carbon-bearing material such as bone meal, leather, or other forms of carbon. The box is covered and sealed with fireclay and is then placed in a furnace where it is maintained at a red heat for hours. At the expiration of the time the box is removed and the pieces are heat treated as is tool steel. The result is a layer of hard tool steel around a soft core of low carbon steel.

Measuring High Temperatures. Hardening temperatures of steel range from 1350 to 1600 degrees. Since it is important to hold each steel within a comparatively narrow range of temperature in order to get the best results, it is therefore necessary to have some means of measuring the temperature. As the steel heats it becomes luminous; the color will depend on the temperature, and will range from a dull-red to white. A well trained eye can estimate the temperature of the steel by watching the colors as they appear.

COLOR OF STEEL AT HIGH TEMPERATURES

752 Degrees F....	Red heat visible only in the dark
885 " " . .	Red, visible in shade
975 " " .. .	Red, visible in the daylight
1077 " "	Red, visible in bright light
1292 " "	Dark red
1472 " " .. .	Dull cherry red
1652 " " ..	Cherry red
1832 " "	Bright cherry red
2012 " "	Orange red
2192 " "	Orange yellow
2372 " "	Yellow white
2752 " "	White, a welding heat

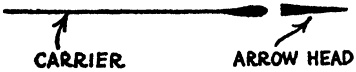
The human eye is subject to variations in vision and color perception, and, although it has been termed by heat treaters as the original pyrometer, it can only be relied upon for rough estimates of temperature. Well equipped heat treating rooms and machine shops will be provided with more accurate means of measuring the temperature of steel.

There are several ways in which high temperatures can be measured. The *optical pyrometer* is used in some cases because of its portability. It is operated on the principle of matching colors. The operator looks through the pyrometer and compares the color of the steel that is being measured with a standard color in the optical pyrometer. Some of the optical pyrometers are equipped with glasses of colors which will blend into the steel if the steel is of the same color. Since the corresponding temperature of each color glass is known, the temperature of the steel or fire is also known. Another form of optical pyrometer uses a small filament which is heated by an electric current. On "fixing" the pyrometer on the piece to be measured the filament will be visible against the background of the steel until the color of the steel and the filament are the same, until they blend together; the temperature of the steel is then read from a built-in indicator on the pyrometer.

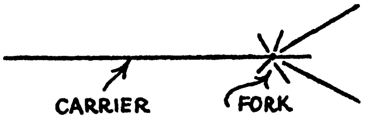
The *thermoelectric pyrometer* uses an interesting electrical phenomenon. The reader is probably aware that an electric battery develops current because two different metals are placed in an acid bath. A somewhat similar thing occurs when two different metals are placed in a bath of heat; the metals used must be strong enough to stand high temperatures, of course.

The electric pyrometer consists of a very sensitive millivoltmeter which has a scale that is graduated to read in degrees of heat instead of electrical units. Two wires extend from the meter to a thermocouple. The thermocouple consists of two wires which are separated by nonconductors of electricity down to their ends. The insulators are usually made of porcelain. The wires are twisted together at the ends and may be brazed or welded for better contact. In fixed installations where the thermocouple is inserted permanently into a furnace, or where any danger of mechanical injury exists, the thermocouple is enclosed in a protecting iron tube to guard against damage from a blow. The electric current that is generated at the thermocouple tip flows back to the pyrometer and is registered as degrees heat. A variety of thermocouples is available for varying temperature ranges. The thermocouple should seldom be used beyond its rated maximum capacity. Temperatures as high as 3000° can be measured, in brief intervals of exposure, with some high-temperature thermocouples.

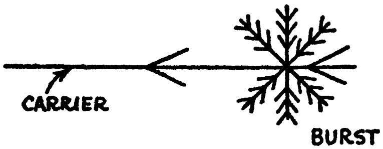
Identification of Ferrous Metals. Many embarrassing mistakes have been made by partially trained mechanics who were unable to tell the difference between different classes of steel. Industrial concerns usually paint an identifying stripe along bars of steel so that the stock men can recognize each kind; however, after the paint has been removed there is no such safeguard. Some mechanics attempt to judge the kind of steel by its weight or ring when dropped on a piece of metal or on the floor, a method which is not too reliable, however.



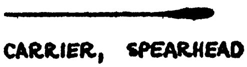
Wrought Iron Spark



Low Carbon Steel; about 20 points



Carbon Steel; about 35 points



Fortunately there is a simple and positive method of identifying different types of steel with quite accurate results. It is known as the *spark test.* Each steel throws a different kind of spark when it is held against a fast moving grinding wheel. The sparks vary in shape, color, action, and in relative quantity. Each spark has recognizable differences in shape. The line of the spark is called the *carrier*. As the carrier nears the end of its travel, or in some cases before, it either dies out or breaks into shapes which are called *forks, bursts, arrow heads,* or *spearpoints.* The lower the carbon content of the steel, the fewer bursts will be present. As the carbon content increases the number of bursts becomes greater. With experience it is possible to estimate the approximate per cent of carbon in the steel. The color of the spark stream is also important. It ranges from a dull red to a bright yellow. The irons throw an orange spark; carbon steels, a bright yellow spark; and tungsten steels, a globular red spark.

The learner is advised to observe the spark formations of steels that are available and whose type is known. Touch the object *lightly* to the grinding wheel, since the spark can be observed and studied just as well if not better when a small stream is being thrown by the wheel. Observe the sparks when a tool is being sharpened and inquire as to what kind of steel the tool is made of.

Hardness Testing. Hardness is a relative term; in general, it means the ability to resist abrasion or cutting. The hardness of metals is measured in several ways in modern industry. The hardness of a material is now specified in numerical terms, although until a comparatively few years ago only rule of thumb methods of testing were available.

A standard test for hardness by the mechanic is the *file test.* A good file, one that is not dull, is pressed against the piece and is pushed across its surface. If the file teeth "bite" the surface the piece is considered soft. If they fail to bite the piece is hard. Of course this method is a comparison between the hardness of the file and of the piece of metal. It has its place and should be used as a rough check. If the file cuts the steel easily, then it may be assumed that the material can be drilled, turned or otherwise machined. For manufacturing purposes, however, where engineering requirements have specified that a certain definite amount of hardness is needed, the file test is useless.

The *Scleroscope* hardness test is made by observing the height of rebound of a steel hammer which is dropped on the steel. The instrument consists of a glass tube which has graduations on its back, a rubber bulb which operates a suction valve pulling the hammer up to the starting position, and a base on which the object to be measured is placed. The hammer, which is a small elongated plunger, is sucked up to the top by suction from the bulb; it is also released by pressing on the bulb. The glass tube must be vertical for accurate results, as the fall of the hammer must not be impeded in any way. The scleroscope is a quick easy test to make, but requires some quickness of eye to avoid error in reading the height of rebound. The piece must be held firmly on the table and the tube bottom must press slightly against the piece.

The *Brinell* hardness test is based on the principle that if a standard steel ball is pressed into the surface of a piece of metal it will make an indentation which will be an indication of the hardness of the material. The machine is essentially a hydraulic press. It is equipped with a holder for steel balls. The steel balls come in two sizes for measuring hard or soft material. The work is held on an anvil and the ball is then moved into position so that it touches the metal. Pressure is then applied and the amount of pressure is read on a gage at the top of the machine. The pressure is raised to 3000 kg. for hard materials; lower pressures are used for the softer materials such as brass. After the required pressure is reached it is held for a few seconds and then released. The diameter of the indentation is measured with a Brinell microscope. The microscope is small and has a low magnifying power. It has a small scale etched in its objective and also a small flashlight bulb and battery illuminate the scale. After the diameter of the impression is obtained, the hardness of the piece is found by consulting a table of hardness numbers which correspond to the diameters obtained with the standard pressures.

The *Rockwell* test involves measuring the depth of penetration made when a standard cone is pressed into the metal. The Rockwell machine uses a system of levers to increase the pressure penetrator and a dial to measure the depth of penetration. There are two types of penetrator for the machine; a small steel ball about 1/16 of an inch in diameter is used for soft material, and a diamond-tipped cone is used for harder materials. The cone is cut to an angle of 120°. The load applied (by weights) is 150 kilograms for hard materials. The pressure applied at the penetrator is greater than the weight of the discs placed on the machine.

The operation of the machine is as follows. The piece is placed on a suitable anvil and is then raised by means of an elevating screw until the penetrator is contacted. A lever at the top of the machine is in a forward position at this time to keep the weight off of the piece. The operator observes the dial as the penetrator makes contact, because a minor load must be applied at this point. The purpose of the minor load is to seat the penetrator and avoid slipping or shock when the major load is applied. After the minor load has been applied, the lever at the top is pushed to the back of the machine and the weight of the load is applied gradually. A dash pot which has a plunger working in oil is used as a shock absorber so that the load will be applied gradually. As the penetrator goes into the surface of the metal the dial needle moves an amount corresponding to the depth. The dial is in effect a linear measuring device which reads in thousandths of an inch. After the load has been fully applied there is a momentary pause, and then the lever is moved forward again. The minor load is still on the piece, and the penetrator is still in place. The reading on the dial after the major load is removed is a measure of the hardness of the piece. This method of testing has been adopted by many manufacturing concerns because of the simplicity of operation and speed of testing.

S.A.E. Steel Code. The Society of Automotive Engineers has devised specifications for steels that meet the needs of industry so well that the system is in general use in industry. The system is in effect a code which gives an indication of the chemical contents of the steel. Most of the code numbers have four digits, each digit indicating the per cent of some element of the steel. The first digit gives the class of steel; the second digit indicates the approximate per cent of the principal alloying element; and the last two figures give the carbon content of the steel:

1—Carbon steels
2—Nickel steels
3—Nickel-chromium steels
4—Molybdenum steels
5—Chromium steels
6—Chrome-vanadium steels
7—Tungsten steels
9—Silico-manganese steels

A steel that is listed as *S.A.E. 1070* would be a carbon steel, since the first digit is 1; since it is a carbon steel there is no alloying element, and the second digit is therefore 0. The last two figures tell us that this steel contains approximately 70 points of carbon. The percentage may vary from 65 to 75. A five-point tolerance is allowed for economy in manufacturing steel. Similarly, a steel listed as *S.A.E. 2330* is a nickel steel with approximately 3% of nickel and 30 points of carbon; or again, *S.A.E. 4340* is a molybdenum steel with 3% molybdenum and about 40 points of carbon; etc.

SELECTED REFERENCES FOR FURTHER STUDY

Broemel, L. Sheet Metal Workers' Manual. Frederick Drake.

Campbell, H. L. Working, Heat Treating and Welding of Steel. J. Wiley & Sons.

Ford Trade School. Metallurgy and Metallography. Henry Ford Trade School.

Herb, C. O. Die-casting. Industrial Press.

Hubert, E. H. (ed.) Manual of Electric Arc Welding. McGraw-Hill Book Co.

Hughes, T. P. Principles of Forging and Heat Treatment of Steel. Burgess Publishing Co.

Keller, J. F. Lectures on Steel and its Treatment. American Society for Steel Treating.

O'Rourke, F. L. Sheet Metal Pattern Drafting. McGraw-Hill Book Co.

Palmer, F. R. Tool Steel Simplified. Carpenter Steel Co.

Rossi, B. E. Welding. McGraw-Hill Book Co.

Simons, E. N. and Gregory, E. The Structure of Steel. Prentice-Hall, Inc.

Spring, LaVerne. Non-Technical Chats on Iron and Steel. F. A. Stokes Co.

Thum, E. E. Elementary Metallurgy. J. Wiley & Sons.

Van Leuven, E. P. Cold Metal Working. McGraw-Hill Book Co.

CHAPTER XIV

ELECTRICAL TRADES

George Enders

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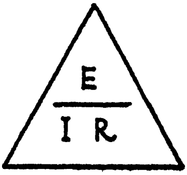
66. ELECTRICAL WIRING

Basic Units. The *volt* is the practical unit of electrical pressure; its symbol is E. Voltage is found by multiplying the amperage by the resistance (ohms); $E=I\times R$. The quantity of electricity is measured in *amperes,* the practical unit of electrical current; its symbol is I. Amperage is found by dividing the resistance (ohms) into the voltage; $I=\frac{E}{R}$. The resistance is the opposition to the flow of electricity—that which tends to hold the current back. It is measured in *ohms;* its symbol is R. Resistance is found by dividing the amperes into the voltage; $R=\frac{E}{I}$.

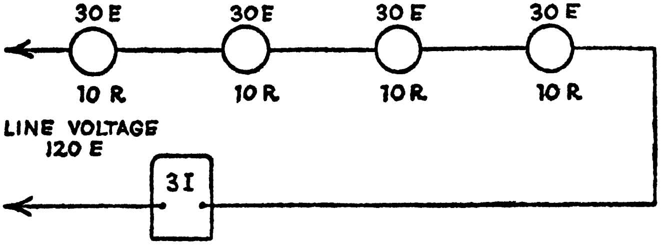
The above equations are known as Ohm's Law. A simple way of remembering these equations is shown in the diagram. To find any equation, cover that which you wish to find: e.g., to find the voltage, cover the E, and IR remains; thus $E=I\times R$. To find the amperes, cover the I, and $\frac{E}{R}$ remains; thus $I=\frac{E}{R}$. To find the resistance, cover the R, and $\frac{E}{I}$ remains; thus $R=\frac{E}{I}$



Fundamental Definitions. *Complete Circuit:* The course electricity travels from one polarity through a current-consuming device and returning to the opposite polarity of the source of supply. *Short Circuit:* The course electricity travels from one polarity *without* going through a current-consuming device or resistance and returning to the opposite polarity of the source of supply. *Grounded Circuit:* When the earth or metallic frame of a piece of equipment is used in lieu of a wire. This type of wiring is used for electric railways and for wiring certain types of low-voltage apparatus.

Ground on a Circuit. This is the condition that exists when a bare conductor accidentally comes in contact with the earth, directly or indirectly, or when a bare conductor comes in contact with the metallic part of a piece of equipment.

Series Circuit. This is the arrangement obtained when a series of pieces is connected successively from the end of one piece of equipment to the end of the next piece to form one continuous path for the current. This type of circuit is used for connecting certain types of equipment, the wiring for burglar alarm systems, and the gong circuit of fire alarm systems. In a series circuit the voltage divides across the pieces so connected and totals the line voltage. The amperage of a series circuit remains the same throughout the entire circuit. The resistance of a series circuit totals the sum of all the resistances connected in series.



The four 10-ohm lamps connected in series to a 120-volt line have 30 volts across each lamp and draw 3 amperes from the line; this can be proven by Ohm's law as follows:

Total Resistance$=R_1+R_2+R_3+R_4=10+10+10+10=40$ ohms.

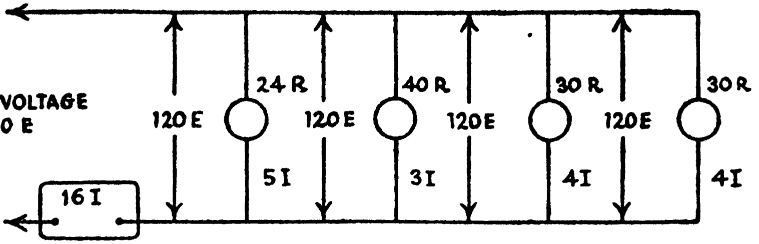
$$I=\frac{E}{R}=\frac{120}{40}=3 \text{ amp.}$$

$$E=I\times R=3\times 10=30 \text{ volts across each lamp.}$$

$$30+30+30+30=120 \text{ volts, line voltage.}$$

Parallel Circuit. This type of circuit offers two or more paths of flow for the current. To connect in parallel means to connect across, i.e., to connect from positive to positive, and from negative to negative. This type of circuit is the most common in the electrical field. It is used for wiring bells, push buttons, and all lighting with a standard voltage. The voltage or pressure across each piece of equipment is the same as the line voltage. The amperage of each piece of equipment totals to the line voltage. The combined or joint resistance is less than the lowest resistance connected in parallel.

In the figure, lamp #1 having 24 ohms resistance, lamp #2 having 40 ohms resistance, and lamps #3 and #4 each having 30 ohms resistance, are connected in parallel to a 120-volt line. Lamp #1 draws 5 amperes,



lamp #2 draws 3 amperes, and lamps #3 and #4 draw 4 amperes each. The ammeter in the main line draws 16 amperes. The voltage across each lamp is 120 volts. This can be proved by Ohm's Law as follows.

(a) If the voltage and resistances are known:

$$\text{Lamp 1: } I=\frac{E}{R}=\frac{120}{24}=5 \text{ amp.}$$

$$\text{Lamp 2: } I=\frac{E}{R}=\frac{120}{40}=3 \text{ amp.}$$

$$\text{Lamp 3: } I=\frac{E}{R}=\frac{120}{30}=4 \text{ amp.}$$

$$\text{Lamp 4: } I=\frac{E}{R}=\frac{120}{30}=4 \text{ amp.}$$

$$\text{Total amperes}=16$$

(b) If the amperage drawn by each lamp and the resistance of each lamp are known:

$$\text{Lamp 1: } E=I\times R=5\times 24=120 \text{ volts.}$$
$$\text{Lamp 2: } E=I\times R=3\times 40=120 \text{ volts.}$$
$$\text{Lamp 3: } E=I\times R=4\times 30=120 \text{ volts.}$$
$$\text{Lamp 4: } E=I\times R=4\times 30=120 \text{ volts.}$$

(c) If the amperage drawn by each lamp and the voltage are known:

$$\text{Lamp 1: } R=\frac{E}{I}=\frac{120}{5}=24 \text{ ohms.}$$

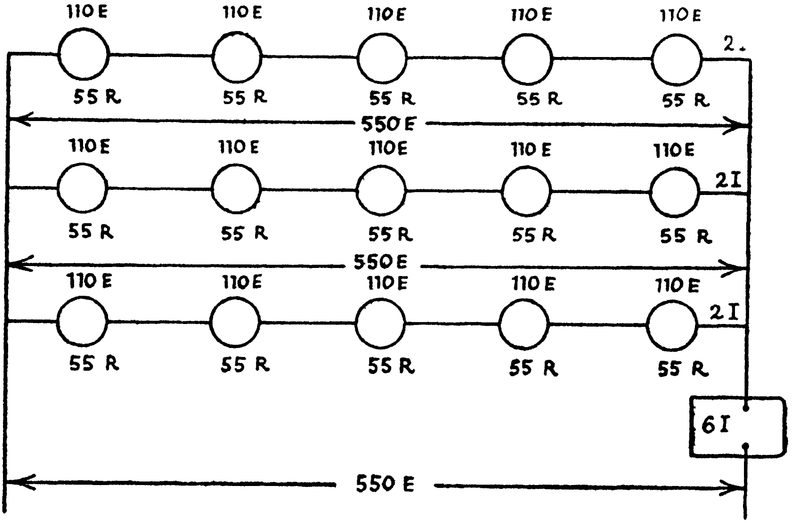
$$\text{Lamp 2: } R=\frac{E}{I}=\frac{120}{3}=40 \text{ ohms.}$$

$$\text{Lamp 3: } R=\frac{E}{I}=\frac{120}{4}=30 \text{ ohms.}$$

$$\text{Lamp 4: } R=\frac{E}{I}=\frac{120}{4}=30 \text{ ohms.}$$

(d) Joint Resistance: $R=\frac{E}{I}=\frac{120}{16}=7.5$ ohms, which is less than the smallest resistance.

Series Parallel Circuit. This type of circuit is a combination of series and parallel circuits, as the term implies. In order to make this connection the equipment has to be divided into groups. The equipment of each group is connected in series, and the groups are connected in parallel. This type of circuit is used for connecting batteries and for wiring lights in trains of electric railways where the railway system operates on 550 volts.



EXAMPLE: In the figure above there are three groups of lamps, each group connected in series and the three groups connected in parallel. The line voltage is 550 volts and the main line draws 6 amperes. Each lamp has a resistance of 55 ohms and the voltage across each lamp is 110 volts. Each group draws 2 amperes. Check by using Ohm's Law.

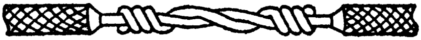
In a series parallel circuit the following characteristics hold true:

1. The voltage divides across each piece connected in series and totals the line voltage.
2. The amperage of the pieces connected in series remains the same.
3. The total resistance is equal to the sum of the resistance connected in series.

4. The voltage of each group connected in parallel remains the same.
5. The amperage of each group connected in parallel increases.
6. The combined or joint resistance is less than the total resistance of any of the groups connected in parallel.

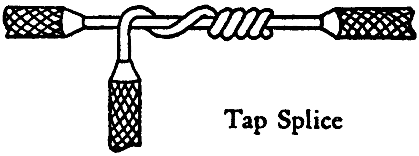
Electrical Joints. One of the most important phases of the electrical field is the selection and making of proper joints and splices. All conductors must be thoroughly clean before making a splice. All joints and splices must be mechanically secure to withstand strain, and electrically tight in order to conduct the electricity. A joint or splice improperly made produces heat and may start a fire. The most commonly used splices are as follows.

1. *Western Union;* used to extend the length of a wire. This splice is made by holding the two conductors with a pair of pliers, and wrapping the free ends around the conductors in opposite directions, towards the insulation.

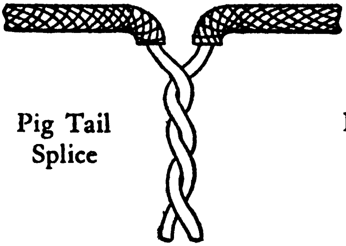


Western Union Splice

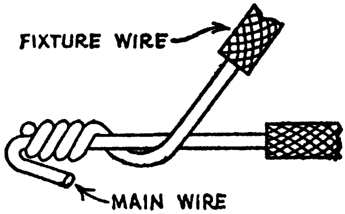
2. *Tee, or Tap splice;* used where a branch wire is to be taken from a main wire. It is made by wrapping the tap wire around the main wire.



Tap Splice



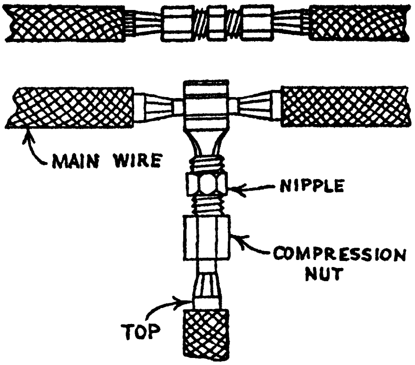
Pig Tail Splice



End Fixture Splice

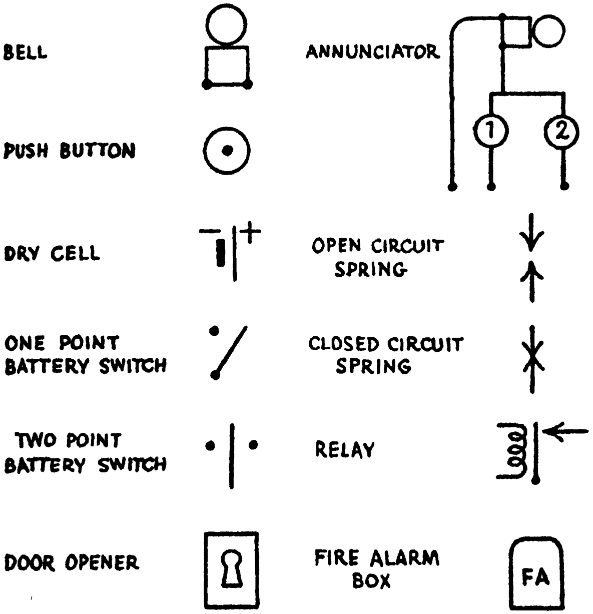
3. *Pig Tail splice;* used for joints in junction and outlet boxes. Pull the wires to be spliced at right angles to the outlet. Hold the insulated end of the wires with the left hand, cross the bare conductors about one inch below the insulation, and twist with the pliers until secure.

4. *End Fixture splice;* used when #18 fixture wire is to be joined to a circuit wire (usually #14). The large wire is bared about 2″, and the fixture wire about 3″. The fixture wire is wrapped around the circuit wire; then the circuit wire is bent back over the joints. Other less commonly used splices are the wrapped cable splices, wrapped tee or tap, and the split cable tee or tap. Solderless connectors are also used to a great extent.

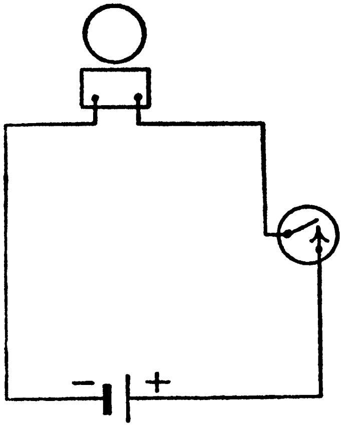


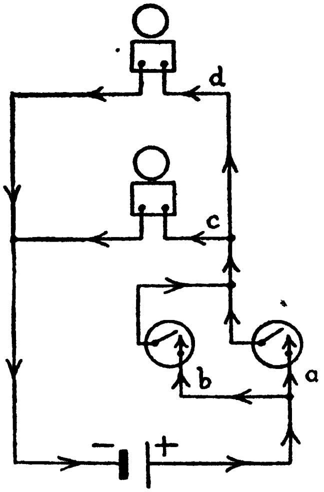
Soldering. All splices and joints used in light and power work must be soldered. The most common devices used are: the alcohol torch, which has a small flame and is used for soldering small splices; the soldering iron, used when an open flame is dangerous and in bench work; and the gasoline torch, which has a large open flame and is used for soldering large cable splices. The following are the steps taken in soldering splices: (1) Clean the splice; (2) Apply the soldering flux; (3) Heat the splice (using one of the above mentioned devices) above the melting point of solder, 375°F.; (4) Apply the solder so that it flows *between* the turns of the splice.

Wiring Systems; Symbols. The following symbols are the conventional designations of the various appliances commonly used in wiring circuits:



Simple Bell Circuit. To wire this circuit, a wire is run from the positive side of the battery to one connection of the push button; this is known as the *button-battery* wire. Another wire is run from the other connection of the push button to one of the terminals of the bell. This wire is known

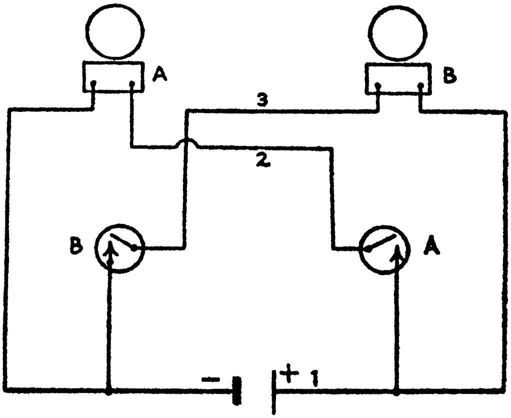


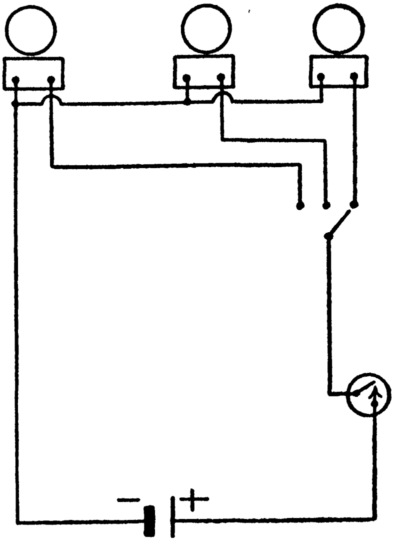


as the *section wire*. To complete the circuit, a bell-battery wire is run from the other terminal of the bell to the negative side of the battery.

Bells and Push Buttons Connected in Parallel. This type of connection is used where bells have to be rung at the same time from more than one location. In this type of circuit the button-battery wire feeds both push buttons. The section wire connects the other terminal of each push button with one terminal of each bell. The bell-battery wire connects the other terminal of each bell with the negative terminal of the battery. Note that there are two paths for the current to flow in the button circuit, one path through *a*, and one path through *b*. In the bell circuit the current divides two ways, through *c* and *d*. When more than one push button is to operate the same piece of equipment, they are connected in parallel.

Return-Call Bell System. This type of circuit is very common and is used where two persons wish to call each other. In this circuit it will be noted that there are only three wires between the stations. The button-battery wire for push button A is also the bell-battery wire for bell B, and the button-battery wire for push button B is also the bell-battery wire for bell A. By making the connections in this manner one wire is eliminated.

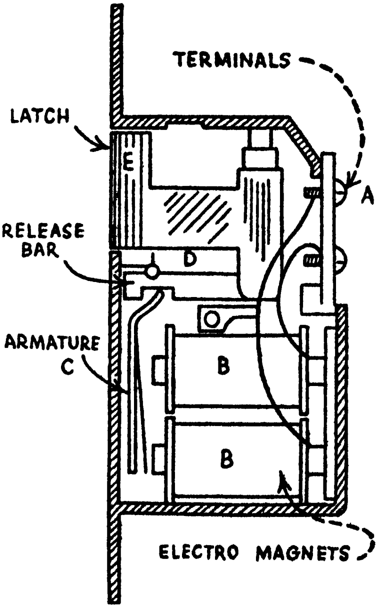




Battery Switches. The essential parts of a battery switch are: the insulated base, the lever and the contacts. The lever connects or disconnects any of the contacts as may be desired. By the use of battery switches, push buttons may be eliminated. As here shown, a three-point battery switch may be so connected that each bell can be rung separately.

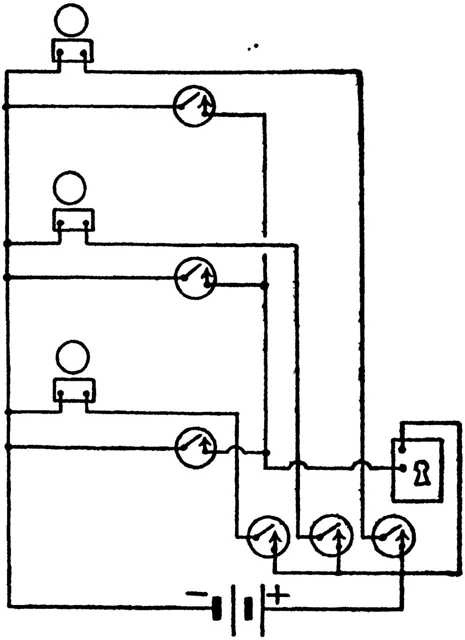
Electric Door Opener. This is a device by which doors may be opened from remote locations. It consists of the following parts: a frame, two electromagnets, a release bar, a latch, an armature and the terminals. There are two types: the mortise and the rim.

When a current is impressed on the terminals (A) the electromagnets (B) become energized and the magnetism attracts the armature at (C). By this action the release bar (D) frees the latch (E) and the door is free to open. Push buttons controlling a door opener are always connected in parallel. The figure shows the wiring diagram for a three-family house with push buttons at the entrance to operate a bell in each apartment; in each apartment there is a push button which operates door opener at the entrance.



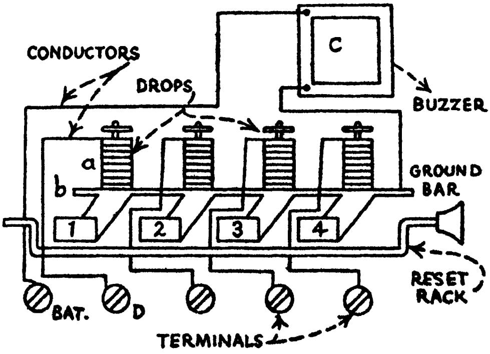
Electric Door Opener

Annunciator Wiring. Annunciator systems are installed to terminate the calls at one location. If several bells or buzzers were installed at one location, confusion would result in determining which bell had rung. On annunciators a buzzer sounds to attract attention and a signal comes into view telling where the call emanated. Annunciators are used in offices, elevators, hospitals, etc.



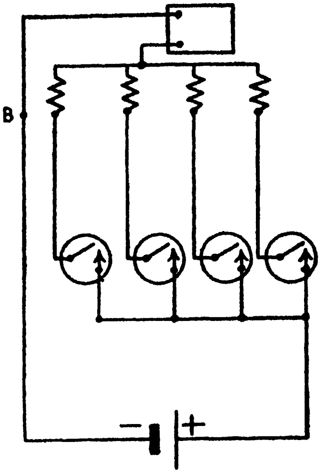
Three-Family House-Bell Job
Complete with Door Opener.

Types of Annunciators. (1) The gravity drop is the most common; here an indicator comes into view by means of gravity. (2) The needle drop, where an arrow turns when a call is made. (3) The electric reset, where a flag comes into view when the call is made. This type of annunciator may be reset by electricity either at the annunciator or from the remote locations. (4) The lamp annunciator, which is similar to the electric reset type, the only difference being that a "bull's eye" lights up instead of a flag coming into view. All annunciators are made either for flush or surface mounting. The size of an annunciator is determined by the number of drops or indicators. The essential parts are: (1) Drop (A), which consists of electromagnet, armature, and indicator; (2) the ground bar (B), one wire from each electromagnet being connected to the ground bar; (3) the buzzer (C), one terminal of which is connected to the ground bar and the other to the battery terminal; (4) the terminals (D), one wire from each electromagnet being connected to these terminals (external wiring is also connected to these terminals); (5) the base or frame to which the drops, ground, bar, buzzer and internal wiring are fastened; (6) the cover;

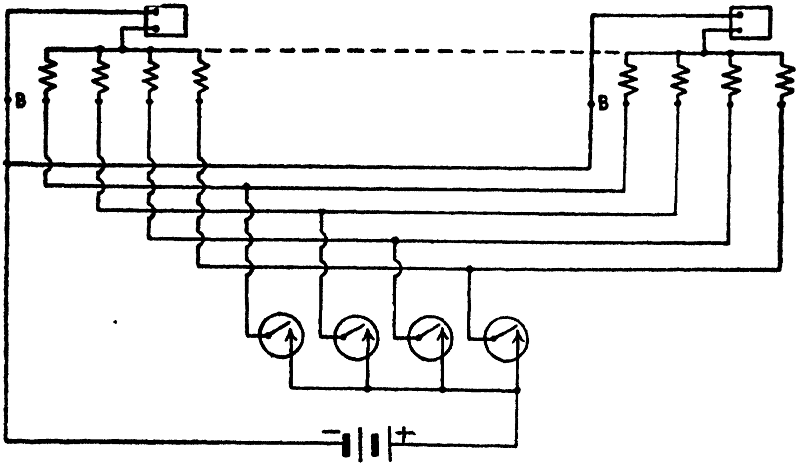


Gravity Drop Annunciator

(7) the reset rack, which is used to return the drops to their original positions after the call has been made.

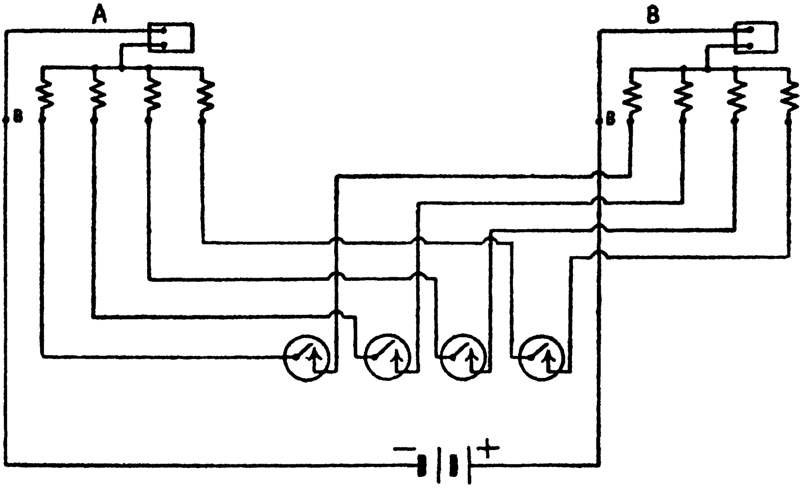


When a button is pressed current flows from the positive side of the battery through the button to the annunciator, through the electromagnet to the ground bar. From the ground bar the current flows to the buzzer and returns to the negative side of the battery. When the circuit is completed the electromagnet becomes energized and attracts the armature, releasing the indicator which then comes into view. At the same time the buzzer sounds, calling attention to the fact that a call has been made. The indicators are returned to their original position by means of the reset rack. Care must be taken to connect the battery to the proper terminal on the annunciator; if the battery is connected to the wrong terminal, two indicators will drop at the same time.

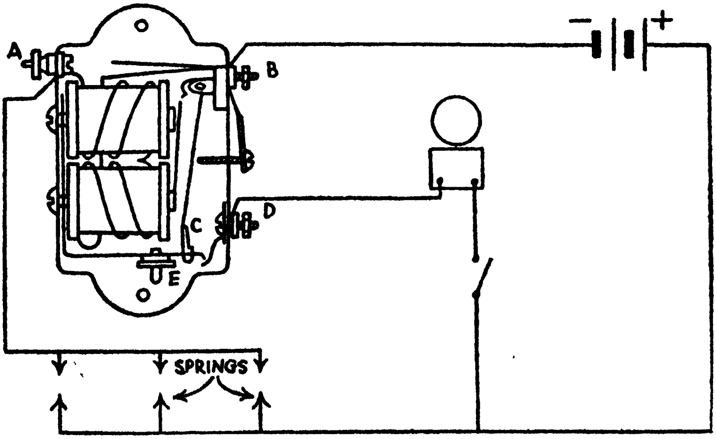


Annunciators Connected in Parallel

Annunciator Connections. When annunciators are connected in parallel care must be taken to use annunciators of the same manufacture and type, because annunciators of the same resistance have to be used when so connecting. A wire is also to be connected to the ground bar of each annunciator, as shown by the dotted line. If these precautions are not taken there will be a back-feed and more than one indicator on each annunciator will fall when a call is made. When annunciators are connected in series, it will be noted that the positive side of the battery feeds annunciator B and the negative side of the battery is connected to annunciator A; the push buttons are connected between the drops of each annunciator.



Annunciators Wired in Series

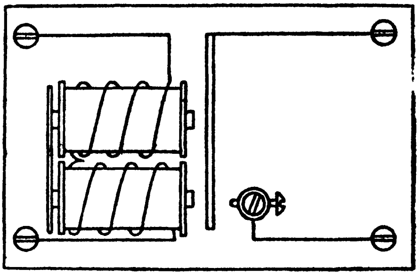


Open Circuit Alarm Using Constant Ringing Attachment

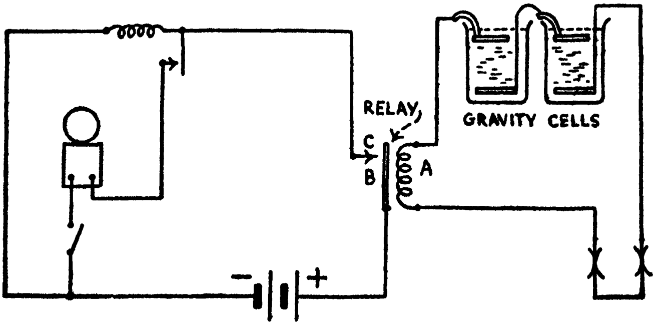
Burglar Alarm Systems. These are a combination of bell and annunciator wiring with several additions. Instead of using push buttons, door springs, window springs, etc., are used. There are two types of burglar alarm systems, open circuit and closed circuit. The disadvantage of the open circuit system is that if a wire is cut or broken the system will not operate; while on a closed circuit system, if a wire is cut or broken in the closed circuit the alarm will be sounded just the same.

A device known as a *constant ringing attachment* is used on both open and closed circuit systems. This device is used to keep the alarm bell ringing continuously until it is reset. The figure shows an open circuit alarm system with a constant ringing attachment. When one of the alarm springs is closed, current flows from the battery through spring to terminal (A) and the electromagnet of the constant ringing attachment to terminal (B), returning to the opposite side of the source of supply. This magnet becomes energized and attracts the armature which makes contact at (C), closing the circuit to the bell. This bell keeps ringing continuously until the spring which was closed is opened and the constant ringing attachment is returned to its normal position terminal (E).

Closed Circuit Burglar Alarm. This type of system is made up of two circuits, the alarm or open circuit, and the spring or closed circuit. A device known as the *relay is* used to connect the alarm circuit when the spring circuit has been broken. The diagram shows the schematic arrangement of a simple closed circuit burglar alarm system. When the spring circuit is closed, current flows from the gravity cells through the springs, which are connected in series, and through the electromagnet (A) of the relay



Relay



Wiring for Closed Circuit Alarm

returning to the opposite side of the gravity cell. When this circuit is completed the electromagnet of the relay is energized and the armature (B) is attached, keeping the contact (C) controlling the alarm circuit open. If a spring is opened or a wire is broken the electromagnet of the relay becomes demagnetized and the armature closes the contact (C) for the alarm circuit. The constant ringing attachment keeps the alarm bell ringing continuously until it is reset and the spring closed.

Power and Light Wiring. Power and light wiring has to be installed in a safe manner for the protection of life from shock or fire. All wiring has to be inspected by the National Board of Fire Underwriters and/or any other governing agencies according to the rules and regulations issued by them. No wires smaller than #14 B&S gage may be used for circuit wiring.

Types of Wires and their Uses. These are tabulated below:

Type	Insulation	Use
R	Code grade rubber	General use
RH	Moisture resistant rubber	General use; wet locations
RP	Performance grade rubber	General use
SNA	Synthetic and felted asbestos	Switchboard wiring
AVL	Impregnated asbestos and varnished cambric, lead sheathed	Wet locations

Splices. Conductors shall be so spliced as to be mechanically and electrically secure and then soldered. The splice shall then be covered with an insulation equal to the insulation of the conductor.

Over-Current Protection. The most common means of over-current protection is the fuse. It is made of a special alloy which melts at certain temperatures and is rated in amperes. A fuse protecting a conductor should not be larger than the ampere rating of the conductor as shown in the Tables below. Plug fuses are made from 0 to 30 amperes. Cartridge fuses are made in different sizes as follows:

0— 30 amp.
31— 60 amp.
61—100 amp.
101—200 amp.
201—400 amp.
401—600 amp.

ALLOWABLE CURRENT-CARRYING CAPACITIES OF CONDUCTORS IN AMPERES

Not More Than Three Conductors in Raceway or Cable

Size AWG MCM	Rubber Type RW Type R	Synthetic Type SN Type RU Rubber Type RPT Type RP	Rubber Type RHT Type RH	Paper Synthetic Type SNA Asbestos Var.-Cam. Type AVB Var.-Cam. Type V	Asbestos Var.-Cam. Type AVA Type AVL	Impregnated Asbestos Type AI	Asbestos Type A
14	15	18	22	23	28	29	32
12	20	23	27	29	36	38	42
10	25	31	37	38	47	49	54
8	35	41	49	50	60	63	71
6	45	54	65	68	80	85	95
5	52	63	75	78	94	99	110
4	60	72	86	88	107	114	122
3	69	83	99	104	121	131	145
2	80	96	115	118	137	147	163
1	91	110	131	138	161	172	188
0	105	127	151	157	190	202	223
00	120	145	173	184	217	230	249
000	138	166	199	209	243	265	284
0000	160	193	230	237	275	308	340
250	177	213	255	272	315	334	372
300	198	238	285	299	347	380	415
350	216	260	311	325	392	419	462
400	233	281	336	361	418	450	488
500	265	319	382	404	468	498	554
600	293	353	422	453	525	543	612
700	320	385	461	488	562	598	668
750	330	398	475	502	585	621	690
800	340	410	490	514	600	641	720
900	360	434	519	556	..	. .	. . .

1000	377	455	543	583	681	730	811
1250	409	493	589	643		. .	. .
1550	434	522	625	698	784	. .	.
1750	451	544	650	733			.
2000	463	558	666	774	839	. . .	. . .

ALLOWABLE CURRENT-CARRYING CAPACITIES OF CONDUCTORS IN AMPERES

Single Conductor in Free Air

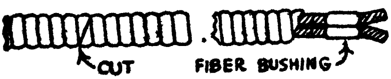
Size AWG or MCM	Rubber Type R	Rubber Type RT	Rubber Type RHT Type RH	Synthetic Type SNA Asbestos Var.-Cam. Type AVB Var.-Cam. Type V	Asbestos Var.-Cam. Type AVA	Impregnated Asbestos Type AI	Asbestos Type A	Slow Burning Type SB Weatherproof Type W Type SPW
14	20	24	29	30	39	40	43	23
12	26	31	37	40	51	52	57	30
10	35	42	50	54	65	69	75	40
8	48	58	69	71	85	91	100	53
6	65	78	94	99	119	126	134	70
5	76	92	110	115	136	145	158	80
4	87	105	125	133	158	169	180	90
2	118	142	170	179	211	226	241	125
1	136	164	196	211	247	264	280	150
0	160	193	230	245	287	306	325	200
00	185	233	267	284	331	354	372	225
000	215	259	310	330	384	410	429	275
0000	248	298	358	383	446	476	510	325
250	280	338	403	427	495	528	562	350
300	310	373	446	480	555	592	632	400
350	350	421	504	529	612	653	698	450
400	380	457	547	575	665	710	755	500
500	430	517	620	660	765	814	870	600

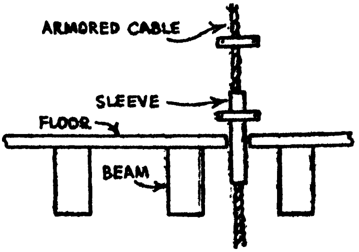
600	480	577	691	738	857	912	970	680
700	525	632	756	813	942	1003	1065	760
750	545	655	785	846	981	1044	1118	800
800	565	680	815	879	1020	1085	1150	840
900	605	728	872	941	....	....	. . .	920
1000	650	782	936	1001	1163	1238	1332	1000
1250	740	890	1066	1131	....		.	....
1500	815	980	1174	1261	1452	..	. . .	1360
1750	890	1070	1282	1370	..		. .	. . .
2000	960	1155	1383	1472	1713	.	. .	1670

More than Three Conductors in a Raceway. The first of the above Tables gives the allowable current capacity for not more than three conductors in a raceway or cable. If the number of conductors in a raceway or cable is from 4 to 6, the allowable current-carrying capacity of each conductor shall be reduced to 80% of the values in the Table. If the number of conductors in a raceway or cable is from 7 to 9, the allowable current-carrying capacity of each conductor shall be reduced to 70% of the values in the Table.

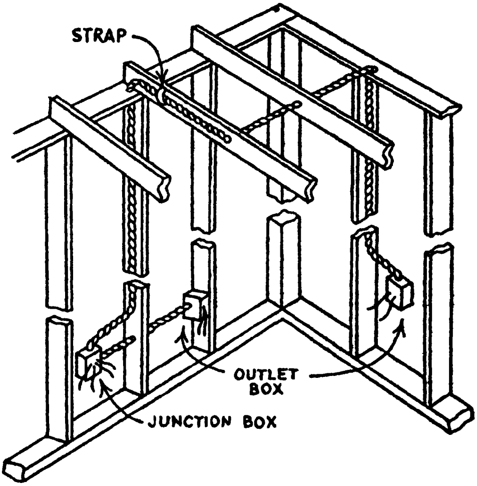
Polarization. This means the identifying of the grounded conductor throughout the entire wiring system. This is done by having the grounded conductor white or natural gray. All conductors of #6 B&S gage and smaller must be identified. The identified conductor should not be broken throughout the entire wiring system except when double pole switching is used.

Flexible Armored Cable. This is also known as BX cable, and contains rubber-covered conductors around which a spiral interlocking steel armor is wound. It is made in one-, two-, three-, and four-conductor in sizes #14 and larger. Type AC may be used for exposed wiring and concealed wiring in dry locations. Type ACL has a lead sheathing over the rubber insulation and under the armor; it is used either for exposed or concealed wiring in damp or wet locations. Cutting flexible armored cable is done with a fine-tooth hacksaw. It is cut diagonally across the spiral armor. Care should be taken not to cut the insulation of the conductors. After the armor has been removed a fiber bushing known as an *anti-short* is placed between the armor and the conductors.





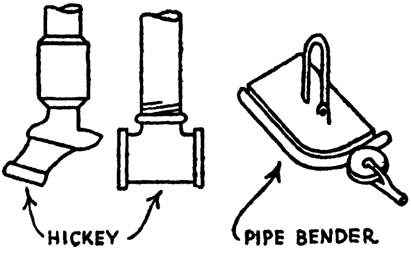
Flexible armored cable is run from outlet to outlet or fitting with no splices in between. It is connected to boxes and fitting with a BX connector unless the box is equipped with a suitable clamping device. When cable is run exposed through floors it has to be protected against mechanical injury and water, which is done by running the cable through a pipe. The accompanying figure shows how armored cable is run when a building is under construction.



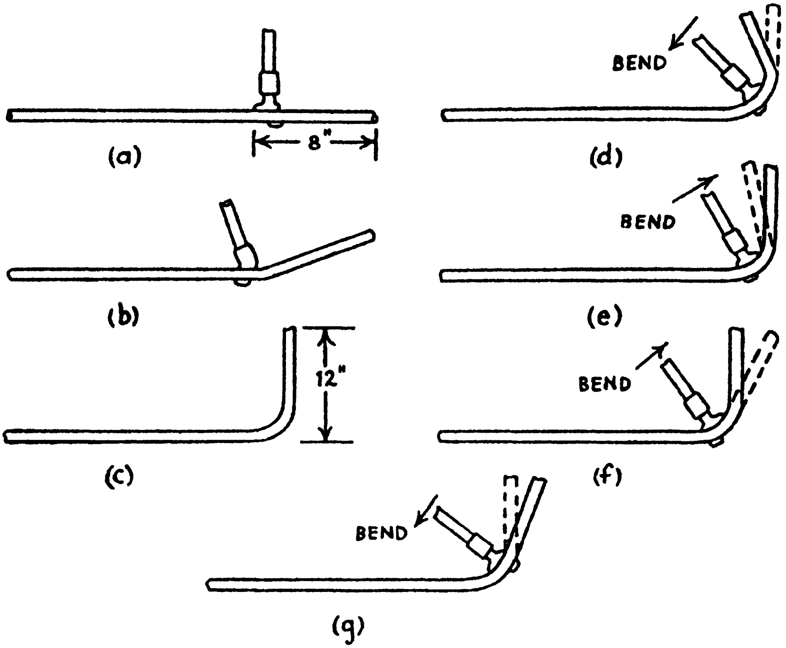
Rigid Iron Conduit Wiring System. In this case the conductors are inserted in iron or steel pipes. The conduit is made in 10′ lengths in regular pipe sizes from ½″ to 6″. All conduit must be smooth on the inside so that the insulation of the wire will not be injured. The most common type of conduit used is coated with a non-corrosive black enamel paint inside and outside, and is used for interior installation. For installations subject to moisture, galvanized or shearadized conduit is used.

Rigid conduit is cut with a hacksaw. After conduit has been cut it has to be reamed to remove the rough edges left from the saw cut. If this is not done these rough edges would tend to cut the insulation of the conductors as they are pulled in. A pipe cutter is not recommended for cutting conduit since it leaves too large a burr on the inner edge of the conduit.

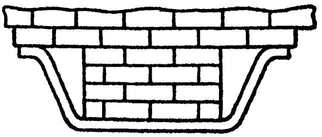
Bending. A device known as a *hickey* is used for bending small size conduit; hickeys are of several types. The hickey is used to make such bends as offset, elbows and saddle. For bending large size conduit a bending machine is used.



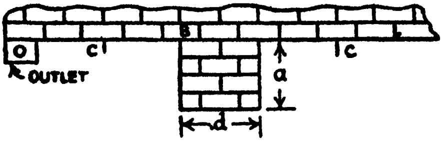
Bends. *Elbow or right angle bend:* To make an elbow, four inches are allowed for half-inch conduit and six inches for three-quarter-inch conduit for the sweep of the bend. If a twelve-inch elbow is to be bent on a length of half-inch conduit, we proceed as follows: Place the conduit on the floor and mark it eight inches from the end. Place the heel of the hickey on the mark, Fig. (a). Place one foot where the bend is to be made and bend the conduit to 45° angle, Fig. (b). Slide the heel of the hickey slightly forward and complete the bend, Fig. (c). If the length of the elbow is too short, place the heel of the hickey as shown in Fig. (d) and bend conduit beyond the 90° angle. Next turn the head of the hickey around and place it on the other side of the bend, Fig. (e), and straighten the vertical piece until it is at right angles with the piece on the floor. To decrease the length of the elbow, the heel of the hickey is placed on the bend nearest the floor and some of the bend removed, Fig. (f). Next place the hickey beyond the bend, Fig. (g), and bend conduit until it is at right angles with the piece on the floor.



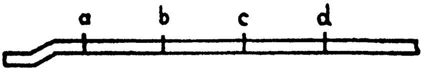
Saddle. In a properly made saddle, the bends should be as short as possible if the conduit is exposed, but the radius of each bend should not be less than the code permits. The plotting and bending of the saddle are suggested by the accompanying figures.



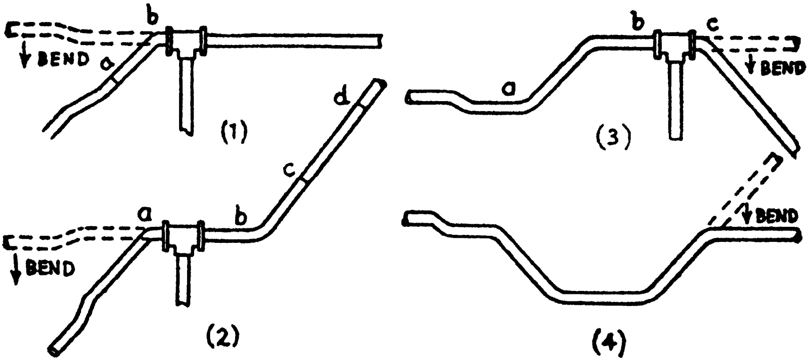
Properly made Saddle



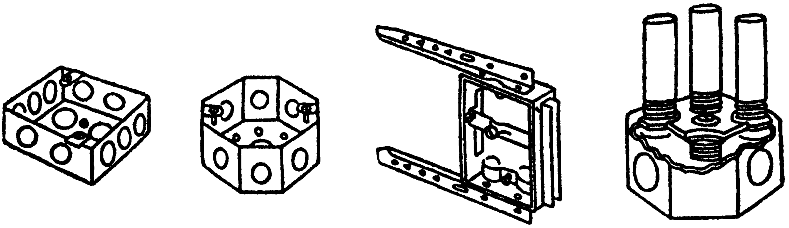
Beam



Plotting Saddle

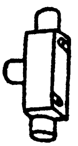


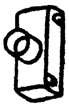
Bending Saddle

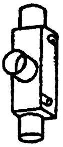


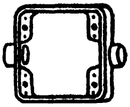
Switch and Outlet Boxes

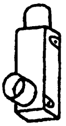
Boxes. Conduit systems contain switch and outlet boxes. The conduits are fastened to these boxes by means of a locknut and bushing. Several types of boxes with locknut and bushing locking the conduit to them are shown below; the locknut and bushing serve not only to keep the conduit fastened rigidly to the box, but also act as a bonding means for grounding.

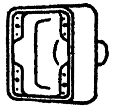


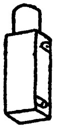


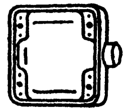












Fittings. Some of the most common fittings used are known as *condulets, unilets, taplets* and *arrowlets;* they are made both threaded and threadless. These fittings are generally used when it is not practical to make bends and where very neat jobs are required. Such fittings increase the cost of the installation considerably.

Pulling the Wires. After the conduit system is completely installed the wires are pulled in. This is done by means of a steel wire, known as a *snake,* which is inserted into the conduit. The wires are then fastened to the snake and pulled into the conduit. Graphite, talc or other approved compounds are sometimes used as a lubricant when pulling in wires. Cleaning agents or lubricants having a deleterious effect on the insulation are not allowable. Conductors in a conduit system have to be continuous from outlet to outlet or fitting; no splices are allowed inside the conduit. Table A shows the number of conductors permitted in a conduit or in tubing:

TABLE A

NUMBER OF CONDUCTORS IN CONDUIT OR TUBING

(One to Nine Conductors Rubber Covered;

Types R, RW, RP, RH and RHT; 600V)

Size of Conductor	*Number of Conductors in One Conduit or Tubing*								
	1	2	3	4	5	6	7	8	9
No. 18	½	½	½	½	½	½	½	½	¾
16	½	½	½	½	½	½	¾	¾	¾
14	½	½	½	½	¾	¾	¾	1	1
12	½	½	½	¾	¾	1	1	1	1¼
10	½	¾	¾	¾	1	1	1¼	1¼	1¼
8	½	¾	1	1	1¼	1¼	1¼	1¼	1½
6	½	1	1¼	1¼	1½	1½	2	2	2
5	¾	1¼	1¼	1¼	1½	2	2	2	2
4	¾	1¼	1¼	1½	2	2	2	2	2½
3	¾	1¼	1¼	1½	2	2	2	2½	2½
2	¾	1¼	2½	2½	2	2	2½	2½	2½
1	¾	2½	2½	2	2	2½	2½	3	3
0	1	1½	2	2	2½	2½	3	3	3

Very often it is economical to run wires of different sizes in the same conduit instead of several conduits. In Table B, column (1) shows the B&S wire size of conductors and column (2) shows the area external transverse in square inches. In Table C, column (1) shows the 40% area of conduit and column (2) shows the size of conduit. The method of finding the proper size conduit is as follows: Add the area external transverse in square inches, column (1) of all the conductors to be run in the one conduit (Table B). Referring to Table C, select from column (2) the number nearest to the total you have found; this is the size required.

Example: What size conduit is needed if the following wires are to be run in one conduit: three No. 2 wires, three No. 0 wires, and three No. 00 wires?

Solution: Area external transverse square inches from Table B:

#2 equals .2 × 3 equals .6
#0 " .3 × 3 " .9
#00 " .35 × 3 " 1.05
2.55 total

From Table C, 40% area column (1), 2.96 is nearest to the above total; by referring to column (2), 3″ conduit is required for the above wires.

TABLE B

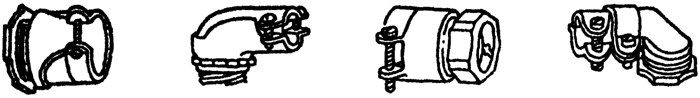
Area External Transverse Sq. In.	Size B&S
.028	14
.033	12
.041	10
.052	8
.11	6
.16	4
.2	2
.26	1
.3	1–0
.35	2–0
.41	3–0
.47	4–0

TABLE C

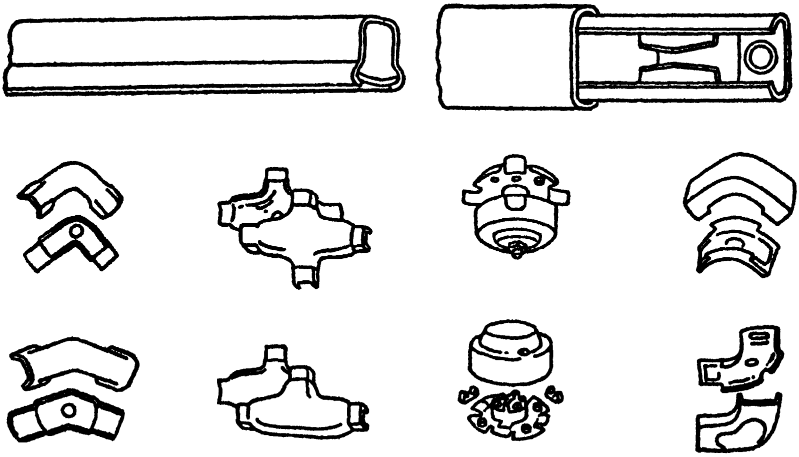
Internal Transverse Area—40% Area	Size Conduit
.12	½″
.21	¾″
.35	1″
.6	1¼″
.81	1½″
1.34	2″
1.92	2½″
2.96	3″
3.95	3½″
5.09	4″

Flexible Conduit. Also called Greenfield conduit, this is similar to armored cable, the only difference being that it contains no conductors. It is made in pipe sizes and comes in lengths ranging from 50 to 250 feet.

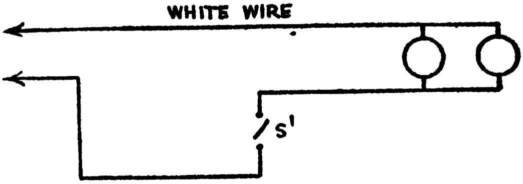
Fittings, of which several are shown below, are used to connect flexible conduit with rigid conduit, boxes and fittings.



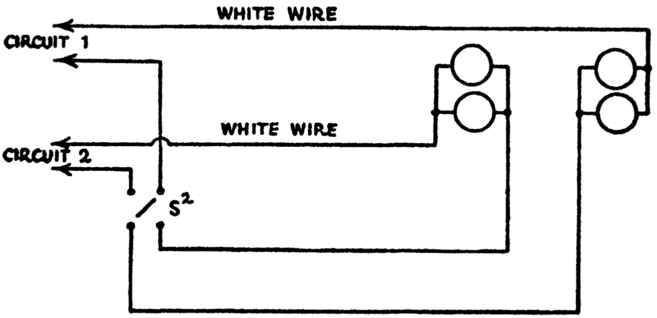
Surface Metal Raceway. This is also known as *metal moulding* and *wire moulding,* and is used only for exposed work. These raceways require special fittings and boxes, and are not interchangeable. Offsets and saddles may be bent with either type raceway, but all other bends or turns require the use of fittings. The most common type of metal moulding is made in two pieces, the base and the cap. When installing, the base is put up first. The wires are then inserted, and, while held in place with fiber clips, the capping is then clamped on. Wire mould is installed in one piece with the exception of the fittings. The wires are not inserted until all the moulding is installed. After the wires are installed the fitting caps are snapped on. Both metal and wire moulding come in different sizes; both types of moulding and some of the fittings used on each are shown herewith.



Switching. *Single pole and double pole.* There are several different types of single and double pole switches, such as *knife, surface snap, flush tumbler, toggles* and *push.* Knife switches are made both fusible and nonfusible, and carry the same ampere-rating as cartridge fuses. The snap, tumbler, toggle and push switches are rated at from 5 amp., 250 volt to 30 amp.—250 volt.

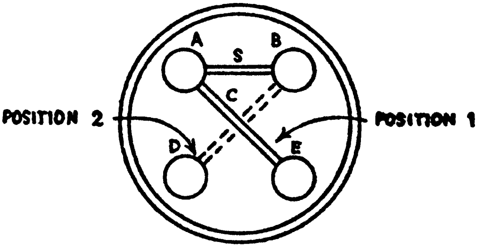


Single Pole Switch Controlling Two Lights



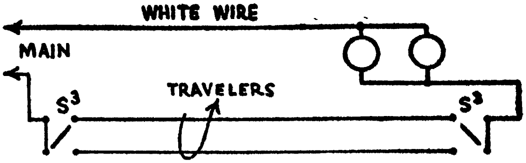
Two Circuits Controlled by Double Pole Switch

Two Location Control. This type of switching is generally known as three-way switching. Lights may be controlled from either of two locations, and it is used in residences, stock aisle lighting, etc. A three-way switch is similar to a single pole double throw switch, but it is never in an "off" position. It has four terminals, two of which are shunted together (S) connecting (A) and (B); the main is connected to this terminal. Terminals (D) and (E) are known as the *travellers.* At position (1) the arm (C) connects terminals (A) and (E). At position (2) the arm (C) connects terminals (B) and (D).



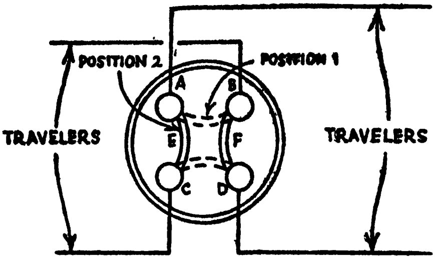
Three-Way Switch

Three Location Control. When lights are to be controlled from three locations, two three-way and one four-way switches are used. The three-way switches are always located on

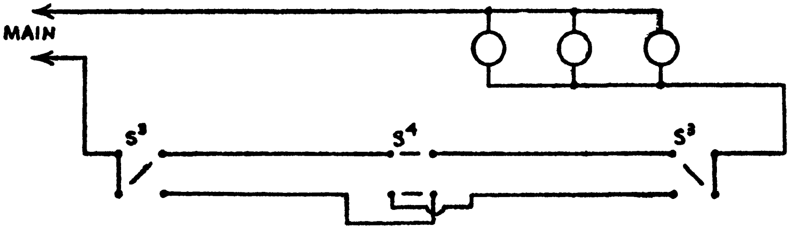


Two Location Control

the ends and the four-way switch in the center. If the lights are to be controlled from more than three locations, four-way switches are added, always being located between the three-way switches. A four-way switch which has four terminals is illustrated. When the switch is in position (1), arm (E) connects terminals (A) and (C), and arm (F) connects terminals (B) and (D). At position (2), arm (E) connects terminals (A) and (B), and arm (F) connects terminals (C) and (D).



Four-Way Switch

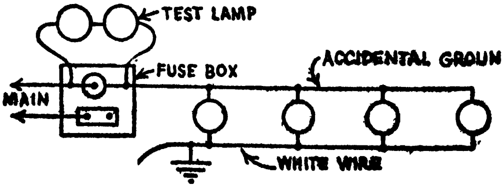


Three Location Control

Testing. One of the most useful devices for the electrician is a set of test lamps. It is made by connecting two pigtail sockets in series and attaching flexible cords to the ends. Bulbs of the same wattage should be used. usually 10-watt.

Testing for a Ground. Disconnect the grounded conductor (white wire) from the cut-out. Remove blown fuse from cut-out and connect test lamp as shown. The test lamps will burn at half brilliancy. Trace the circuit for a ground; when the test lamps go out, the ground has been located.

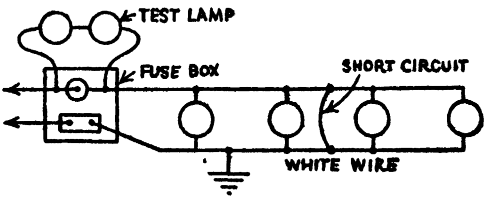
Testing for a Short Circuit. Disconnect all lamps on the circuit. Remove blown fuse and connect test lamps as shown. The lamps will burn at half brilliancy. Trace



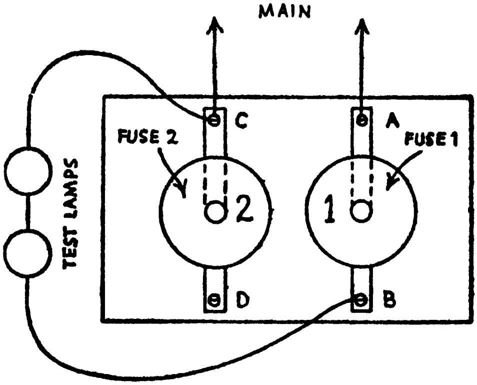
Testing for Ground

the circuit for a short. When the lamps go out, the short circuit has been found.

Testing for a Blown Fuse. Connect the test lamps to terminals (A) and (C). If both lamps light, it will show that there is current to the fuses. To test fuse (1), connect one lead of the test lamps to terminal (C) and the other lead to terminal (B) of the cut-out. If the bulbs of the test lamps light, the fuse is good; if not, the fuse is blown. To test fuse (2), connect one lead of the test lamps to terminal (A) and the other lead to terminal (D) of the cut-out.



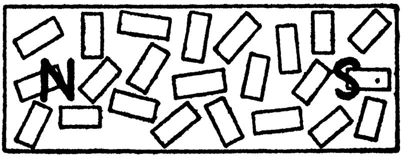
Testing for Short Circuit



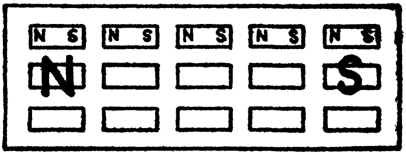
Testing for Blown Fuse

67. DIRECT CURRENT MACHINERY

Electromagnetism. A permanent magnet is one which is magnetized at all times. The molecular theory is that a bar of steel is composed of molecules, and each molecule becomes a separate magnet. When the steel is not magnetized these molecules are disarranged and irregular; but when magnetized, the molecules arrange themselves in straight lines, each one becoming an individual magnet with a north and south pole.

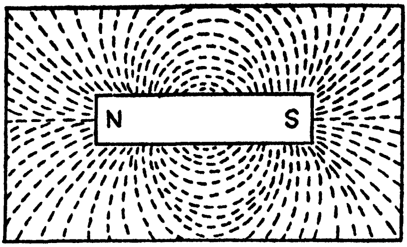


Unmagnetized Bar



Magnetized Bar

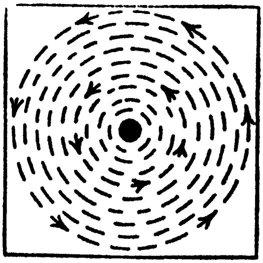
Field About a Magnet. If a bar magnet is covered with a piece of glass, and iron filings are then sprinkled over the glass, it will be noted that the iron filings take a definite pattern or arrangement; this arrangement is known as the *magnetic field*. It is assumed that the magnetic field or force emanates from the north pole, travels through space to the south pole.



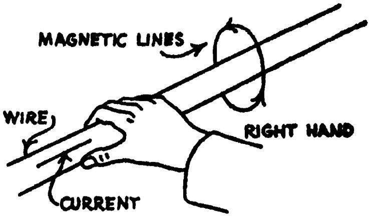
Magnetic Difference between Iron and Steel. Place a bar of steel in a coil of wire connected to electricity. Test the amount of magnetism with iron filings. Remove the steel from the coil and test the magnetism with iron filings. Repeat the test using a piece of soft iron. The soft iron will be found to have more magnetism than the steel when inserted in the coil, but it will have very little magnetism when removed from the coil. The magnetism that remains is known as *residual magnetism* and plays an important role in the generation of electricity.

Electromagnetism. Place a wire carrying direct current into a container of iron filings and you will note that the iron filings adhere to the entire length of the wire. When the circuit is broken the filings will drop from the wire. Electromagnetism differs from permanent magnetism in that the magnetism around the conductor exists only so long as there is a current flowing through it. Electromagnetism was discovered by the Danish scientist Oersted a century and a quarter ago. In a way it is a simple enough phenomenon, yet it has led to amazing developments. Without it, modern electrical engineering, with its endless practical applications, could not exist.

Directions of Lines of Force Around a Wire Carrying Current. Pass a wire carrying current vertically through the center of a piece of cardboard. Sprinkle iron filings on the cardboard and tap the cardboard lightly. It will be found that the filings shape themselves into circles, showing that the magnetic flux takes a circular route around the wire. If the current flows through the vertical wire in an upward direction it will be found (by using a compass) that the magnetic flux flows from left to right.

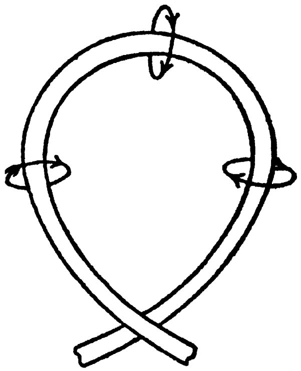


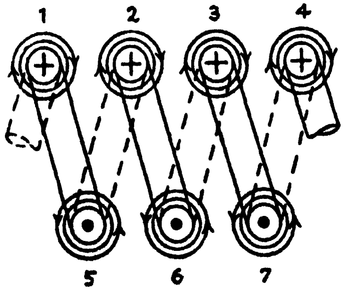
Right Hand Rule for Direction of Magnetic Field Around Wires. If the direction of the current in a straight wire is known, the direction of the magnetic field may be found, and vice versa. *Rule:* Grasp the wire with the right hand, with the thumb pointing in the direction of current flow; the fingers will then point in the direction of the magnetic lines of force.

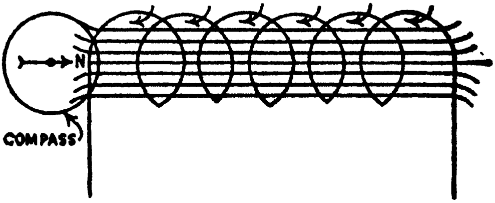


Right Hand Rule

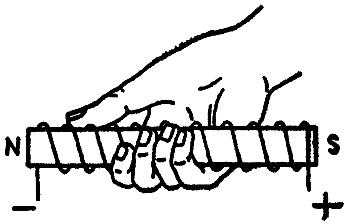
Bend a wire carrying current into a loop. By applying the thumb rule it will be found that the lines of force which encircle the wire enter the same face of the loop and come out the opposite face. If several loops are placed together to form a small coil, most of the lines of force will thread the entire coil, returning along the outside to the other end. This is explained by studying the figure below. The current enters 1, 2, 3, 4, and comes out 5, 6, 7. If 1 and 2 are close together, the magnetic field on the right side of 1 will neutralize the magnetic field on the left side of 2 because the magnetic fluxes are in opposite directions; this is true of any two adjacent turns. This compels the magnetic lines to pass through the entire coil. This is the same as a bar magnet, the end from which the lines come out being the north pole, and the end in which the lines enter the south pole.





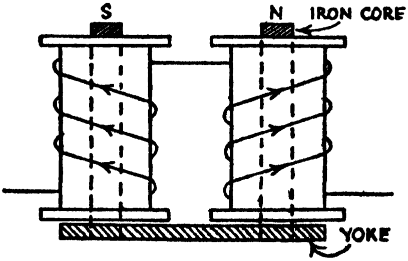


Thumb Rule for Coil. Grasp the coil with the right hand so that the fingers point in the direction of the current flow. The thumb which is horizontal will then point to the north pole.



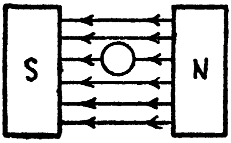
Electromagnets. Wind a number of turns of insulated wire around a steel bar and send current through it. Plunge the bar into iron filings. It will be noted that most of the filings will cling to the ends. Test for polarity with a compass. Disconnect the current and the iron filings will fall. Remove the bar of steel and repeat. It will be noted that: (1) The polarity remains the same. (2) Magnetism occurs only when current is flowing through the coil. (3) More magnetism is produced when the steel bar is in the coil. Replace the steel bar with a soft iron bar and test the magnetism. It will be found that the soft iron bar will attract more iron filings than when the steel bar was inserted in the coil or when nothing was in the coil. When a piece of hard steel is brought near an electromagnet the lines of force from the electromagnet are diverted from their natural course and converge into the steel. More lines of force pass through the space occupied by the steel than when the space was occupied by air. The capacity of any substance for conducting magnetic lines of force is called its permeability. If a piece of soft iron were substituted for the steel, many more lines of force would pass through the space occupied by the soft iron. We can therefore state that the permeability of iron is greater than steel or air.

Construction and Use of Electromagnets. An electromagnet is made by winding many turns of wire around a soft iron core. Most electromagnets are in the shape of a "U," or horseshoe. Two separate coils are wound in opposite directions, the wires from each coil being connected together. The two cores are fastened together by means of a yoke. The reason for the wires on each coil being wound in opposite directions is that the limbs or poles are of opposite polarity (right hand rule). Electromagnets have many practical uses, such as in electric bells, annunciators, door openers, telegraph sounders, relays and lifting devices.

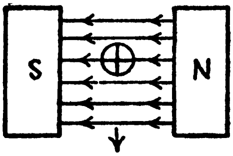


Circuit Breaker. The main use of a circuit breaker is to disconnect a circuit when an excessive amount of current flows through the circuit. It consists of a coil of heavy wire, wound into a solenoid which is connected in series with the circuit. As the current increases in the circuit and becomes excessive the solenoid becomes energized, attracting a plunger which trips a spring and opens the circuit. Each disconnecting device is made up of two contacts, one carbon and one of copper leaves. The carbon makes contact first, taking the load and arc, after which the copper leaves make contact. The reason for using the carbon contacts is that they are easy to replace at low cost. Circuit breakers may be constructed with a "time limit" device which delays the tripping for a short period. They may also be equipped with a "no voltage" device for opening the circuit when the voltage falls below a predetermined point, with a "reverse current" arrangement which opens the circuit when there is a reversal of polarity. This is desirable when running generators in parallel or when charging storage batteries.

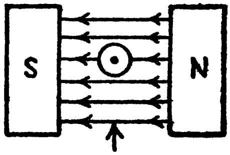
The Generator. A generator is a machine which changes mechanical energy into electrical energy. It consists fundamentally of a number of conductors which rotate in a magnetic field. The generator operates on the principle of cutting lines of force which induces an e.m.f. in the conductor, as shown in figure (a). Move the conductor downward, cutting the lines of force at right angles, and an e.m.f. flowing away from you will be induced in the conductor, as in (b). Cut the lines of force at right angles in an upward direction, and an e.m.f. will be induced in the conductor, flowing towards you, as in (c). If the conductor is moved horizontally no lines of force will be cut; therefore no e.m.f. will be induced in the conductor. From the foregoing it can be concluded that in order to induce an e.m.f. in a conductor we must have a magnetic field and motion. The direction of the e.m.f. may be found by applying the following rule, which is known as *Fleming's Right Hand Rule:* Extend the thumb, the forefinger and the middle finger of the right hand at right angles to each other, as in (d); point the forefinger in the direction of the magnetic flux, the thumb in the direction of the conductor's motion, and the middle finger will then point to the direction of the induced e.m.f. The amount of e.m.f. produced is controlled by the following: (1) rate of motion,



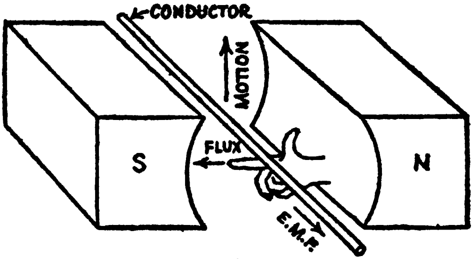
(a)



(b)

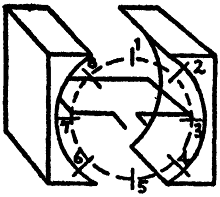


(c)

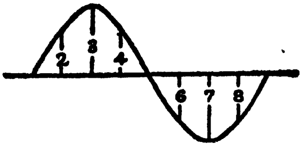


(d)

(2) amount of magnetic flux, and (3) number of conductors cutting the flux.

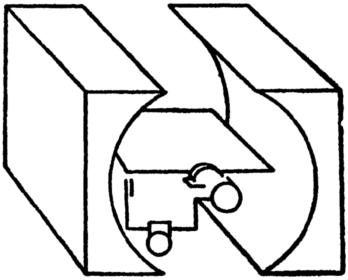


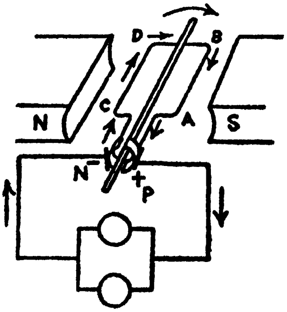
(a)



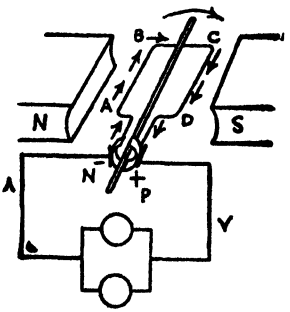
(b)

Single Loop Generator. Consider a single coil which can be rotated at a constant speed through a uniform magnetic field. When the coil is in position (1), no e.m.f. is induced because the coil is parallel with the lines of force, and therefore none are being cut. At position (2) some lines of force are being cut at a slight angle and an e.m.f. is being induced as shown in (b). When the coil is in position (3) it is cutting the maximum lines of force and the greatest e.m.f. is being induced. At position (4) fewer lines are being cut and the induced e.m.f is equal to (4). When the coil reaches position (5) it is again parallel with the lines of force and therefore no e.m.f. is being generated. At (6) the coil is coming under a pole of opposite polarity, therefore the induced e.m.f. is in the opposite direction. At (7) the induced e.m.f. is at its maximum *in the opposite direction,* diminishing at (8), and zero when it completes the cycle of (1). The alternating e.m.f. generated is brought to the external circuit by means of two slip rings fastened to, but insulated from, the shaft, and insulated from each other. Brushes resting on the slip rings transfer the e.m.f to the external circuit.



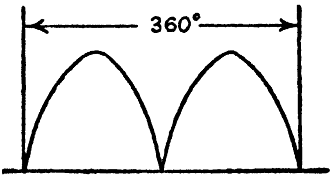


(a)



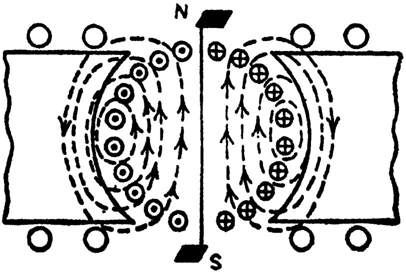
(b)

Commutation. If direct current is desired the slip ring is replaced by a single ring divided in half, the halves insulated from each other and also from the shaft. Each end of the coil is then soldered to one of the halves of the commutator. The performance of the commutator is to collect all the positive e.m.f. under one brush and all the negative e.m.f. under the other brush. By studying figure (a) the act of commutation can be traced. When the coil AB rotates past the S pole and e.m.f. flows from B to A (right hand rule) to brush P, which is positive, through the external circuit to brush N, the negative poles complete the circuit through coil CD; fig. (b) shows the loop after it has made one-half a revolution. AB is now passing the N pole and the induced e.m.f. is flowing in the opposite direction from A to B. However, the positive brush is not making contact with this part of the loop, but is connected to the negative brush N. Current flows to the external circuit from brush P, which is still positive even though the current in the coil has been reversed. Brush P is always the positive terminal and brush N always the negative terminal. An e.m.f. shown in fig. (a) is now generated by the use of a commutator. By studying fig. (c) it will be noted that while the e.m.f. is in one direction it is nevertheless fluctuating. This e.m.f. can be made steady by adding more coils to the armature and also increasing the number of field poles.

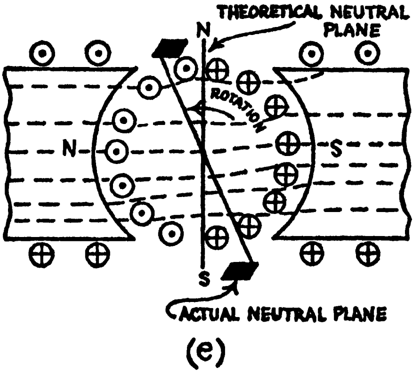


(c)

Sparkless Commutation. When the simple loop of armature is in position (1) above, the conductor is not cutting any lines of force, therefore no e.m.f. is being generated. This position is known as the theoretical neutral or commutating plane, and is the point where the brushes are located. Consider fig. (d), which shows a current from an outside source being applied to the armature with the current flowing toward you through the left hand coils and away from you in the right hand coils. A magnetic flux will be produced with an N pole at the top and an S pole at the bottom. In fig. (e) there is current flowing in both the fields and the



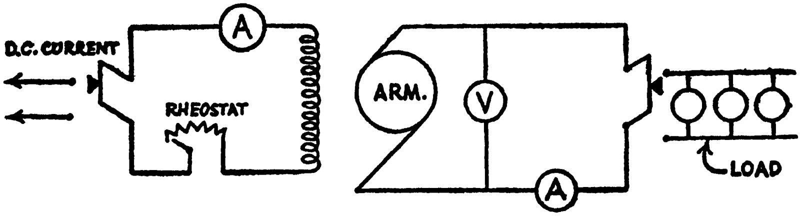
(d)



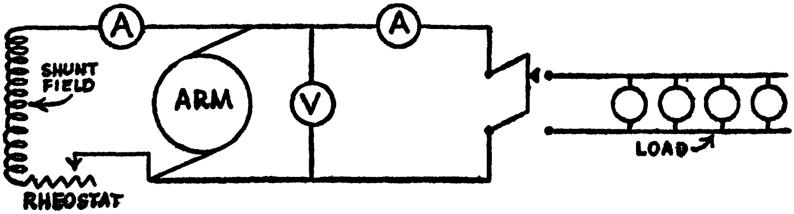
(e)

armature in the direction indicated. The combination of these two fluxes distorts the field as shown in fig. (e) and therefore changes the location of the neutral or commutating plane ahead of rotation. This position changes as the current in the armature changes; therefore the position of the brushes has to be changed as the load varies. The brushes have to be moved ahead (toward rotation of armature) for an increase in armature current and back for a decrease in armature current. By keeping the brushes in the exact neutral point there will be no sparking at the brushes.

Energizing the Fields. The field coils of generators are energized by either of two methods. (1) *Energized from an Outside Source.* When this method is employed they are connected to a direct current entirely independent from the generator. All alternating current generators have to be separately excited. Very often this is done by connecting a small direct current generator to the shaft of the driving machine. (2) *Self-Excited Field Coils.* It will be remembered that when an electromagnet with a soft iron core is disconnected from the source of supply a portion of the magnetism re-

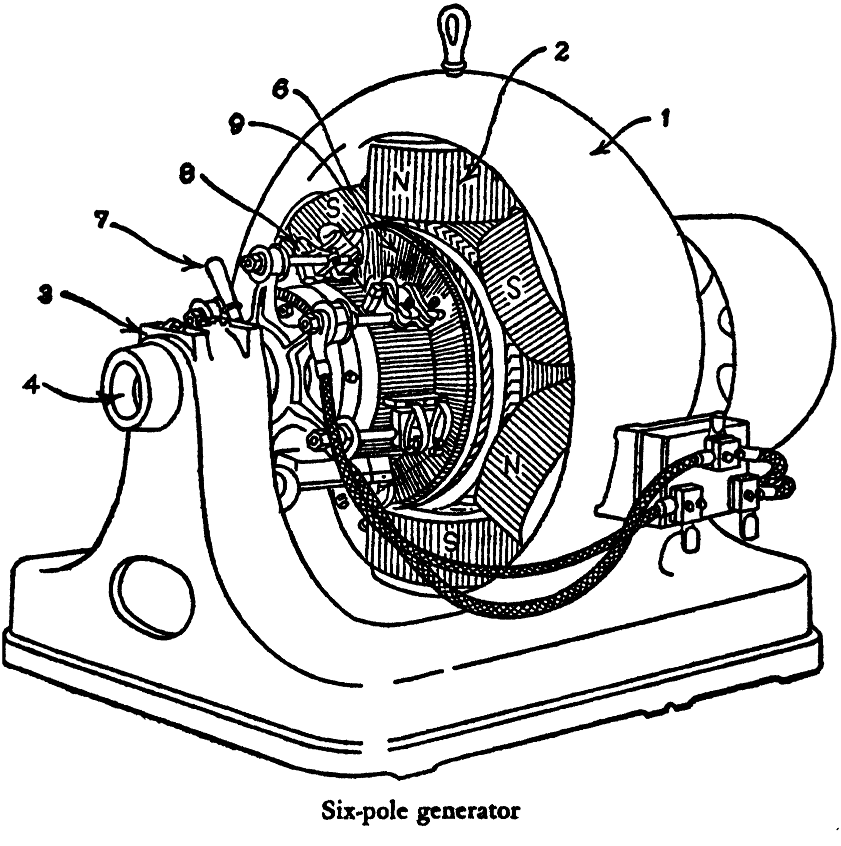


Separately Excited Generator

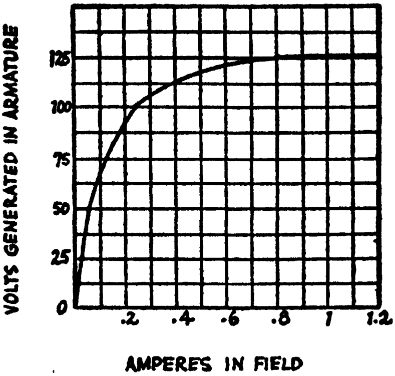


Self-Excited Shunt Generator

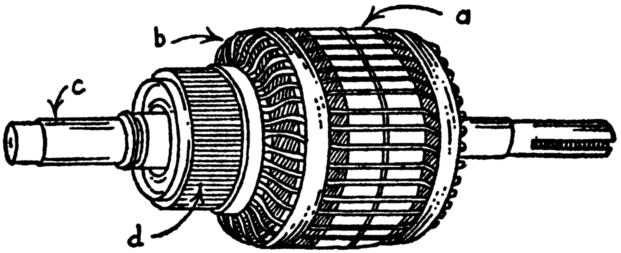
mains and is known as *residual magnetism*. This is also true of the field coils of a generator, since they are really large electromagnets. To have a generator self-excited the fields are connected directly across the armature or brushes. As the armature is rotated and brought up to speed, the conductors of the armature are cutting the weak lines of force supplied by the residual magnetism of the fields and a low e.m.f. is generated. By having the fields connected across the armature, the generated e.m.f. is applied to the field. This increases the amount of magnetism and therefore a greater e.m.f. is generated. This process keeps repeating until the saturation point of magnetism is reached and the voltage reaches its maximum point. The generated voltage is controlled by connecting a resistance known as a *field rheostat* in series with the field. This rheostat controls the amount of magnetism. The graph shows relation of field current to the generated voltage.



Six-pole generator



Direct Current Generator Voltage Curve

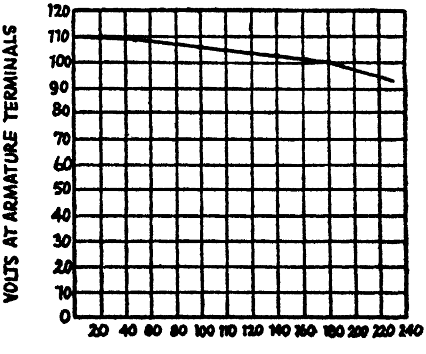


Armature

Parts and Description of Generator. The above figures show the generator and its parts. (1) *Frame.* This is made of a solid piece of iron to which other parts of generator are fastened or connected. (2) *Fields.* (a) *Shunt field,* consisting of many turns of small wire wound around a soft iron core, known as the pole pieces. The pole pieces are made of many thin pieces of metal known as lamenations. The reason for the pole pieces being lamenated is to cut down eddy currents which produce heat. Every generator has at least two field coils, one north pole and one south pole. This type of generator is known as a bi-polar machine. The field coils may be increased by twos. When connecting the field coils of generators having more than two poles, the adjacent poles always have to be of opposite polarity, found by the use of a compass. (b) *Series field,* consisting of a small number of large wires wound on a lamenated iron core. The series field is always connected in series with the armature. Both the shunt and the series fields are bolted to the frame of the generator. (3) *Heads* (2). These are made of cast iron which fasten to the frame and form the end plates of the generator. Each head is made with an oil well for supplying lubrication to the armature shaft. (4) *Bearings* (2). These are generally made of bronze; one bearing is fastened to each head. (5)

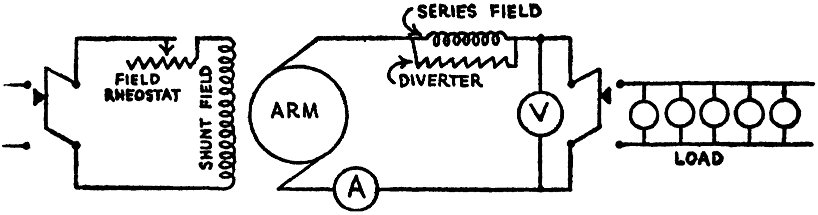
Oil Rings. Bronze rings which go over armature shaft and transport the oil from the oil well to the armature shaft and bearing. (6) *Armature.* This is a rotating piece which fits between the pole pieces and rests in bearings which are in the heads of the generator. The armature is made up of the following parts. (a) *Slotted core* made of lamenated iron into which the winding is placed. (b) *Armature winding,* i.e., the conductors which are placed in the core slots. The winding may be either hand wound or form wound. When hand wound, the conductors are wound directly into the slots of the armature. When form wound the conductors are wound into coils and taped. The coils are then placed into the armature slots. Before the winding is put on the armature the slots are insulated with "fish paper" and "empire cloth." (c) *Shaft,* made of steel to which the armature core and commutator are fastened; the shaft fits into bearings which are fastened to the heads. (d) *Commutator,* consisting of separate segments of copper insulated from each other, fastened to, but insulated from, the shaft. The ends of the armature coils are soldered to the commutator. The commutator has twice the number of segment bars as there are slots in the armature core. (7) *Rocker Arm.* Piece to which brush holder studs are fastened; it is fastened to the commutator end of the head. The rocker arm is movable for adjustment of the brushes. (8) *Brush holder and studs.* These are parts which hold the brushes; they are fastened to rocker arm. The brush holder studs are insulated from the rocker arm. (9) *Brushes.* The brushes are made of carbon and fit into the brush holders. They rest on the commutator and carry the generated e.m.f. from the armature to the terminals which feed the external circuit.

Characteristics of the Generator. There are three types of generators: Shunt, Compound and Series. *Shunt Generator.* This type of generator has the fields connected across the armature. The field rheostat is used to control the field magnetism, which in turn controls the voltage output. As the load is increased, resistance has to be cut out of the rheostat to strengthen the field. The ammeter is connected in series with the load and records the amount of amperes being drawn. The voltmeter is connected across the brushes and records the generated voltage. As the load of the external circuit is increased the speed and the generated voltage decrease. The voltage may be brought up to its proper pres-

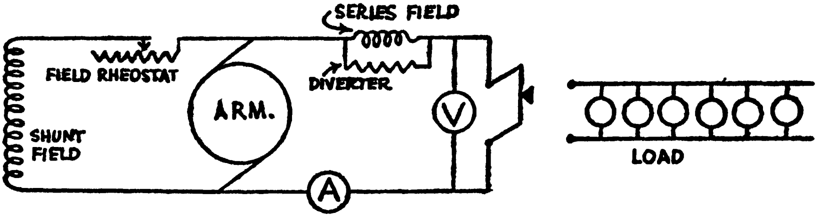


LOAD-VOLTAGE CURVE; SHUNT GENERATOR

sure by (1) increasing the speed of the prime mover or (2) by increasing the strength of the magnetic field. Shunt generators are used where the load is constant, such as when charging batteries.



Compound Generator Separately Excited



Self-Excited Compound Generator

Compound Generators. This type of machine is similar to a shunt generator, but in addition to the shunt field it has a series field. The series field consists of a few turns of heavy conductors of low resistance and is connected in series with the armature. A compound generator may be either separately or self-excited. The above diagrams show the wiring connections of a separately-excited machine and the connections for a self-excited machine. Care must be taken to be sure that the current flowing in the series field is in the same direction as that flowing in the shunt field. When so connected these are said to be connected *accumulatively*. If the series field is opposing or "bucking" the shunt field, they are said to be *differentially* connected. The method of testing whether a machine is connected accumulatively is: (1) Connect the fields at random and take the voltmeter reading. (2) Change the direction of current flow in one of the fields and take the voltmeter reading. (3) The higher of the two readings is the accumulative connection. The reason that the voltmeter reading is higher when so connected is that when the current flow is in the same direction in both fields, the magnetic lines of force are in the same direction and therefore increase. By increasing the magnetic lines of force the conductors are cutting more magnetism, therefore the generated voltage is higher.

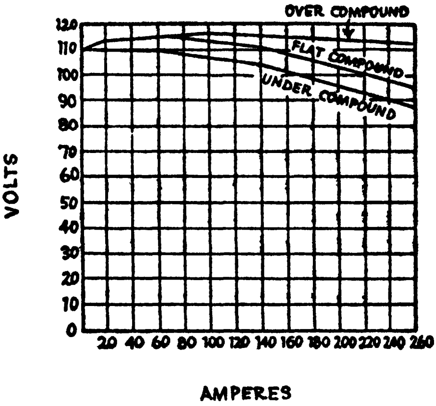
Compound Generators may be over compounded, flat compounded, under compounded or differentially compounded. These different types of compounding are controlled by the number of turns in the series field. This is done by connecting a resistance across the field. On small generators german silver wire is used, while on large generators cast iron grids are used. The type of compounding desired is usually done at the factory.

Over Compounding. This is the most common way of connecting compound generators. With this type of connection the full load voltage is higher than the no-load voltage. It is used to compensate the volt-drop loss in distribution. The greater the distance the load is from the generator, the higher the percentage of over compounding. Generators are over compounded as low as 3% where the load is near the point of distribution and as high as 10% where the load is a great distance from the point of distribution.

Flat Compounding. This type of connection is used when the no-load voltage and the full load voltage are to be the same.

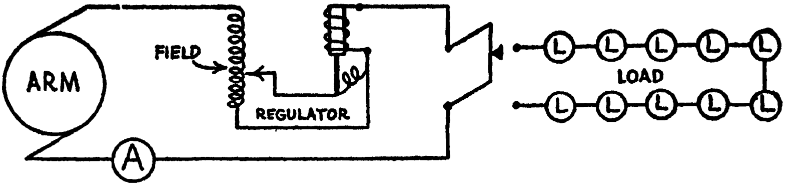
Under Compounding. With this type of connection the full load voltage is lower than the no-load voltage and is very seldom used except for supply power for welding.

Differentially Compounding. With this type of connection the voltage drops very rapidly with the increase of the load. The reason for this is that the series field is "bucking" the shunt field. The graph shows the characteristics of the various types.

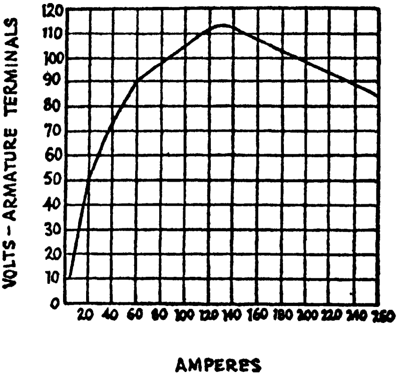


Load Voltage Curve: Compound Generators

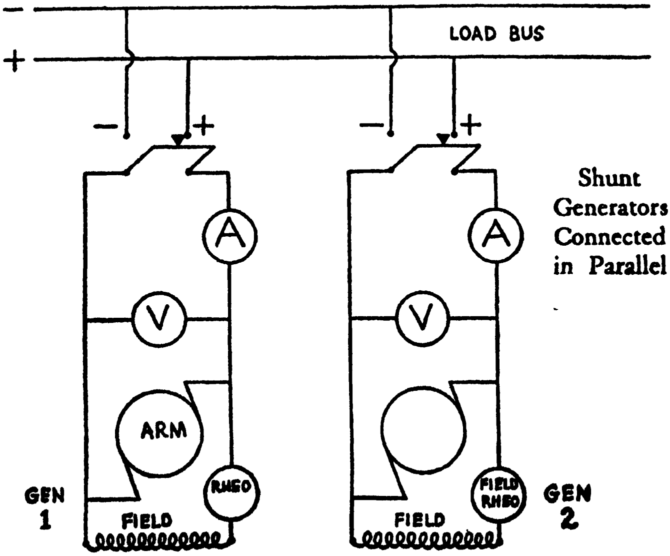
Series Generator. This type of generator is used for series street lighting and where a constant current has to be supplied. They are also known as constant current generators. For the proper operation a regulator must be connected to the line to prevent a fluctuation of the current. A very popular type of regulator for this use is a carbon pile rheostat operated by a solenoid. It must be understood that this type of machine is so called not because it tends to maintain a constant current, but because it is used in a circuit whose current must be kept constant. The field of a series generator consists of a few turns of heavy wire and is connected in series with the armature.



Connecting Shunt Generators in Parallel. To connect shunt generators in parallel, machines of the same voltage must be used. The machine to be paralleled is brought up to speed and the voltage regulated by means of the field rheostat to that of the generator in operation, or a trifle higher. After the voltage is correct the switch is then closed. Care must be taken to keep the voltage of each machine the same because if one runs faster than the other it will tend to take more of the load. When generators are connected in parallel the voltage remains the same as one machine,



Load Voltage Curve: Series Generator

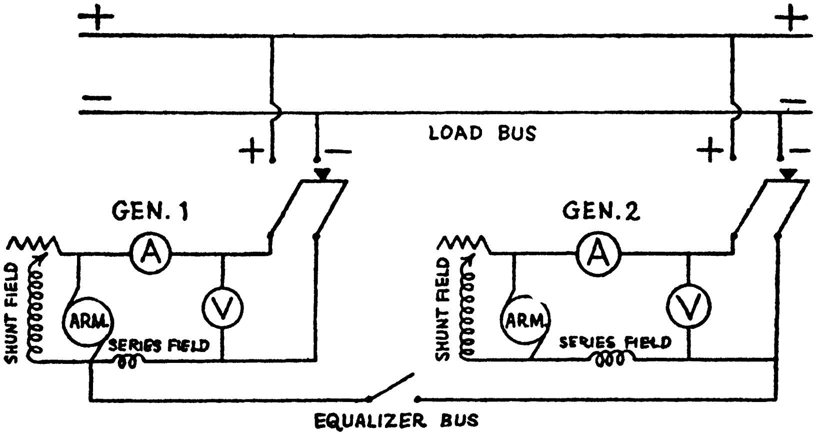


Shunt Generators Connected in Parallel

but this capacity increases to the total of both generators. To shut down generator No. 2, resistance is added to the machine until no current is flowing through it. The "load" switch is then opened and the generator stops running.

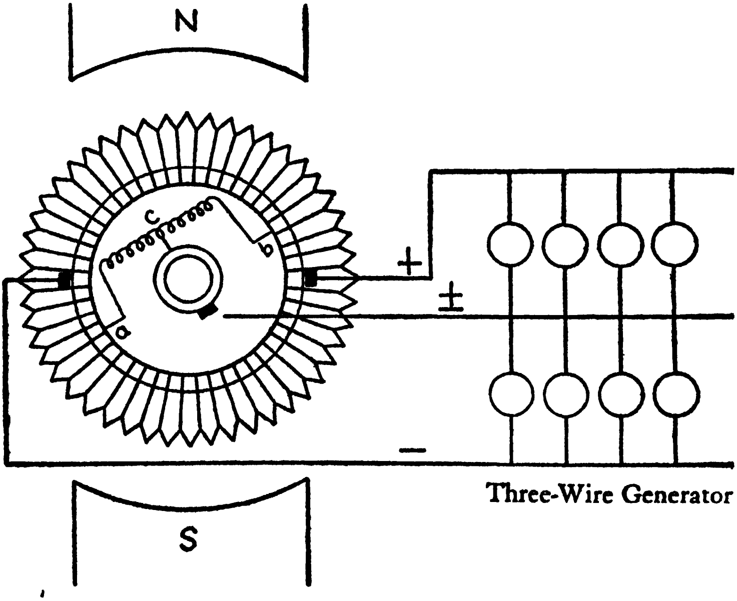
Shunt Generators may also be connected in series. When so connected the voltage will equal the voltage of both machines and the capacity will be equal to that of the smallest generator.

Connecting Compound Generators in Parallel. More care must be taken when paralleling compound generators than shunt generators. A wire must be run from the brush to which the series field is connected of each machine and is known as the *equalizer bar.* The reason for this equalizer is to keep the load balanced in the event of one of the machines slowing up. The method of paralleling is to bring the free machine up to a voltage equal to that of the machine in operation by adjusting the field rheostat. Even though the readings of both machines are the same, the terminal voltage of the loaded machine is higher than that of the free machine, due to the I-R drop of the series field of the loaded machine. The equalizer switch is then closed. The unbalanced voltage from the generators causes a current to flow through the equalizer bar around the series field of generator No. 2 to the external circuit. The voltage of machine No. 2 is again regulated to that of machine No. 1. The load switch of machine No. 2 is then closed. In general practice a three-pole switch is used, the equalizer bar being connected to the middle blade which is longer than the two outside blades. With this arrangement the middle blade makes contact first and breaks contact last. Assuming that the speed of either generator is running slow, the voltage of one machine will then be lower



Compound Generator Connected in Parallel than that of the other; current will therefore flow through the equalizing bar, strengthening the series field and thus raising the voltage. To shut down one of the generators the load switch is opened first and then the equalizing switch. After this the voltage is lowered by inserting resistance in the shunt field after which the field switch may be opened and the speed cut.

Three-Wire Generator. This type of machine generally delivers 115 volts from either of the two outside wires and the center or neutral wire and 230 volts between the outside wires. The same results may be obtained by connecting two generators in series, the center or neutral connection being made on the wire connecting the two machines together. This method of producing a three-wire system is more costly than using a three-wire generator. One type of three-wire generator has a low resistance wound on an iron core fastened to and connected to the armature as shown. A wire is connected to the center of this coil and run to a slip ring and brush on the armature shaft. On a two-pole machine the coil is connected as shown at (a) and (b) and the generated e.m.f. is alternating, of a small value due to the inductance of the coil. The center tap (c) will have a voltage equal to half of the outside brush terminals. If the load on each side of the outside wires is equal, no direct current will flow through the coil or neutral wire. If more current flows through one outside wire than through the other, the neutral wire will then carry the unbalanced current.



Three-Wire Generator

Commercial Ratings of Direct Current Generators. Generators are rated for size in kilowatt (KW) output. This means that a 200 volt—100 KW machine can maintain a constant load of 500 amperes without heating;

$$I = \frac{KW \times 1000}{E} = \frac{100 \times 1000}{200} = 500 \text{ amp.}$$

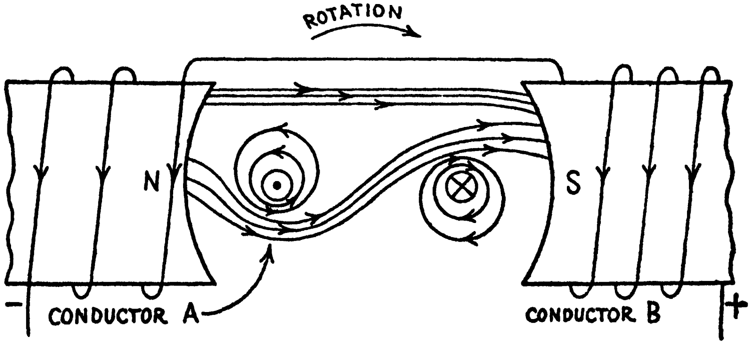
Generator Losses. There are two kinds of losses in a generator: (1) mechanical, and (2) electrical. Mechanical losses are those which are caused by the friction of the bearings and of the brushes on the commutator. The electrical losses are: wattage lost in the armature, in the field, in brush contacts, in the field rheostat, and losses due to eddy currents.

Efficiency of the Generator. The electrical meaning of efficiency is the ratio of useful work or output of an electrical device to the power supplied to it. When the efficiency of a machine is given it is always understood that the rating given is for full load. To find the efficiency of a generator divide the voltage output of the generator by the voltage plus the mechanical and electrical losses, which together equal input; or

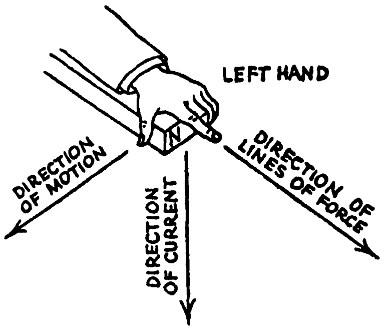
$$Efficiency = \frac{Output}{Input}.$$

Direct Current Motors. An electric motor is a machine that changes electrical energy into mechanical energy. The parts are the same as a generator, namely: frame, armature, which consists of the core, winding and commutator; field coils, both series and shunt; heads or end plates; bearings; rocker arm; brush holders and brushes.

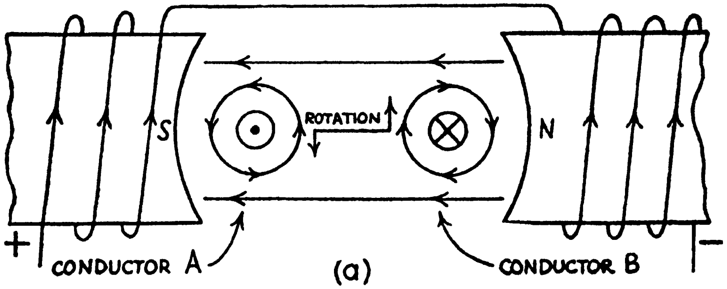
Rotation. The figure shows a north and south pole carrying current with magnetism flowing from north to south. In Conductor (A) of the loop between the pole pieces the current is flowing toward you with the flux around the conductor in a counterclockwise direction. The current in conductor (B) is flowing away from you with the flux in a clockwise direction. It will be noted that the lines of force are in the same direction and are being crowded at the bottom of conductor (A) and the top of



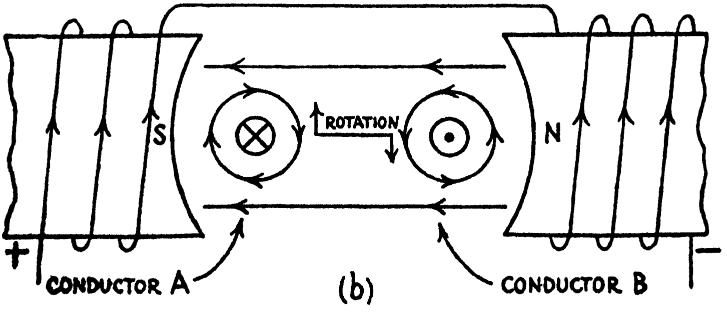
conductor (B). This crowding of the flux pushes conductor (A) in an upward direction and conductor (B) of the loop in a downward direction, and thus causes the loop of the armature to rotate in a clockwise direction. The direction of rotation may also be found by using Fleming's Left Hand Rule. *Rule:* Point the index finger, middle finger and thumb at right angles to each other. The index finger points in the direction of the flux, the middle finger in the direction of the current flowing in the armature conductor and the thumb will point in the direction of motion. To change the direction of rotation of a direct current motor, the direction of current flow is changed in either the field or armature circuits, but never in both. In the preceding diagram the rotation of the armature is in a clockwise direction. Consider figure (a) below: the current in the field circuit has been changed and therefore the polarity of the poles. The magnetic lines of force are now being crowded at the top of conductor (A) and



Fleming's Left Hand Rule



(a)



(b)

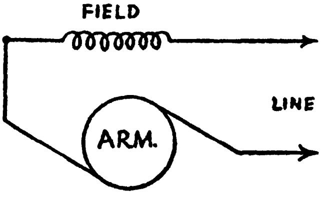
the bottom of conductor (B). This crowding of the magnetic lines of force pushes the conductor away from the crowding effect and the armature will rotate in a counterclockwise direction. In figure (b) the current through the fields is in the same direction as in (a), but the current through the armature conductor has been changed. The lines of force are now crowding conductor (A) at the bottom and conductor (B) at the top. The conductors move at right angles away from the crowding and the rotation is again clockwise.

Sparkless Commutation. In order to have practically no sparking at the brushes, commutating interpoles are located between the field poles. These poles are so connected that they induce in the short circuited armature coil a voltage opposite and as nearly equal to the voltage resulting from the decaying magnetic lines around the short circuited armature coil. All modern direct current motors are equipped with interpoles.

Counter E.M.F. As the armature of the motor rotates it generates an e.m.f. in the opposite direction to that being supplied to the motor and is known as counter e.m.f. As the armature is of very low resistance (about .01 ohms) this counter e.m.f. acts as a resistance in the armature circuit. To test the counter e.m.f. of a motor, open the motor switch when the motor is running, and at the same instant connect a voltmeter across the brushes of the motor. It will be found that the meter will show a voltage even though the main switch is open. The voltage will decrease as the speed of the motor decreases.

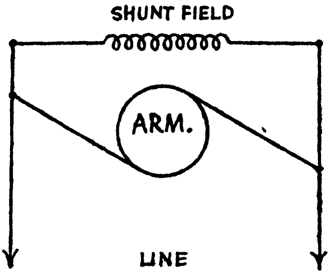
Characteristics of Direct Current Motor. There are three types of direct current motors: (1) *Series motor,* used where a heavy tractive force with variable speed is required, such as in cranes; (2) *Shunt motor,* used where constant speed and load are required; and (3) *Compound motor,* used where constant speed and variable load are required.

Series Motor. This motor should never be started without a load. The field is wound with a few turns of heavy wire and is connected in series with the armature. As the motor starts, a very large current will flow through the armature and field due to the high torque (the tendency to turn). As the speed of the motor increases, the counter e.m.f. resulting from the generator action reduces the armature current. By having the armature and field connected in series, the current flowing through the armature will also flow through the field. This lessening of the field current will weaken the field and increase the speed of the motor. This increase in speed will continue indefinitely until the armature is destroyed from centrifugal force unless the motor is properly loaded. For this reason a series motor should never be belt driven.



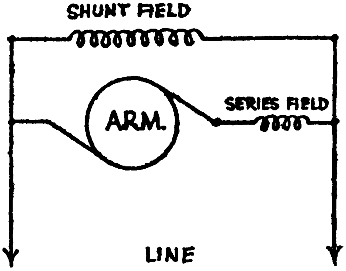
Series Motor Connection

Shunt Motor. The field coils of this motor are made of many turns of very small wire. The field draws very little current, but has a very strong magnetism. The field of this type of motor is connected in parallel with the armature. The speed varies between 5 and 10% between no load and full load. Since the field is connected across the line, the magnetism remains the same at all times. As the current in the armature increases the torque increases. This type of motor is used where constant speed and a nonvariable load are required.



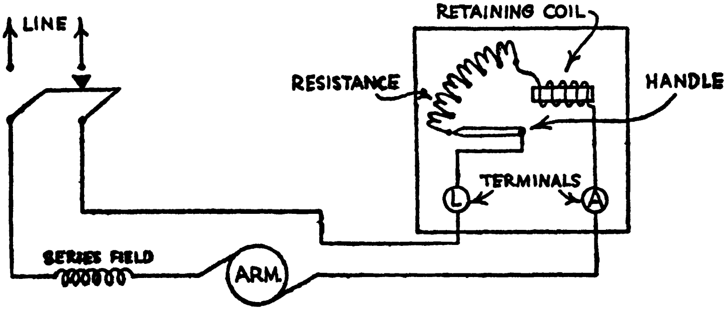
Shunt Motor Connection

Compound Motor. This type of motor has the combined characteristics of both the series and the shunt motor. In addition to the shunt field, a series field is wound on the pole pieces. This consists of a few turns of heavy conductor and is connected in series with the armature. If the magnetism of both the series and shunt fields is in the same direction, they are cumulatively connected; but if one of the fields is opposing or "bucking" the other, they are differentially connected. The method of testing whether they are cumulatively connected is as follows: 1. Connect the motor as shown. 2. Disconnect the shunt field and start the motor, running it as a series motor and note the rotation. 3. Reconnect the shunt field. 4. Short out the series field and start the motor, running it shunt-connected. Note the rotation. 5. If the rotation is in the same direction for both tests, they are cumulatively connected. If the direction of rotation is opposite, the motor is differentially connected. If the motor is differentially connected and it is desired to have it cumulatively connected, change the direction of the current in either the series or shunt field. If the motor is cumulatively connected and the load is increased, the current in the armature and series field will increase. This increase of current in the series field will strengthen the field and increase the torque of the motor. If the motor is differentially connected an increase of current in the armature and series field will also strengthen the magnetism of the series field. Because the series field is connected in opposition to the shunt field it will reduce the magnetic lines. If the load keeps increasing the magnetism of the series field will overcome the magnetism of the shunt field and the motor will come to a complete stop or possibly change rotation.



Compound Motor Connection

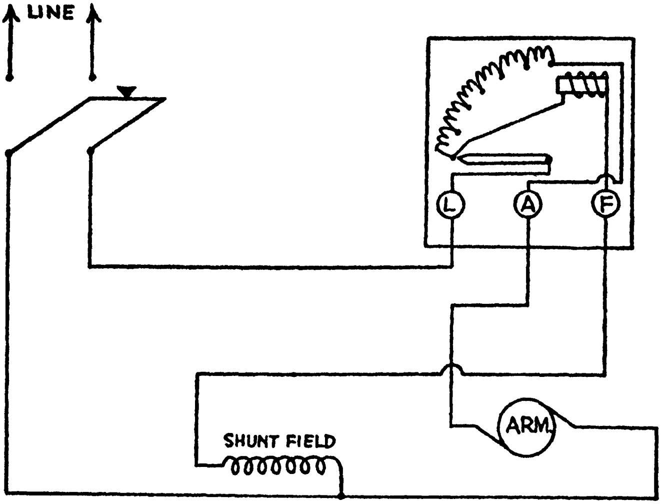
Starting Rheostats. Because the armature has a very low resistance, a resistance has to be connected in series with it for starting purposes. According to the code all direct current motors of ½ h.p. and larger have to be equipped with a starting rheostat. This resistance is mounted in a separate box with taps connected to contact buttons on the face of the rheostat. A movable arm with spring tension makes contact with the contact buttons. The resistance is cut out by moving the arm to the right as the speed and counter e.m.f. of the motor increase. A holding magnet keeps the arm in the running position as long as current is flowing through it. This holding magnet is known as a no-voltage release. Rheostats are rated in h.p. and must correspond to the horsepower of the motor.



Two-Point Starting Box

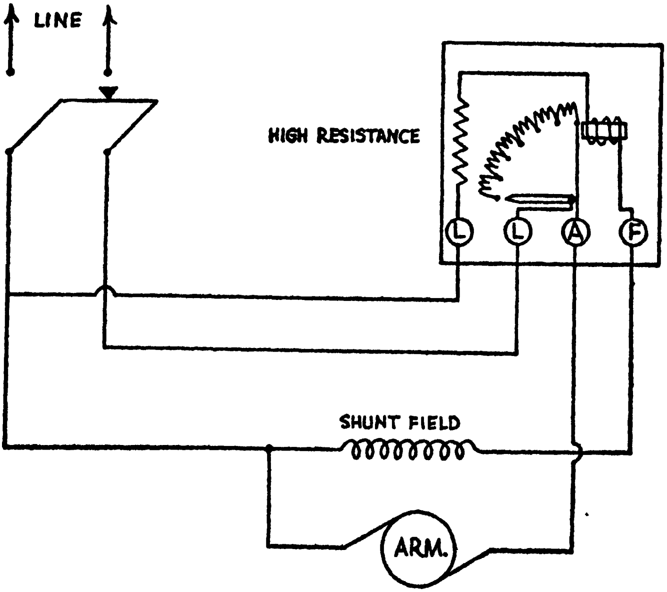
Manual Starting Rheostats. (*a*) *Two-Point Starter.* The resistance and the holding magnet are connected in series. If there should be a power failure the holding magnet would lose its magnetism and the arm would return to the off position due to the spring. To start the motor the arm of the starter is moved to the first contact button. As the motor picks up speed the arm is moved to the right until all the resistance is cut out and the motor is connected directly across the line.

(*b*) *Three-Point Rheostat.* The next figure shows a three-point starting rheostat which is used to start shunt and compound motors. It has three terminals for connecting to the external circuit, one terminal for a line wire, one that connects to the shunt field, and one that connects to the armature. It will be noted that there is a connection from the first contact button to one side of the holding magnet and from the other side of the holding magnet to the terminal marked "field." As the arm is brought to the first position it not only cuts the entire resistance in series with the armature but also magnetizes the holding magnet and connects the field directly across the line. This is very important because if a motor runs without a field it will race and eventually tear itself apart from centrifugal force.



Three-Point Starting Box

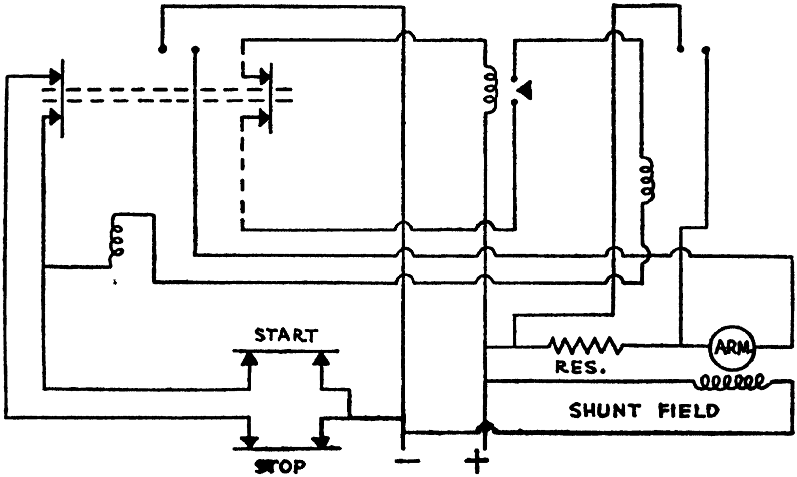
(*c*) *Four-Point Rheostat.* The only difference between this type rheostat and the three-point rheostat is that the three-point rheostat has the holding magnet connected in series with the field while the holding mag-



Four-Point Starting Box

net of the four-point rheostat is connected directly across the line. Also, on the four-point rheostat, both line wires have to be connected to the rheostat. All rheostats have to be enclosed in an iron box to meet code requirements.

Automatic Starting Rheostats. Automatic control is becoming more important as it makes possible the starting and stopping of machinery by merely pressing a button located on the machine. It also plays an important role in starting pumps, compressors, etc. by means of tank switches, pressure switches, thermostats, etc. One of the most common is the definite time starter. Other types are the magnetic time starter and the counter e.m.f. starter.



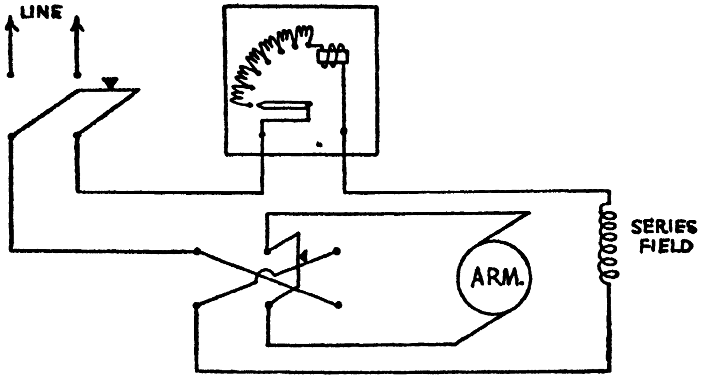
Automatic Starter

Reversing D.C. Motors. Many machines have to run in both directions; therefore the motors have to do likewise. A double throw switch is used to accomplish this. As stated before, rotation can be changed by changing the direction of the current in either the armature or the field. In general practice the armature method is used because there is no danger of having an open field circuit due to the double throw switch not being closed tightly. The following figure shows the wiring connections for reversing a series motor.

Fig. (a) shows the wiring connections for reversing a shunt motor using a three-point starting rheostat.

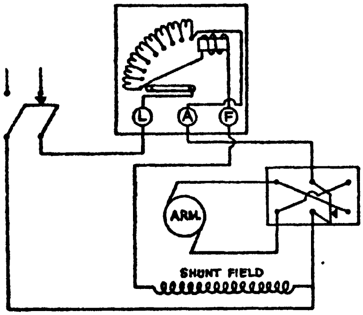
Fig. (b) shows the wiring connections for reversing a shunt motor using a four-point starting rheostat.

Fig. (c) shows the wiring connections for reversing a compound motor using a three-point starting rheostat.

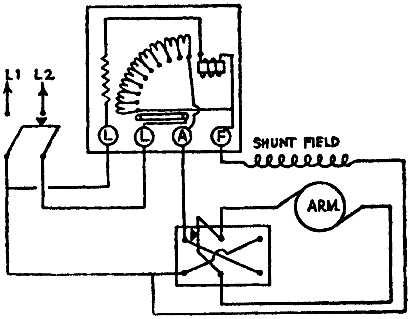


Reversing Series Motor

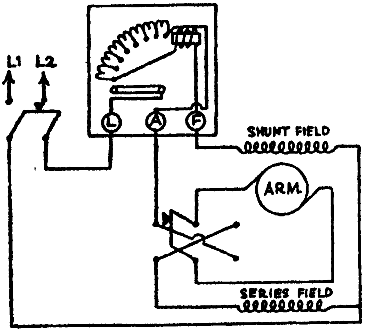
Fig. (d) shows the wiring connections for reversing a compound motor using a four-point starting rheostat.



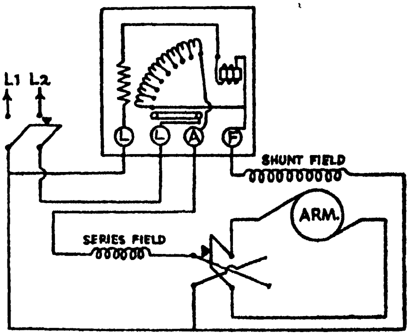
(a) Reversing Shunt Motor—
3-Point Starter



(b) Reversing Shunt Motor—4-Point
Starter



(c) Reversing Compound Motor—
3-Point Starter



(d) Reversing Compound Motor—
4-Point Starter

Speed Control of Direct Current Motors. (1) *Series Motor.* To vary the speed of a series motor a resistance is connected in series with the armature and field. Care must be taken to install the proper size resistance to avoid burning out and overheating. (2) *Shunt and Compound Motor.* To reduce the speed below the name-plate reading a resistance of the proper size is connected in series with the armature. In general practice this resistance is incorporated in the starting rheostat. One type has a ratchet and arm arrangement on the starting box arm and a holding magnet which allows the starting arm to remain on any contact button desired. It is also so arranged that in the event of a power failure the holding magnet will become demagnetized and return the arm to the "off" position. When the speed is reduced below the name-plate reading, the horsepower is reduced in proportion to the speed. To increase the speed above the name-plate reading of the motor, resistance is connected in series with the shunt field. Care must be taken in selecting the proper resistance. When ordering a rheostat with field resistance, the horsepower rating, type, speed, field current and percentage of speed increase must be given to the manufacturer. This resistance is also built in as part of the starting rheostat, but has two rows of contacts, one for starting the motor and one for cutting resistance in series with the field. On most rheostats this is accomplished with the rheostat arm. The motor is first started in the usual manner by moving the starter arm from left to right until all the armature resistance has been cut out and the motor is running at full speed. The rheostat arm (which is made in two sections, one of which is held by the retaining magnet) is then moved to the left, cutting resistance on the second row of buttons in series with the shunt field. Motors may be increased 50% above normal speed.

On automatic rheostats this speed control may be either manual or automatic. If automatic, the resistance for speed control is operated by a small auxiliary motor which is operated by push buttons. This type of control is used on newspaper presses, the control station being located around the press. These control stations have the following buttons: stop, start, jog, increase, decrease and safety.

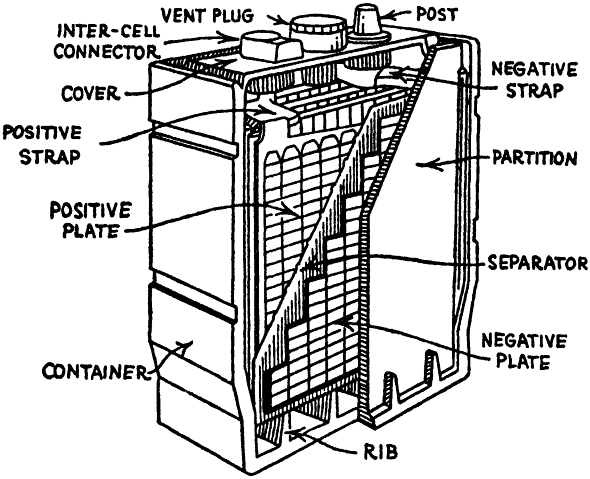
Motor Faults and Corrections. 1. *Sparking at the brushes:* (a) Brushes not set at the commutating plane. Rocker arm should be adjusted until proper position is found. (b) Not enough pressure on brushes. Tighten brush spring. (c) Brushes vibrate and chatter. Make sure that the brushes fit brush holder and commutator properly. (d) High, low or loose commutator bar. Repair commutator. (e) Loose connection between armature coil and commutator. Solder lead from armature coil to commutator. (f) Commutator worn in ridges. Turn down commutator in lathe. (g) Armature coil short circuited. Armature should be rewound, but in case of emergency the shorted coil may be cut out. (h) Commutator segments short-circuited. Reinsulate commutator, or remove dirt and grease from between commutator bars. (i) Overload.

2. *Motor races:* Either an open circuit in the field, or field is short circuited. Rewind field or repair same if possible.

3. *Armature hits pole pieces:* Worn bearings. Replace or rebabbitt.

4. *Motor overheats:* (a) Overload. (b) Bearings too tight. (c) End plates not put on properly.

Lead Storage Battery. The storage battery, like any other type of battery, is a device by which chemical energy is converted into electrical energy. Unlike other batteries, however, it can be recharged after it has run down, so that it can be used over and over again until the plates finally disintegrate.



Sectional View of Lead Storage Battery, Showing Construction Details

The most commonly used type is the *lead storage cell,* which consists essentially of two plates immersed in dilute sulphuric acid. The positive plate is a lead grid packed with lead peroxide (PbO_2). The negative plate is a lead grid packed with spongy lead (Pb). When the cell is furnishing current, that is, when it is being discharged, the positive plate loses oxygen and becomes coated with lead sulphate ($PbSO_4$). The negative plate also becomes coated with lead sulphate. The chemical reaction which takes place is given by the following equation:

$$PbO_2+Pb+2H_2SO_4\rightarrow 2PbSO_4+2H_2O+\text{Electric Energy}$$

As the two plates become coated with lead sulphate, the strength of the battery decreases, and it must, therefore, be recharged. In charging the cell an electric current is passed through it in a direction opposite to that of the discharging process. The reverse of the chemical reaction described above takes place. That is, the positive plate loses its sulphate ions and becomes coated with lead peroxide, while the negative plate loses its sulphate ions and becomes spongy lead again. The chemical equation is:

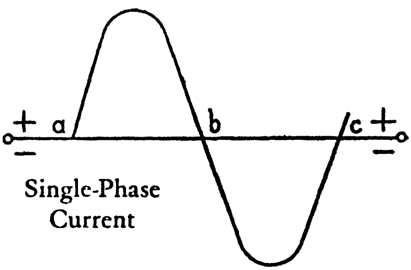
$$2PbSO_4 + 2H_2O + \text{Electrical Energy} \rightarrow PbO_2 + Pb + 2H_2SO_4$$

The storage battery is widely used on automobiles for starting, lighting, and ignition, in trucks, in radio and telephone work, in stand-by and reserve units for power plants, etc. The voltage of a single storage cell is about 2 volts. Batteries ordinarily used in automobiles consist of either 3 cells in series giving 6 volts, or 6 cells in series, giving 12 volts.

In the *Edison storage cell,* which is also quite commonly used, the negative plate has pure iron and the positive plate nickel peroxide packed in nickel plated steel frames; the electrolyte is potassium hydroxide. Although this type of cell is much lighter than the lead storage cell, its voltage on discharge is only about 1.2 volts, while that of the lead cell is 2 volts.

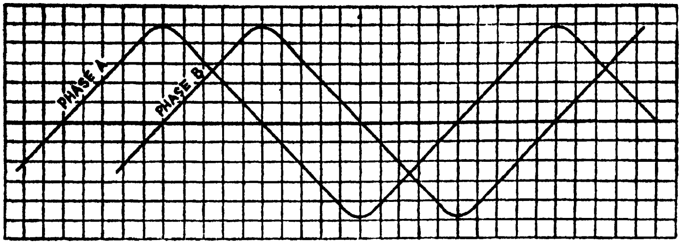
68. ALTERNATING CURRENT MACHINERY

Single-Phase Current. Alternating current keeps changing its polarity and voltage continuously. In the diagram, the horizontal line represents zero voltage; above the line indicates positive voltage, and below the line, negative voltage. When the voltage is generated it starts at zero, increasing in the positive direction until it reaches its maximum point and then decreases to zero. This type of electricity is known as *single-phase*. The course the electricity travels from *a* to *b* is one alternation, and from *b* to *c* one alternation. From *a* to *c* is one *cycle*. Therefore two alternations equal one cycle. The number of cycles completed in one second is known as the *frequency*. The standard frequency is 60 cycles, which means that the electricity makes 120 alternations or changes of polarity in one second. By having the frequency at this rate the alternating changes are so fast that the voltage is constant and shows no fluctuation. Occasionally 25-cycle current is used for power work, but it is not suitable for lighting purposes since a flickering of the light becomes noticeable. When ordering equipment it is very important that the desired frequency be specified.



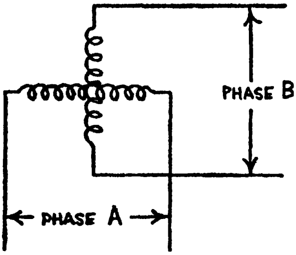
Single-Phase Current

Two-Phase Alternating Current. This type of electricity is comprised of two distinct circuits, namely, phase A and phase B, which are ninety electrical degrees apart, as shown in the graph of a two-phase current. When phase A reaches its maximum point in the positive direction, phase B is only starting from zero; when phase B reaches its maximum point phase A is at zero. In a two-phase four-wire system, the voltage of each

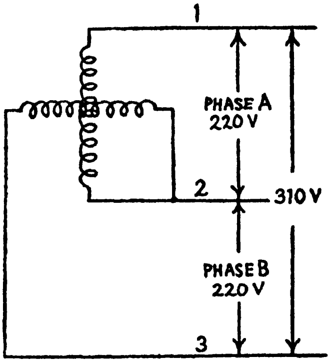


Two-Phase Current

phase is the same, and there is no voltage between any wire of phase A and any wire of phase B. In a two-phase three-wire system, the voltage



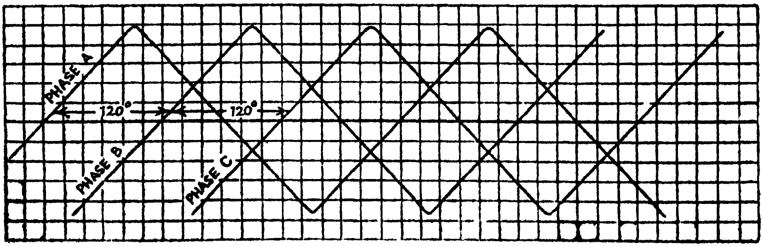
Two-Phase 4-Wire System



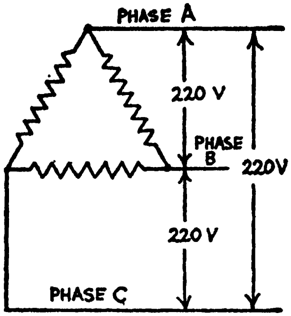
Two-Phase 3-Wire System

between the phases is the same, but the voltage between the outside wires (1 and 3) is 1.4 ($=\sqrt{2}$) times as large as the phase voltage.

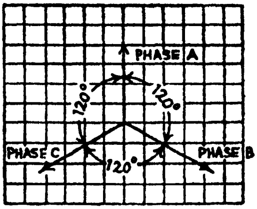
Three-Phase. This type of electricity has three separate windings 120 electrical degrees apart, phases A, B, and C, with the voltage between any two phases the same. There are two methods of connecting three-phase



Three-Phase Current

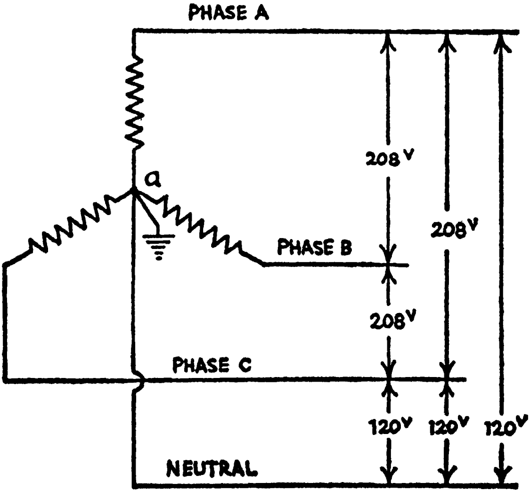


Three-Phase Delta Connection

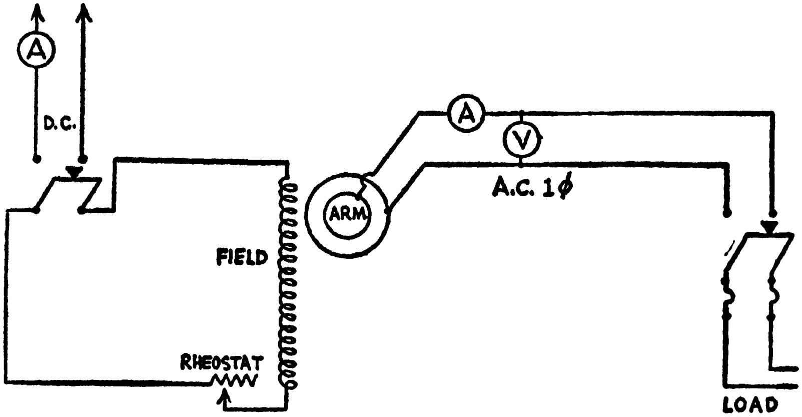


Three-Phase Current

electricity. One method has the windings connected in "delta" fashion. This type of connection may be used for power or light, but not for both. In the other method the windings are "star" connected. By connecting a wire known as the neutral at the point where phases A, B and C are joined, the star connection may be used for both power and light. This connection is the most commonly used. The voltage between any two phases is the same, but the voltage between any phase and the neutral equals the voltage between the phases divided by 1:73 ($=\sqrt{3}$).



Three-Phase 4-Wire Star Connection

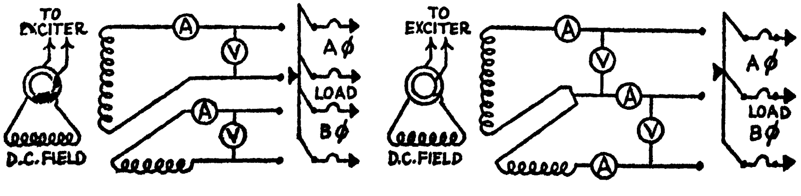


Single-Phase Alternator-Stationary Field (Separately Excited)

Generators. The principle of generation of alternating current is the same as that of direct current, except that alternating current generators have to be separately excited. This is done in general practice by having a small DC generator, known as an *exciter,* mechanically connected to the driving machine to supply direct current to the fields of the generator.

Single-Phase Generator. This type of generator has one winding terminating at two slip rings fastened to the shaft, but insulated from the shaft and from each other. Brushes making contact with the slip rings carry the generated e.m.f. to the terminals and external circuits. Care must be taken to have the generator running at the proper speed in order to have the correct frequency. If the generator is running slow the frequency will be low, and conversely if it is running fast. The speed is controlled by regulating the driving machine. To regulate the voltage output, resistance is either cut in or out of the field circuit by means of a field rheostat; the wiring connections of a separately excited single phase generator are as shown.

The Two-Phase Generator. This generator has two separate windings which are 90 electrical degrees apart. Each winding or phase is brought



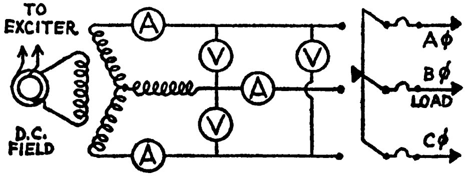
FOUR WIRE THREE WIRE

Two-Phase Alternator-Revolving Field

out to a separate pair of slip rings mounted on the shaft. The frequency and voltage are controlled in the same way as in single phase machines. The two-phase three-wire system may have one wire of each phase connected internally, or it may be connected externally by means of a jumper.

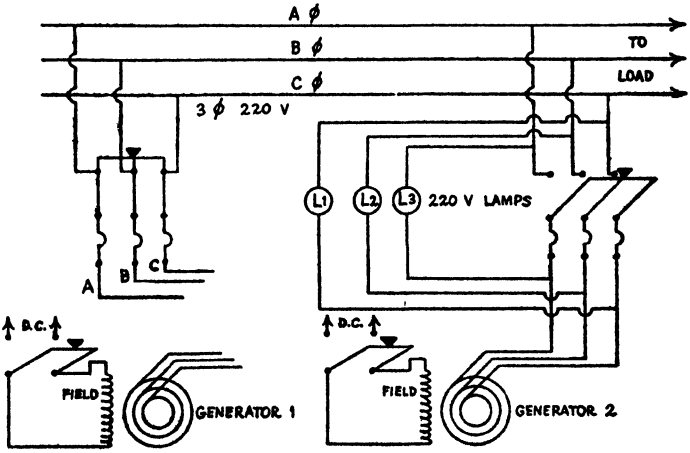
The Three-Phase Generator. This has three separate windings which are 120 electrical degrees apart. These windings may be connected internally either "Star" or "Delta" fashion by the manufacturer, or the six leads may be brought to a terminal block on the generator and the "Star" or "Delta" connections made externally.

The generator losses and efficiency are the same as those of direct current generators. All alternating current generators are rated in Kilovolt-amperes (KVA), which are the volts multiplied by the amperes divided by one thousand; or $KVA = \frac{E \times I}{1000}$.



Three-Phase Star-Connected Alternator-Revolving Field.

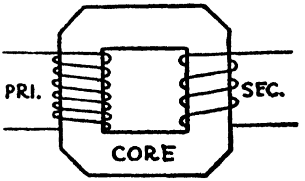
Synchronizing Generators. When two or more alternating current generators are connected in parallel, the method of connecting the generator to be paralleled to the load is known as *synchronizing*. The accompanying diagram shows the wiring connections for synchronizing two three-phase generators. Before the load switch of generator #2 can be thrown in, the frequency and voltage of machine #2 must be identical with that of



Paralleling Three-Phase Generators

machine #1. This is done with the use of lamps (L_1, L_2, L_3) which are connected across the load switch of each phase of generator #2. The generator is brought up to voltage regulated to that of generator #1. The speed is then adjusted to a point where all the lamps vary in brilliancy and go entirely out very slowly. If one lamp should follow the other in brightness the generator is not properly phased. To correct this change the wires of one of the phases. When all the lamps are out it signifies that generator #2 is in "step" or synchronized with generator #1. The load switch of generator #2 is closed when all the lamps are out. To disconnect generator #2 from the line open the load switch.

Transformers. A transformer is a piece of equipment by which the voltage of an alternating current may be raised or lowered. For distribution purposes it is far more economical to transmit alternating current than direct current. By the use of transformers the voltage may be raised at the point of distribution and lowered at the point of consumption, thereby saving copper and also eliminating or reducing line losses. As the voltage is raised the amperage is decreased in direct proportion, and therefore a wire of less carrying capacity may be used.



Construction and Principles of Operation. A simple transformer has a coil of wire known as the *primary* winding, wound around an iron core and connected to the line. A second coil, known as the *secondary* winding, is also wound on the iron core. This winding is independent and has no electrical connection with the primary winding. When current is impressed on the primary winding, a magnetic field exists around this coil. Because the current is alternating, the magnetism is also constantly changing, thus inducing an e.m.f. in the secondary winding. The same amount of magnetic flux cuts both the primary and secondary windings; therefore the voltage induced per turn is the same. The amount of voltage induced in the secondary winding depends on the ratio of the number of turns on the secondary winding to the number of turns on the primary winding. If we wish to increase the voltage ten times, the secondary winding would then have to have ten times the number of turns as the primary winding. If the voltage were to be reduced ten times, then the secondary winding would have one-tenth the number of turns that the primary winding had.

Example: A transformer having 1000 turns of wire is connected to 100 volts. How many turns must the secondary winding have to increase the voltage to 1000 volts?

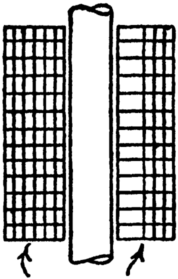
$$\frac{\text{Primary voltage}}{\text{Primary turns}}=\frac{\text{Secondary voltage}}{\text{Secondary turns}},\ \text{or}\ \frac{E_p}{N_p}=\frac{E_s}{N_s}\ .$$

$$\frac{100}{1000}=\frac{1000}{x}$$

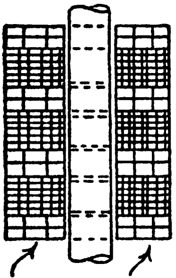
$$100x=1{,}000{,}000$$

$$x=10{,}000$$

There are several different methods of installing the windings. In one method the windings are cylindrical in form, and are placed one inside the other with the proper insulation between them. In the other method the coils are built-up in flat sections called pancake coils, and are sandwiched together.



Cylindrical Type



Pancake Type

As in all electrical equipment the electrical losses produce heat Small transformers of 5 KVA and smaller are air cooled; i.e., the heat produced by the transformer is carried off by the surrounding cooler air. On larger transformers oil is used for cooling. The warm oil keeps rising to the top of the transformer which causes a circulation. The oil serves a dual purpose as it is also used to act as an insulator between the primary and secondary windings.

Transformer Losses and Efficiency. The efficiency of a transformer is the ratio of the power output on the secondary side to the power input on the primary side of the transformer. This method is not very satisfactory due to the high efficiency of transformers. A more accurate method is to measure the core and copper losses separately with a wattmeter. The core loss is measured by placing a wattmeter in the primary circuit with the secondary open. To measure the copper loss, connect a wattmeter in the primary circuit with the secondary coils short-circuited through an ammeter. Enough voltage is applied to the primary side of the transformer so that the amperes flowing through the secondary coils is equal to the full-load rating of the transformer. The losses being known, the efficiency is found by dividing the output plus the core and copper losses into the output:

$$\text{Efficiency}=\frac{\text{Output}}{\text{Output}+\text{Copper loss}+\text{Core loss}}\ .$$

Example: If a transformer has a copper loss of 200 watts and a core loss of 125 watts when it is delivering its rated output of 10 KW, what is its efficiency?

$$E=\frac{Output}{Output+Cu\ loss+Core\ loss}$$

$$\mathrm{E}=\frac{10{,}000}{10{,}000+200+125}$$

$$\mathrm{E}=\frac{10{,}000}{10{,}325}=96.8\%$$

Transformer Connections. The accompanying figure (*a*) shows a single phase transformer with the secondaries made up of two coils. It will be noted that the secondary coils are connected in series and the voltage is 220 v. If the secondary coils were connected in parallel, Fig. (*b*), the voltage would be 110 v., but the capacity would be twice as much as when the coils were connected in series. Care must be taken when connecting the coils in parallel so that the wires of the same polarity are connected together. If the secondary coils are not connected in the proper manner they are said to be "bucking" and will burn out the coils. The method of testing so as to prevent bucking is as follows: Connect one wire from coil A and one wire from coil B together, Fig. (*c*). Connect a test lamp between the other wire of coil A and coil B. With current on the primary side of the transformer, the lamp C will not light if the coils are properly connected. If the coils are improperly connected lamp C will light because of the difference of potential.

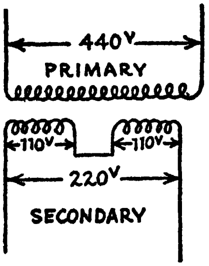


Fig (a)

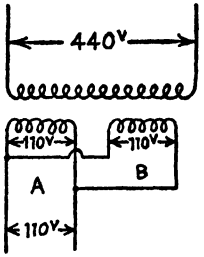


Fig. (b)

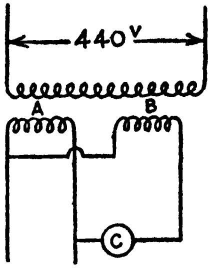
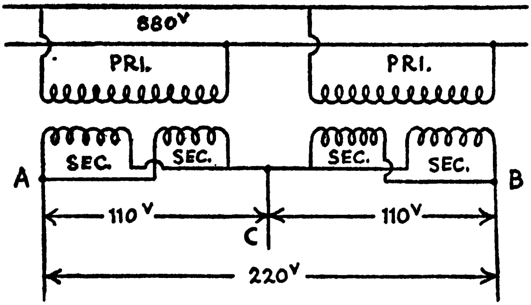


Fig. (c)

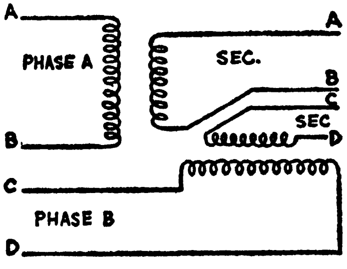
Single-Phase Three Wire. The following diagram shows two single phase



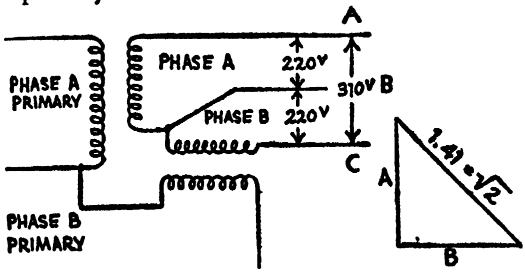
Single Phase Three Wire Transformer Connections

transformers so connected that either 110 v. or 220 v. may be obtained on the secondary side. The secondary coils of transformers I and II are first connected in parallel, and then each transformer is connected in series. The outside wire A and B of each transformer clearly supplies 240 volts; a tap made at C will supply 120 volts between A and C of transformer I, and 120 volts between B and C of transformer II. The primary side of each transformer is connected in parallel with the 440-volt main line. If the main line were 880 volts, the primary coils would be connected in series with one wire of each transformer connected to the 880 volts.

Two-Phase Transformer Connections. Because a two phase circuit is similar to two single phase circuits it has to be treated as such when connecting. The primary side of phase one has to be connected to one phase of the main, and the primary side of phase two has to be connected to the second phase of the main. The secondary side will also be two phase four wire. The voltage between each phase will be the same, but there will be no voltage between the phases since there are no electrical connections between them. However, it is possible to eliminate a wire both on the primary and secondary sides of the transformer; this is done by connecting B and C together on each side of the transformer. With this type of connection the voltage between the phases is the same, but the voltage between A and D is 1.41 ($1.41=\sqrt{2}$) times the



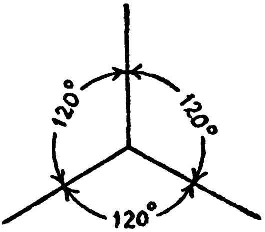
Two-Phase Four-Wire Transformer Connections

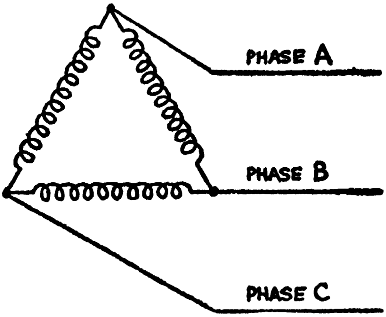


Two-Phase Three-Wire Transformer Connections phase voltage. The amperes that flow through the common or middle wire B is also 1.41 times the amount that flows through the wires A and D.

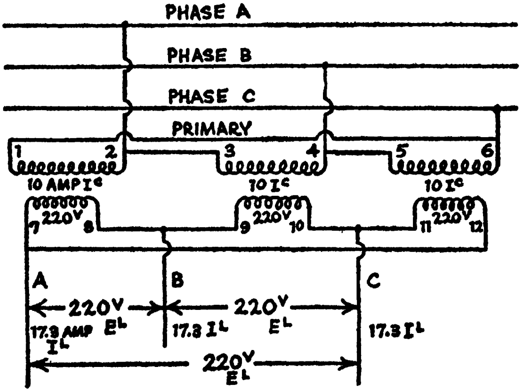
Single phase equipment can be connected on either phase of a two phase system, regardless of whether the two phase system is three or four wire. When connecting single phase equipment on either phase of a two phase system care must be taken that the voltage of the equipment is the same as that of the phases.

Three-Phase Delta Transformer Connections. Three single phase transformers, or one transformer with the three phases encased in a single iron core, may be used. In a three phase circuit the electromotive force is 120 degrees behind the preceding phase and 120 degrees ahead of the following phase. In any three phase circuit the sum of the voltage must be zero or there would be current flowing through the coils when no load is con-

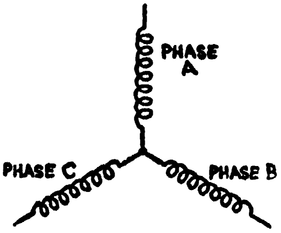




Three-Phase Delta Connection



Three-Phase Delta Connection



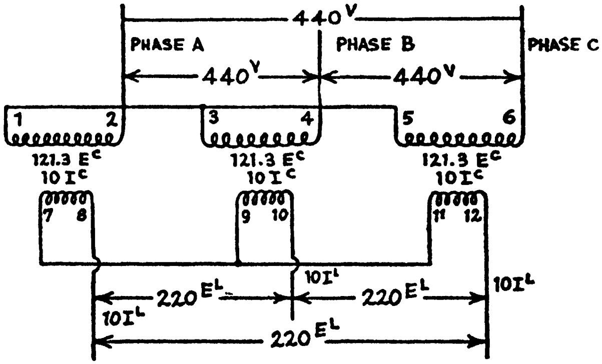
Three-Phase Star Connection (also called Y connection) nected. When three transformers are connected in delta the line voltage equals the coil voltage ($E_c=E_e$) and the voltage between any of the phases is the same. When a load is connected, the current in the respective line wires is 1.73 ($=\sqrt{3}$) times that of the current flowing in the corresponding coils ($I_e=I_c\times1.73$). The figure shows three transformers connected in delta on both the primary and secondary coils. On the primary side:

Terminals 2 and 3 connect to phase A
" 4 " 5 " " " B
" 1 " 6 " " " C

On the secondary side: Terminals 7 and 12 are connected and brought out to line wire, phase A. Terminals 8 and 9 are connected and brought out to the line wire, phase B. Terminals 10 and 11 are connected and brought out to the line wire, phase C.

Three-Phase Star or Y Transformer Connections. With this type of connection the coil amperes equals the line amperes, I_c equals I_e, but the line voltage is 1.73 ($\sqrt{3}$) times higher than the coil voltage; or, $E_e=E_c\times 1.73$. This type of connection is used for transmission.

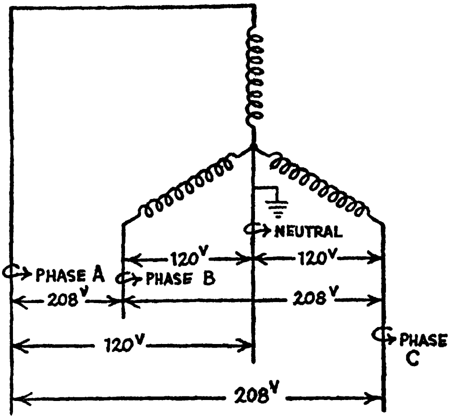
The method of connecting these transformers is also shown. On the primary side, terminals 1—3—5 are all connected together to form the center connection; terminals 2—4—6 are connected to the respective phase wires A—B—C. On the secondary side of the transformer, terminals 7—9—11 are connected together; terminals 8—10—12 are connected to phase wires A—B—C, respectively. The voltage is the same between phases AB—BC and AC. For economy the utility companies have adopted the three-phase four wire system. With this system both power and light may be taken from the same lines. The transformers are star-connected, with the fourth wire connected to the center connection. On the secondary side of the transformer, 208-volt, three-phase power is taken from phase A—B and



Three Phase Star Connection

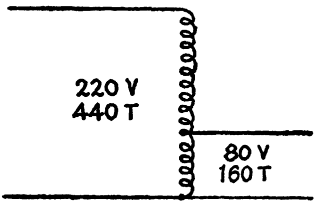
C, and 120-volt single-phase lighting is taken from the neutral and from any of the phases. Regardless of the unbalance of the phases, the voltage between any phase and the ground or neutral wire never exceeds 120 volts.

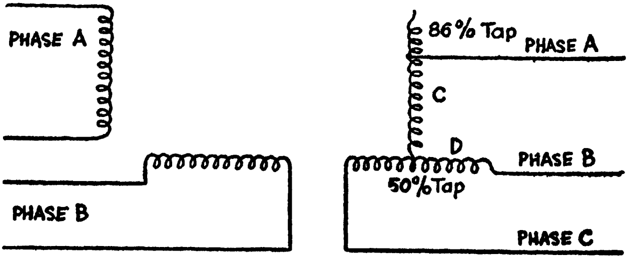
Single-phase power equipment may be connected to any one of the three phases, whether star- or delta-connected, provided that the voltage of the equipment is the same as that of the transformers. Two-phase motors can never be connected to a three-phase circuit.



Three Phase Four Wire Star Connection

Auto-Connected Transformer. For economy, transformers are sometimes wound with one winding which serves both primary and secondary circuits. These transformers may be either step up or step down, but are never used on voltages over 440 volts. This type of transformer is used for starting motors with reduced voltage, mercury arc rectifiers, and mercury arc lamps.



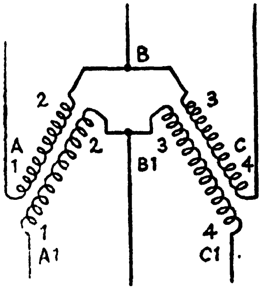


Scott Transformer Connection

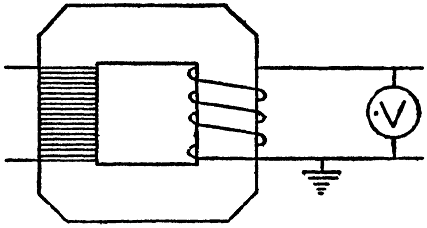
Scott-Connected Transformer. At times it is essential to convert a two-phase current to a three-phase, or vice versa. This is accomplished by the use of a Scott-connected transformer. It will be noted that coil C has a tap placed at a point 86% of that coil and is phase A. The other end of coil C is tapped at 50% or midway to coil D; phases B and C are the ends of coil D.

Open Delta Connection. It is possible to keep a three-phase system in operation even though a transformer on one of the phases has burned out by connecting the two remaining transformers, as shown, in a type of connection known as "Open Delta," with characteristics of both star and delta connections. Only 58% of the load that was connected to the three transformers can be connected on this type of connection. If the open delta connection is to be permanent the KVA rating must be increased 16% at unity power factor.

Current Potential Transformers. Alternating current relays and meters should never be connected to high voltages, but should be linked to them by means of a current or potential transformer which is also known as an *instrument transformer.* By using this type of transformer the instruments are insulated from high voltages but will read the high voltage, amperes, power, etc., correctly. By the use of this transformer instruments have been standardized to 120 volts for potential transformers and 5 amperes for current transformers. The primary side of the transformer is wound so that it may be connected to the high voltage; the secondary side of the transformer is so wound that it will deliver 120 volts. The low voltage side should always be grounded to eliminate static and insure safety to anyone using them.



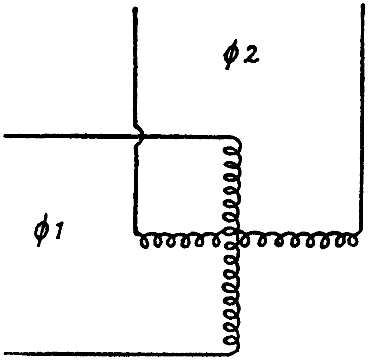
Open Delta Connection



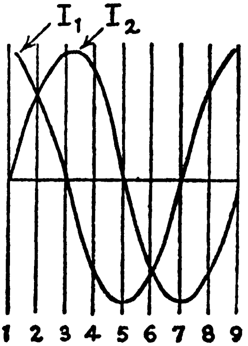
Potential Transformer

Alternating Current Motors: *Polyphase Induction Motor.* The induction motor is the most common type of alternating current motor because it is simple and rugged in design. It consists principally of two units, the *stator* and the *rotor.* The stator consists of a number of slots in the frame into which the winding is placed. The rotor consists of a laminated cylinder with slots in its surface. The winding which is placed in these slots may be of two types. One type, known as the *squirrel cage,* is made up of copper bars, placed in the slots and connected together at each end by a copper ring. The connections between the copper bars and rings are generally made by an electric weld. The other type of rotor, known as a *slip ring,* or wound rotor, consists of a winding placed in the slots, which is similar to the stator winding. This winding is usually star connected with the open ends brought out to slip rings fastened on the shaft. Brushes are connected to the slip rings and connected to a star connected rheostat. The purpose of this rheostat is for starting the motor and for speed control.

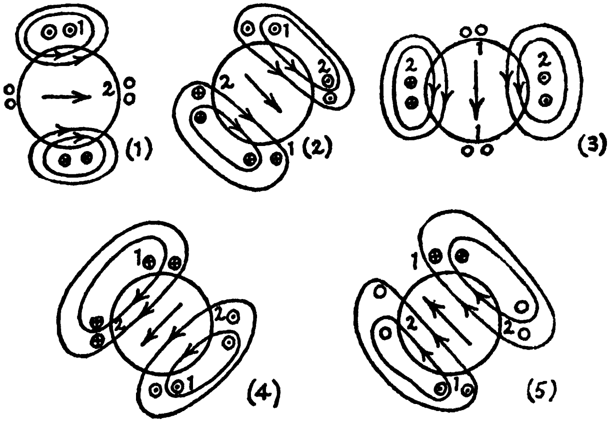
Rotating Magnetic Field. The stator of an induction motor has no projecting poles because the winding is embedded in slots. Fig. (*a*) shows a two phase stator winding. When the winding is energized with a two phase source, the currents vary as shown by I_1, and I_2, Fig. (*b*). At instant (1) the current is zero in phase 2 and maximum in phase 1. With a current flowing as shown in (1) a flux is set up to the right as indicated by the arrow. At (2) it is still in the same direction in phase 1, and an equal current flows through phase 2. This establishes a resultant flux of the same strength 45 degrees clockwise from position (1). At (3) phase 1 has decreased to zero and the current in



(a) Winding



(b) Current

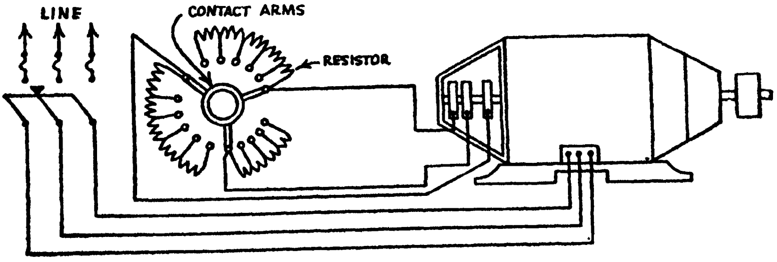


(c) Rotating Field

phase 2 has increased to a maximum. This establishes a flux downward. At (4) the current in phase 1 has reversed and has the same value as that of phase 2. This establishes a flux clockwise from position (3). At (5) the current in phase 1 is at a maximum and in phase 2 it is zero, establishing a flux to the left. The successive instants considered in Fig. (*b*) indicate that rotary magnetic field of constant strength is produced which makes a complete revolution per cycle. The magnetic field of a two pole stator winding makes 60 revolutions per second, or 3600 revolutions per minute when supplied by 60 cycle current. To lower the speed it is necessary to increase the number of poles in the stator winding.

Torque. When the stator is connected to a polyphase source of supply the rotating magnetic field induces a current in the short circuited rotor winding in a direction parallel to the rotor axis and the reaction of the magnetic flux of the rotor conductor against the rotating field is therefore in the correct direction to produce rotation. The speed with which the rotor revolves is less than that of the rotating magnetic field, because if the rotor speed were the same as the revolving field, there would be no cutting of the rotor conductor by the field and therefore no current nor reaction in the rotor conductors to produce mechanical power or torque by the rotor. The ratio difference between the rotor speed and revolving magnetic field is known as the "slip" and is usually expressed as a percentage. The polyphase induction motor with a squirrel cage rotor has the advantage of being simple in design and rugged in construction, of self-starting under a load, of exerting powerful torque, and of having practically constant speed at all loads. It is particularly suitable in places where there is inflammable material as this type of motor requires no collector rings or brushes, and therefore cannot spark.

Method of Changing Rotation. In order to change rotation it is necessary to reverse the direction of the rotation of the field. This is done by the interchange of the connection of the leads of one phase in a two phase motor and interchanging any two line leads in a three phase motor.

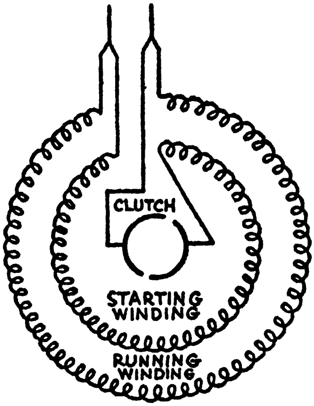


Three-Phase Motor-Wound Rotor and Resistor

Single Phase Motors: ***Induction Motor.*** Single phase induction motors have a very limited use because they are not self-starting; they have to be started from an external source. This fact is exemplified by a polyphase motor, which does not start with one phase open, but will continue to run if started. When a single phase is applied to the winding, a single magnetic field is set up, and no torque is developed; therefore the rotor will not rotate. When the rotor is turned, the rotor bars cut lines of force and a magnetic field results about these bars which is 90 degrees out of phase with the main field. This is similar to a two phase circuit and a rotating field results, developing a torque. The motor will continue to run as long as the current is flowing.

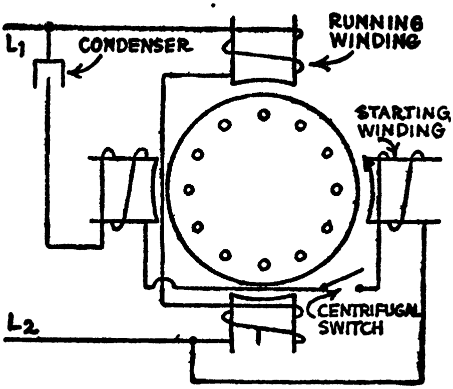
Split-Phase Motor. This type of motor has a starting and running winding and is self-starting. These windings produce the same condition as a two phase motor when it starts. The running winding is low-resistance while the starting winding is high-resistance. The starting winding is placed 90 degrees from the main winding. The starting winding is connected in series with a centrifugal starting switch and is connected in parallel with the running winding. When the motor is started, the starting winding remains in circuit until the motor comes up to 75% speed and then the centrifugal switch is opened. The difference in impedance of the circuit produces a two phase current that produces a rotating magnetic field. This develops the torque needed to accelerate the motor.

Impedance is the total opposition to the flowing of an alternating current; its symbol is "Z": $Z=\frac{E}{I}$; $E=Z\times I$; $I=\frac{E}{Z}$. In DC work we consider only resistance, but in AC work there are other factors. The total of the other factors can be called *impedance.* The power factor is the cosine of the angle of lead or lag; it also equals the true watts divided by the apparent watts. True watts are found by the use of a wattmeter. Apparent watts are found by taking the product of the volts × amperes. This motor has a good starting torque, but must be brought up to speed quickly and is used for general purposes. To reverse rotation, either the starting or the running leads are changed.



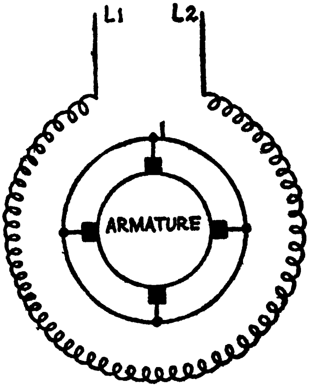
Single-Phase Split-Phase Motor

Single Phase Capacitor Induction Motor. This type motor consists of two windings, namely, starting and running windings, and is similar to the split phase motor. These windings are 90 degrees apart, and are connected in parallel with the main line. A condenser of the proper size is connected in series with the starting winding and cut out switch. Placing the condenser in the starting winding makes the starting winding current lead the main winding current by 90 degrees. The starting torque of this motor is very high because the currents in the two phases are 90 degrees apart. It has the same starting torque as that of a three phase motor and is used for refrigerators, compressors, etc. To reverse this type motor change either the starting or running leads.



Single Phase Capacitor Motor

Single Phase Repulsion Induction Motor. This type of motor has a very high starting torque. It has a stator winding the same as any squirrel cage motor, and the rotor is wound like a DC motor. The brushes are not connected to the supply line, but are short circuited. There is no electrical connection between the rotor and stator except by magnetic induction. If we study this motor we see that at any instant the induced current in the rotor is opposite in direction to the applied voltage of the stator. If the brushes are 15 degrees off the pole center the rotor will produce a high torque. When the rotor comes up to speed, a centrifugal switch short-circuits the rotor winding and the motor will run as an induction machine. To reverse this type of motor the brushes have to be shifted. This type of motor is used on compressors, pumps, conveyors, etc., and ranges in size from ⅛ h.p. to 15 h.p. In some cases the brushes are lifted when in the "run" position; in others they are not.



Single Phase Induction Repulsion Motor

When figuring the size of feeders for motors total the full load current of all the motors to be connected to the feeder plus 25% of the largest full load current.

Example: Determine the size of conductor, the motor running overcurrent protection, the branch circuit protection, and the feeder protection for: 1–25 H.P. squirrel cage induction motor and 2–30 H.P. wound rotor induction motors on a 220 v. 60 cycle supply.

The full load current of the 25 H.P. motor is 64 amperes; this requires a #2 type R wire. The full load current of the 30 H.P. motor is .77 amperes and requires a #0 wire.

The feeder size will be 77 plus 77 plus 64 plus (25% of 77) 19 equals 237 amperes; this would require 500,000 CM conductor. The conduit size would be 3″.

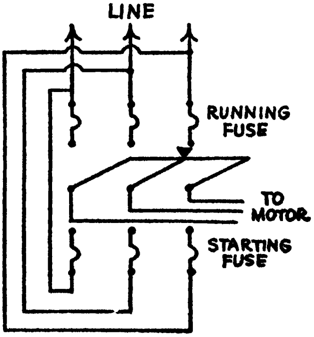
The following tables show the full load current, size of fuse for running protection, size of branch circuit fuse, wire size, and pipe size for single and three phase motors.

Single-Phase

	Full Load Current		For running protection Amp. NEC fuse		Branch Circuit Fuses		Wire Size		Pipe Size	
H.P.	115v	230v	115v	230v	115v	230v	115v	230v	115v	230v
⅙	3.34	1.67	6	3	15	15	14	14	½″	½
¼	4.8	2.4	8	4	15	15	14	14	½″	½
½	7	3.5	10	6	25	15	14	14	½″	½
¾	9.4	4.7	15	8	35	15	14	14	½″	½
1	11	5.5	15	8	35	20	14	14	½″	½
1½	15.2	7.6	20	10	45	25	12	14	½″	½
2	20	10	25	15	60	30	10	14	½″	½
3	28	14	35	20	90	45	8	12	¾″	½
5	46	23	60	30	150	80	4	8	1¼″	¾
7½	68	34	90	45	225	110	1	6	1½″	1¼
10	86	43	110	60	300	125	00	4	2″	1¼

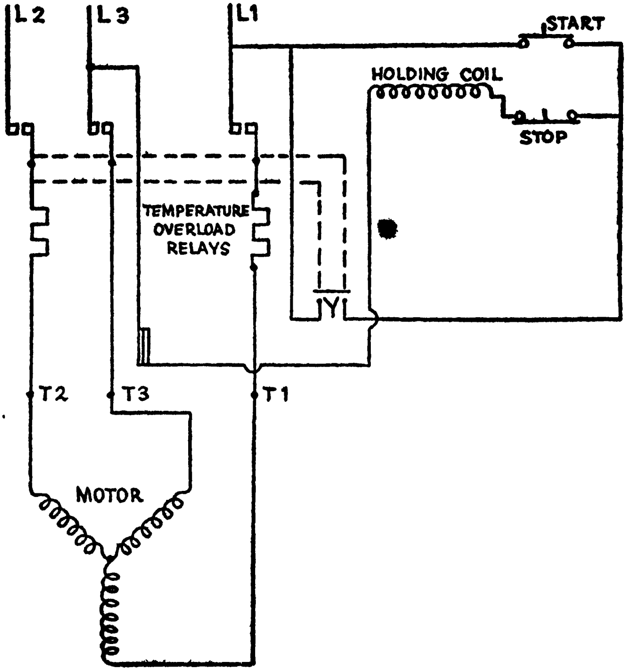
		Three-Phase			
	Motor Full Load Current	For running protection Max. Rating NEC fuses	Max. rating Branch Circuit Fuses	Wire Size	Pipe Size
H.P.	220				
½	2.5	3	15	14	½
¾	2/8	3	15	14	½
1	3.3	6	15	14	½
1½	4.7	8	15	14	½
2	6	8	20	14	½
3	9	12	30	14	½
5	15	20	45	12	½
7½	22	30	70	8	1
10	27	35	90	8	1
15	38	50	125	5	1¼
20	52	70	175	3	1¼
25	64	80	200	2	1½
30	77	100	250	0	2
40	101	125	300	000	2½
50	125	175	400	0000	2½
60	149	200	450	0000	2½
75	180	250	600	400,000	3
100	246	300	500	700,000	3½
125	310	375	600	1,250,000	4½
150	360	450	600	1,750,000	5

Starters. When a squirrel cage induction motor is connected to the line the motor draws a large current in starting (from four to six times the running current). The condition is the same as if the primary side of a transformer were connected to the line with the secondary coil short circuited. The starting current decreases as the motor speed increases. Small induction motors may be protected by fuses, but if the fuses are large enough to carry the rush of current in starting, the motor would have no protection when it reached full speed. Hence a double throw switch is often used; one side of the switch is fused for starting and

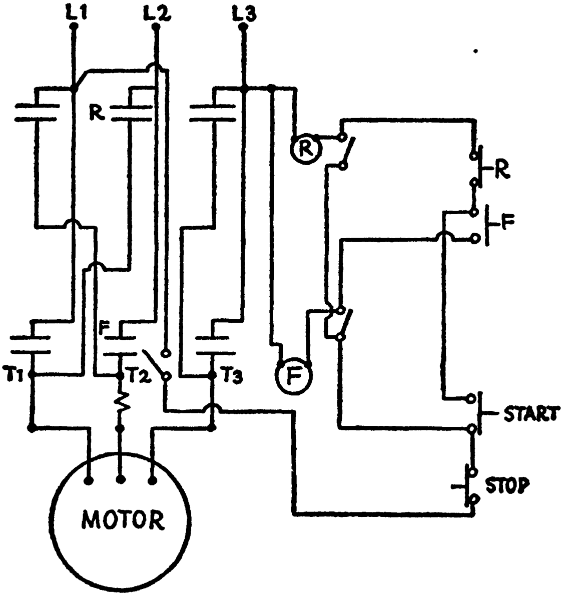


the other side for running. To start the motor the switch is thrown on the side with the large fuses to take the inrush of current. When the motor reaches full speed the switch is thrown to the running position and the motor is protected from overload by fuses of the proper size.

The most common and modern method of starting squirrel cage motors is with a magnetic across the line starter. This consists of a three pole magnetically operated contactor with two thermal overload relays, and is started and stopped by a "start" and "stop" push button. When the "start" button is closed, current flows from line L_1 through the start and stop button, through holding coil, and returns to line L_3. When the circuit is completed the holding coil becomes magnetized and closes the main contactor, throwing the circuit across the line. When the line circuit is closed the gap *Y* is bridged, placing the holding magnet directly across the line. To stop the motor, the "stop" button is pressed, opening the holding magnet circuit which in turn releases the main contactor. If for



Across the Line Starter

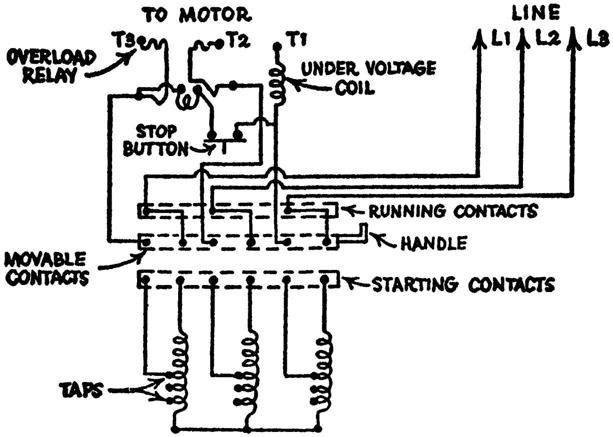


Reversing Controller

any reason the motor becomes overloaded, or the starting current prevails for too long a period, the thermal relays become heated, due to the amperes flowing through them, expand, and open the holding magnet circuit, thereby stopping the motor. After a short interval (enough time for the thermal metal to cool) the thermal overload may be reset and the motor again started.

Reversing Controller. To operate some machines it is necessary to run them in both directions. This is done in a three phase induction motor by a controller which changes the connections of two of the three line wires to the motor. One button is provided for each direction and one for stopping the motor.

Compensator Starter. Compensators are generally used when starting large squirrel cage induction motors. They use auto-transformers, star connected with each transformer, having three taps for voltage regulation. When the handle is in the start position the auto-transformer is connected across the line and supplies a reduced voltage to the motor. As soon as the motor comes up to speed the handle is thrown to the "run" position, connecting the motor directly to the line. Under-voltage protection is provided by a retaining magnet.



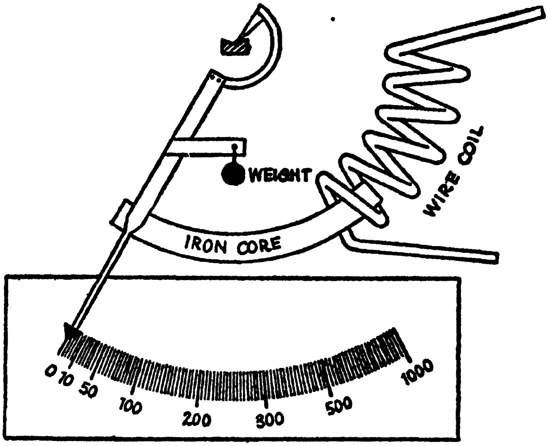
Compensator-Starter, Three-Phase

69. MEASURING INSTRUMENTS

Direct Current Ammeter. The ammeter is used to measure the quantity of electricity and is always connected in series with the line. Ammeters are classified according to their construction into two types: (1) fixed coil and moving iron type, and (2) fixed permanent magnet and movable coil type. All ammeters have a very low resistance and are so constructed that they are free from outside magnetic influences. They are also made in two classes: (1) portable type and (2) switchboard type.

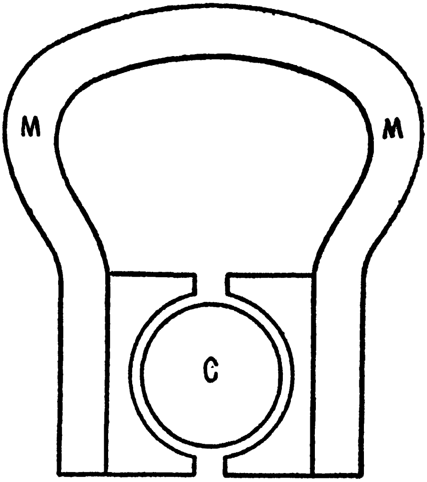
Moving Iron Type Ammeter. This type of instrument has a stationary coil of heavy wire shaped into an arc of a circle; a small iron core bent in the same arc is suspended with one end free so that it may be drawn into the coil. A pointer is attached to the soft iron core and passes over the scale. The needle is controlled by a fixed weight. When the ammeter is connected in series, the line current flows through the coil, magnetizing it and drawing the soft iron core into the coil. The needle which is attached to the coil moves along the calibrated scale showing the number of amperes being drawn from the line. This type of instrument can only be used in a vertical position.

Moving Coil Type Ammeter. The most prominent parts of this type instrument are: a permanent magnet (M) and movable coil (C). The



Direct Current Moving Iron Type Meter

movable coil is made of several turns of small wire insulated and wound on a rectangular aluminum frame. The coil is pivoted on jeweled bearings and is so arranged that a current may flow through its windings. By the method of mounting, only rotary action is possible. This motion is opposed by two springs, which also serve as conductors to carry the current to and from the movable coil winding. The movable coil, even though it contains no iron, has all the properties of a magnet when current is flowing through it and therefore will have polarity. As it lies within a strong magnetic field it will tend to move due to the action between the electromagnetic effect created by the current carried in the coil and the magnetism of the permanent magnet, causing these two magnetic fields to establish a certain relation of their respective lines of force. When a current is sent through the movable coil which has a pointer fastened to it, a deflection occurs, which corresponds to the strength of the current. The pointer moves over a calibrated scale and indicates the amount of current flowing. Since this type of instrument is really a D'Arsonval galvanometer, a shunt has to be connected across the movable coil to take most of the current. The shunt is made of a combination of materials, forming a special resistance alloy that does not change

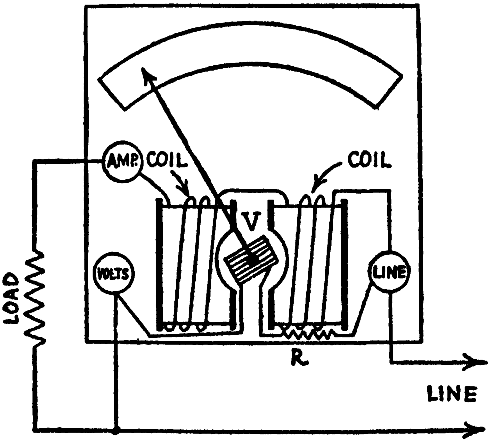


Direct Current Moving Coil Type Meter

in resistance as its temperature changes. The lead wires to the instrument and shunt are calibrated and should never be changed, as this would throw the meter out of calibration.

Voltmeter. The purpose of the voltmeter is to measure the amount of electrical pressure impressed. A voltmeter is always connected across or in parallel with the line to be measured. The types and the parts of the voltmeter are the same as the ammeter, the only difference being that the resistance of the voltmeter is high while the resistance of the ammeter is low. The voltmeter has a fixed resistance connected in series with the movable coil. Because of the uniformity of these instruments they can be made with more than one scale range. For instance, in a voltmeter having a range of 300 volts, it is only necessary to make a tap in the series resistance to have it read 150 volts. In most cases such resistances are encased in the voltmeter case, but external multipliers may be supplied to measure very high voltages.

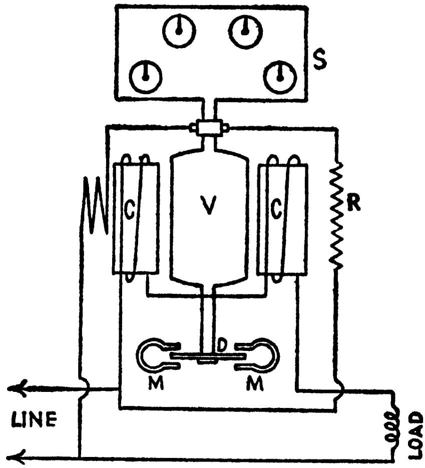
Direct Current Wattmeter. The wattmeter is used to measure the amount of power that a piece of apparatus is using. On direct current it is equal to the volts multiplied by the amperes. Instead of a permanent horseshoe magnet (as in the voltmeter and the ammeter) to create the flux on the movable coil, the wattmeter has two coils, each consisting of a few turns of large wire having a low resistance. These coils are connected in series with one side of the line. The movable coil (V), which consists of many turns of small wire having a high resistance, is connected in series with a high resistance (R) across the line. Current flows through the stationary coils only when there is current flowing through the external circuit. Therefore it is only when current is flowing through the external circuit that a magnetic field will be produced to act on the movable coil (V). The voltage of most direct current circuits is practically constant, therefore the strength of coil (V) will be practically constant. The current, however, varies as the resistance of the circuit;



Direct Current Wattmeter

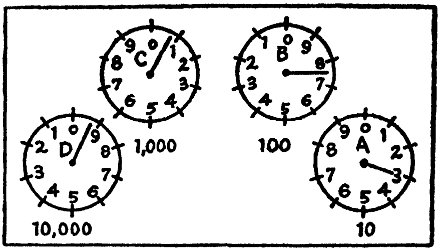
therefore the strength of the lines of force will vary in the stationary coils and the coil (V) will move accordingly. Care should always be taken to be sure that the voltage and current ratings of the instrument are of the proper capacity; if they are smaller than the current and voltage to be measured, the instrument will be burned out.

Thomson Watt-Hour Meter. This type of instrument is used to measure the amount of power used over any period of time. It is the product of the volts, amperes and the time in hours. It reads either watt-hours or kilowatt-hours. This meter consists of two field coils (C) connected in series in the line to produce a magnetic field to act upon a movable coil (V). This movable coil is constructed in the same way as the armature of a direct current motor except that the winding is placed over a very light insulated form instead of an iron core. The movable element or armature is connected in series with a resistance (R) across the line. The top end of the shaft engages a registering system (S) which is so geared that the pointers will register watt-hours or kilowatt-hours consumed, depending upon the capacity of the meter. An aluminum disc (D) is fastened to the bottom of the shaft and rotates between two permanent horseshoe magnets (M) known as *drag magnets*. These magnets are used to adjust the meter. When the magnets are moved out towards the edge of the disc, the meter will be slowed up; when moved towards the center of the disc the speed will be increased. When current flows through coils (C) the armature will move in proportion to the product of the current in coils (C) and the voltage impressed on armature (V). As the load is increased more current flows through coils (C) which in turn speeds up the armature (V). Watt-hour meters are rated in amperes.



Direct Current Watt-hour Meter

Reading the Watt-Hour Meter. The register of the watt-hour meter consists of four dials. When dial (A) makes one complete revolution, the pointer on dial (B) moves to one. When dial (B) makes one complete revolution, the pointer on dial (C) moves to 1. When dial (C) makes one complete revolution, the pointer on dial (D) moves to 1. In deciding the reading of a pointer, the pointer on the dial to the right must be noted, because, unless the pointer at the right has reached or passed "0" (i.e., completed a revolution), the one at the left has not completed the division on which it may appear to rest. It is always best to read a meter from left to right. The reading shown in the figure is 9073. The number of kilowatts consumed is the difference between the previous reading and the present reading.



Watt-hour Meter Dial

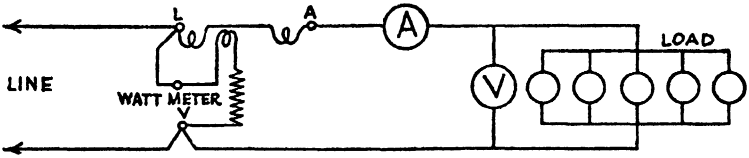
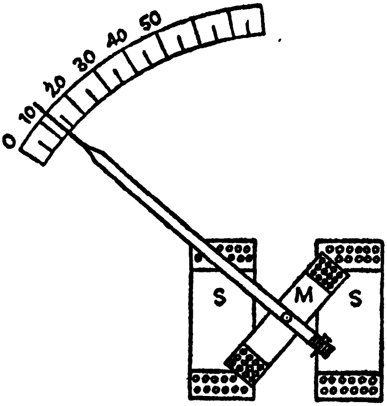


Diagram Showing Connections of Wattmeter, Ammeter and Voltmeter

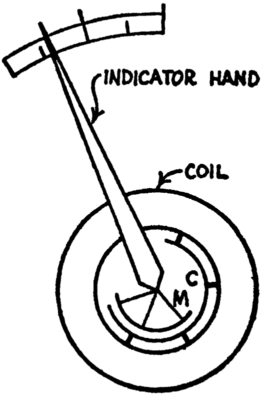
Alternating Current Ammeter: Electrodynamometer Type. The construction of this type of meter consists of two stationary coils (S) and a movable coil (M). The indicating hand is fastened to coil (M). Coils (M) and (S) are connected in series by means of two spiral springs. When the coils carry current, coil (M) tends to turn in a clockwise direction, due to the fact that the flux of coil (M) tends to line up with the flux in coil (S). If the current through the coil is reversed, the torque developed still remains in a clockwise direction. For this reason, this type of instrument may be used on alternating current, since the rate of change of the polarity is proportional to the frequency. The scale on this type of instrument is not uniform because the torque varies as the *square* of the current (as in the case of a series motor). The scale divisions at the start end of



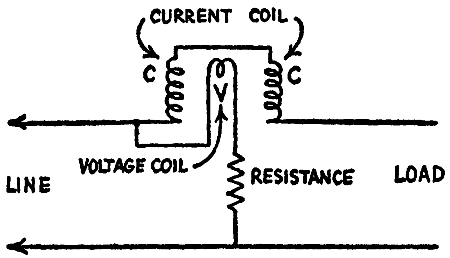
Electrodynamometer Type Alternating Current Ammeter the meter are small and cannot be read accurately, while those at the finish end of the scale are large and can be read very accurately. This instrument is usually dampened by means of an aluminum vane which is fastened to the same shaft as the indicating hand and fits closely in an enclosed box. When the coil (M) moves, the vane also moves and air is forced from one side of the vane to the other side, giving a damping effect. This instrument is like the direct current movable coil type with the exception that the coils (S) are used instead of the permanent magnets.

Alternating Current Voltmeter: Iron Vane Type. This type instrument has a soft iron vane (M) formed in the shape of a cylinder and mounted on a shaft. The indicating hand is fastened to this vane and moves over the scale. Another vane (F), which is wedge-shaped and slightly longer than vane (M), is fastened alongside of the first vane. A stationary coil (C), made up of a large number of turns of small wire, is connected in series with a resistor. This coil encircles both vanes (M) and (F). When the instrument is connected in the circuit, current through the coil sets up a field which magnetizes the two vanes. The upper as well as the lower edges of the vanes are of the same polarity; hence these vanes repel each other, regardless of which way the current flows through the coil. The repulsion causes the movable vane (M) to move away from the stationary vane (F), which in turn moves the indicating hand, registering the voltage, which is fastened to the movable vane (M).

Alternating Current Wattmeter. This instrument is generally of the dynamometer type, like the alternating current ammeter, but the coils (C) are connected in series with the load to carry the load current and are therefore known as *current coils*. The movable coil (M) is connected across the line in series with a high resistance (R). The current through this coil is in proportion to the line voltage and is therefore known as the *voltage coil.* Since the voltage circuit has practically no reactance and a high resistance, its current and flux can therefore be considered as in phase with the line voltage. At unity power factor the flux of coil (C) and the flux of (M) are in phase and maximum deflection occurs for given values of amperes and volts. When the amperes lag behind the voltage, only that component of the flux of coil (C) which is in phase with the flux in coil (M) is effective in producing a torque. Since the fluxes of coil (C) and coil (M) are respectively proportional to the amperes and volts, the torque is proportional to the volts and amperes,



Iron Vane Type Alternating Current Voltmeter true watts. This instrument is therefore calibrated in watts or kilowatts to indicate the power supplied to the load. On alternating current with an inductive load, the product of the volts and the amperes equals "apparent watts." True watts can only be found by the use of a wattmeter. To find the power factor, the true watts are divided by the apparent watts. A polyphase wattmeter employs two single phase meters combined into one. The two voltage coils are mounted on one shaft and rotate in their respective current coils. The total power is read on one scale.



Alternating Current Wattmeter

70. ELECTRICAL ILLUMINATION

Definitions. *Foot-candle.* The amount of light from a standard candle one foot away.

Candlepower. A measure of light from a lamp, based on the light from a standard candle.

Lumen. The light from a standard candle passing through a square centimeter of space one centimeter from the source of illumination.

Light Flux. The amount of illumination produced upon a surface which is intercepted by the unit area of that surface. If sixty lumens of light flux fall upon a surface of twenty square feet, the illumination is sixty divided by twenty, or three lumens per square foot. A more common term would be three *foot-candles.*

Watts per Candle. Term used to denote the specific consumption of a lamp in watts per mean horizontal candle-power produced.

Reflection Factor. The amount of light reflected from a surface.

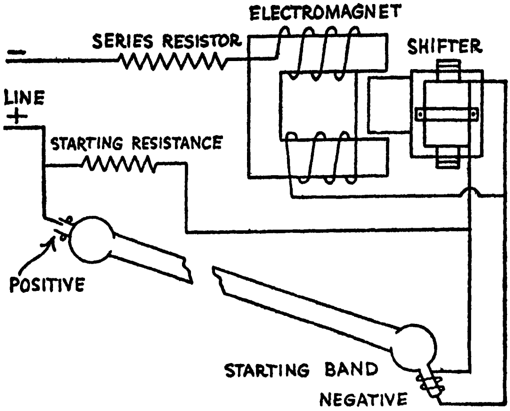
Incandescent Lamp. An incandescent lamp is one in which the light is produced by heating a solid conductor of high resistance, known as the *filament,* to a point of incandescence. This filament is hermetically sealed in a vacuum or in an inert gas within a glass bulb. The lead-in wires are contained in a glass tube with the ends of the wires fastened to two metal contacts making the base of the lamp.

Carbon Lamp. Absorbent cotton is dissolved in zinc chloride and hydrochloric acid which forms a mass a little thicker than molasses. It is then forced under pressure through a die and into a vessel containing alcohol, which causes it to set and harden, forming a thread-like filament. It is then washed and wound on a drum. After it has been dried and gaged it is cut into suitable lengths and wound up on forms to produce the proper shape. The filament is then treated with a graphite-carbon coating by flashing it in an atmosphere of hydrocarbon vapor. The metalized carbon filament is produced by placing the carbon thread in an electric furnace, where it acquires both the appearance and electrical characteristics of metal. At the present time very few carbon lamps are used in the United States.

Tungsten Filament Lamp. This metal comes in the form of a powder and is pressed into ingots which are heated to a white heat in an electric furnace in an atmosphere of hydrogen, thus making them good electrical conductors. The ingots are heated a number of times until they become small enough to be drawn; the drawing process takes place through diamond dies while the metal is very hot. When the filament is the proper size it is fastened to the lead-in wires and the bulb exhausted of air to a good vacuum. These lamps, known as "Mazda" lamps, have a high efficiency and give a much whiter light than the carbon lamp.

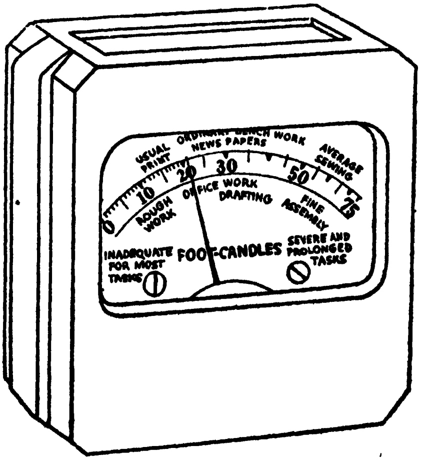
Gas-Filled Lamp. This lamp has a pear-shaped bulb extending from a long glass stem to which the base is fastened. The filament is wound in a coil of very small diameter and fastened in the center of the bulb by radial supporting arms. The bulb is filled with nitrogen gas which permits the tungsten wire filament to operate at a higher temperature and increases the efficiency of the lamp. The gas also reduces the rate of evaporation of the filament; the hot gas currents tend to carry any evaporated particles into the stem of the bulb where they are deposited without interfering with the light output. The average life of the gas-filled lamp and of the Mazda lamp is about 1000 hours.

Cooper-Hewitt Mercury Vapor Lamp. This lamp derives its light from the vapor of mercury, which, with the passage of electricity through it, causes incandescence. The lamp consists of a glass tube containing mercury vapor at a low pressure, with a piece of iron at the positive terminal and a pool of mercury at the negative terminal. The modern lamps are automatically started. The starting device, known as a "shifter," consists of a small glass bulb containing mercury and mounted on a pivoted frame which is actuated by an electromagnet. The shifter has a resistance in series and is connected in parallel with the tube. When the lamp is turned on, current flows from the positive terminal through the starting resistance, electromagnet, shifter and series-resistance, returning to the negative terminal of the source of supply. The electromagnet becomes energized and causes the shifter to rotate; this separates the mercury in the shifter and causes an interruption of the current flowing through it. A momentary high voltage, induced by the electromagnet, is impressed upon the tube between the negative electrode and the starting band. This band acts as a condenser and a hot spot of electric discharge is formed on the mercury pool, causing a current to flow through the lamp tube. The shifter is held in the open circuit position. If the lamp does not ignite with the first operation, the above process is repeated until the lamp does light. This lamp gives off a greenish-blue light and is used very successfully in all photographic processes.



Cooper-Hewitt Mercury Vapor Lamp

Measurement of Light. The light meter is used to measure the amount of illumination in foot-candles on any working plane. The General Electric light meter is one of several types available. This meter is 2¼″ square, direct reading, and graduated from 0 to 75 foot-candles across the scale. It also has notations on the face as to the amount of light necessary for different types of work. A light-sensitive cell is located on the top surface at right angles to the scale face. This arrangement is convenient for taking readings which avoid the observer's shadow. It also permits the meter to be used to measure reflection factors of ceilings and walls, as well as the transmission factor of all types of glass and translucent materials.



General Electric Light Meter

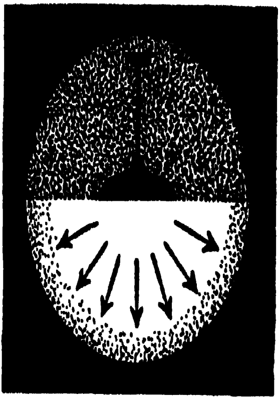
Reflection Factor. To measure the reflection factor of a wall, place the light cell against the wall, drawing it back slowly about two to four inches until a constant reading is obtained. Note the reading. Turn the meter around and place it against the wall with the face of the cell turned away from the wall and take a second reading. Divide the first reading by the second reading and the result will be the approximate *reflection factor.*

Transmission Factor. To find the approximate transmission factor of all types of glass and other translucent materials, divide the reading that is taken when the sample of glass is placed over the cell by the reading taken with the glass removed; the quotient is the *transmission factor.*

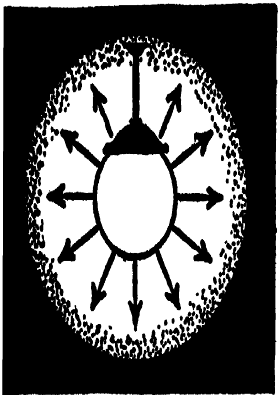
Recommended Foot-Candles:

Auditoriums	5	foot-candles
Automobile Showrooms	20	" "
Banks		
Lobby	15	" "
Offices	20	" "
Drafting Rooms	30–50	" "
Hangars—Aeroplane	10	" "
Hospitals		
Laboratories	20	" "
Lobby and Reception Room	10	" "
Operating Room	20	" "
Hotels		
Lobby	10	" "
Dining Room	5	" "
Kitchen	20	" "
Guestrooms	10	" "
Corridors	2	" "
Writing Rooms	20	" "
Libraries—Reading Room	20	" "
Offices—Bookkeeping, Typing, Accounting	30	" "
Desk Work	20	" "
Designing	30–50	" "
Reading Blueprints	30	" "
Filing and Index Reference	20	" "
Restaurants—Dining Area	10	" "
Schools		
Class and Study Rooms	20	" "
Corridors and Stairways	5	" "
Gymnasium	20	" "
Large Cities—Stores		
General Interior Lighting	20	" "
Show Case	50–100	" "
Show Window	200	" "

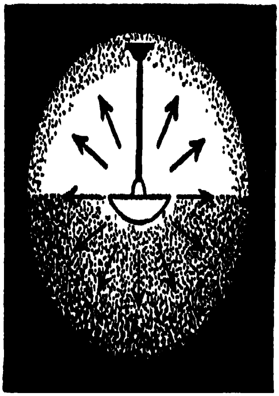
Lighting Systems and Methods. *Direct lighting* is defined as any system in which all the light on the working surface is essentially downward and comes directly from the lighting units. The methods may range from spot-lights to deep bowl and dome reflectors. Semi-direct lighting refers to systems where most of the light on the working plane comes from the lighting unit, but where a considerable part of the light comes from the reflection of light from the ceiling. Semi-indirect lighting is any system in which some light, usually from 5% to 25%, is transmitted directly downward, but over 50% of the light is transmitted upward. Indirect lighting refers to systems where all the light is upward and reflected from large ceiling areas. This light is soft and free from glare.



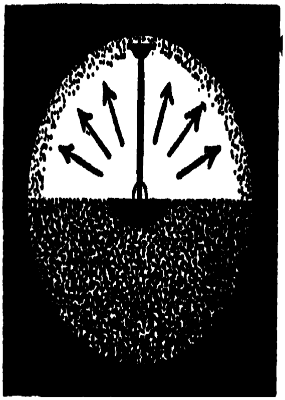
Direct Lighting



Semi-direct Lighting



Semi-indirect Lighting



Indirect Lighting

Table A shows the spacing between outlets and area per outlet.

TABLE A. ALLOWABLE SPACING BETWEEN LIGHT SOURCES

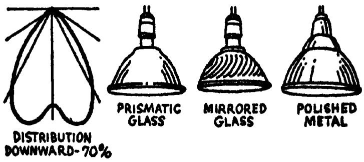
Ceiling Height (Or Height in the Clear)	Spacing Between Outlets		Spacing Between Outside Outlets and Wall		Approximate Area per Outlet (At Usual Spacings)
	Usual	Maximum (For Units at Ceiling)	Aisles or Storage Next to Wall	Desks, Workbenches, etc., Against Wall	
(Feet)	(Feet)	Not more than		Not more than	(Square Feet)
8	7	7½	Usually one-half actual spacing between units	3	50–60
9	8	8		3	60–70
10	9	9		3½	70–85
11	10	10½		3½	85–100
12	10–12	12		3½–4	100–150
13	10–12	13		3½–4½	100–150
14	10–13	15		4–5	100–170
15	10–13	17		4–5	100–170
16	10–13	19		4–6	100–170
18	10–20	21		4–6	100–400
20 and up	18–24	24		5–7	300–500

Table B shows the mounting height of reflectors and fixtures for direct, semi-direct, semi-indirect and indirect lighting.

TABLE B. MOUNTING HEIGHT OF LIGHT SOURCES

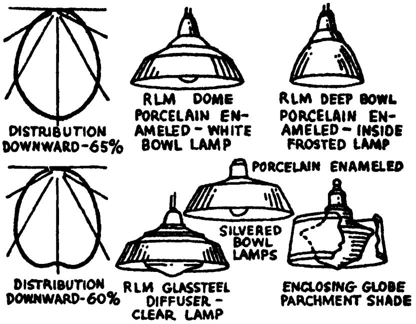
DIRECT AND SEMI-DIRECT LIGHTING UNITS				SEMI-INDIRECT AND INDIRECT LIGHTING	
Actual Spacing Between Units	Distance of Units from Floor Not Less Than	Desirable Mounting Height in Industrial Interiors	Desirable Mounting Height in Commercial Interiors	Actual Spacing Between Units	Recommended Suspension Length (Top of Bowl to Ceiling)
(Feet)	(Feet)			(Feet)	(Feet)
7	8	12 feet above floor if possible—to avoid glare, and still be within reach from stepladder for cleaning.	The actual hanging height should be governed largely by general appearance, but particularly in offices and drafting rooms, the minimum values shown in the second column should not be violated.	7	1–3
8	8½			8	1–3
9	9			9	1–3
10	10			10	1½–3
11	10½			11	2–3
12	11			12	2–3
14	12½	Where units are to be mounted much more than 12 feet it is usually desirable to mount the units at ceiling or on roof trusses.		14	2½–4
16	14			16	3–4
18	15			18	3–4
20	16			20	4–5
22	18			22	4–5
24	20			24	4–6

Factors Influencing the Selection of Lighting Equipment. *Concentrating Type.* This type has a light-output range of 68% to 75%. The source of brightness is low at normal angles of view because of the large shielding angle. The reflected glare is severe, due to the reflected source of brightness. This type of reflector is used for high bay lighting where the reflectors are at some distance away from the working plane.



High Bay Open Reflectors

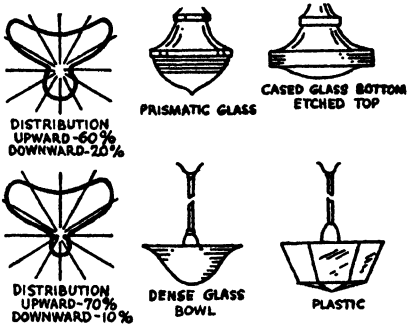
Distributing Types. These are exemplified by the porcelain enamel RLM dome and deep bowl type of reflectors which have a light output ranging from 63% to 68%. The source of brightness is fairly high, but is good for moderate levels of general illumination. When these reflectors are equipped with bowl enamel lamps the reflected glare is not too severe. These reflectors are used for many industrial lighting requirements. By using a reflector equipped with an opal glass bowl, known as a Glassteel reflector, or by using a silver bowl lamp, the light output range is cut down by approximately 5%, but the reflected glare is low. It is a very desirable reflector for industrial use.


DISTRIBUTION DOWNWARD-65%
RLM DOME PORCELAIN ENAMELED - WHITE BOWL LAMP
RLM DEEP BOWL PORCELAIN ENAMELED - INSIDE FROSTED LAMP
PORCELAIN ENAMELED
SILVERED BOWL LAMPS
DISTRIBUTION DOWNWARD-60%
RLM GLASSTEEL DIFFUSER - CLEAR LAMP
ENCLOSING GLOBE PARCHMENT SHADE

Distributing Type Open Reflectors

Large Area Diffusing Reflectors

Semi-Indirect Lighting. The closed bowl, prismatic glass, and cased glass bowls have a light range output of from 75% to 85%. The source of brightness is satisfactory and the reflected glare is low. Enclosed units have the advantage of slower depreciation and easy maintenance. These units are commonly used for lighting in stores and offices. The open glass bowl and plastic types have a light range output of 70%–80% with a very low source of brightness. The reflected glare is very low, but the depreciation and maintenance factors are appreciable since the bowl is open and readily accumulates dust, grease, etc.

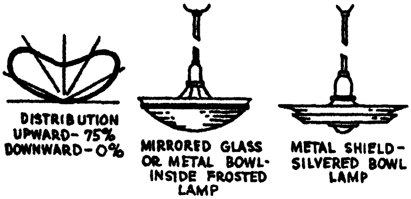

DISTRIBUTION UPWARD-60% DOWNWARD-20%
PRISMATIC GLASS
CASED GLASS BOTTOM ETCHED TOP
DISTRIBUTION UPWARD-70% DOWNWARD-10%
DENSE GLASS BOWL
PLASTIC

Enclosed Translucent Bowls

Open Top Translucent Bowls

Indirect Lighting. All the light from this type of fixture is thrown upward to the ceiling. The light output range is from 60% to 75%. This type of lighting has a very high rating from the standpoint of source brightness, reflected glare, and shadows. Wide-angle distribution makes for a more uniform brightness, but the utilization of light is lower because all the light is directed upward and then reflected from the ceiling downward.

The Fluorescent Lamp. This is the most modern lamp and has many advantages over Mazda or gas-filled lamps because (1) it delivers more


DISTRIBUTION
UPWARD-75%
DOWNWARD-0%
MIRRORED GLASS OR METAL BOWL-INSIDE FROSTED LAMP
METAL SHIELD-SILVERED BOWL LAMP

Shallow Bowl Reflectors and Shields

lumens per watt, (2) it has a longer life. The parts consist of the lamp, starter switch, and transformer or ballast unit. Inside the starter switch is a small glass tube known as a *glow relay switch* which contains either neon or helium gas, under a regulated pressure, and a bimetallic strip which arches down and nearly touches a contact point. These glow relay switches are of different sizes. The FS-2 is for 15- and 20-watt lamps; FS-4 is for 30- and 40-watt lamps; and FS-6 is for 100-watt lamps. When the current is turned on in a 20-watt lamp (using an FS-2 starter), the voltage is about 80 volts; this produces an arc at the end of the bimetallic strip of the glow relay switch. The current that flows through the arc also flows through each of the double-coiled tungsten-wire electrodes in each end of the fluorescent lamp. The heat of the arc causes the bimetallic strip to bend and make contact at its end. When this contact is made the arc ceases and there is no longer any heating effect; the strip then cools and straightens out. This opens the circuit in the series connections through the lamp electrodes. These electrodes have been conducting at twice the normal operating current for a period of three or four seconds, becoming heated to a yellow incandescence, and to a temperature at which they should remain throughout the burning period of the lamp.

Each electrode is a tungsten filament which has a white oxide coating baked on it. A rich emission of electrons, produced by the heat, travels down the bore of the lamp at great speed. The electrons sustain the arc stream between the electrodes. If the starting voltage were applied to the electrodes when cold there would be a resistance to the establishment of the arc which would ruin the arc. At the instant the glow relay opens, the reactance transformer or ballast relay delivers a reactance "kick" of about 500 volts. This surge of voltage insures an arc inside the lamp and gives the first flash of brightness throughout the tube. If the arc continues it will keep the electrodes at the proper temperature for the continuation of electron emissions and will pass current which is limited by the automatic action of the ballast unit. Each arc tends to run away with itself because it tends to get hotter, and with lowered resistance the current would increase until the unit was destroyed. One of the chief functions of the ballast is to hold down the current to a predetermined value. This is done by the design of the ballast. If the arc fails to continue within the lamp, the glow relay switch repeats its operation every few seconds until the lamp lights.

CHARACTERISTICS OF FLUORESCENT LAMPS

Lamp Watts	Bulb Size		Approximate Lumens	Life Hours
6	9″ x ⅝″	white	180	750
14	15″ x 1½″	"	460	1500
15	18″ x 1″	"	615	2500
20	24″ x 1½″	"	900	2500
30	36″ x 1″	"	1450	2500
40	48″ x 1½″	"	2100	2500
65	36″ x 2⅛″	"	2100	2000
100	60″ x 2½″	"	4200	2000

Basis of Color. The glass tube is coated with a fluorescent material or phosphor. This material has nothing to do with phosphorus, but consists of mineral compounds such as shown in Table A. After the air has been exhausted from the coated tube with the electrodes sealed in, it is filled with argon gas and a few drops of mercury. The argon gas aids the flow of electrons and permits the making of the arc when the lamp is cool. The mercury vapor becomes ionized in the arc steam and gives out the majority of its energy in the invisible ultra-violet radiation of wave lengths. It also emits mercury lines in the visible spectrum. This radiation is absorbed in the phosphor and is transformed into visible light. The fluorescent lamp is not as sensitive to line-drop in light output as the Mazda filament lamp.

TABLE A

Minerals That Produce Color

Phosphor	Color of Light
Magnesium Tungstate	Bluish White
Zinc Beryllium Silicate	Yellow White
Cadmium Borate	Pink
Calcium Tungstate	Blue
Zinc Silicate	Green
Double Coating	Red
Double Coating	Gold

Candle Power. The initial candle power drops during the first 100 hours of burning, but this is worked out of the lamp during the seasoning period at the factory. There is a slight dimming at about 70% of average life, but this dimming is less than 15%. The fluorescent burns equally well in any position and will withstand normal vibration; also, it burns cool. It burns best at about 80° F, but may fail to start in temperatures below freezing.

Table C shows the line amperes, wattage of the ballasts, and number of lamps per 15-ampere circuit.

TABLE B

Operation of Fluorescent Lamp

Lamp Size		Operating Volts	Operating Amps.
15W	T- 8	56	.3
15W	T-12	49	.33
20W	T-12	62	.35
30W	T- 8	103	.34
40W	T-12	108	.41
65W	T-17	50	1.35
100W	T-17	72	1.45

TABLE C

Lamp			Line Amp.	Ballast Watt	No. of lamps per 15-amp. circuit
Two	15W	7- 8	.35	9.0	86
Two	20W	T-12	.43	9.0	70
Two	30W	T- 8	.66	14.5	46
Two	40W	T-12	.86	17.5	34
Two	100W	T-17	2.05	35.0	14
One	15W	T- 8	.18	4.5	83
One	20W	T-12	.23	4.5	65
One	30W	T- 8	.37	10.0	40
One	40W	T-12	.50	13.0	30

SELECTED REFERENCES FOR FURTHER STUDY

Avery, A. H. Fractional Horse-power Motors. Pitman Publishing Corp.

Braymer, D. H. & Roe, A. C. Rewinding Small Motors. McGraw-Hill Book Co.

Burns, E. E. Electricity, a Study of First Principles. D. Van Nostrand & Co.

Croft, T. W. Practical Electricity. McGraw-Hill Book Co.

Dawes, C. L. Industrial Electricity. 2 vol. Direct Current. McGraw-Hill Book Co.

Jones, E. W. Essentials of Applied Electricity. Bruce Publishing Co.

Nadon, J. M. & Gelmine, B. J. Industrial Electricity. D. Van Nostrand Co.

Richter, H. P. Practical Electric Wiring. McGraw-Hill Book Co.

Richter, H. P. Practical Electricity and House Wiring. F. J. Drake & Co., Chicago

Slack, E. P. Elementary Electricity. McGraw-Hill Book Co.

Timbie, W. H. Industrial Electricity; Direct Current. J. Wiley & Sons.

Uhl, A., et al. Interior Electric Wiring and Estimating. American Technical Society

Weber, W. B. Job Instruction Sheets on Practical Electricity. J. Wiley & Sons.

Willoughby, G. A. House Wiring; for Trade Preparatory and Trade Extension Classes. J. Wiley & Sons.

GLOSSARY OF SCIENTIFIC AND TECHNICAL TERMS

NOTE. *The reader should also refer to the INDEX for the large number of technical and trade terms which are more fully explained in the body of the text; also, for a few terms that are listed in both places.*

ABRASIVE. A hard substance, such as emery, carborundum, etc., which, when fractured, will have many sharp cutting edges; used in industry for surfacing and finishing.

ADAPTER. Any device for holding a machine part, such as an arbor or a cutter, which would not otherwise fit into the spindle, etc.

ADHESION. The tendency of objects which consist of different or dissimilar materials to stick together.

ADIABATIC. Any process or condition in which no heat enters or leaves during a change in volume.

AËRATION. An artificial, mechanical method of mixing air and water for the purpose of purifying the water; increases palatability.

AGGREGATE. The solid particles constituting the major part of the volume of concrete mixture.

AMALGAM. An alloy formed by mixing mercury with one or more other metals.

AMORPHOUS. Without definite structure; not crystalline.

ÅNGSTRÖM. A unit of length, generally used to express the wave length of light; equals 0.00000001 cm., or 10^{-10} meter.

ANHYDRIDE. An oxide which unites with water to form an acid; such oxides are usually non-metallic.

ANHYDROUS. Having no water in its composition; refers usually to salts or crystals.

ANNEALING. Heating followed by slow cooling while in the solid state.

ANNUNCIATOR. An electrical device for showing a number, a letter or a name when a bell is rung or a button is pushed.

ANODE. The positively charged pole in any electrical apparatus.

APRON. A protecting cover enclosing a mechanism or part of a machine.

AQUA REGIA. A mixture of HCl and HNO_3 in the ratio of 3:1; commonly used as a solvent for ores and metals.

AQUEOUS. Refers to a water solution.

ASPIRATOR. A form of air pump, attached to a water faucet, and used to remove air from a vessel.

ASSAY. To determine quantitatively the constitution of any material, usually a metal or an ore.

AUTOGENOUS WELDING. The process of fusing or uniting metals by the use of intense heat from a gas flame, but without hammering or the use of pressure.

BACKING SAND. Sand used to fill flasks after facing-sand has been used to cover the pattern.

BAFFLE PLATE. Any projection or partition placed in such a way as to divert the flow of a liquid, vapor or gas, causing it to follow a prearranged path.

BEARING. Any part of a machine or structure which transmits a load to its support.

BEDDED-IN MOLD. Any mold, the bottom half of which is made right in the sand of the foundry.

BELLCRANK. A bent lever with two arms, usually at right angles or approximately so, and pivoted where the two arms are joined.

BENZENE. An organic compound, benzol (C_6H_6), used as a solvent; not to be confused with benzine.

BEVEL. A sloping part or surface; an oblique or slanting cut; a tool for setting of angles.

BINDER. Any connecting piece, e.g., in carpentry, a tie-beam. Also, any material used to hold or cement other materials together, generally by cohesive action of some sort.

BINOMIAL. An algebraic expression consisting of only two terms, i.e., two quantities to be added or subtracted.

BLOWHOLE. A bubble of air or gas space imbedded in a solid at a time when the solid was molten.

BLOWN CASTING. Any casting containing blowholes.

BOILER-SCALE. The incrustation formed on the inside of a boiler, due to impurities in the water.

BOOSTER. Any piece of electrical equipment inserted in series in a circuit to increase the voltage of that circuit.

BOSS. The enlarged part of a shaft on which a wheel is keyed, or the end, where it is coupled to another shaft.

BOT. Clay wedge used in a cupola to stop the hole through which the metal is run.

BRACKET. A piece projecting from a wall, etc., in such a way as to support, or help support, a shelf or ledge.

BRAKE HORSEPOWER. The mechanical output, available at the shaft, of any engine, motor or turbine.

BRIMSTONE. Commercial form of sulfur; cylindrical molds, called roll sulfur.

BRINE. A prepared saline solution, such as an aqueous solution of sodium or calcium chloride.

BUFFING. The process of securing a very fine surface on a metal object by the use of soft wheels of felt or cotton, with an appropriate abrasive.

BURNISHING. The process of obtaining a very smooth finished surface on a metal by compressing its outer layer through the application of rollers or steel balls. It is accomplished by rotating barrels in which the piece to be burnished and the burnishing material are rotated; the repeated impacts of the burnishing material produce the desired effect.

BURR. The rough edge or ridge remaining on a metal piece after cutting, milling, punching, etc.

BUS-BAR. The common connection joining two or more generators, and from which one or more feeders draw their supply; usually made of tubing or flat bars of copper.

BUSHING. A metallic circular piece used to reduce the size of a hole or opening.

CALCINATION. The process of subjecting a material to a high temperature, but below its fusion point, either to cause it to lose water or volatile matter, or to become more friable, or to become oxidized or reduced.

CALIBRATION. An accurate comparison of any measuring instrument with an accepted standard, especially to determine the errors of the instrument.

CALORIMETER. Any apparatus used for measuring the quantity of heat; especially used in determining the specific heat of materials, the heat of combustion of fuels, or the calorific value of food, etc.

CAM. A mechanical device which converts rotating motion into straight line motion.

CAMBER. A slightly arched or convex shape.

CARBIDE. Calcium carbide (CaC_2); used in the making of acetylene.

CARRIAGE. Any mechanical contrivance which moves and carries some part of a machine; a supporting framework.

CARRIAGE BOLT. A common type of bolt with one end threaded for a distance of approximately twice its diameter, and carrying a square nut; the other end has an oval cross-section head. Under the head there is a square section which makes it possible to tighten the bolt without having it turn in the work.

CATHODE. The negative pole, or lower potential electrode of an electrical cell or device.

CAULKING. The act of stopping up, or filling in, any seam or crevice.

C.G.S. SYSTEM. The metric system of physical measurements, based on the centimeter-gram-second units of length, mass, and time, respectively.

CENTRIFUGE. A driving machine with perforated holes, which, revolving at high speed, dries the material by utilizing centrifugal force; the water is literally thrown out.

CHAMFER. The surface formed by bevelling off a square edge or corner equally on both sides.

CHATTERING. The formation of nicks or ridges on a piece of work that is being turned, milled, planed, or ground.

COAGULATION. The conversion of a liquid into a soft solid mass as a result of chemical action.

COHESION. The tendency of objects which consist of the same or similar material to stick together.

COLLAR. A ring or a band around something, usually near the top of a round piece; a shoulder.

COLLINEAR. Points which lie in the same straight line.

COLOGARITHM. The logarithm of the reciprocal of a number; colog $N = -\log N$.

COMPARATOR. An apparatus which projects the enlarged shadow of a screw thread or gear profile upon a standard screen to determine errors, tolerance, etc.

CONDULET. A fitting used in connecting electrical conduits.

CONSTANT. A quantity whose numerical value remains the same at all times.

CO-PLANAR. Points or lines all lying in the same plane; used also with respect to forces.

CORE. The sand facsimile of the interior part of a casting.

CORE BOX. A box in which a core is rammed up and shaped.

CORROSION. The formation, over a period of time, of an oxide on the surface of a metal; not to be confused with erosion.

COTTER PIN. A split pin of metal used for fastening machine parts, especially for locking nuts, etc., subject to vibration.

COUNTERSHAFT. An intermediate shaft driven by the main shaft when motion is transmitted to machinery.

CRANKSHAFT. The main shaft in a gas engine, connecting the pistons and transmitting the power to the driving mechanism.

CRITICAL PRESSURE. The pressure of a vapor at the temperature above which the vapor can exist only as a gas; the pressure at the critical temperature.

CRITICAL TEMPERATURE. The temperature at which, for any pure substance, the liquid and its vapor cannot be distinguished; the maximum temperature at which a substance can remain in the liquid state; the temperature at which a change in molecular structure takes place.

CROCUS. Iron oxide (Fe_3O_4), or crocus martus; used as an abrasive.

CROSSHEAD. A machine part having reciprocating motion, and attached to a connecting rod; a necessary part in a double-acting engine.

CRUCIBLE. A small pot, made of clay or graphite, and used for melting small amounts of a metal.

CUBIC. Any algebraic expression or function involving the third power (but not higher than the third) of a variable.

CYCLIC. Anything which runs successively through a complete cycle or period; periodic.

CYCLOID. The geometric curve obtained by tracing the successive positions of a point on a circle as the circle rolls along a straight line.

DADO JOINT. A common type of joint used in woodworking and patternmaking, particularly for joining the ends of boxes or partitions.

DAMPING. The act of checking motion, usually by friction, such as diminishing the amplitude of vibration.

DEHYDRATE. To remove water from a substance by means of chemical action.

DELIQUESCENCE. Absorption of water by a substance when exposed to air; the opposite of efflorescence.

DEW-POINT. The temperature at which air would be saturated by the moisture actually in it.

DIAMAGNETIC. Having the property of being repelled by a strong magnet.

DIELECTRIC. A substance which acts as an electric insulator.

DIELECTRIC CONSTANT. The ratio of the capacity of a condenser with a given substance as dielectric to the capacity of the same condenser with air as dielectric.

DIFFUSION. The spreading out of a substance, as a liquid or gas, and consequent intermingling with other substances.

DILATOMETER. A device for measuring and recording volume changes in steel when subjected to heat treatment.

DISPERSION. The act of scattering in many directions, as of molecules, or light rays, or sound waves, etc.

DRILL ROD. Rods made of high carbon tool steel in the same sizes as drills up to ½″ diameter; the diameter of drill rod is finish-ground to size.

DYNAMOMETER. A device for measuring the amount of power developed, transmitted, or absorbed by any mechanism or machine.

EBONITE. Hard rubber, or vulcanite; treated rubber, with from 25 to 50% sulfur added.

ECCENTRIC. Not centrally located. Also applied to a rotating member which transforms rotating motion to straight line motion.

EFFLORESCENCE. Loss of water on exposure to air; refers to solid substances primarily.

ELLIPSOGRAPH. A mechanical instrument for drawing an ellipse on a drawing board automatically.

ELLIPSOID. A solid object having a geometric shape such that any section through the center is an ellipse.

EMERY WHEEL. A wheel whose periphery is supplied with a suitable artificial abrasive, and which is used for grinding or polishing.

ENTRAIN. To drag along or carry with, as when steam carries droplets of water vapor with itself.

EROSION. Any process by which material is slowly worn away by physical or chemical means, or both.

EUTECTIC. A mixture of two solid metals (alloy) having a minimum melting-point as compared with other percentage compositions of the two metals.

EXPANSION JOINT. Provision made in long pieces of metal, usually piping, to allow for increase and decrease in length due to changes of temperature.

EXTENSOMETER. A measuring instrument used to determine the elongation of a material when studying its elastic limit, etc.

EXTRUSION. The process of expelling or squeezing out, as when a paste is forced out of a soft tube through a small aperture.

FACING. Materials used in the foundry for painting the surface of a finished mold to obtain a smooth skin on the casting.

FACING SAND. Sand used to form the faces of a mold.

FALSE ODDSIDE. A permanent oddside made of plaster or some other material.

FEEDING. Assisting metal to run into a mold to compensate for contraction when the metal cools.

FEELER GAGE. A set of flat blades of metal ranging from .0015″ to .025″ in thickness, and used separately or in combination to measure small distances between machine parts.

FETTLING. Cleaning up, trimming and finishing a casting after it has been taken out of the sand.

FIXATIVE. Any substance which serves to make another substance less volatile or non-volatile.

FLANGE. Any projecting flat rim, or collar, used to strengthen a machine part, to guide it, to fasten it, or to facilitate its attachment to another part.

FLASH POINT. The temperature at which the vapor given off from an oil or liquid fuel will ignite.

FLOW (CREEP). The gradual continuous distortion of a material under continued load, usually at higher temperatures.

FLOW-OFF GATE. A channel cut from the mold to the riser.

FLUME. An artificial channel for a stream of water to be applied to some industrial use.

FLUORESCENT. The property of emitting light of one wave length as a result of absorbing light of a different wave length.

FLUTE. A groove; in machine shop work, grooves on drills, taps, reamers, etc., which not only provide cutting edges, but also space to permit removal of chips and shavings.

FLUX. A substance which is used to cover hot metal to prevent oxidation while soldering or welding.

FLYWHEEL. A heavy wheel not connected with anything except its axle; maintains even running of an engine because of its great momentum.

FOCUS. A point at which lines or rays converge or gather together.

FOIL. Metal which has been hammered or rolled into a very thin sheet.

FREQUENCY. The rate of recurrence, as, for example, of a vibration.

FRIABLE. Easily crumbled in the fingers.

FUSIBILITY. The ease with which a material is melted.

FUSION. The melting of a solid, i.e., the act of becoming liquid as a result of change in temperature; not to be confused with solution, or dissolving.

GALVANOMETER. An instrument for determining the direction and intensity of an electric current.

GASKET. A ring or other shape of flat, compressible material, such as leather, rubber, paper, etc., inserted between two metallic surfaces to prevent leakage of any gaseous or liquid material.

GATE. A channel which allows a metal to enter a mold.

GIRDER. Any beam supported at each end upon piers, pedestals, or walls, and supporting a superstructure.

GIRDLE. A belt; a zone; an encircling rim or projection.

GONIOMETER. An instrument for measuring the angles of crystals.

GUSSET PLATE. A metal plate used to hold or strengthen two or more pieces at an angle.

HELIX. A mathematical curve in 3-dimensional space, exemplified by the "roller" spring in a window shade, or a so-called (incorrectly) spiral staircase.

HINGE. Any device which permits or enables one part to turn on another.

HOMOGENEOUS. Having the same properties at all points.

HYGROMETER. Any instrument for measuring humidity.

HYGROSCOPIC. Readily absorbing any retaining moisture.

HYSTERESIS. The mechanical work done when a substance suffers repeated magnetization and demagnetization; it is dissipated in the form of heat, and is called the hysteresis loss.

IMPEDANCE. The resistance of an inductive circuit to an alternating current.

INDICATOR DIAGRAM. A graphic representation of the work done in an engine cylinder (steam or internal combustion) during a single stroke of the piston.

INFINITE. Without limit in extent of number or magnitude; unending.

IN-GATE. A channel cut from the bottom of the runner into the mold; used where runner does not enter the mold direct.

INTERPOLATION. A mathematical process for finding "in between" values in a table of values of a variable.

INTERSTICES. Empty spaces or holes between things, or between parts of a body, as, for example, in a lattice-work or crystalline structure.

ISINGLASS. A gelatinous material obtained and prepared from the bladder of sturgeon, and used in making cements; not to be confused with mica.

ISOTHERMAL. Any process or change of condition in a system which occurs without a change of temperature.

ISOTROPIC. Having the same properties in all directions at any given point.

JACKET. An outer covering for anything, usually a cylindrical part, such as a pipe, boiler, engine cylinder, condenser tube, etc.

JAW-CHUCK. A holding device used for drilling, boring, etc.

JIG. A frame or device for holding a piece of work and at the same time guiding the tool which is to operate on the work.

JOIST. A horizontal timber in a ceiling or floor.

KEEN. Refers to the angle of slope of the supporting sides of a cutting edge; see SHARP, with which it is not to be confused.

KINDLING TEMPERATURE. The lowest temperature at which combustion will take place.

KNIFE-EDGE. A thin-edged device used to support a bar or beam and which facilitates its rotation.

LAPPING. A refined method of using an abrasive for the purpose of reducing size by minute amounts, or for giving a very smooth finish.

LEACHING. Allowing a liquid to percolate through a solid for the purpose of dissolving or removing soluble constituents; similar to lixiviation.

LEVIGATION. The process of grading very fine abrasives by flotation.

LINE-SHAFT. The shaft which drives the machinery in a shop, or a part of a shop, by means of pulleys and belts.

LITHARGE. Plumbous oxide, or oxide of lead (PbO), used in making lead glass and pottery glazes.

LITHOPONE. A mixture of $BaSO_4$ and ZnS; used as pigment for paints and as a filler in rubber compounds.

LIXIVIATION. The process of separating soluble from insoluble substances by dissolving the soluble material in water or other suitable solvent.

LUTE. A tenacious clay or cement used to stop an orifice or make a joint air-tight.

MANOMETER. An apparatus for measuring the pressure of a fluid (gas or liquid).

MAT. To entangle together in a thick mass; e.g., the microscopic intermingling of crystals or fibers.

MENISCUS. The convex or concave surface of a column of liquid, caused by capillarity.

MICRON. A unit of length equal to 10,000 Ångströms, or 0.001 millimeter.

MINERAL WOOL. A fibrous preparation made from blast-furnace slag; used as a fireproofing and heat-insulating material.

MODIFICATION. A patented process used in producing light-weight aluminum alloys such as Alpax, Elektron, etc.

MODULUS. A ratio or index number.

MORTISE. A space hollowed out of a piece, usually wood, to receive a tenon.

MUFFLE FURNACE. A special type furnace in which the combustion gases are very carefully kept away from the heated work.

MULTIPLE MOLDS. Molds which are stacked on top of each other and cast through a single runner.

NEEDLE VALVE. A special type of valve provided with a long, slender, tapering point rather than the usual disc.

NIPPLE. A short piece of standard pipe threaded at each end.

OAKUM. Material used by plumbers when packing certain types of joints, or by shipbuilders when caulking seams or stopping leaks.

OCCLUSION. The infiltration and permeating of a gas into a porous solid, with attendant condensation of the gas on the surfaces of the pores.

ODDSIDE. Support used for supporting a pattern while the drag is being rammed up.

ODOMETER. A measuring instrument attached to the wheel of a vehicle to determine the distance traveled by the vehicle.

ODONTOMETER. An instrument for testing the profile and accuracy of gear teeth.

OILDAG. A variety of graphite suspended in oil, prepared especially for lubricating purposes.

OILSTONE. A natural stone used for sharpening woodworking tools.

OPAQUE. The property of absorbing all unreflected incident light and acting as a perfect shield or screen.

ORIFICE. An opening or aperture which serves as a mouth of a tube, a vent, or a pipe.

OSCILLOGRAM. A graphic record of the continued readings of an oscillograph.

OSCILLOGRAPH. A special type of galvanometer constructed so as to show the variations in an alternating current.

PACKING. Any material used to prevent leakage of water, steam, air, etc., in pistons, pumps and the like.

PANTOGRAPH. A mechanism consisting of an arrangement of linkages admitting of enlargement or reduction of a drawing, etc., but preserving the angular dimensions of the original.

PARALLAX. The amount of apparent displacement of an object due to the real displacement of the observer; a common cause of error in reading some types of instruments.

PEENING. The hammering of one side of a metal plate or bar for the purpose of expanding the metal by indenting and so compressing it.

PEINS. Tools used in cold-riveting machines.

PENSTOCK. A sluice for restraining or regulating the flow from a head of water formed by a pen.

PERCUSSION. The act of delivering a sharp blow.

PERIODIC. That which is repeated in successive equal intervals of time.

PERMEABILITY (MAGNETIC). The relative degree of conductivity for magnetic lines of force of a substance as compared to air.

PIEZOMETER. An instrument for measuring the pressure in a pipe containing a liquid.

PINION. A small cog-wheel or gear, the teeth of which engage with those of a larger one.

PLANISHING. A process for giving metals a fine finished surface, by rubbing or rolling with a hardened tool, or sometimes with blows; very similar in principle to burnishing.

PLUG FUSE. A safety device used in an electrical circuit.

PLUMBAGO. A special form of graphite, also known as "black lead"; used in foundry work, and as a lubricant for rope drives when mixed with wax.

POPPET VALVE. Any valve having a conical surface which engages a conical seat having the same angle.

POROSITY. The ratio of the volume of the interstices of a material to the volume of its mass.

PORT. Any passageway in a mechanical device, such as in a valve or cylinder.

POTENTIOMETER. An accurate electrical measuring instrument used for the laboratory determination of current, voltages, and resistance.

POWDER METALLURGY. A comparatively new method of producing metallic articles by putting together powdered metals, with or without bonding agents, and then applying heat and pressure.

PRINT. A wooden projection put on to a pattern to furnish supports for the cores in a mold.

PRUSSIC ACID. Trade name for hydrocyanic acid (HCN).

PYKNOMETER. A glass instrument used for the precision determination of the specific gravity of liquids.

PYRENE. Trade name for carbon tetrachloride (CCl_4), used in certain types of commercial fire extinguishers.

PYROMETER. Any type of instrument used for measuring temperatures of from approximately 500° F or above.

QUENCHING. Rapid cooling by immersion in a liquid, gas or solid.

QUICKSILVER. The common name for mercury.

QUILL PUNCH. A piercing type of punch, used when the stock is large in comparison to the punch.

RACE. A groove in a ball bearing in which the balls run.

RACK. Any framework; specifically, a piece designed to engage and receive a worm gear.

RADIAN. A unit of angle measure, defined as a central angle whose arc is equal in length to the radius of the circle; πradians$=180°$, or 1 radian$=57.29578°$.

RADICAL. An indicated root of any number or quantity; e.g., $\sqrt{10}$, $\sqrt[3]{R}$, etc.

RADICAL. Any group of atoms held together and acting as a unit; e.g., ammonium radical, NH_3, or ethyl radical, C_2H_5, etc.

RED LEAD. Lead tetroxide (Pb_3O_4); when mixed with linseed oil, it is used as a lute in plumbing and gas fitting. Also used as a color for paints.

REFRACTORY. Any material capable of resisting change of shape, weight or physical properties when subjected to high temperature.

RISER. A channel from a mold used to draw off foreign matter, or to assist in feeding casting as it cools.

ROSIN. A resin obtained by the distillation of turpentine oil from crude turpentine; used as a flux in soldering and brazing metals.

ROTOR. A part of a machine that rotates within a stationary housing or outer part.

ROUGE. Finely divided iron oxide (Fe_3O_4); used as a pigment and as a polishing agent.

RUBBLE. Rough stone of irregular size and shape; used in building retaining walls, dams, etc.

RUNNER. The channel down which the metal is poured into a mold.

SADDLE. A machine part that is capable of sliding by virtue of its being mounted on the ways of a bed or other guiding surfaces.

SAL AMMONIAC. Ammonium chloride, NH_4Cl; used in ordinary wet batteries.

SEDIMENTATION. The process of separating finer particles of a material from the coarser particles by simply stirring up in water and allowing the heavier particles to settle out.

SEGER CONES. Triangular pyramids (about 2″ high) of specially prepared clays and feldspars, used in measuring the high temperatures of furnaces, ovens, etc.

SET-SCREW. A screw, usually hardened, which is used to lock a machine part in position by pressure on the point.

SHARP. Refers to the total vanishing of the supporting sides of a cutting edge; a tool that is sharp is not necessarily keen, and vice versa.

SINTERING. A form of heat treatment used in powder metallurgy, resulting in the formation of a porous alloy.

SLEEVE. A short tube, or hollow cylindrical piece; usually made of metal.

SLICK. To make smooth, as when troweling a sand surface.

SLUDGE. Mud from a drill hole in boring; also, muddy sediment in a steam boiler.

SLURRY. A thin watery mixture, such as liquid mud, cement, mortar, etc.

SNAGGING. Removing gates, fins, sprues or other projections from a casting by filing, grinding or chipping.

SPANDREL. A fillet.

SPANNER WRENCH. A wrench adapted to the shoulders or milled slots of cylindrical nuts.

SPINDLE. Any cylindrical machine part which admits of rotation.

SPINNING. A process of forming shapes by revolving sheet-metal while applying pressure.

SPIRAL. A plane curve having a constantly increasing radius of curvature; not to be confused with a helix, which is a space curve.

SPIRIT LEVEL. A device consisting of a closed glass tube containing a liquid and an enclosed air bubble; used for determining whether a surface is horizontal.

SPRIG. A small wooden or metal peg used to strengthen a mold or repair damage to a mold.

SPROCKET. Toothed-wheels used in chain drives; the chain links mesh with the teeth.

SPRUE. A piece of wood or metal used to make a hole through which molten metal will flow into pouring basin and mold.

STAGGERING. An arrangement by which bolts, holes, etc., are deliberately placed so as to be out of line to some extent.

STAY BOLT. A bolt used to support metal sheets; threaded throughout, inserted in tapped holes, and both ends riveted over.

STEEL WOOL. A preparation of thin steel shavings, having sharp edges; used as an abrasive.

STEP BOLT. A bolt similar in shape to a carriage bolt, except that it has a flatter head.

STOVE BOLT. An ordinary bolt, with slotted head and a hexagonal or square nut at the threaded end.

STRIKE. A bar of wood or metal used to remove surplus sand from a molding-box.

STRUT. Any structural member whose length is very much greater than its diameter, width, or depth.

STUD BOLT. A cylindrical piece having a thread at each end; generally used in fastening heavy machine parts.

STUFFING BOX. A cylindrical chamber used in connection with pistons to prevent leakage.

SUBLIMATION. The process of vaporizing a solid substance and condensing the vapors to again form the solid directly, without passing through an intermediate liquid state.

SUGAR OF LEAD. Lead acetate, $Pb(C_2H_3O_2)_2$; used in dyeing.

SWEATING. Process of soldering without the use of a soldering iron; also, the condensation of moisture from the air when chilled below the dew-point.

TACHOMETER. A device used to measure the speed in revolutions per minute of rotating wheels, pulleys, shafts, etc.

TEMPLATE. Any flat sheet, so marked or cut out to be used as a pattern or guide, generally for the purpose of duplication.

TENON. A projection fashioned on the end or side of a piece of wood or other material, to fit into a corresponding cavity (the mortise) of another piece so as to form a secure joint.

THERMOCOUPLE. An arrangement of two dissimilar metal wires joined together at one end. When the junction is heated, an electric current flows and may be measured so as to indicate the temperature.

TOGGLE JOINT. A special type of link mechanism which yields great pressure.

TOMPION. A plug for stopping an aperture.

TORSION. A twisting motion or force.

TORUS. A circular or ring-shaped solid having a circular cross-section; an anchor-ring.

TRANSLUCENT. The property of transmitting light, but scattering it so that objects cannot be seen clearly as a result.

TRANSPARENT. Capable of transmitting a large part of the incident light without scattering it.

TRIPOLI. Very finely powdered silica, or silicon dioxide, SiO_2; when mixed with stearic acid, the mixture, called tripoli powder, is used for polishing nickel plating.

TRUNNION. A pivot or journal designed to permit swiveling of a piece of apparatus or equipment.

TUYERE. The nozzle used to introduce a blast of air into a furnace or a forge.

UNION. A threaded cylindrical piece used to connect pipes.

VALVE. Any device that controls the amount of flow of a liquid or gas through a pipe or other vessel by restricting its passage mechanically.

VARIABLE. A quantity whose numerical value is subject to change; the relationship between variables is often expressed by a mathematical formula.

VECTOR. A straight line having a given length and a specified direction; used to represent graphically a force, velocity, etc.

VENT. An aperture or opening in an object, which serves as an outlet or means of escape for air, liquid or other material from the interior of the object.

VENTURI METER. A gage used to measure the flow of water in pipes.

VISCOSITY. Internal friction of a fluid; rate at which a fluid yields to a deforming force.

VISE. A clamping device generally consisting of two jaws which may be brought together by means of a screw.

VOIDS. Empty spaces between particles or parts of a mixture, as in concrete.

WASTER. A faulty or rejected casting.

WATER GLASS. An aqueous solution of sodium silicate ($Na_2SiO_3 \cdot H_2O$); miscellaneous uses as a fixative, vehicle and preservative.

WEIR. A device used to measure the flow of a stream in cubic feet per minute.

WHEATSTONE BRIDGE. A simple and well-known device for measuring electrical resistance.

WHITE LEAD. Basic lead carbonate, $2PbCO_3 + Pb(OH)_2$; used in making paints.

WOOD ALCOHOL. Methyl alcohol (CH_3OH); a useful solvent, not to be confused with grain alcohol (or ethyl alcohol), C_2H_5OH.

APPENDIX

TABLES FOR REFERENCE

TABLE I

THE CHEMICAL ELEMENTS

Element	Symbol	Atomic Number	Atomic Weight	Element	Symbol	Atomic Number	Atomic Weight
Actinium	Ac	89	226.7	Molybdenum	Mo	42	96.0
Alabamine	Ab	85	227.00	Neodymium	Nd	60	144.27
Aluminum	Al	13	26.97	Neon	Ne	10	20.183
Antimony	Sb	51	121.76	Nickel	Ni	28	58.69
Argon	A	18	39.944	Nitrogen	N	7	14.008
Arsenic	As	33	74.91	Osmium	Os	76	191.5
Barium	Ba	56	137.36	Oxygen	O	8	16.0000
Beryllium	Be	4	9.02	Palladium	Pd	46	106.7
Bismuth	Bi	83	209.00	Phosphorus	P	15	31.02
Boron	B	5	10.82	Platinum	Pt	78	195.23
Bromine	Br	35	79.916	Polonium	Po	84	210.0
Cadmium	Cd	48	112.41	Potassium	K	19	39.096
Calcium	Ca	20	40.08	Praseodymium	Pr	59	140.92
Carbon	C	6	12.00	Protactinium	Pa	91	231.0
Cerium	Ce	58	140.13	Radium	Ra	88	226.05
Cesium	Cs	55	132.91	Radon	Rn	86	222.0
Chlorine	Cl	17	35.457	Rhenium	Re	75	186.31
Chromium	Cr	24	52.01	Rhodium	Rh	45	102.91
Cobalt	Co	27	58.94	Rubidium	Rb	37	85.44

Columbium	Cb	41	92.91	Ruthenium	Ru	44	101.7
Copper	Cu	29	63.57	Samarium	Sm	62	150.43
Dysprosium	Dy	66	162.46	Scandium	Sc	21	45.10
Erbium	Er	68	167.64	Selenium	Se	34	78.96
Europium	Eu	63	152.0	Silicon	Si	14	28.06
Fluorine	F	9	19.00	Silver	Ag	47	107.880
Gadolinium	Gd	64	157.3	Sodium	Na	11	22.997
Gallium	Ga	31	69.72	Strontium	Sr	38	87.63
Germanium	Ge	32	72.60	Sulfur	S	16	32.06
Gold	Au	79	197.2	Tantalum	Ta	73	180.88
Hafnium	Hf	72	178.6	Tellurium	Te	52	127.61
Helium	He	2	4.002	Terbium	Tb	65	159.2
Holmium	Ho	67	163.5	Thallium	Tl	81	204.39
Hydrogen	H	1	1.0078	Thorium	Th	90	232.12
Illinium	Il	61	146.00	Thulium	Tm	69	169.4
Indium	In	49	114.76	Tin	Sn	50	118.70
Iodine	I	53	126.92	Titanium	Ti	22	47.90
Iridium	Ir	77	193.1	Tungsten	W	74	184.0
Iron	Fe	26	55.84	Uranium	U	92	238.14
Krypton	Kr	36	83.7	Vanadium	V	23	50.95
Lanthanum	La	57	138.92	Virginium	Vi	87	224.0
Lead	Pb	82	207.22	Xenon	Xe	54	131.3
Lithium	Li	3	6.940	Ytterbium	Yb	70	173.04
Lutecium	Lu	71	175.0	Yttrium	Y	39	88.92
Magnesium	Mg	12	24.32	Zinc	Zn	30	65.38
Manganese	Mn	25	54.93	Zirconium	Zr	40	91.22
Mercury	Hg	80	200.61				

TABLE II

DECIMAL EQUIVALENTS
of
Millimeters and Fractions of Millimeters

1/100 mm.=.0003937 inch

mm.	inches	mm.	inches	mm.	inches
1/50=	.00079	26/50=	.02047	2=	.07874
2/50=	.00157	27/50=	.02126	3=	.11811
3/50=	.00236	28/50=	.02205	4=	.15748
4/50=	.00315	29/50=	.02283	5=	.19685
5/50=	.00394	30/50=	.02362	6=	.23622
6/50=	.00472	31/50=	.02441	7=	.27559
7/50=	.00551	32/50=	.02520	8=	.31496
8/50=	.00630	33/50=	.02598	9=	.35433
9/50=	.00709	34/50=	.02677	10=	.39370
10/50=	.00787	35/50=	.02756	11=	.43307
11/50=	.00866	36/50=	.02835	12=	.47244
12/50=	.00945	37/50=	.02913	13=	.51181
13/50=	.01024	38/50=	.02992	14=	.55118
14/50=	.01102	39/50=	.03071	15=	.59055
15/50=	.01181	40/50=	.03150	16=	.62992
16/50=	.01260	41/50=	.03228	17=	.66929
17/50=	.01339	42/50=	.03307	18=	.70866
18/50=	.01417	43/50=	.03386	19=	.74803
19/50=	.01496	44/50=	.03465	20=	.78740
20/50=	.01575	45/50=	.03543	21=	.82677
21/50=	.01654	46/50=	.03622	22=	.86614
22/50=	.01732	47/50=	.03701	23=	.90551
23/50=	.01811	48/50=	.03780	24=	.94488
24/50=	.01890	49/50=	.03858	25=	.98425
25/50=	.01969	1=	.03937	26=	1.02362

10 mm.=1 centimeter=0.3937 inch
10 cm.=1 decimeter =3.937 inches
10 dm.=1 meter =39.37 inches
25.4 mm.=1 English inch

TABLE III

TAPERS AND ANGLES

Taper per Foot	Included ∠			With Center Line ∠			Taper per Inch	Taper per Inch from Center Line
	Deg.	Min.	Sec.	Deg.	Min.	Sec.		
⅛	0	35	48	0	17	54	.010416	.005203
3/16	0	53	44	0	26	52	.015625	.007812
¼	1	11	36	0	35	48	.020833	.010416
5/16	1	29	30	0	44	45	.026042	.013021
⅜	1	47	24	0	53	42	.031250	.015625
7/16	2	5	18	1	2	39	.036458	.018229
½	2	23	10	1	11	35	.041667	.020833
9/16	2	41	4	1	20	32	.046875	.023438
⅝	2	59	42	1	29	51	.052084	.026042
11/16	3	16	54	1	38	27	.057292	.028646
¾	3	34	44	1	47	22	.062500	.031250
13/16	3	52	38	1	56	19	.067708	.033854
⅞	4	10	32	2	5	16	.072917	.036456
15/16	4	28	24	2	14	12	.078125	.039063
1	4	46	18	2	23	9	.083330	.041667
1¼	5	57	48	2	58	54	.104666	.052084
1½	7	9	10	3	34	35	.125000	.062500
1¾	8	20	26	4	10	13	.145833	.072917
2	9	31	36	4	45	48	.166666	.083332
2½	11	53	36	5	56	48	.208333	.104166
3	14	15	0	7	7	30	.250000	.125000
3½	16	35	40	8	17	50	.291666	.145833
4	18	55	28	9	27	44	.333333	.166666
4½	21	14	2	10	37	1	.375000	.187500
5	23	32	12	11	46	6	.416666	.208333
6	28	4	2	14	2	1	.500000	.250000

TABLE IV

AMERICAN NATIONAL COARSE AND FINE THREAD DIMENSIONS AND TAP DRILL SIZES

$$p = \text{pitch} = \frac{1}{\text{No. thrd. per in.}}$$

$$d = \text{depth} = p \times .649519$$

$$f = \text{flat} = \frac{p}{8}$$

Size	Threads per Inch			Outside Diameter Inches	Pitch Diameter Inches	Root Diameter Inches	Tap Drill Approx. 75% Full Thread	Decimal Equivalent of Tap Drill
	NC	NF	NS					
0		80		.0600	.0519	.0438	3/64	.0469
1	. . .	. .	56	.0730	.0614	.0498	54	.0550
1	64	...	...	.0730	.0629	.0527	53	.0595
1	....	72	....	.0730	.0640	.0550	53	.0595
2	56	...	....	.0860	.0744	.0628	50	.0700
2	....	64	....	.0860	.0759	.0657	49	.0730
3	48	...	....	.0990	.0855	.0719	47	.0785
3	....	56	...	.0990	.0874	.0758	45	.0820
4	....	....	32	.1120	.0917	.0714	45	.0820
4	....	....	36	.1120	.0940	.0759	44	.0860

4	40			.1120	.0958	.0795	43	.0890
4		48		.1120	.0985	.0849	42	.0935
5			36	.1250	.1070	.0889	40	.0980
5	40			.1250	.1088	.0925	38	.1015
5		44		.1250	.1102	.0955	37	.1040
6	32			.1380	.1177	.0974	35	.1065
6			36	.1380	.1200	.1019	34	.1110
6		40		.1380	.1218	.1055	33	.1130
8			30	.1640	.1423	.1207	30	.1285
8	32			.1640	.1437	.1234	29	.1360
8		36		.1640	.1460	.1279	29	.1360
8			40	.1640	.1478	.1315	28	.1405
10	24			.1900	.1629	.1359	25	.1495
10			28	.1900	.1668	.1436	23	.1540
10			30	.1900	.1684	.1467	22	.1570
10		32		.1900	.1697	.1494	21	.1590
12	24			.2160	.1889	.1619	16	.1770
12		28		.2160	.1928	.1696	14	.1820
12			32	.2160	.1957	.1754	13	.1850
¼	20			.2500	.2175	.1850	7	.2010

Size	Threads per Inch			Outside Diameter Inches	Pitch Diameter Inches	Root Diameter Inches	Tap Drill Approx. 75% Full Thread	Decimal Equivalent of Tap Drill
	NC	NF	NS					
¼		28		.2500	.2268	.2036	3	.2130
5/16	18			.3125	.2764	.2403	F	.2570
5/16		24		.3125	.2854	.2584	I	.2720
⅜	16			.3750	.3344	.2938	5/16	.3125
⅜		24		.3750	.3479	.3209	Q	.3320
7/16	14			.4375	.3911	.3447	U	.3680
7/16		20		.4375	.4050	.3726	25/64	.3906
½	13			.5000	.4500	.4001	27/64	.4219
½		20		.5000	.4675	.4351	29/64	.4531
9/16	12			.5625	.5084	.4542	31/64	.4844
9/16		18		.5625	.5264	.4903	33/64	.5156
⅝	11			.6250	.5660	.5069	17/32	.5312
⅝		18		.6250	.5889	.5528	37/64	.5781
¾	10			.7500	.6850	.6201	21/32	.6562
¾		16		.7500	.7094	.6688	11/16	.6875
⅞	9			.8750	.8028	.7307	49/64	.7656
⅞		14		.8750	.8286	.7822	13/16	.8125

7/8	..	..	18	.8750	.8389	.8028	53/64	.8281
1	8			1.0000	.9188	.8376	7/8	.8750
1		14		1.0000	.9536	.9072	15/16	.9375
1 1/8	7			1.1250	1.0322	.9394	63/64	.9844
1 1/8		12		1.1250	1.0709	1.0168	1 3/64	1.0469
1 1/4	7			1.2500	1.1572	1.0644	1 7/64	1.1094
1 1/4		12		1.2500	1.1959	1.1418	1 11/64	1.1719
1 3/8	6			1.3750	1.2667	1.1585	1 7/32	1.2187
1 3/8		12		1.3750	1.3209	1.2668	1 19/64	1.2969
1 1/2	6			1.5000	1.3917	1.2835	1 11/32	1.3437
1 1/2		12		1.5000	1.4459	1.3918	1 27/64	1.4219
1 3/4	5			1.7500	1.6201	1.4902	1 9/16	1.5625
2	4 1/2			2.0000	1.8557	1.7113	1 25/32	1.7812
2 1/4	4 1/2			2.2500	2.1057	1.9613	2 1/32	2.0313
2 1/2	4			2.5000	2.3376	2.1752	2 1/4	2.2500
2 3/4	4			2.7500	2.5876	2.4252	2 1/2	2.5000
3	4			3.0000	2.8376	2.6752	2 3/4	2.7500
3 1/4	4			3.2500	3.0876	2.9252	3	3.0000
3 1/2	4			3.5000	3.3376	3.1752	3 1/4	3.2500
3 3/4	4			3.7500	3.5876	3.4252	3 1/2	3.5000
4	4			4.0000	3.3786	3.6752	3 3/4	3.7500

TABLE V

Letter Sizes of Drills

Diameter Inches	Decimals of 1 Inch	Diameter Inches	Decimals of 1 Inch
A 15/64	.234	N	.302
B	.238	O 5/16	.316
C	.242	P 21/64	.323
D	.246	Q	.332
E 1/4	.250	R 11/32	.339
F	.257	S	.348
G	.261	T 23/64	.358
H 17/64	.266	U	.368
I	.272	V 3/8	.377
J	.277	W 25/64	.386
K 9/32	.281	X	.397
L	.290	Y 13/32	.404
M 19/64	.295	Z	.413

TABLE VI

Decimal Equivalents of the Numbers of Twist Drills and Steel Wire Gage

No.	Size of No. in Decimals	No.	Size of No. in Decimals	No.	Size of No. in Decimals	No.	Size of No. in Decimals
1	.2280	21	.1590	41	.0960	61	.0390
2	.2210	22	.1570	42	.0935	62	.0380
3	.2130	23	.1540	43	.0890	63	.0370
4	.2090	24	.1520	44	.0860	64	.0360
5	.2055	25	.1495	45	.0820	65	.0350
6	.2040	26	.1470	46	.0810	66	.0330
7	.2010	27	.1440	47	.0785	67	.0320
8	.1990	28	.1405	48	.0760	68	.0310
9	.1960	29	.1360	49	.0730	69	.0292
10	.1935	30	.1285	50	.0700	70	.0280
11	.1910	31	.1200	51	.0670	71	.0260
12	.1890	32	.1160	52	.0635	72	.0250
13	.1850	33	.1130	53	.0595	73	.0240
14	.1820	34	.1110	54	.0550	74	.0225
15	.1800	35	.1100	55	.0520	75	.0210
16	.1770	36	.1065	56	.0465	76	.0200
17	.1730	37	.1040	57	.0430	77	.0180
18	.1695	38	.1015	58	.0420	78	.0160
19	.1660	39	.0995	59	.0410	79	.0145
20	.1610	40	.0980	60	.0400	80	.0135

INDEX

Note: *The reader should also refer to the GLOSSARY for additional technical terms not listed in the INDEX.*

Abnormal boiling point, 492
Abnormal freezing point, 492
Abrasives, 591–597, 722
Absolute temperature, 402
Acceleration, 370
Accuracy, degree of, 27
Acetylene, 557
Acetylene welding, 815
Acids, 494
Acme screw thread, 620
Adhesion, 343
Algebra, 61–105
  addition in, 65
  equations, 74, 77, 85, 87, 98, 105
  formulas, 72, 78
  multiplication in, 66
  signed numbers, 63–65
  substitution, 62
  symbols in, 61
Allotropism, 503
Alloy steels, 584, 585, 827
Alloys, 586–590
Alloys, die casting, 802
Alternate interior angles, 131
Alternating current, 879
Alternating current generators, 879–884
Alternating current motors, 891–900
Altitude, 140, 223
Aluminum, 547
American National screw thread, 611
American Standard pipe thread, 622
Ammeter, 900, 904
Ammonia, 511
Aneroid barometer, 358
Angle-bisector, 161, 167
Angle measurement, 126
Angle of taper, 249
Angles, 129, 141, 155, 161, 167
Annealing, of steel, 822
Annunciators, 836
Anthracite, 554
Anvil, 804
Anvil tools, 805
Apparent elastic limit, 449
Apron, lathe, 685
Aralac, 598
Arbor mill, 712
Arc, 155, 163, 194
Archimedes' law, 351
Arc welding, 817
Area, 179, 182, 190
  of circle, 186, 193
  of cone, 215
  of cylinder, 214
  of ellipse, 195
  of fillet, 194
  of frustum, 215
  of parallelogram, 191
  of prism, 213, 214
  of pyramid, 214
  of rectangle, 191
  of rectangular solid, 199, 202, 213
  of regular hexagon, 189, 193
  of regular polygon, 193, 263
  of rhombus, 191
  of ring, 194
  of sector, 194
  of segment, 195
  of similar figures, 225
  of sphere, 216
  of square, 191
  of trapezoid, 193
  of triangle, 192, 262
Armored cable, 844
Asbestos, 597
Atmospheric pressure, 355–358
Atomic numbers, 473
Atomic structure, 474, 475

Atomic weight, 483
Atoms, 471
Auto-connected transformer, 890

Babbitt metals, 589
Bakelite, 599
Band saw, for metal-cutting, 732
  for woodworking, 763
Barometer, 358, 359
Bases, 494
Battery switches, 836
Beams, 458–466
  flexure of, 462
  loads, 458
  strength of, 460
  types of, 458, 461
Bearing metals, 588, 589, 659
Bearings, 657–661
Bell circuits, 835
Bell coil, 631
Belting, 80, 627–630
Bench knife, 735
Bessemer process, 577
Bevel gears, 637, 652–656
Bevel protractor, 748
Bevel square, 749
Bisector of an angle, 161
Bituminous coal, 553
Blast furnace, 529
Bleaching powder, 522
Blown fuse, testing for, 853
Boiling, 408
Bolts and nuts, 623–625
Bolts and nuts, formulas, 78
Boring, wood, 742
Bottle method, for specific gravity, 354
Boyle's law, 359
Brakes, hydraulic, 350
Brass, 586, 587
Brazing, 816
Bricks, 570
Briggs Standard pipe thread, 622
Brinell hardness, 438, 826
Brittleness, 441
Broaching, 731
Bronze, 586, 587
Brown & Sharpe taper, 607
Buoyancy, 350
Burglar alarm system, 840
Burning, 468, 469
Butt weld, 811
Buttress screw threads, 619
Calcium carbonate, 519
  hydroxide, 520
  oxide, 520
  sulfate, 521
Calipers, 36, 37, 744
Candlepower, 416, 906
Cantilever beam, 458
Capillarity, 343
Carboloy, 591
Carbon lamp, 906
Carborundum, 593
Case hardening, 822
Cast iron, 531, 574, 575
Cast steel, 583
Castings, 752, 756
Celluloid, 597
Cement, 567
Cemented carbide, 591
Cementite, 819
Center head, 748
Centigrade scale, 398
Central angle, 155
Centrifugal force, 376
Centripetal force, 376
Chamfer, 778
Change of state, 407
Characteristic, of a logarithm, 106
Charles' law, 403
Chasing, 705, 708
Chemical changes, 467
Chemical compounds, 470
Chemical equations, 488–490
Chemical equilibrium, 514
Chemical reactions, 523
Chisels, 670, 679
Chisels, wood, 736, 771
Chlorine, 498
Chord, 154, 157
Chromium steel, 585
Chucks, lathe, 687
Circle, 153, 186, 226
  area of, 186, 193
  circumference of, 163, 193
  circumscribed, 159
  inscribed, 161
  tangent to, 160, 172
Circuit breaker, 856
Circumference, 163, 193
Circumscribed circle, 159
  hexagon, 177
  pentagon, 178
  square, 177
Clay, 569

Clearance, 673, 678
Clearance angle, 678
Cleft weld, 812
Coal gas, 557
Code, for steels (S.A.E.), 827
Coefficient of elasticity, 451, 453
Coefficient of expansion, 400, 443, 444
Coefficient of friction, 389
Co-function, 235
Cohesion, 343
Coil of belting, 631
Collets, 689
Colloids, 491
Combustion, 468, 469, 479
Commercial discount, 46
Commutation, 859
Compensator-starter, 899
Component forces, 241
Compound gears, 633
Compound generator, 864
Compound motor, 872
Compound rest, 701
Compression, 446
Concrete, 567
Concrete, strength of, 454
Condensation, 408, 410
Condenser, electrical, 425
Conduction, heat, 404, 444
Conductor, electrical, 423
Conduit wiring, 845–850
Cone pulleys, 627
Cone, right circular, 211
  area of, 211
  frustum of, 211
  volume of, 211
Congruence, 145
Constructions, geometric, 167–178
Convection, 405
Cooper-Hewitt lamp, 907
Copper, 534–540
  refining of, 536
  electroplating, 539
Core box, 789, 791
Corrosion, 479
Corundum, 592, 595
Cosine of an angle, 234, 237
Cosines, law of, 256
Counter E.M.F., 871
Cracking of petroleum, 561
Creep of metals, 442
Crucible steel, 582
Cube of a number, 68
Cubical expansion, 444
Cupola furnace, 575
Current electricity, 426–435
Cutting angle, 679, 680
Cutting tools, 677
Cylinder, 203, 205, 214, 216
  area of, 204
  volume of, 205, 214, 216

Dardalet screw threads, 619
Dead center, 686
Decimal equivalents, 24, 25
Decimals, 18–24
  addition, 19
  conversion, 23, 24
  division, 22
  multiplication, 21
  reading, 18
  subtraction, 19
Definite proportions, law of, 470
Deflection of beams, 462
Deformation, 447
Degrees, 127
Delta connection, 881, 888, 891
Density, 341
Dependence, 96, 103
Depreciation, 45
Depth of cut, formula for, 79
Destructive distillation, 551, 553
Diamond, 591
Die casting, 801
Die-casting alloys, 802
Dies, 675
Direct current generators, 857–869
Direct current motors, 869–878
Direct lighting, 910
Direct proportion, 54
Direct variation, 100
Distance, 131
Distillation, 410
Dividers, 743
Dogs, lathe, 688, 689
Door opener, electric, 836
Dow metal, 590
Draft, 753
Drawing, 807
Draw knife, 742
Drill clearance, 673
Drilling, 673
Drilling on a lathe, 704
Drills, 671
Drop hammer, 813
Dry cell, 427, 541

Ductility, 441
Duralumin, 590

Edison storage cell, 879
Efficiency, 389
Efficiency, of generator, 869
Elastic limit, 449
Elasticity, 440
Elasticity, modulus of, 451, 453
Electric steel, 582
Electric welding, 817
Electric wiring, 829–853
Electrical instruments, 430
Electrical units, 427
Electrification, 424
Electrochemical series, 524
Electrodynamometer, 904
Electrolytes, 493
Electrolytic dissociation, 493
Electromagnet, 422, 856
Electromagnetic induction, 434, 853, 856
Electromagnetism, 421, 422
Electromotive force, 428
Electrons, 472
Electroplating, 539
Electroscope, 424
Ellipse, 164
  area of, 190, 195
  construction of, 337
  perimeter of, 164, 195
Embossing, 800
Emery, 592
Emulsions, 491
Empirical curves, 97
Endurance of materials, 453
Energy, 392
  conservation of, 395
  dissipation of, 395
  electrical, 432
  kinetic, 393
  potential, 393
  transformation of, 394
Equations, 74–82
  exponential, 105, 115
  graphs of, 98–105
  quadratic, 87
  radical, 85
  simple, 74
  solving, 74–77
Ethyl gas, 562
Evaporation, 409, 410
Expansion, by heat, 399, 443
  coefficient of, 400, 443
  of gases, 402
  of liquids, 401
  of solids, 399, 401, 443
  of water, 402
Explosion, 478
Exponential equations, 105, 115
Exponential function, 105
Exponents, 68–72
Exponents and logarithms, 112

Factor of safety, 455, 456
Factors, in algebra, 67
Fahrenheit scale, 398
Fatigue of metals, 453
Files, 666
File test, for hardness, 825
Filing, 666
Fillet, 188, 194, 205
Finishing, of casting, 755
  of pattern, 757
Fire extinguisher, 508
Flaws, in metals, 443
Fleming's rule, 434, 870
Flexible conduit, 849
Flexure of beams, 462
Floating bodies, 351
Fluorescent lamp, 913–916
Flux, light, 906
Flux, magnetic, 420
Follow rest, 689
Foot-candle, 906
Forces, 360–369
  composition of, 361, 362
  concurrent, 361, 363
  graphic representation of, 361
  moment of, 366
  parallel, 368
  resolution of, 363, 365
Forging, 803–809
Formulas, 72–82, 94
  evaluating, 72, 85, 94
  shop, 78–82
  transforming, 75, 77
Fourth proportional, 222
Fractions, 10–17, 42
  addition, 13
  division, 16
  multiplication, 16
  per cents, 42
  reduction, 12
  subtraction, 13
Frasch process, 502
Freely falling bodies, 373

Freezing, 407
French curves, 300
Friction, 387–389
Frustum, of cone, 211, 215
  of pyramid, 209, 215
Fuels, 551–562
Fullering, 805, 808
Functions, 96–106
  exponential, 105
  hyperbolic, 102
  linear, 98
  parabolic, 100
  rate of change, 103

Gage blocks, 37–40
Galalith, 598
Galvanized iron, 542
Garnet, 592
Gases, 355–360
Gas-filled lamp, 907
Gear cutting, 636–657
Gear formulas, 80
Gears, 631-635
Gear speeds, 631
Gear teeth, 639–643
Gear-tooth gage, 646
Generator efficiency, 869
Generator losses, 869
Generator ratings, 869
Generators, A.C., 879–884
Generators, D.C., 857–869
Geometric constructions, 167–178
German silver, 586, 588
Glass, 571–574
Gouge, 737, 770
Graphs, 95–106
Greenfield conduit, 849
Grinding, 719–722
Grinding wheels, 722
Ground, electric, 830, 852

Hacksaw, 670
Hand forging, 803, 807
Hard water, 519, 521
Hardness, 437, 825
Heat, 396–412
  measurement of, 397, 407
  nature of, 396
  quantity of, 406
  sources of, 396
  transmission of, 404–406
Heat effect, electrical, 431
Heating systems, 405
Heat treatment of steel, 818–824
Helical gear, 638
Helve hammer, 814
Herringbone gears, 638
Hickey, 845
High-speed steel, 585
High temperature measurements, 823
Hollow cylinder, 205, 216
Hooke's law, 448
Horsepower, 81, 82, 391
Hydraulic press, 349
Hydrochloric acid, 501
Hydrogen, 479–483
Hydrogen chloride, 500
Hydrogen sulfide, 504
Hydrogenation, 483
Hydrometer, 354
Hyperbolic function, 102
Hypotenuse, 146

Identification of ferrous metals, 824
Illumination, electric, 909
Inclined plane, 384
Index head, 717
Index of refraction, 416
Indexing, 717–719
Indirect lighting, 910, 913
Induction, electrical, 424
  electromagnetic, 434, 853, 856
  magnetic, 418
Induction motor, 891, 894, 900
Inertia, 339, 375
  moment of, 463
Inscribed angle, 156
  circle, 161
  hexagon, 177
  square, 176
Insulation, electrical, 423
Insulation, heat, 404
Internal stresses, 446, 447
Interpolation, 111
Inverse proportion, 56
Inverse-square law, 417
Inverse variation, 102
Involute curve, 642
Ionization, 494
Ionized atoms, 483
Iron, 528–532, 574
Isometric drawings, 332–338

Jarno taper, 607
Jig borer, 732
Jump weld, 811

Keys, 661
Kilowatt, 432
Kindling temperature, 477
Kinetic energy, 393
Knurling, 703

Lap weld, 811
Lathe, 680–693
  apron, 685
  attachments, 686
  centers, 686, 693
  chucks, 687
  dogs, 689
  feeding mechanism, 683
  follow rest, 689
  gears, 689
  spindle sleeve, 687
  steady rest, 688
  tools, 691
  turning, 693
  turret, 690
Lathe turning, 693–700
Lathe, wood-turning, 768
Layout tools, 677
Layout work, 676
Lead, 543
Lead storage cell, 878
Left-hand rule, 870
Length of belts, 630
Lenz's law, 435
Lever, 379
Light, 412–417
  illumination, 412, 416
  reflection, 413, 414
  refraction, 414, 416
Lighting fixtures, 911–914
Lighting systems, 910
Lignite, 553
Linear expansion, 400, 443
Linear function, 98
Lines and angles, 125
Liquids, mechanics of, 343–353
Live center, 686
Logarithms, 106–116
  finding a logarithm, 107
    " " number, 111
    " " power, 114
    " " product, 112, 113
    " " quotient, 113
    " " root, 114
Lumen, 906
Machine forging, 813
Machine tools, 723–733, 763–766
Machines, simple, 377
  mechanical advantage of, 378
Magnalium, 590
Magnetic field, 854
Magnetic induction, 418
Magnetism, 417–422
Malleability, 441
Mandrel, 694
Manganese steel, 584
Mantissa, 106
Marking gage, 750
Mass, 342
Measurements, units of, 340
Measures, 1–10
  linear, 3
  metric, 8
  standard, 2
  surface, 4
  volume, 5
  weight, 7
Measuring instruments, 27–40
Mechanical advantage, 378
Mechanical drawing, 297–338
  conventions, 303–308
  instruments, 298–301
  isometric drawings, 332–338
  lettering, 301, 302
  three-view drawings, 323–332
  two-view drawings, 309–322
Median, 140
Melting, 407
Mercury vapor lamp, 907
Metric standard screw thread, 617
Metric system, 8–10, 341
Mica, 597
Micrometer, 30–33
Mid-join, 151
Milling, 708–719
Milling cutters, 711–716
Modulus of elasticity, 451
Moh's scale, 438
Molecular weight, 485
Molecules, 471
Molybdenum steel, 585
Moment of a force, 366, 367
Moment of inertia, 463
Momentum, 375
Monel metal, 590
Morse taper, 606

Motion, 369–376
  accelerated, 370
  falling bodies, 373
  laws of, 374–376
  uniform, 370, 371
Motor trouble, 877
Motors, A.C., 891–900
Motors, D.C., 869–878
Moulding, wire, 850
Multiple proportions, law of, 471

National screw thread, 611
Natural functions, 234
Natural gas, 556
Negative exponents, 70
Neutralization, 495
Neutrons, 474
Newton's laws of motion, 375
Nickel, 546
Nickel steel, 584
Nitric acid, 516
Nitrogen, 509–511
Norbide, 596

Oblique triangles, solution of, 255–262
Ohm's law, 428, 430, 829
Open-hearth furnace, 579
Optical pyrometer, 823
Outlet boxes, 847
Over-current protection, 841
Oxidation, 475–479, 482
Oxy-acetylene torch, 477
Oxygen, 475

Pack hardening, 822
Panel gage, 749
Parabolic function, 100
Parallel circuits, 429, 830, 832
Parallel lines, 130, 132
Parallelogram, 152
  area of, 179
Parting, 702
Pascal's law, 348
Patterns, one-piece, 774
  segment, 784, 787
  split, 758
  stave, 791
  two-piece, 778, 781
Peat, 552
Penumbra, 413
Percentage, 40–48
  finding the base, 44
  finding the per cent, 40
  finding the rate, 43
Perimeter, of circle, 193
  of ellipse, 195
  of fillet, 194
  of parallelogram, 191
  of rectangle, 191
  of regular hexagon, 193
  of regular polygon, 193
  of rhombus, 191
  of square, 191
  of trapezoid, 193
  of triangle, 192
Permeability, electrical, magnetic, 420
Permutite, 522
Perpendiculars, 132, 168
Petroleum, 558–562
Pig iron, 531, 574
Pigtail splice, 833
Pinning, 669
Pipe threads, 622
Planer, 728–731
Plane, wood, 738
Planimeter, 190
Plasticity, 441
Plastics, 597–601
Polarization, 844
Polygons, 152
  area, 182, 189, 193
  circumscribed, 177
  inscribed, 176
  regular, 162, 189, 193
Polyphase induction motor, 891
Potential, electrical, 426
Potential energy, 393
Power, 391
  electrical, 432
  transmission of, 392
Powers and roots, 68–72
Precision measurements, 27–40
Pressure angle, 643
Pressure, liquid, 344–350
Prisms, 200, 213, 214
  area of, 202, 213, 214
  volume of, 202, 213, 214
Producer gas, 556
Projections, 132, 241
Proportion, 53–58, 220
  direct, 54
  inverse, 56
Proportional limit, 448
Protons, 475
Protractor, 128
Pulley speeds, 628

Pulleys, 57, 80, 382, 625–630
Punch press, 795
Punch press dies, 797–801
Pyramid, 208, 214, 229
  area of, 208, 214
  frustum of, 209, 215
  volume of, 208, 214
Pyrometers, 823
Pythagorean theorem, 146

Quadilateral, 152
Quenching, 820

Raceway, 850
Rack and pinion, 637
Radiation, heat, 406
Radical equations, 85
Radioactivity, 472
Rate of change, 103
Ratio and proportion, 48–58, 220
Reaction, 376
Reamer, 674
Rectangle, 153, 179, 191
Rectangular solids, 198, 202, 213
  area of, 199, 202, 213
  volume of, 199, 202, 213
Reduction, by hydrogen, 482
Reflection factor, 906, 908
Reflection, of light, 413, 414
Refraction, of light, 414–416
Refrigeration, 411, 515
Regular polygons, 162, 177, 189, 193, 225
  area of, 180, 262
Repulsion motor, 895
Resiliency, 440
Resistance, electrical, 430
Reverberatory furnace, 532
Reversible reactions, 514
Rheostats, 873
Rhombus, 153
  area of, 180, 191
Right hand rules, 854
Right triangle, 139, 146, 151, 223, 252
  solution of, 240
Right triangle rule, 146
Ring, 187, 194
Rockwell hardness test, 826
Roots and powers, 68–72
Rubber, 602

Saddle, bending, 846
S.A.E. screw thread, 612
S.A.E. steel code, 827
Sander, 766
Saws, wood, 739–741
Scale, 745
Scale drawings, 51–741
Scleroscope, 439, 825
Scott-connected transformer, 890
Scraping tool, 772
Screw, 386
Screw thread formulas, 79, 80
Screw threads, 610–625
  depth of, 614
  measuring, 616
  standards, 610
  terminology, 612
Sector, 187, 194
Segment, 188, 195
Segments, dividing a line, 169, 221
Series circuit, 429, 830, 832
Series generator, 865
Series motor, 871
Shadows, 413
Shaper, 723–727
Shear, 446
Shop formulas, 78–82
  belting, 80
  bolts and nuts, 78
  depth of cut, 79
  energy and power, 81
  gears, 80
  horsepower, 81, 82
  pulleys, 57, 58, 80, 81
  screw threads, 79, 80
  work, 81
Short circuit, 852
Shouldering, 809
Shoulder turning, 702
Shrinkage, 751
Shunt generator, 866
Shunt motor, 872
Silicon steel, 585
Similarity, 145, 219, 221, 224
Sine of an angle, 234, 237
Sine bar, 244
Sines, law of, 255
Single-loop generator, 858
Single-phase current, 879
Single-phase generator, 882
Single-phase motor, 894, 895
Sizing tool, 771
Slide rule, 116–123
Slip ring motor, 891

  dium, 495
  icarbonate, 497
  carbonate, 497
  chloride, 496
  hydroxide, 497
Soldering, 816, 834
Solid figures, 198–216
  cone, 211, 215
  cylinder, 203, 205
  frustum, 209, 215
  prism, 200, 202, 213, 214
  pyramid, 208, 214
  rectangular solid, 198, 199, 216
  sphere, 211, 216
lids of revolution, 227
lubility, 491
Solutions, 490–494
Spacing holes on a circle, 247
parking, of motor, 877
arkless commutation, 859, 871
ecific gravity, 352–354
ecific heat, 407
elter, 816
here, 211, 229
  rea of, 212, 216
  olume of, 212, 216
ndle sleeve, 687
ices, electric, 833, 841
ne, 661
t patterns, 758–763
t-phase motor, 894
keshave, 741
ntaneous combustion, 479
welding, 817
ng collets, 689
  gears, 648
re, 153
  ea of, 179, 191
  cumscribed, 177
  scribed, 176
re, carpenter's, 746
re of a number, 68
re root, 82–93
  approximation, 82
  computation, 83, 84
  logarithms, 115
  slide rule, 122
  tables, 83, 89
re screw thread, 619
rel cage motor, 891, 897
  connection, 881, 889
    , of A.C. motors, 897
    rheostat, 873

States of matter, 340
Static electricity, 422
Steady rest, 689
Steel, 528, 576–585
Stellite, 590
Step cone pulleys, 627
Stiffness, 442, 463
Storage battery, 878
Strength, of concrete, 454
  of materials, 452–454, 456
  of timber, 454
Stress-deformation, 450
Sulfur, 501–504
Sulfur dioxide, 505
Sulfuric acid, 506
Surface gage, 750
Surface tension, 343
Suspensions, 491
Swage, 805
Swaging, 800
Switches, electric, 836, 850
Synchronising, of generators, 883

Tangent of an angle, 234
Tangent to a circle, 160, 172–174
Tap drills, 675
Taper gage, 701
Tapers, 249, 605–610
Taper turning, 699
Tap splice, 833
Taps and dies, 675
Temperature, 397
  Absolute, 402
  scales, 398
Tempering, of steel, 820
Tension, 446
Terms, in algebra, 67
Testing, electrical, 852
Testing of materials, 457
Testing, of motors, 877
Tetrahedron, regular, 209
Thermal conductivity, 444
Thermit process, 528, 550
Thermit welding, 816
Thermoelectric pyrometer, 824
Thermometer, 398, 399
Thomson watt-hour meter, 903
Thread cutting, 705
Three-phase current, 880
Three-phase generator, 883
Three-wire generator, 868
Thumb rule, 855
Timber, strength of, 454

Tin, 545
Tolerance, 24
Torque, of A.C. motor, 893
Torricelli's experiment, 356
Trammel points, 751
Transformer, connections, 886
  efficiency of, 885
  losses in, 885
Transformers, 884–891
Transmission factor, of light, 909
Transversals, 130
Trapezoid, 153
  area of, 181, 193
Triangle, 136–151
  angle sum, 142
  area of, 180, 262
  equilateral, 150, 175
  isosceles, 150, 175
  right, 139, 146, 151, 223, 252
Trigonometric functions, 234, 255
Trip hammer, 814
Trusses, 137
Try square, 747
Tungsten filament lamp, 907
Turret lathe, 690–693
Two-phase current, 879
Two-phase generator, 882

Ultimate strength, 450
Umbra, 413
Uniform motion, 370, 371
Unit stresses, 447
Upsetting, 809
U.S.S. screw thread, 612

Valence, 485
Vanadium steel, 585
Variables, 95, 103
Variation, 94–106
  direct, 100
  inverse, 102
Velocity, 370
Vernier instruments, 33–36
Voltaic cell, 426
Voltmeter, 902, 905
Volume, of cone, 211, 214
  of cylinder, 205
  of fillet, 205
  of frustum, 210, 214
  of prism, 202, 213, 214
  of pyramid, 209, 214
  of rectangular solid, 199, 202, 216
  of sphere, 212, 216

Water, electrolysis of, 479
  synthesis of, 481
Water gas, 554
Watt, 432
Watt-hour meter, 903
Wattmeter, 902, 905
Wedge, 386
Weight, 342
Welding, 810-817
Western Union splices, 833
Wheel and axle, 381
Whitworth screw thread, 618
Wire moulding, 850
Work, 377, 379, 391
  units of, 377
Working stresses, 455
Wormgear, 635, 637
Worm screw thread, 621
Wound rotor, 891
Wrought iron, 532

X-rays, 472

Yield point, 450
Young's modulus, 451, 453

Zinc, 540